

# A Support-Operator Method for Rupture Modeling

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SCEC Code Validation Workshop  
Feb 12, 2007

## Discretization Method

On a curvilinear mesh, form operators that obey a discrete analog of the adjoint relation between gradient and divergence. Based on the the Support Operator method of Shaskov.

# Derivation

$$f(\xi) = \sum_{\alpha, \beta, \gamma=0}^n N_{\alpha\beta\gamma}(\xi) F_{\alpha\beta\gamma}$$

$$\nabla f = \frac{\partial f}{\partial \xi} \cdot \mathbf{J}^{-1}$$

$$(\mathbf{D}_i F)_{jkl} \approx \partial_i f(\xi = 0) \quad \text{One-point quadrature}$$

or

$$(\mathbf{D}_i F)_{jkl} \approx \frac{1}{V_{jkl}} \int_V \partial_i f \, dV \quad \text{Mean stress}$$

# Adjoint Relation

$$\int_V f \partial_i w_i dV + \int_V w_i \partial_i f dV = \int_S f w_i dS_i$$

$$\sum_{j,k,l=1}^{m,n,p} F_{jkl} (\mathcal{D}_i \mathbf{W}_i)_{jkl} = \sum_{j,k,l=1}^{m-1,n-1,p-1} -(\mathbf{D}_i F)_{jkl} (\mathbf{W}_i)_{jkl}$$

# PML Absorbing Boundaries

$$\dot{g}_{ij} + d_j g_{ij} = \partial_j v_i$$

$$w_{ij} = \lambda \delta_{ij} \sum_k g_{kk} + \mu (g_{ij} + g_{ji})$$

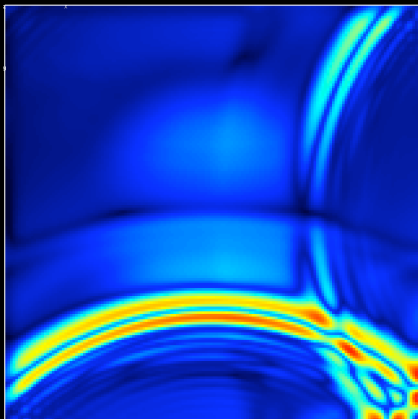
$$\dot{p}_{ij} + d_j p_{ij} = \partial_j w_{ij}$$

$$f_j = \sum_i \dot{p}_{ij}$$

$$\dot{v}_i = \frac{f_i}{m}$$

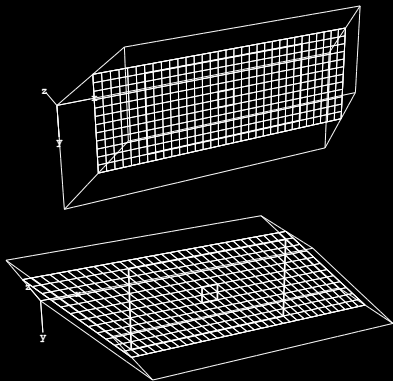
note: summation convention not used here.

# Point Source

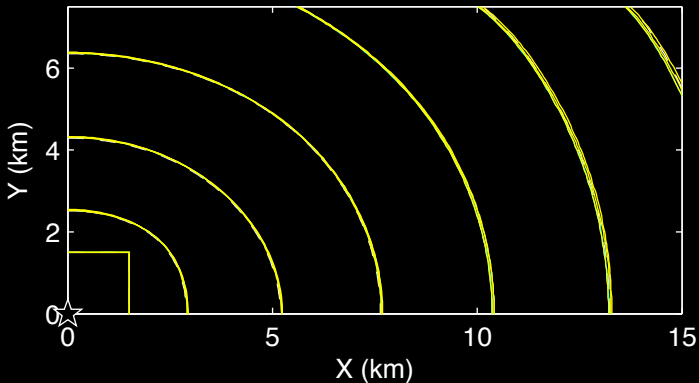


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## Problem ver. 3



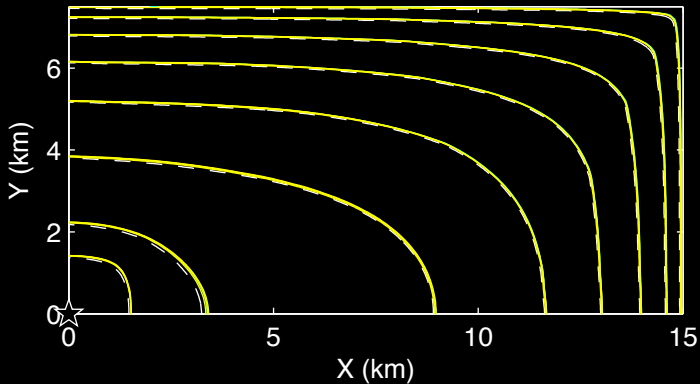
## Rupture Time



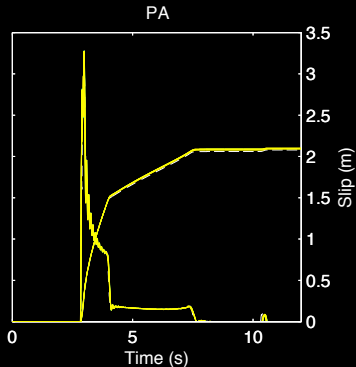
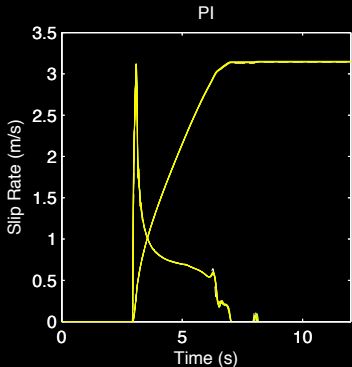
white: 50m rectangular, blue: 100m rectangular, yellow: 100m sheared



# Slip



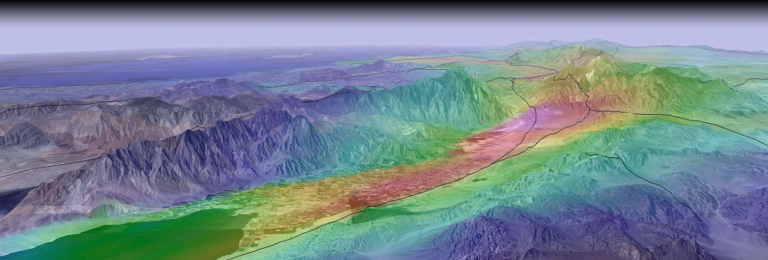
white: 50m rectangular, blue: 100m rectangular, yellow: 100m sheared



white: 50m rectangular, blue: 100m rectangular, yellow: 100m sheared  
 PI: mode II point, PA: mode III point

# Challenges for large problems

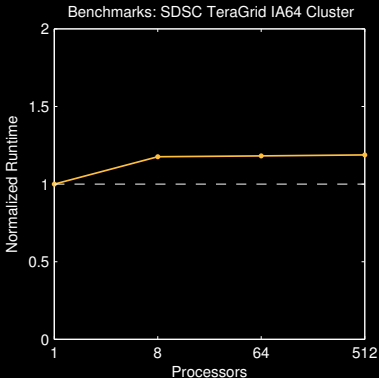
- Large propagation distances
- Non-planar faults
- Topography
- Spatial resolution for rupture dynamics
- Frequency resolution for seismic hazard



# Portability

- Linux: Intel, GNU Fortran 95, MPICH2
- Solaris: Sun Workshop Fortran 95
- SDSC DataStar: IBM Power4+, XL Fortran
- SDSC Teragrid: IA64, Intel Fortran 95, MPICH-GM
- SDSU Babieca Cluster: Intel Xeon, PGI Fortran 95, MPICH-GM

# Scalability



Weak scaling where problem size is proportional to number of processors, and perfect scaling is a horizontal line.

## Summary of Capabilities

- Support-Operator method on curvilinear meshes
- Rupture Dynamics on non-planar faults
- PML absorbing boundary
- Portable and scalable

## Future work

- Attenuation