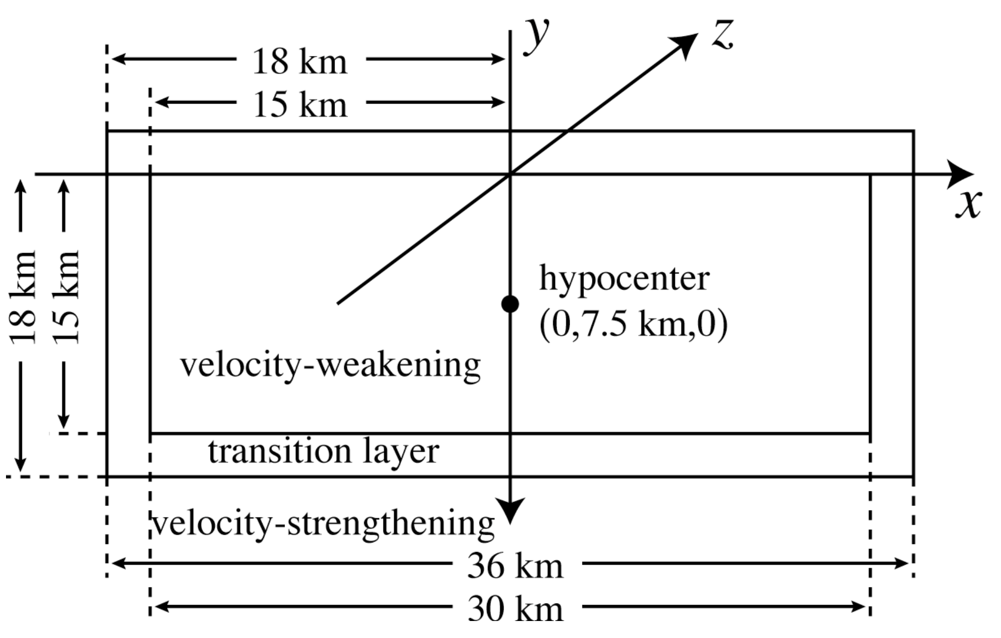
SCEC Code Validation: TPV103
Rate-and-State Friction, Slip Law, Strong Rate-Weakening Whole-Space

Model Geometry: A planar fault lies in an isotropic, linear elastic whole-space. The material on either side of the fault is characterized by its density ρ, *S*-wave speed c_s, and *P*-wave speed c_p. The properties are given in the table below and are constant everywhere in the medium.

ρ	c_s	c_p
2670 kg/m^3	3.464 km/s	6 km/s

To simplify later expressions, a coordinate system will be adopted in which the fault is the plane $z = 0$, with the hypocenter located at $(x_0, y_0) = (0, 7.5 \text{ km})$. This is shown in the figure below. The central portion of the fault, $-W < x < W$, $0 < y < W$, with $W = 15$ km, is velocity-weakening. A transition layer of width $w = 3$ km in which the frictional properties continuously change from velocity-weakening to velocity-strengthening surrounds the central velocity-weakening region of the fault. Outside of the transition region, the fault is velocity-strengthening.



Friction Law: Let $\boldsymbol{\tau} = (\tau_x, \tau_y)$ be the shear traction vector (specifically, the traction exerted by the positive side of the fault on the negative side), the magnitude of which is $\tau = \sqrt{\tau_x^2 + \tau_y^2}$, and let σ be the normal stress acting on the fault, taken to be positive in compression. In terms of the components of the stress tensor σ_{ij}, $\tau_x = \sigma_{zx}$, $\tau_y = \sigma_{zy}$, and $\sigma = -\sigma_{zz}$. Let $\boldsymbol{V} = (V_x, V_y)$ be the slip velocity vector, the magnitude of which is

$V = \sqrt{V_x^2 + V_y^2}$, and let $\boldsymbol{\delta} = (\delta_x, \delta_y)$ be the slip vector. Slip is defined as the displacement discontinuity across the fault: $\delta_i = u_i(x,y,0^+) - u_i(x,y,0^-)$ $(i = x,y)$, where $u_i(x,y,z)$ is the displacement field. Likewise, $V_i = v_i(x,y,0^+) - v_i(x,y,0^-)$ $(i = x,y)$, where $v_i(x,y,z)$ is the particle velocity. Finally, let ψ be the state variable on the fault. The shear traction is always equal to the shear strength of the fault, which is the product of the friction coefficient and normal stress:

$$\tau = f(V,\psi)\sigma . \tag{1}$$

The friction coefficient is a function of V and ψ:

$$f(V,\psi) = a \text{ arcsinh}\left[\frac{V}{2V_0}\exp\left(\frac{\psi}{a}\right)\right]. \tag{2}$$

The state variable evolves according to the equation

$$\frac{d\psi}{dt} = -\frac{V}{L}\left[\psi - \psi_{ss}(V)\right], \tag{3}$$

$$\psi_{ss}(V) = a \ln\left\{\frac{2V_0}{V}\sinh\left[\frac{f_{ss}(V)}{a}\right]\right\}. \tag{4}$$

$f_{ss}(V)$ is the steady state friction coefficient, which depends on V and the friction law parameters f_0, V_0, a, b, f_w, and V_w:

$$f_{ss}(V) = f_w + \frac{f_{LV}(V) - f_w}{\left[1 + (V/V_w)^8\right]^{1/8}} \tag{5}$$

with a low-velocity steady state friction coefficient

$$f_{LV}(V) = f_0 - (b-a)\ln(V/V_0). \tag{6}$$

The slip velocity vector points in the direction of the shear traction vector:

$$\boldsymbol{\tau}/\tau = \mathbf{V}/V . \tag{7}$$

The friction law parameters are given in the table below. Note that with the exception of a and V_w, they are uniform on the fault.

f_0	V_0	$a(x,y)$	b	L	f_w	$V_w(x,y)$
0.6	10^{-6} m/s	$0.01+\Delta a(x,y)$	0.014	0.4 m	0.2	0.1 m/s$+\Delta V_w(x,y)$

To stop the rupture, the friction law changes from velocity-weakening in the rectangular interior region of the fault to velocity-strengthening sufficiently far outside this region. The transition occurs smoothly within a transition layer of width $w = 3$ km. Outside the transition layer, the fault is made velocity-strengthening by increasing *a* by $\Delta a_0 = 0.01$ and V_w by $\Delta V_{w0} = 0.9$.

The changes in *a* and V_w, which are added to the values of *a* and V_w in the velocity-weakening interior of the fault, are

$$\Delta a(x,y) = \Delta a_0 [1 - B(x;W,w)B(y - y_0;W/2,w)] \quad (8)$$

$$\Delta V_w(x,y) = \Delta V_{w0} [1 - B(x;W,w)B(y - y_0;W/2,w)], \quad (9)$$

in which

$$B(x;W,w) = \begin{cases} 1, & |x| \leq W \\ \frac{1}{2}\left[1 + \tanh\left(\frac{w}{|x| - W - w} + \frac{w}{|x| - W}\right)\right], & W < |x| < W + w \\ 0, & |x| \geq W + w \end{cases} \quad (10)$$

is a mathematically smooth version of the boxcar function (meaning that *B* and all of its derivatives are continuous).

Initial Conditions: At $t = 0$, the fault is everywhere sliding in the horizontal direction with initial velocity $V = V_{ini}$. The initial shear stress on the fault, which is also horizontal, is τ_{ini}, the normal stress is σ_{ini}, and the initial value of the state variable is $\psi_{ini}(x,y)$. Note that the initial state variable is spatially variable, but the initial velocity and stresses are uniform. This is because the initial conditions must be self-consistent, in the sense that they must satisfy (1) and (2). Since the friction law parameter *a* is spatially variable, then, in order for the initial velocity and stress fields to be uniform, ψ_{ini} must also be spatially variable. The values of the initial conditions are given in the table below.

V_{ini}	τ_{ini}	σ_{ini}	$\psi_{ini}(x,y)$
10^{-16} m/s	40 MPa	120 MPa	0.563591842632738 + $\Delta\psi(x,y)$

From equations (1) and (2), it follows that

$$\psi_{ini}(x,y) = a \ln\left[\frac{2V_0}{V_{ini}} \sinh\left(\frac{\tau_{ini}}{a\sigma_{ini}}\right)\right]. \quad (11)$$

In the medium surrounding the fault, the only nonzero stresses are the horizontal shear stress and the normal stress component acting on the fault; these values are uniform and identical to those on the fault:

$$\sigma_{zx}(x,y,z)=\tau_{ini} \text{ and } \sigma_{zz}(x,y,z)=-\sigma_{ini} \text{ at } t=0. \tag{12}$$

The medium is initially moving with equal and opposite horizontal velocities of $V_{ini}/2$ on the two sides of the fault:

$$v_x(x,y,z)=\begin{cases} V_{ini}/2, & z>0 \\ -V_{ini}/2, & z<0 \end{cases} \text{ at } t=0. \tag{13}$$

Displacement in the medium and slip on the fault are measured from zero at $t=0$.

Nucleation Method: Starting at $t=0$, rupture is nucleated by imposing a horizontal shear traction perturbation (i.e., a perturbation to τ_x) that depends on both space and time. The particular form is such that the perturbation smoothly grows from zero to its maximum amplitude $\Delta\tau_0$ over a finite time interval *T*, and is confined to a finite region of the fault of radius *R*. The perturbation is mathematically smooth in time and space (i.e., the function and all derivatives are continuous). Specifically, the perturbation is

$$\Delta\tau(x,y,t)=\Delta\tau_0 F\left(\sqrt{(x-x_0)^2+(y-y_0)^2}\right)G(t), \tag{14}$$

in which

$$F(r)=\begin{cases} \exp\left(\dfrac{r^2}{r^2-R^2}\right), & r<R \\ 0, & r\geq R \end{cases} \tag{15}$$

and

$$G(t)=\begin{cases} \exp\left[\dfrac{(t-T)^2}{t(t-2T)}\right], & 0<t<T \\ 1, & t\geq T \end{cases}. \tag{16}$$

The perturbation is radially symmetric, with the radial distance away from the hypocenter along the fault given by $r=\sqrt{(x-x_0)^2+(y-y_0)^2}$. The nucleation parameters are given in the table below.

$\Delta\tau_0$	*R*	*T*	(x_0,y_0)
45 MPa	3 km	1 s	(0,7.5 km)

SCEC Code Validation: TPV104
Rate-and-State Friction, Slip Law, Strong Rate-Weakening Half-Space

This problem is identical to TPV103, but the fault is embedded in a half-space rather than a whole-space. In terms of the coordinate system defined for TPV103, the half-space is the region $y > 0$. The plane $y = 0$ is a free surface.

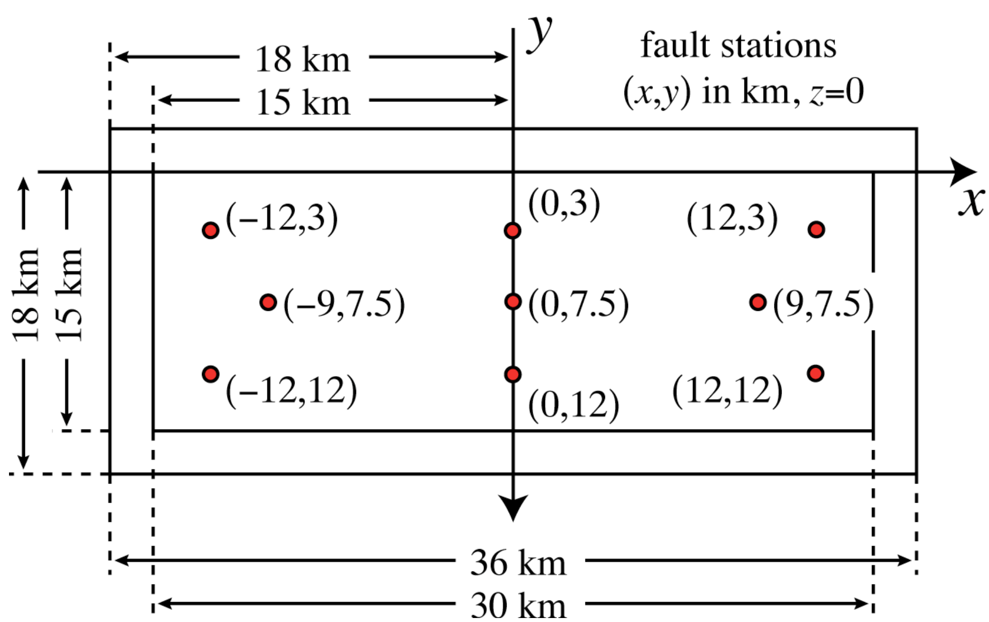
SCEC Code Validation: TPV103 and TPV104
Rate-and-State Friction, Required Output

For both TPV103 and TPV104, the following data should be submitted to the code validation website (where instructions for the data file formats can be found):

Time Histories of Fields at Fault Stations:

Report the complete time histories from $t = 0$ to $t = 12$ s of both components of slip (δ_x and δ_y) and slip velocity (V_x and V_y), all tractions (τ_x, τ_y, and σ), and the state variable (ψ) at each of the following nine stations on the fault:

x (km)	0	0	0	9	12	12	−9	−12	−12
y (km)	3	7.5	12	7.5	3	12	7.5	3	12



Rupture Front Arrival Times:

Report the rupture front arrival time at all points within the velocity-weakening portion of the fault $(-W < x < W,\ 0 < y < W)$, where $W = 15$ km. The rupture front arrival time is defined as the time at which the slip velocity, V, first exceeds 1 mm/s.

Time Histories of Fields at Free Surface Stations (TPV104 only):

Report the complete time histories from $t = 0$ to $t = 12$ s of all components of displacement and particle velocity at each of the following six stations on the free surface:

x (km)	0	0	12	12	–12	–12
z (km)	9	–9	6	–6	6	–6