A Finite Volume Approach for Modeling Rupture Dynamics

Collaboration between SDSU and INRIA / CNRS

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What has been accomplished?

1. Conception and validation in 2D of the FV dynamic rupture model. (Benjemaa et al., 2006, submitted to Geophys. J. Int.)
2. Implementation of the 3D elastic wave propagation parallel code.
3. Implementation of the Kostrov self-similar fixed-velocity circular crack.

Where do we stand?

1. Testing 3D wave propagation accuracy of the finite volume solver.
2. Implementing the 3D spontaneous rupture model in the wave propagation parallel code.
3. Exploring tetrahedral mesh generators to discretize models with both real free surface and fault geometries.

Finite Volume Model

Velocity-Stress Hyperbolic System

$$\rho \frac{\partial v_i}{\partial t} = \tau_{ij,j} + f_i$$
$$\frac{\partial \tau_{ij}}{\partial t} = \lambda v_{k,k}\delta_{ij} + \mu(v_{i,j} + v_{j,i})$$

Integration over a Control Volume

$$\int_{T_i} \Lambda_i \vec{W}_t = \int_{\partial T_i} \vec{\mathcal{F}}(\vec{W})\,\vec{n}\,dS$$

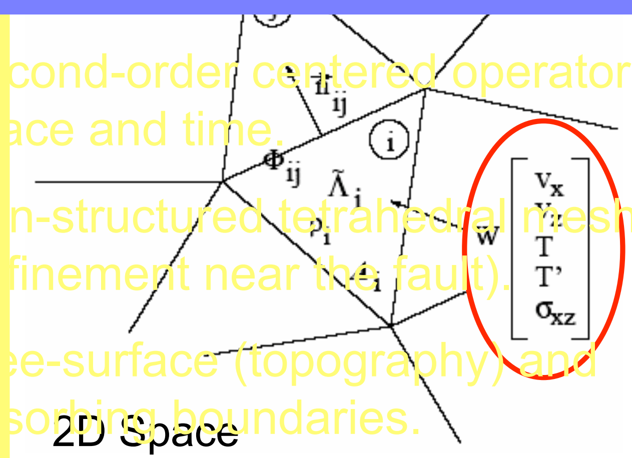
For 2D Space we have

$$\vec{W} = {}^t(v_x, v_z, T, T', \sigma_{xz})$$
$$\Lambda = \mathrm{diag}\,(\rho, \rho, \frac{1}{\lambda+\mu}, \frac{1}{\mu}, \frac{1}{\mu})$$

Quite Fast Centered Scheme

Variables Given by Boundary Fluxes

$$\mathcal{A}_i\,\Lambda_i\,(\vec{W}_t)_{\mathcal{T}_i} = \sum_{\mathcal{T}_j \in V(\mathcal{T}_i)} \Phi_{ij}$$



3D Numerical Code Features:

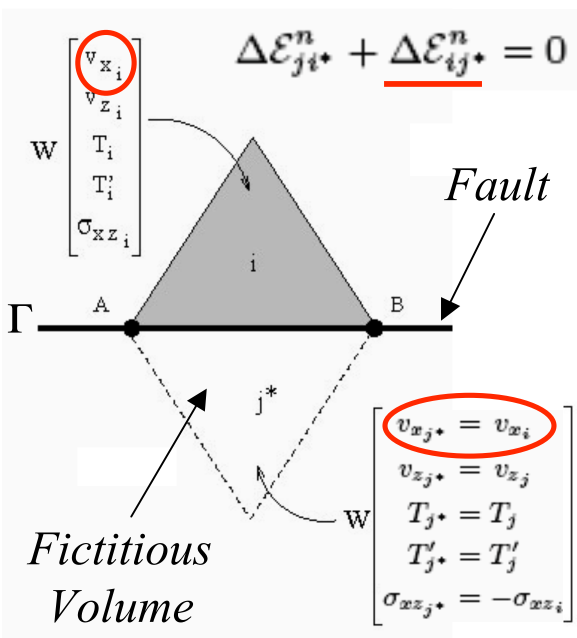
1. Second-order centered operators in space and time.
2. Non-structured tetrahedral mesh (refinement near the fault).
3. Free-surface (topography) and absorbing boundaries.

Dynamic Rupture Boundary Conditions

Mechanical Energy

$$E = \underbrace{\int_{\Omega} \frac{1}{2} \rho \parallel \vec{v} \parallel^2}_{Kinetic} + \underbrace{\int_{\Omega} \frac{1}{2} {}^t\vec{\sigma}\, \tilde{\Lambda}\, \vec{\sigma}}_{Elastic}$$

Fictitious Volume Variables



Energy Conservation Through the Fault

$$\Delta\mathcal{E}^n = \sum_{i,j \subset \Gamma} \Big[\sigma_{xz_i} v_{x_j} + (T_i - T_i{}') v_{z_j} + \sigma_{xz_j} v_{x_i} + (T_j - T_j{}') v_{z_i} \Big] n_{z_{ij}} = 0$$

Friction Condition on Shear Stress Fluxes

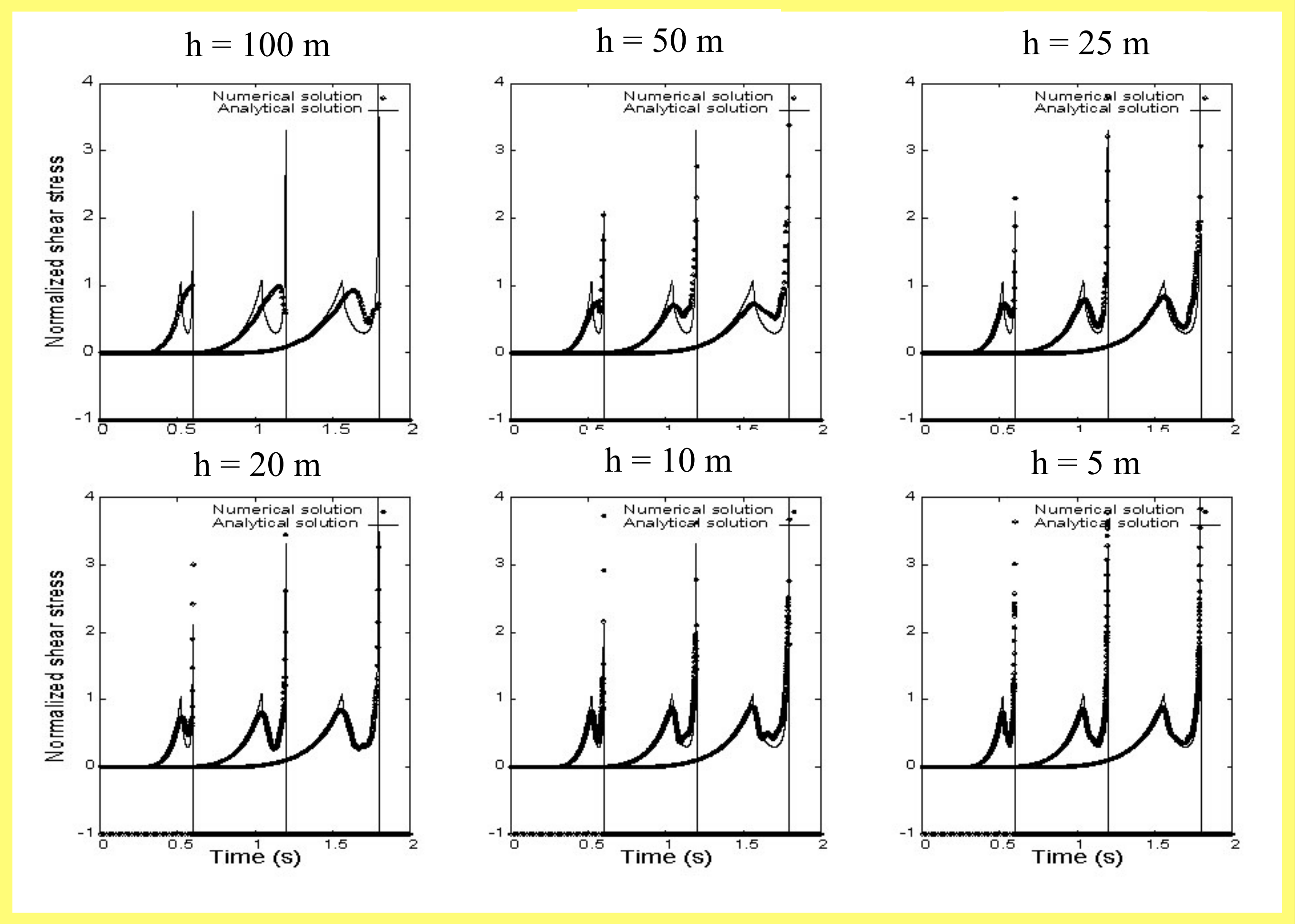
$$\sigma_{xz_{j^*}} = -\sigma_{xz_i} + 2g$$

Conclusive Remark:

Fault boundary conditions imply stress conditions on the fault segment and free tangential velocity discontinuity.

Influence of Unstructured Mesh Refinement

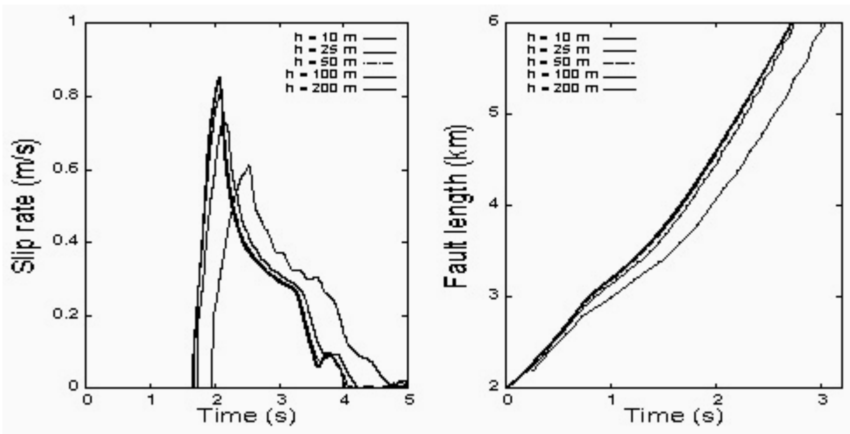
On-Fault Mesh Refinement: Self-Similar Constant-Velocity Crack



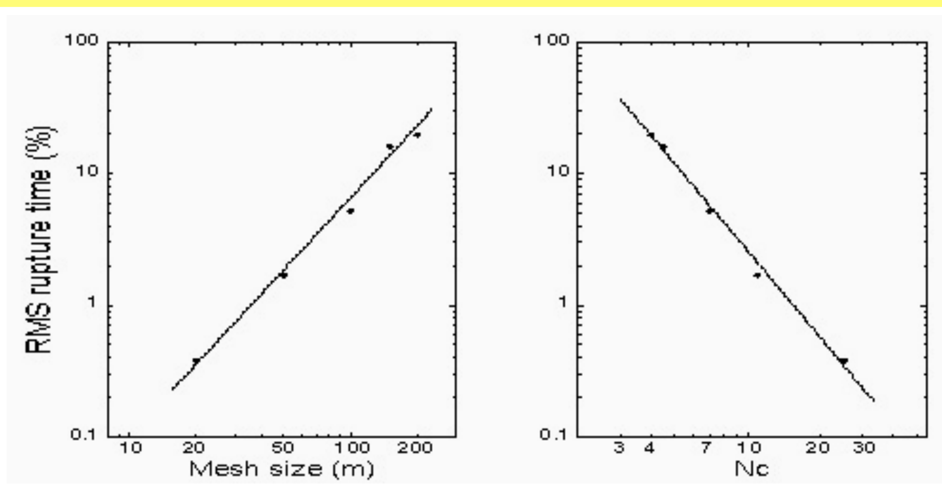
Kostrov Exact Solution (solid) *vs*. Finite Volume Solution (symbols)

Spontaneous Rupture Convergence Analysis

Fault Solutions Convergence



Convergence Rate and Cohesive Zone (Nc)



Influence of Unstructured Mesh Refinement

Spontaneous Rupture Propagation with Slip-Weakening Friction Law.

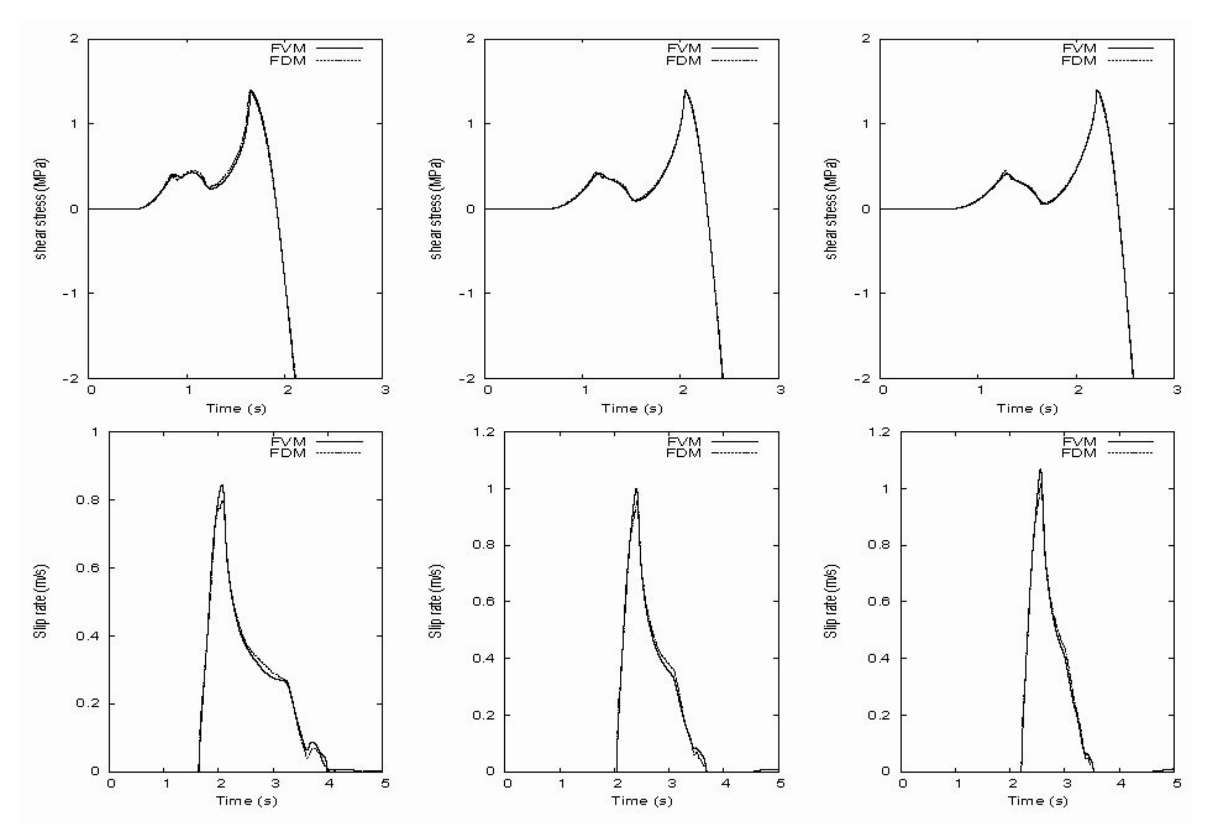
Conclusive Remarks:

1. Power law convergence toward the finest solution.
2. Grid size h=50m along the fault yield error about 1%.
3. Approximately eight fault segments inside the breakdown cohesive zone.

Comparison with a Finite Difference Approach

(Slip-Weakening Spontaneous Rupture)

Finite Volumes (solid) *vs*. Finite Difference (dashed)

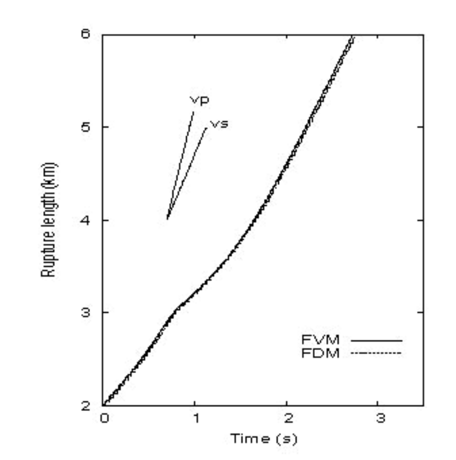


Fault Points Time Histories: Shear Stress and Slip-Rate

(Finite Difference Solutions: Cruz-Atienza and Virieux, 2004)

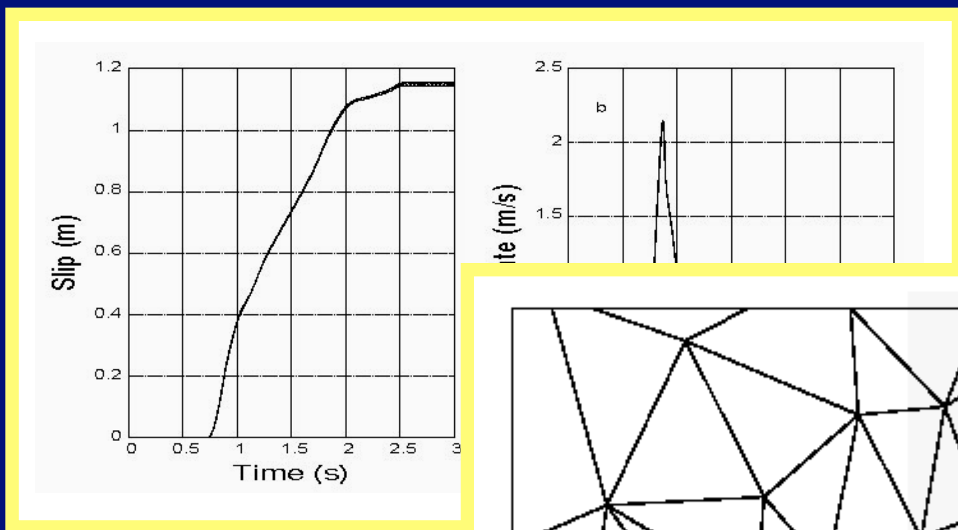
Model Validation

Rupture Fronts Position



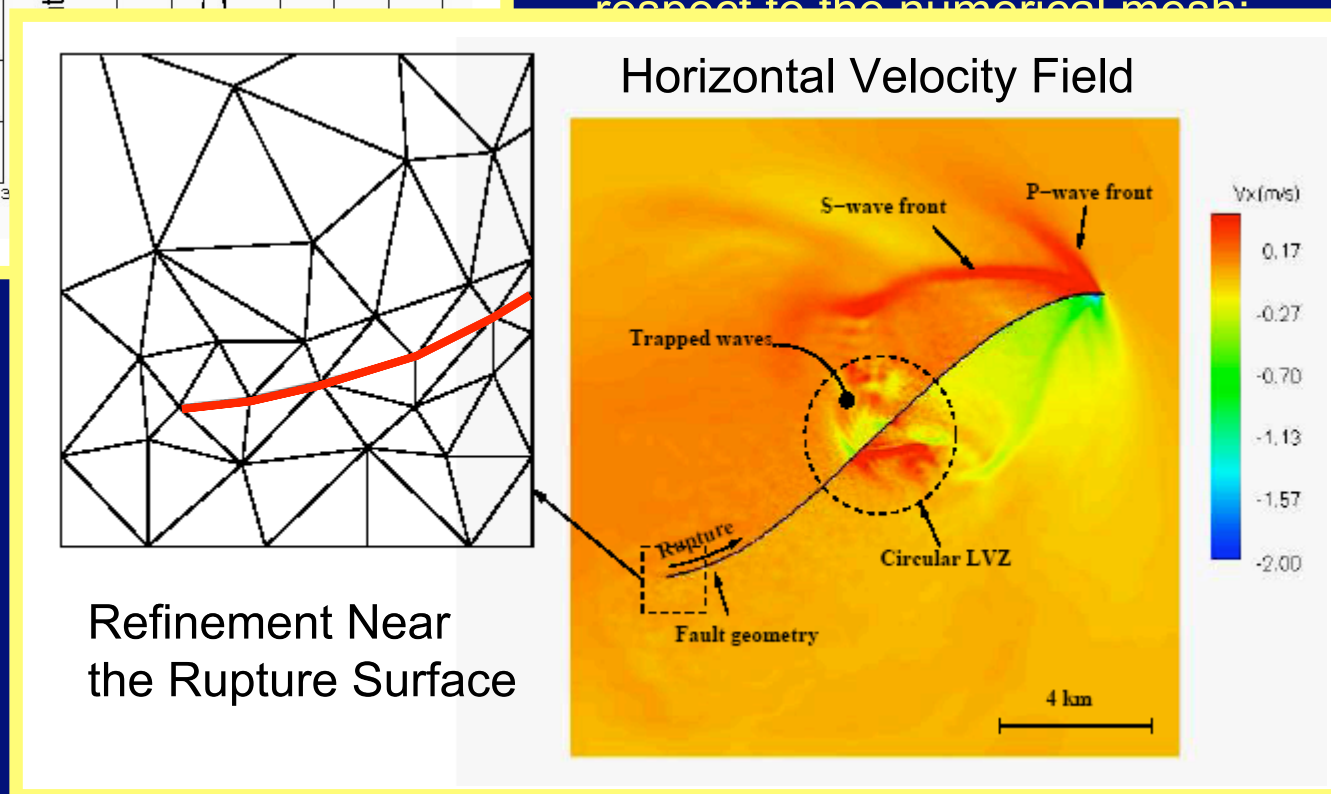
Non-Planar Rupture in Heterogeneous Medium

Solution Invariance with Fault Orientation



Point-Values for different planar fault orientations with respect to the numerical mesh:

Slip-Weakening Non-Planar Spontaneous Rupture

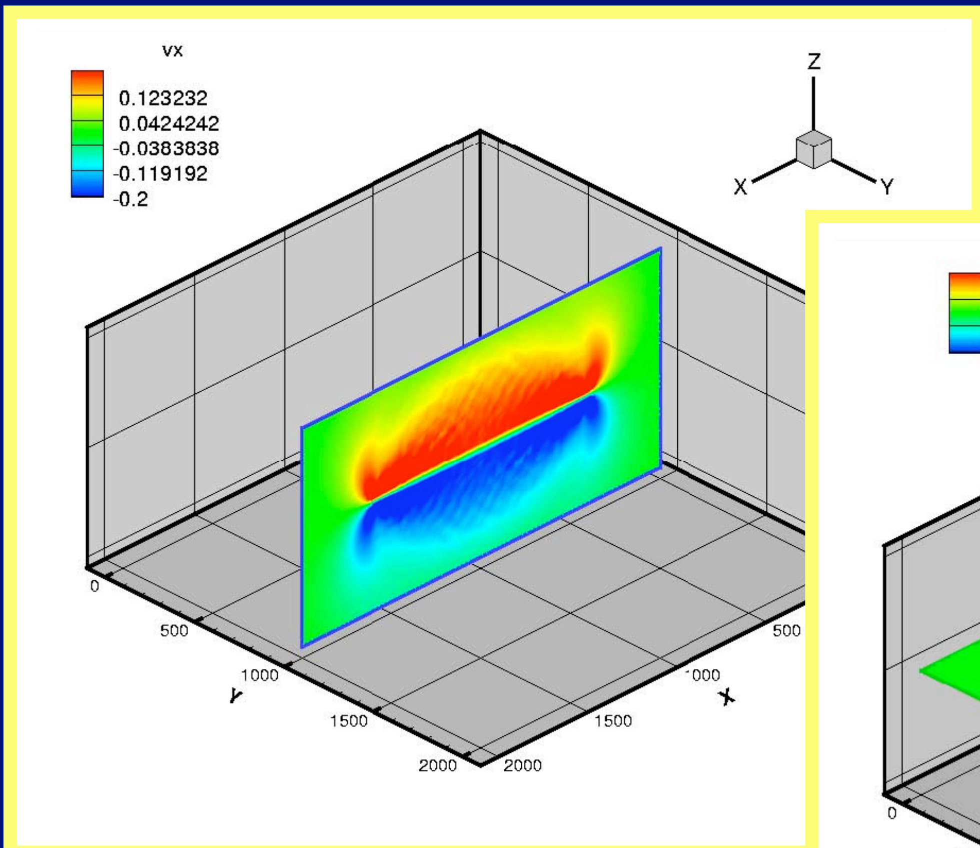


Refinement Near the Rupture Surface

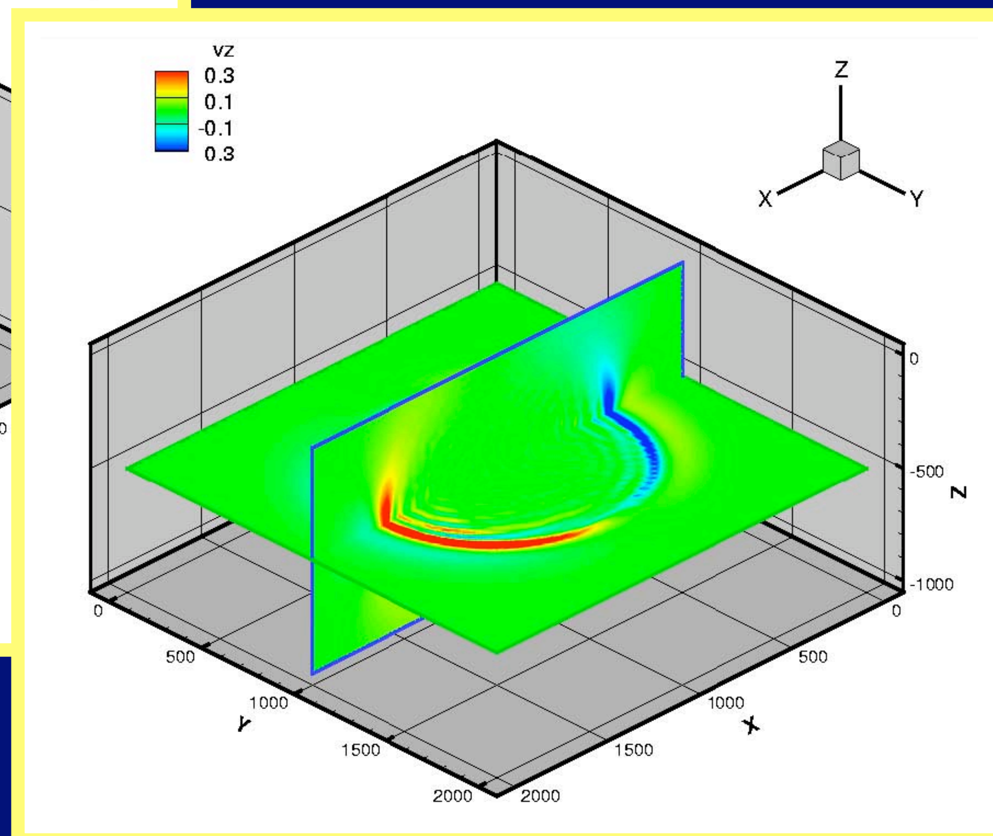
3D Self-Similar Constant Velocity Circular Crack

Preliminary results (Benjemaa et al., 2006, AGU Fall Meeting)

Fault-Parallel Velocity Field



Fault-Normal Velocity Field



Conclusions

1. We presented a new and efficient Finite Volume technique for modeling rupture dynamics in heterogeneous media with non-planar fault geometries.

2. Rupture boundary conditions are applied by imposing shear stress fluxes on the thin fault surface (weak conditions) and by allowing fault-parallel velocity-field free discontinuity.

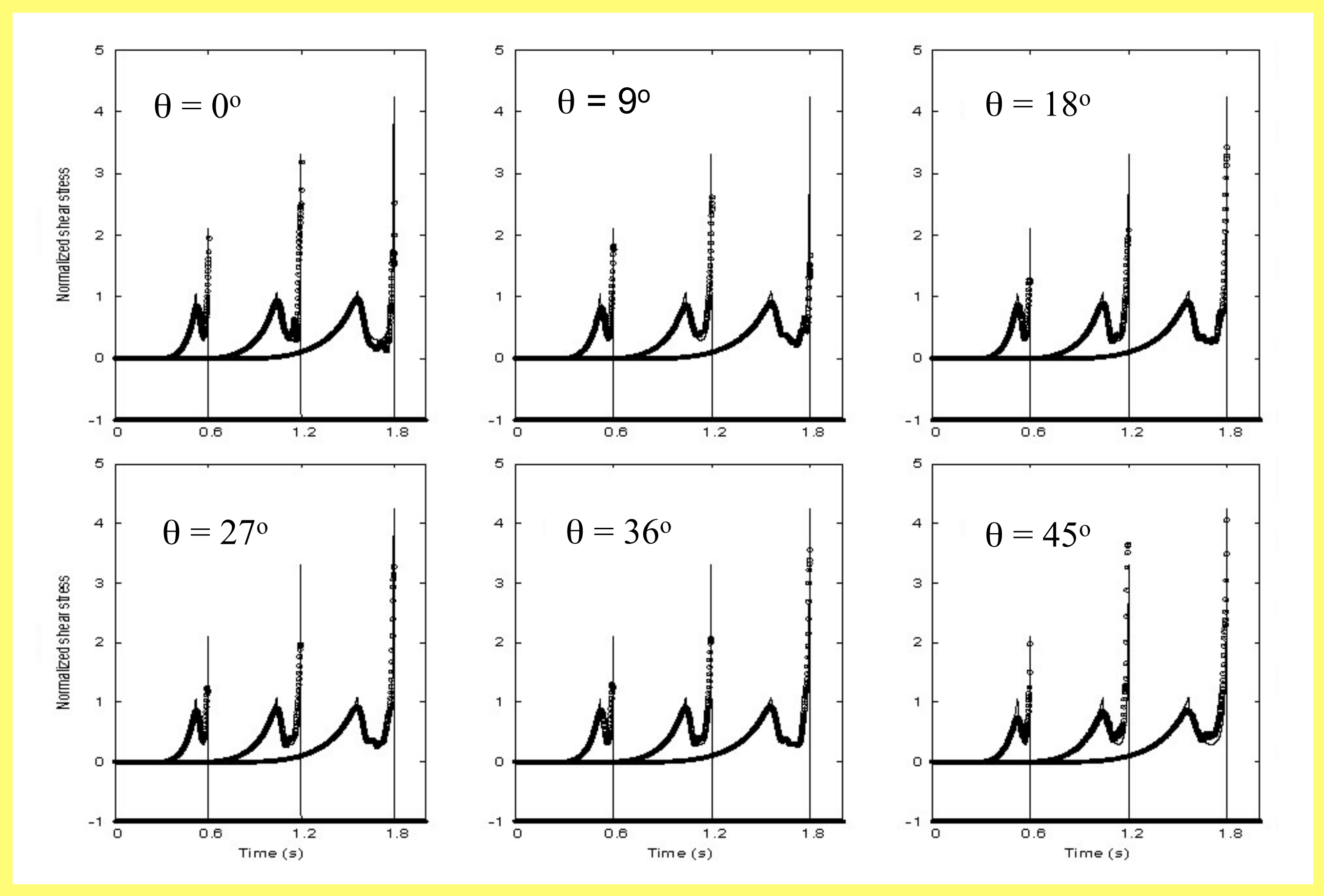
3. The slip-weakening spontaneous rupture model exhibit a power law convergence toward the “real” solution. Grid sizes smaller or equal than 50 m along the fault yielded errors smaller than 1%.

4. The 3D dynamic rupture parallel code may benefit of free surface topography and local mesh refinements around non-planar faults.

The end

Invariance of Solutions with Fault Rotation

On-Fault Mesh Refinement: Self-Similar Constant-Velocity Crack



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3D Self-Similar Constant Velocity Circular Crack

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