3D Rupture Dynamics Code Validation Workshop:

- Finite difference method
- Conventional grid
- Split-node for fault plane

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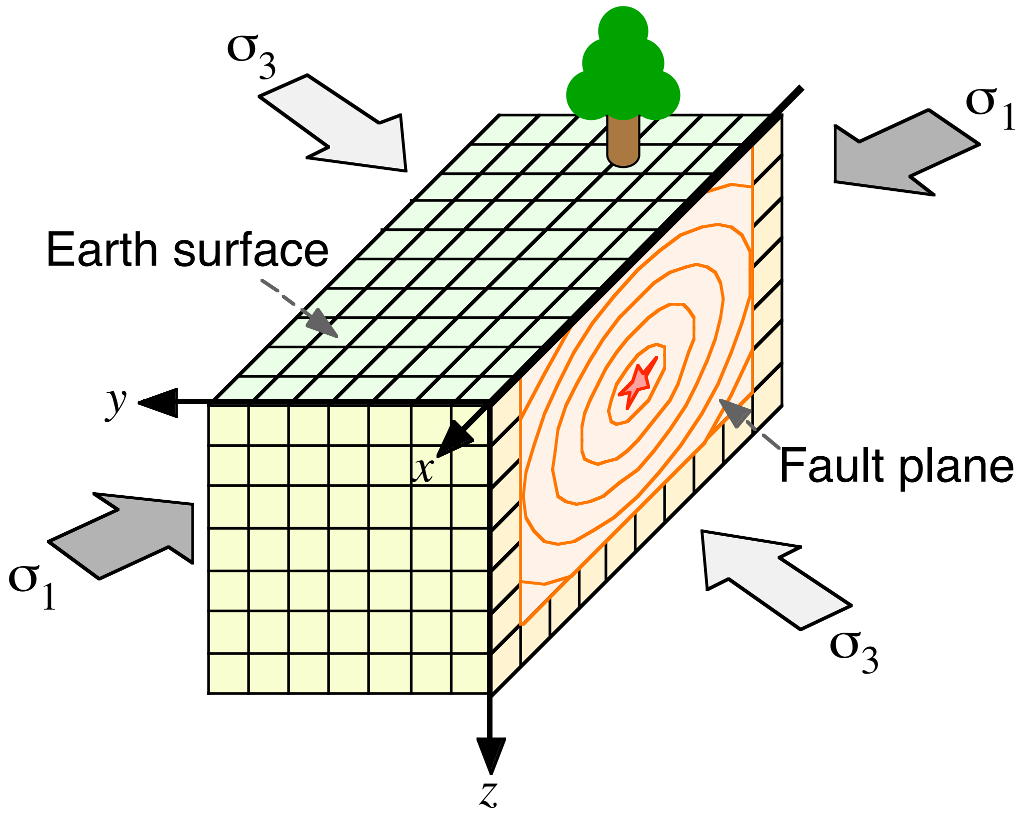
3-D model for calculation



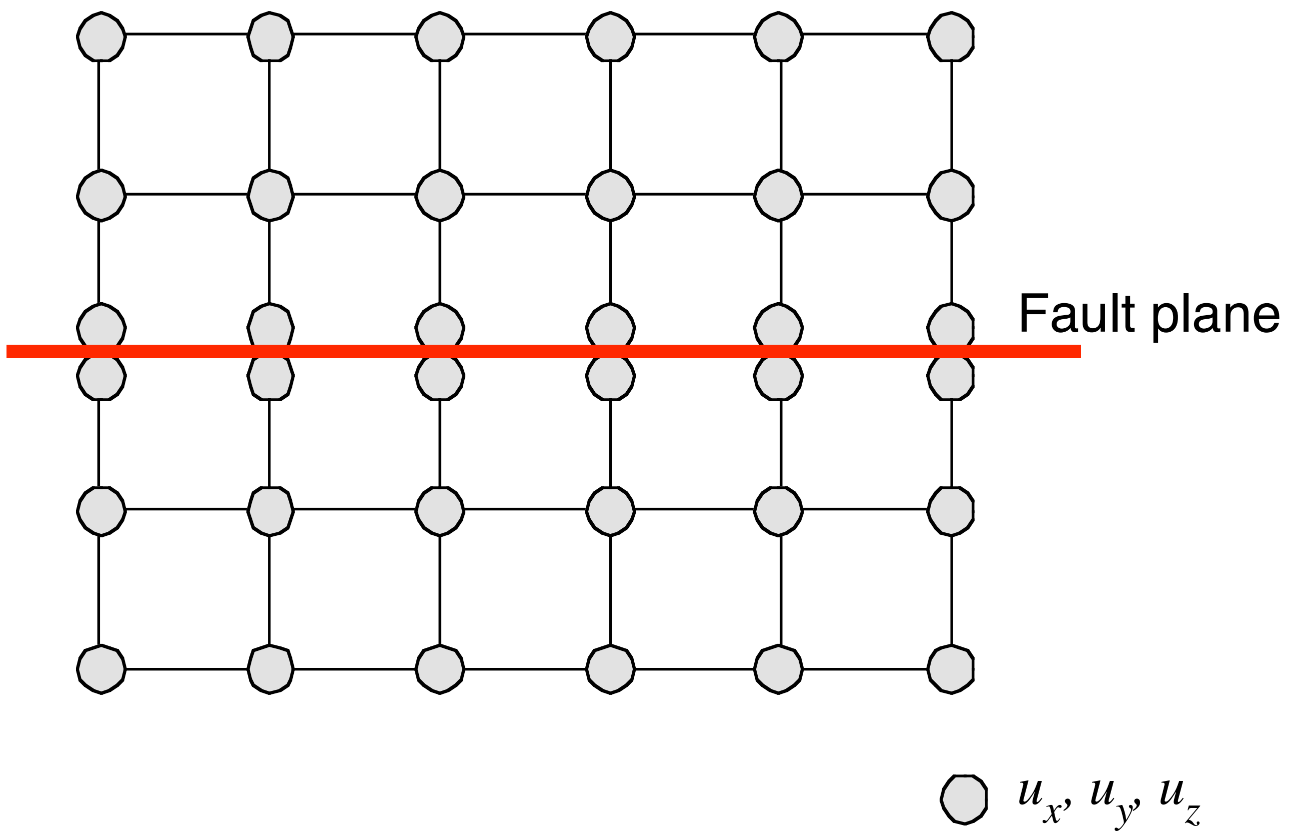
$\Delta x = 0.1$ km
$\Delta t = 0.01$ s

$\sigma_1 = 190$ MPa $\quad\Leftrightarrow\quad$ $\tau_{yy}^0 = 120$ MPa

$\sigma_3 = 50$ MPa $\quad\quad\quad\quad$ $\tau_{xy}^0 = 70$ MPa

Finite difference method

Conventional grid
Central 2nd-order finite-difference
Split-node for fault



Wave equations

$$\rho \frac{\partial^2 u_x}{\partial t^2} = (\lambda + 2\mu)\frac{\partial^2 u_x}{\partial x^2} + \mu \frac{\partial^2 u_x}{\partial y^2} + \mu \frac{\partial^2 u_x}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 u_y}{\partial x \partial y} + (\lambda + \mu)\frac{\partial^2 u_z}{\partial z \partial x}$$

$$\rho \frac{\partial^2 u_y}{\partial t^2} = \mu \frac{\partial^2 u_y}{\partial x^2} + (\lambda + 2\mu)\frac{\partial^2 u_y}{\partial y^2} + \mu \frac{\partial^2 u_y}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 u_z}{\partial y \partial z} + (\lambda + \mu)\frac{\partial^2 u_x}{\partial x \partial y}$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = \mu \frac{\partial^2 u_z}{\partial x^2} + \mu \frac{\partial^2 u_z}{\partial y^2} + (\lambda + 2\mu)\frac{\partial^2 u_z}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 u_x}{\partial z \partial x} + (\lambda + \mu)\frac{\partial^2 u_y}{\partial y \partial z}$$

Initial conditions

$$u_x = u_y = u_z = 0$$

$$\frac{\partial u_x}{\partial t} = \frac{\partial u_y}{\partial t} = \frac{\partial u_z}{\partial t} = 0$$

Stresses on fault

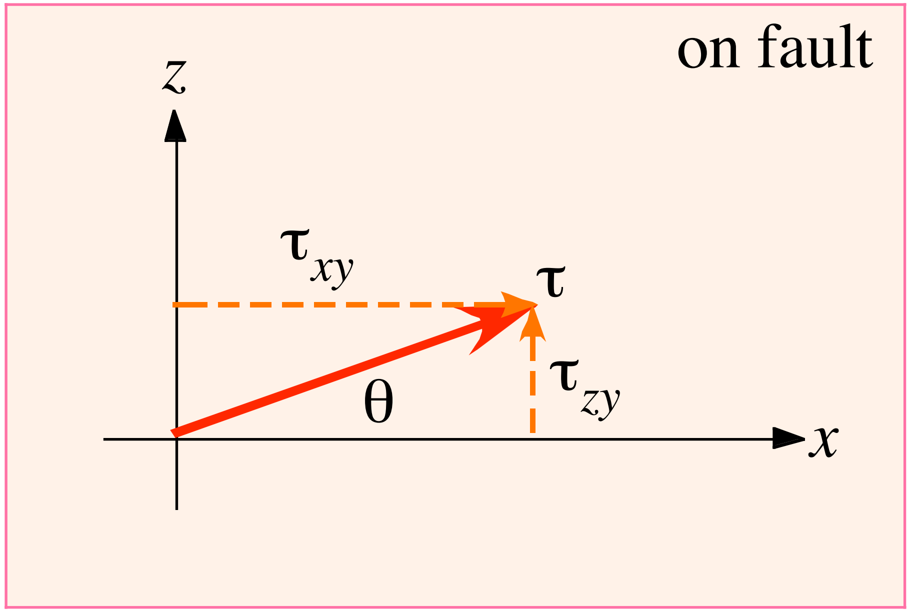
$$\tau_{xy} = \tau_{xy}{}^{0} + \mu \frac{\partial u_x}{\partial y} + \mu \frac{\partial u_y}{\partial x}$$

$$\tau_{zy} = \mu \frac{\partial u_y}{\partial z} + \mu \frac{\partial u_z}{\partial y}$$

$$\tau_{yy} = \tau_{yy}{}^{0} + \lambda \frac{\partial u_x}{\partial x} + (\lambda + 2\mu) \frac{\partial u_y}{\partial y} + \lambda \frac{\partial u_z}{\partial z}$$

Total shear stress

$$\tau = \tau_{xy} \cos\theta + \tau_{zy} \sin\theta$$



Boundary conditions on fault

Before rupture $u_x, u_y, u_z = \text{continuous}$

$\tau_{xy}, \tau_{yy}, \tau_{zy} = \text{continuous}$

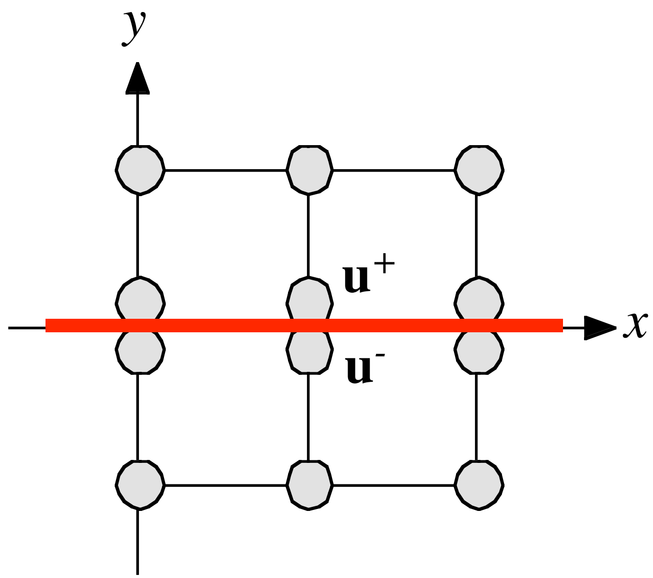
Rupture criterion $\tau \geq \mu_s \tau_{yy}$

After rupture $u_y = \text{continuous}$

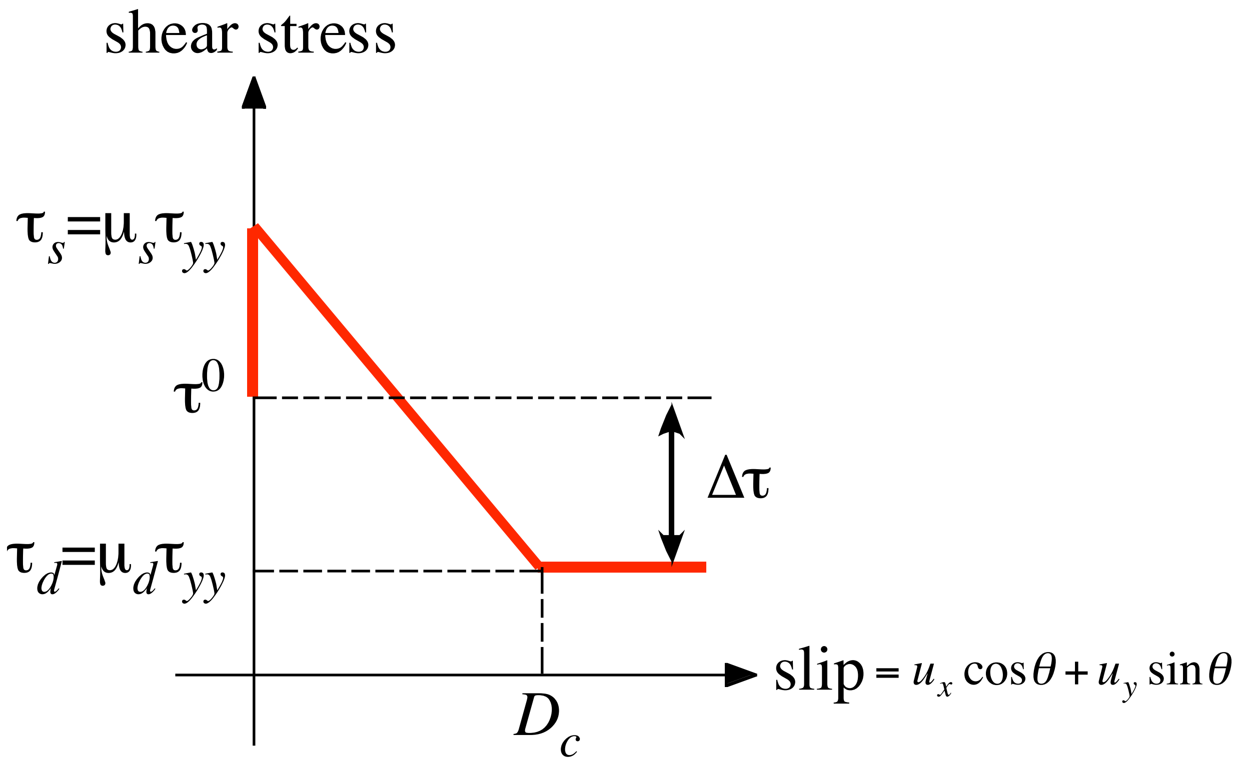
$\tau_{xy}, \tau_{yy}, \tau_{zy} = \text{continuous}$

$\tau = \text{slip} - \text{weakening}: \quad \Delta\tau_{xy} = \Delta\tau \cos\theta$

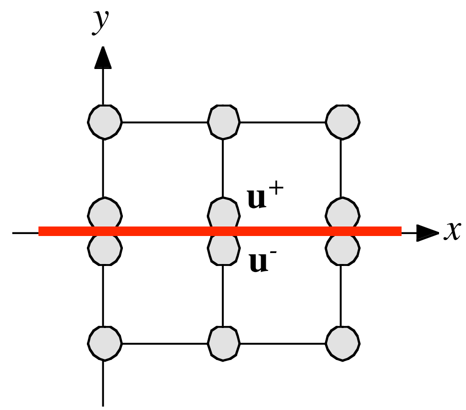
$\Delta\tau_{zy} = \Delta\tau \sin\theta$



Slip-weakening law



Stresses on fault



$$\tau_{xy} = \mu \frac{\partial u_x}{\partial y} + \mu \frac{\partial u_y}{\partial x}$$

Using Taylor expansion

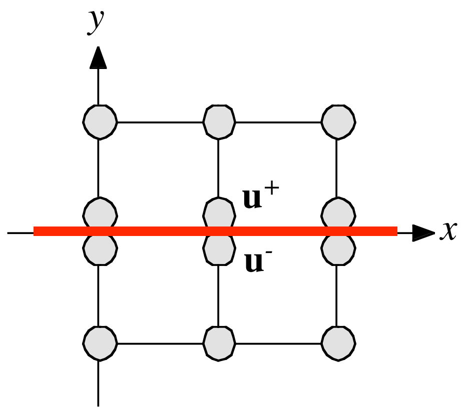
$$u_x(x, y+1, z, t) = u_x(x, y+0, z, t) + \frac{\partial u_x^+}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 u_x^+}{\partial y^2}(\Delta y)^2$$

$$\Rightarrow \frac{\partial u_x^+}{\partial y} = \frac{u_x(x, y+1, z, t) - u_x(x, y+0, z, t)}{\Delta y} - \frac{\Delta y}{2}\frac{\partial^2 u_x^+}{\partial y^2}$$

$$u_x(x, y-1, z, t) = u_x(x, y-0, z, t) - \frac{\partial u_x^-}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 u_x^-}{\partial y^2}(\Delta y)^2$$

$$\Rightarrow \frac{\partial u_x^-}{\partial y} = \frac{u_x(x, y-0, z, t) - u_x(x, y-1, z, t)}{\Delta y} + \frac{\Delta y}{2}\frac{\partial^2 u_x^-}{\partial y^2}$$

Stresses on fault



$$\tau_{xy}^{+} = \mu \frac{\partial u_y^{+}}{\partial x} + \mu \left(\frac{u_x(x, y+1, z, t) - u_x(x, y+0, z, t)}{\Delta y} - \frac{\Delta y}{2} \frac{\partial^2 u_x^{+}}{\partial y^2} \right)$$

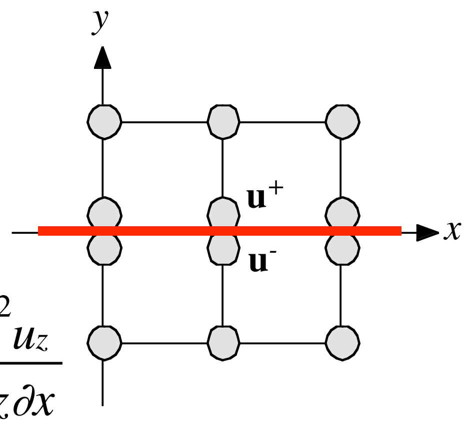
$$\tau_{xy}^{-} = \mu \frac{\partial u_y^{-}}{\partial x} + \mu \left(\frac{u_x(x, y-0, z, t) - u_x(x, y-1, z, t)}{\Delta y} + \frac{\Delta y}{2} \frac{\partial^2 u_x^{-}}{\partial y^2} \right)$$

u_y, τ_{xy} = continuous

$$\Rightarrow \quad 2\tau_{xy} = 2\mu \frac{\partial u_y}{\partial x} + \mu \left(\frac{u_x(x, y+1, z, t) - u_x(x, y+0, z, t)}{\Delta y} + \frac{u_x(x, y-0, z, t) - u_x(x, y-1, z, t)}{\Delta y} \right)$$

$$- \mu \frac{\Delta y}{2} \left(\frac{\partial^2 u_x^{+}}{\partial y^2} - \frac{\partial^2 u_x^{-}}{\partial y^2} \right)$$

Stresses on fault



Using wave equation

$$\rho\frac{\partial^2 u_x}{\partial t^2}=(\lambda+2\mu)\frac{\partial^2 u_x}{\partial x^2}+\mu\frac{\partial^2 u_x}{\partial y^2}+\mu\frac{\partial^2 u_x}{\partial z^2}+(\lambda+\mu)\frac{\partial^2 u_y}{\partial x\partial y}+(\lambda+\mu)\frac{\partial^2 u_z}{\partial z\partial x}$$

$$\Rightarrow \frac{\partial^2 u_x}{\partial y^2}=\frac{1}{\mu}\left\{\rho\frac{\partial^2 u_x}{\partial t^2}-(\lambda+2\mu)\frac{\partial^2 u_x}{\partial x^2}-\mu\frac{\partial^2 u_x}{\partial z^2}-(\lambda+\mu)\frac{\partial^2 u_y}{\partial x\partial y}-(\lambda+\mu)\frac{\partial^2 u_z}{\partial z\partial x}\right\}$$

u_y = continuous

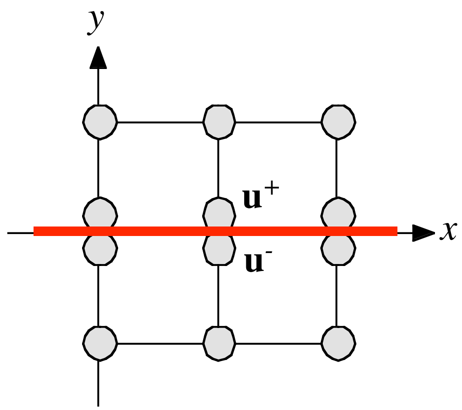
$$\Rightarrow \frac{\partial^2 u_x^{+}}{\partial y^2}-\frac{\partial^2 u_x^{-}}{\partial y^2}=$$

$$-\frac{1}{\mu}\left\{(\lambda+2\mu)\left(\frac{\partial^2 u_x^{+}}{\partial x^2}-\frac{\partial^2 u_x^{-}}{\partial x^2}\right)+\mu\left(\frac{\partial^2 u_x^{+}}{\partial z^2}-\frac{\partial^2 u_x^{-}}{\partial z^2}\right)+(\lambda+\mu)\left(\frac{\partial^2 u_z^{+}}{\partial z\partial x}-\frac{\partial^2 u_z^{-}}{\partial z\partial x}\right)\right\}$$

$$\tau_{xy}=\mu\frac{\partial u_y}{\partial x}+\mu\frac{u_x(x,y+1,z,t)-u_x(x,y-1,z,t)}{2\Delta y}$$

$$+\frac{\Delta y}{4}\left\{(\lambda+2\mu)\left(\frac{\partial^2 u_x^{+}}{\partial x^2}-\frac{\partial^2 u_x^{-}}{\partial x^2}\right)+\mu\left(\frac{\partial^2 u_x^{+}}{\partial z^2}-\frac{\partial^2 u_x^{-}}{\partial z^2}\right)+(\lambda+\mu)\left(\frac{\partial^2 u_z^{+}}{\partial z\partial x}-\frac{\partial^2 u_z^{-}}{\partial z\partial x}\right)\right\}$$

Displacements on fault



$$\tau_{xy} = \mu \frac{\partial u_x}{\partial y} + \mu \frac{\partial u_y}{\partial x} = -\Delta\tau_{xy}$$

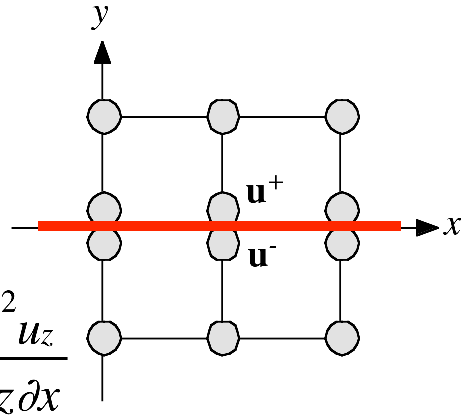
Using Taylor expansion

$$u_x(x, y+1, z, t) = u_x(x, y+0, z, t) + \frac{\partial u_x^+}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 u_x^+}{\partial y^2}(\Delta y)^2$$

$$\Rightarrow \frac{\partial u_x^+}{\partial y} = \frac{u_x(x, y+1, z, t) - u_x(x, y+0, z, t)}{\Delta y} - \frac{\Delta y}{2}\frac{\partial^2 u_x^+}{\partial y^2}$$

$$-\Delta\tau_{xy} = \mu \frac{\partial u_y^+}{\partial x} + \mu \left\{ \frac{u_x(x, y+1, z, t) - u_x(x, y+0, z, t)}{\Delta y} - \frac{\Delta y}{2}\frac{\partial^2 u_x^+}{\partial y^2} \right\}$$

Displacements on fault



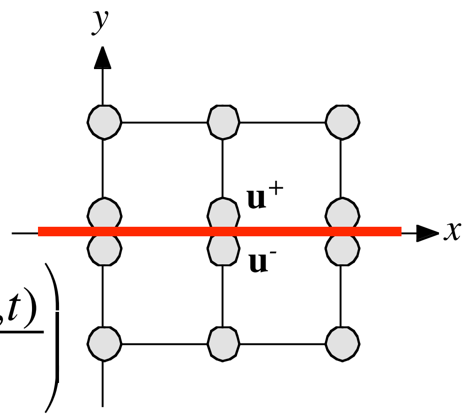
Using wave equation

$$\rho\frac{\partial^2 u_x}{\partial t^2} = (\lambda+2\mu)\frac{\partial^2 u_x}{\partial x^2} + \mu\frac{\partial^2 u_x}{\partial y^2} + \mu\frac{\partial^2 u_x}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u_y}{\partial x\partial y} + (\lambda+\mu)\frac{\partial^2 u_z}{\partial z\partial x}$$

$$\Rightarrow \frac{\partial^2 u_x}{\partial y^2} = \frac{1}{\mu}\left\{\rho\frac{\partial^2 u_x}{\partial t^2} - (\lambda+2\mu)\frac{\partial^2 u_x}{\partial x^2} - \mu\frac{\partial^2 u_x}{\partial z^2} - (\lambda+\mu)\frac{\partial^2 u_y}{\partial x\partial y} - (\lambda+\mu)\frac{\partial^2 u_z}{\partial z\partial x}\right\}$$

$$-\Delta\tau_{xy} = \mu\frac{\partial u_y^+}{\partial x} + \mu\frac{u_x(x,y+1,z,t) - u_x(x,y+0,z,t)}{\Delta y} - \frac{\Delta y}{2}\left\{\rho\frac{\partial^2 u_x^+}{\partial t^2} - (\lambda+2\mu)\frac{\partial^2 u_x^+}{\partial x^2} - \mu\frac{\partial^2 u_x^+}{\partial z^2} - (\lambda+\mu)\frac{\partial^2 u_y^+}{\partial x\partial y} - (\lambda+\mu)\frac{\partial^2 u_z^+}{\partial z\partial x}\right\}$$

Displacements on fault



$$\rho \frac{\partial^2 u_x^+}{\partial t^2} = -\frac{2}{\Delta y}\Delta\tau_{xy} - \frac{2}{\Delta y}\left(\mu \frac{\partial u_y^+}{\partial x} + \mu \frac{u_x(x,y+1,z,t) - u_x(x,y+0,z,t)}{\Delta y}\right) + (\lambda+2\mu)\frac{\partial^2 u_x^+}{\partial x^2} + \mu \frac{\partial^2 u_x^+}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u_y^+}{\partial x \partial y} + (\lambda+\mu)\frac{\partial^2 u_z^+}{\partial z \partial x}$$

Replaced by FDM equation

$$u_x(x,y+0,z,t+1) = 2u_x(x,y+0,z,t) - u_x(x,y+0,z,t-1) + \frac{(\Delta t)^2}{\rho}\left\{-\frac{2}{\Delta y}\Delta\tau_{xy} - \frac{2}{\Delta y}\left(\mu \frac{\partial u_y^+}{\partial x} + \mu \frac{u_x(x,y+1,z,t) - u_x(x,y+0,z,t)}{\Delta y}\right) + (\lambda+2\mu)\frac{\partial^2 u_x^+}{\partial x^2} + \mu \frac{\partial^2 u_x^+}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u_y^+}{\partial x \partial y} + (\lambda+\mu)\frac{\partial^2 u_z^+}{\partial z \partial x}\right\}$$