Slip between Dissimilar Materials: Instabilities, Ill-posedness, and Implications for Numerical Modeling

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with lots of help from and discussions with

Jim Rice, Harvard

Shuo Ma, Stanford

Susana Custodio and Ralph Archuleta, UCSB

Bimaterial Problems (TPV6-8)

material 1:
reduced density/wave speeds

$\rho_1,\ c_{s1},\ c_{p1}$

right-lateral slip $\delta(x,t)$

material 2:
original parameters

$\rho_2 = 2670\ \mathrm{kg/m^3}$
$c_{s2} = 3.464\ \mathrm{km/s}$
$c_{p2} = 6\ \mathrm{km/s}$

y

x

original initial stresses (nucleation by 3 km wide overstressed asperity):
τ^{init}=70 MPa (81.6 MPa)
σ^{init}=120 MPa

original slip-weakening parameters:
f_s=0.677
f_d=0.525
D_c=0.4 m

"positive" → propagation direction

Stability of Steady Sliding at Constant f :

"Positive Direction" [Ranjith and Rice, 2001]

Add Fourier mode perturbation e^{ikx+st} to steadily sliding half-spaces obeying $\tau=f\sigma$ with constant f. Solve for $s(k)$.

Unstable if $\mathrm{Re}(s)>0$, damped if $\mathrm{Re}(s)<0$ (exponential growth/decay in time). Propagating if $\mathrm{Im}(s)$ is nonzero.

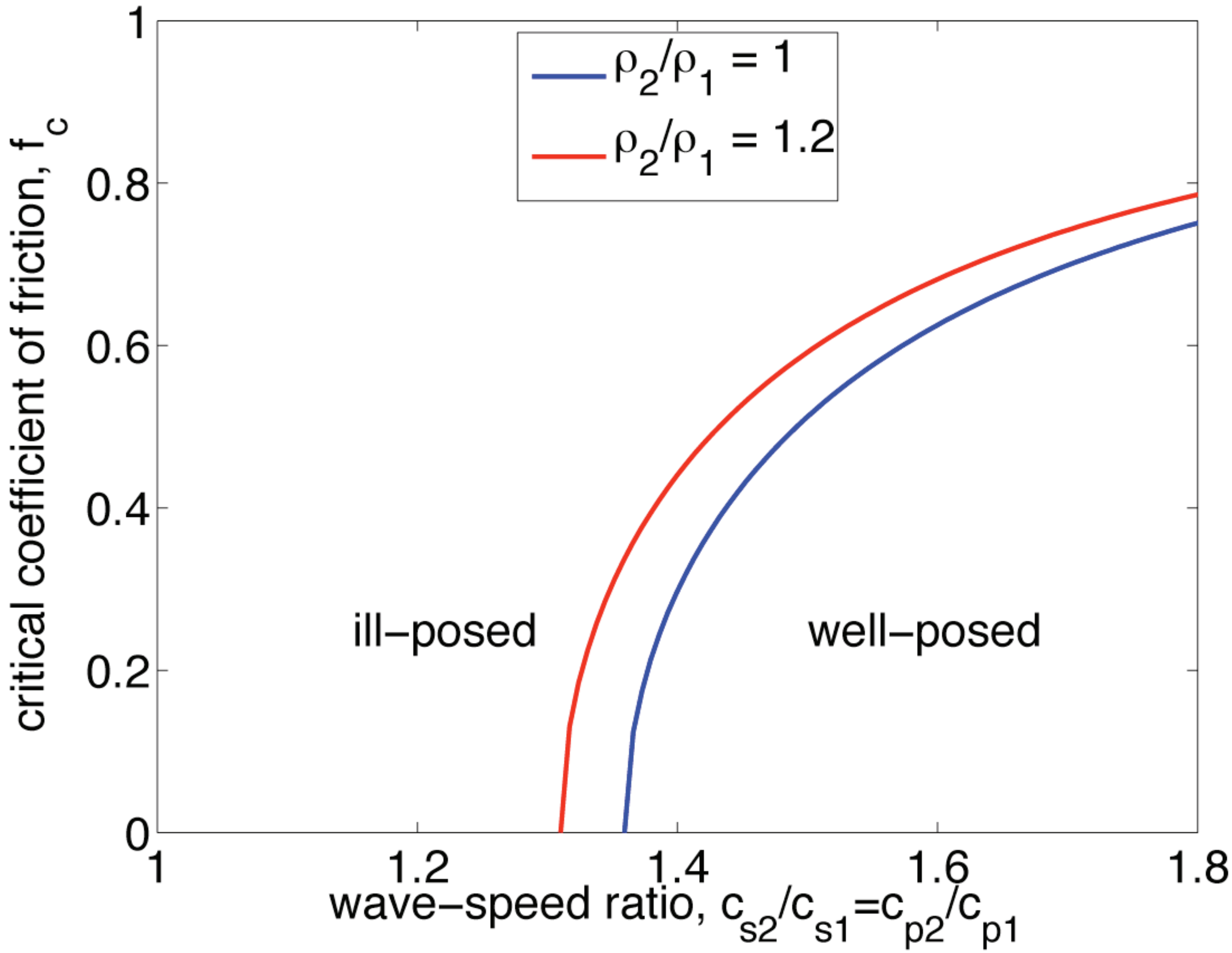
For f=0, perturbations are generalized Rayleigh waves (one propagating in each direction) when wave-speed contrast less than ~30-40% (seismically relevant case):

$$e^{ikx+st} = e^{ik(x \pm ct)} \quad \text{(neutrally stable, } s=\pm ikc\text{)}$$

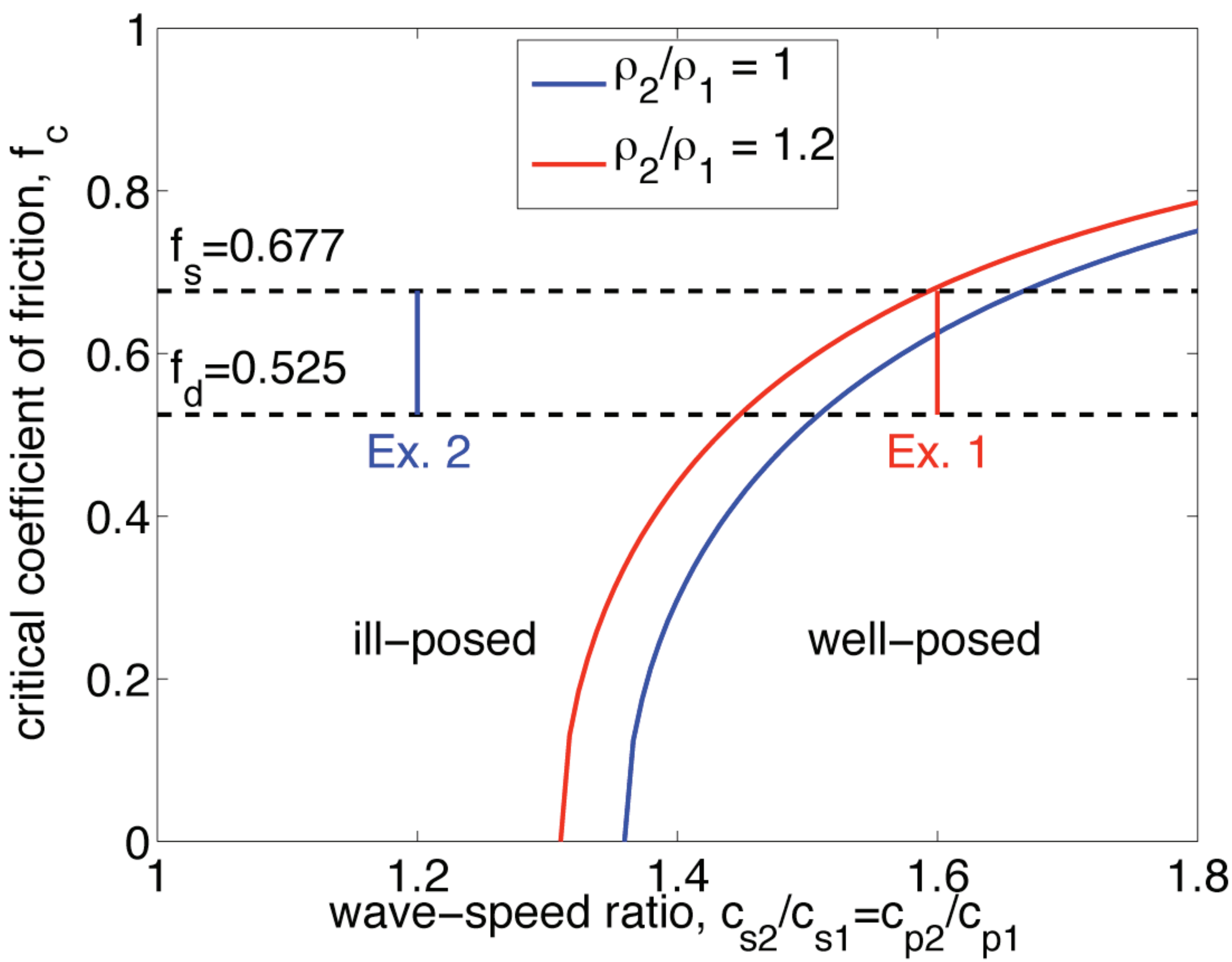
For nonzero f, wave in "positive" direction unstable (normal stress decreases) and wave in "negative" direction damped. Growth/decay rate is proportional to wavenumber $k \Rightarrow$ ill-posed.

Stability of Steady Sliding at Constant f :

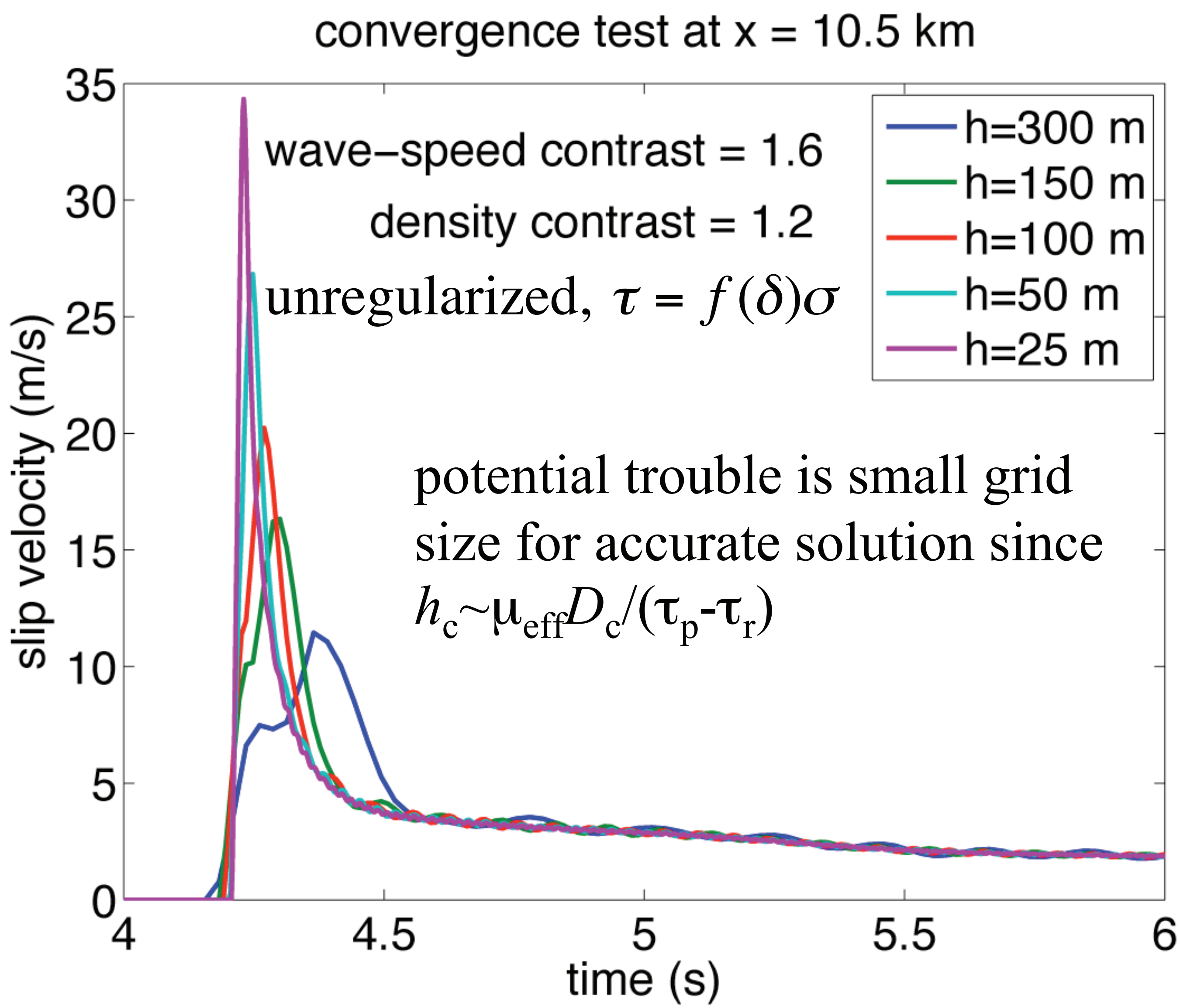
Stable for Large Contrasts and $f<f_c$ [Ranjith and Rice, 2001]



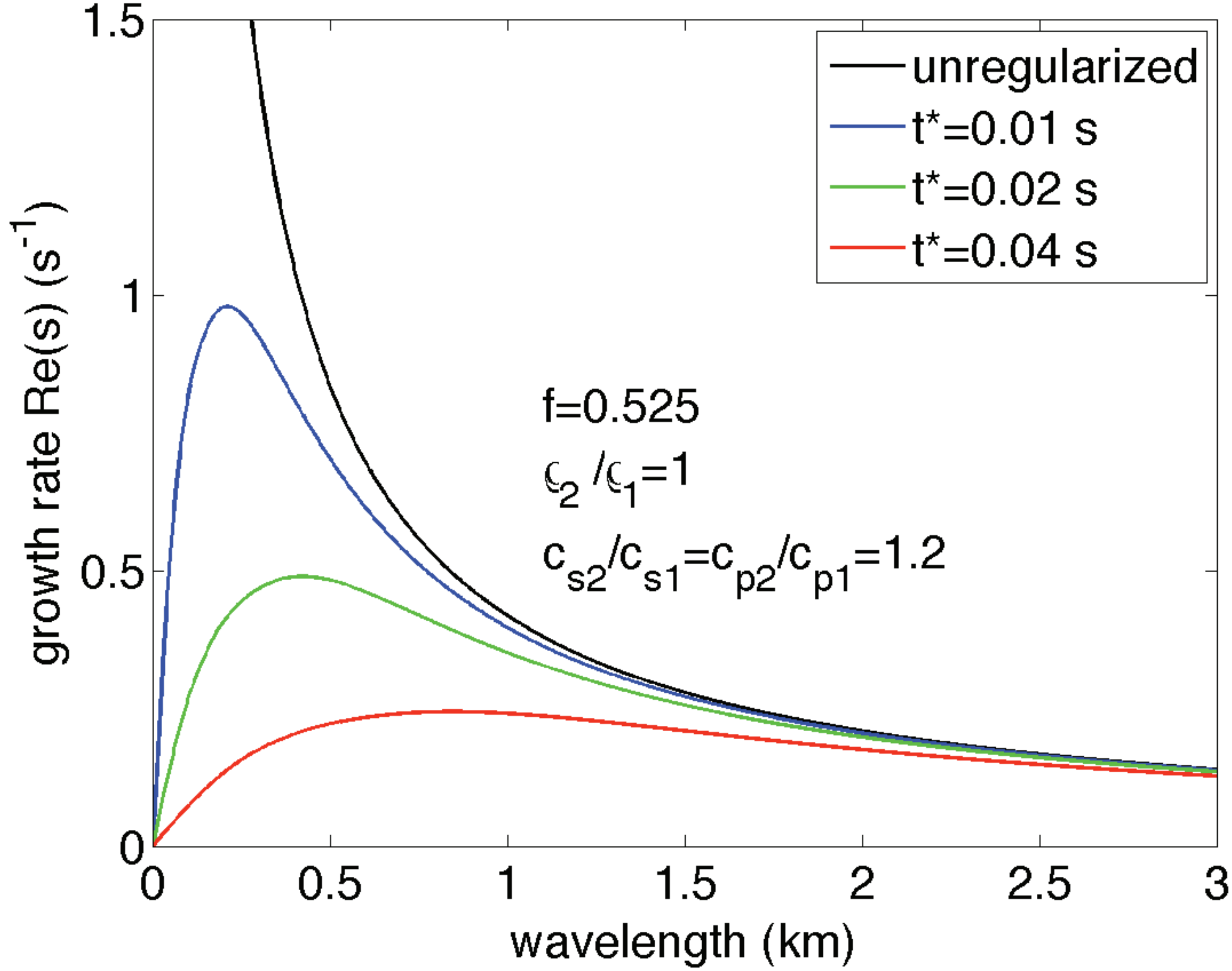
Two Regimes:
1. Well-posed (TPV6), 2. Ill-posed (TPV7/8)



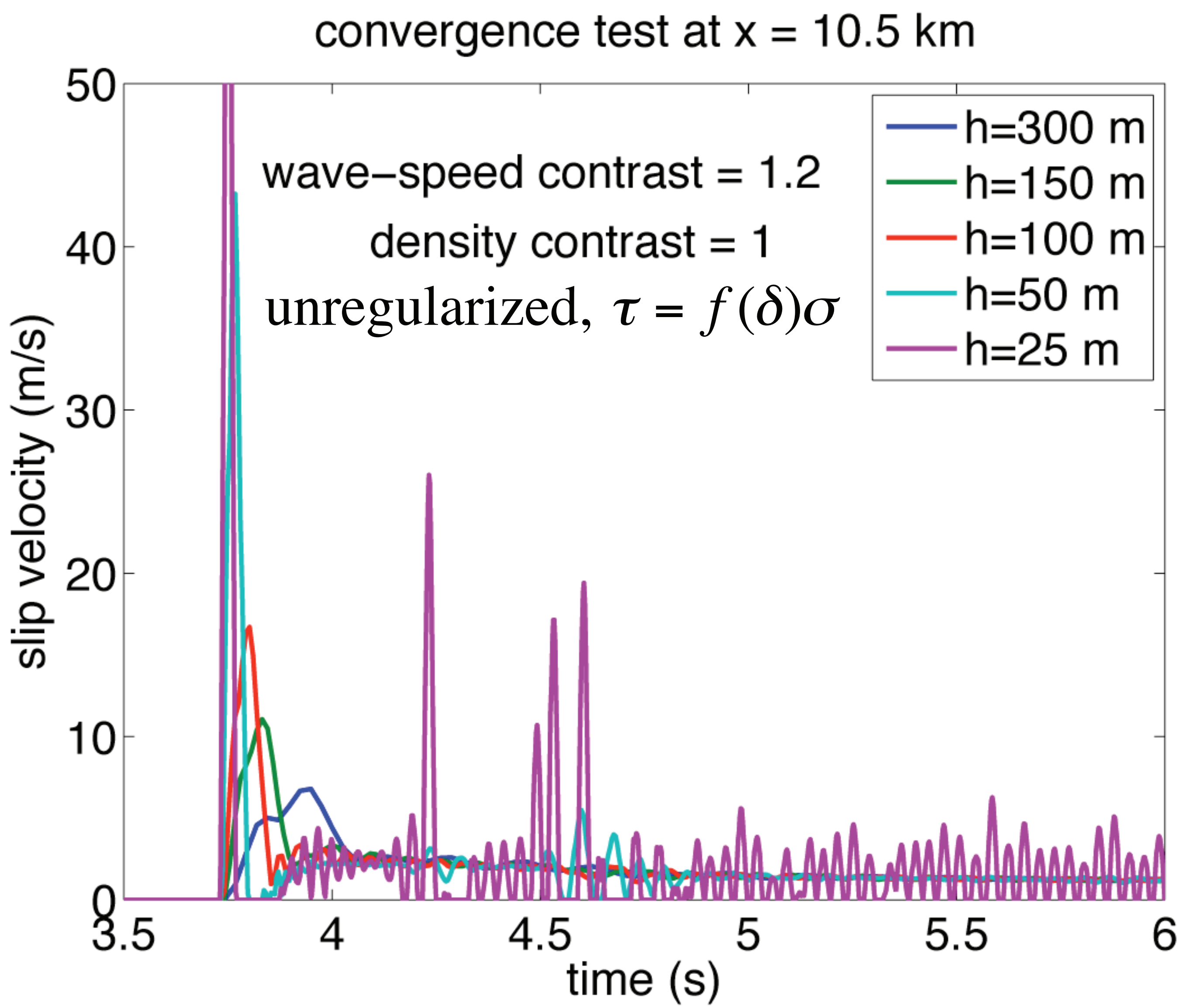
Ex. 1: Well-posed (TPV6)



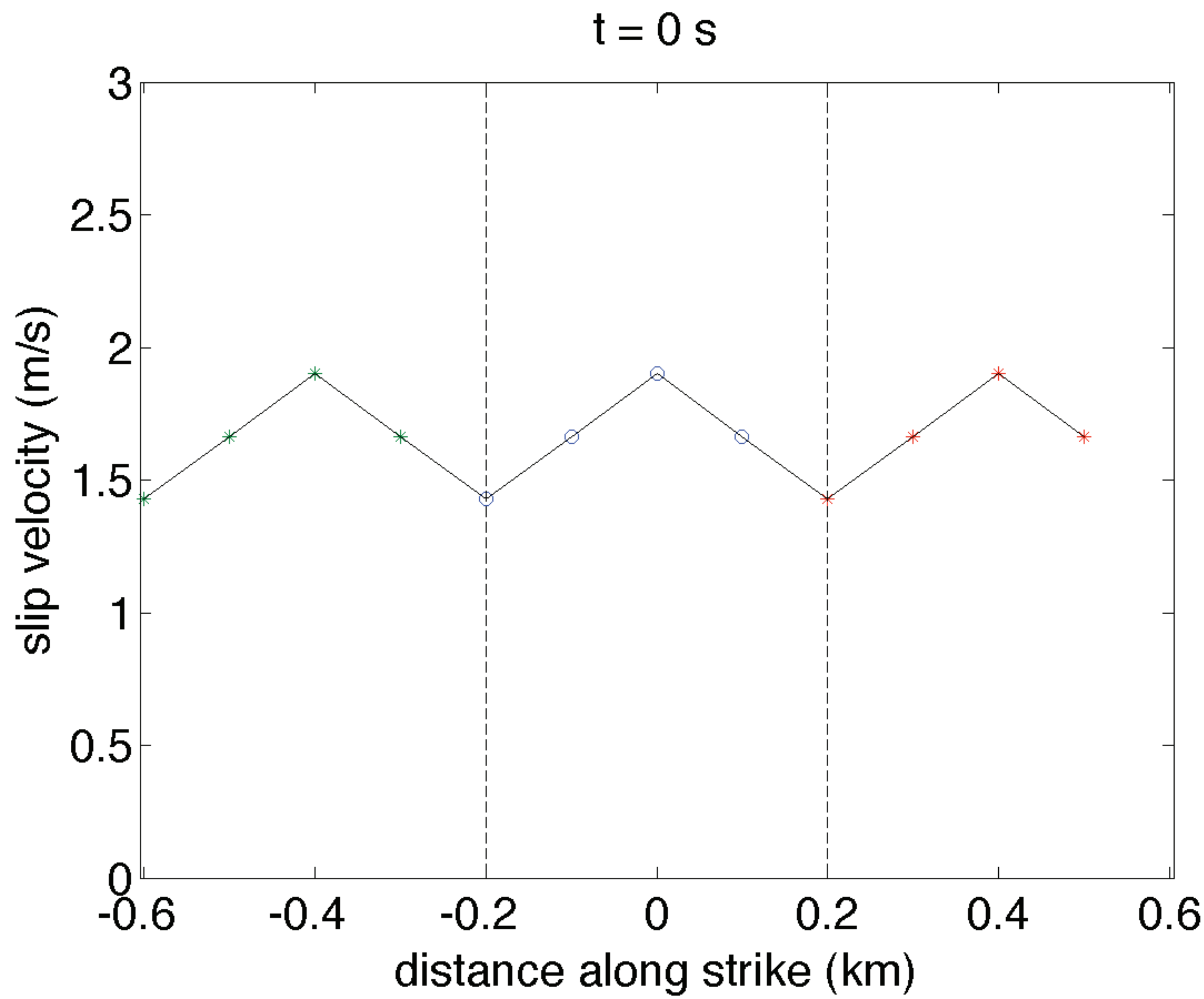
Unstable Region: Growth Rate Increases with Decreasing Wavelength Unless Regularized

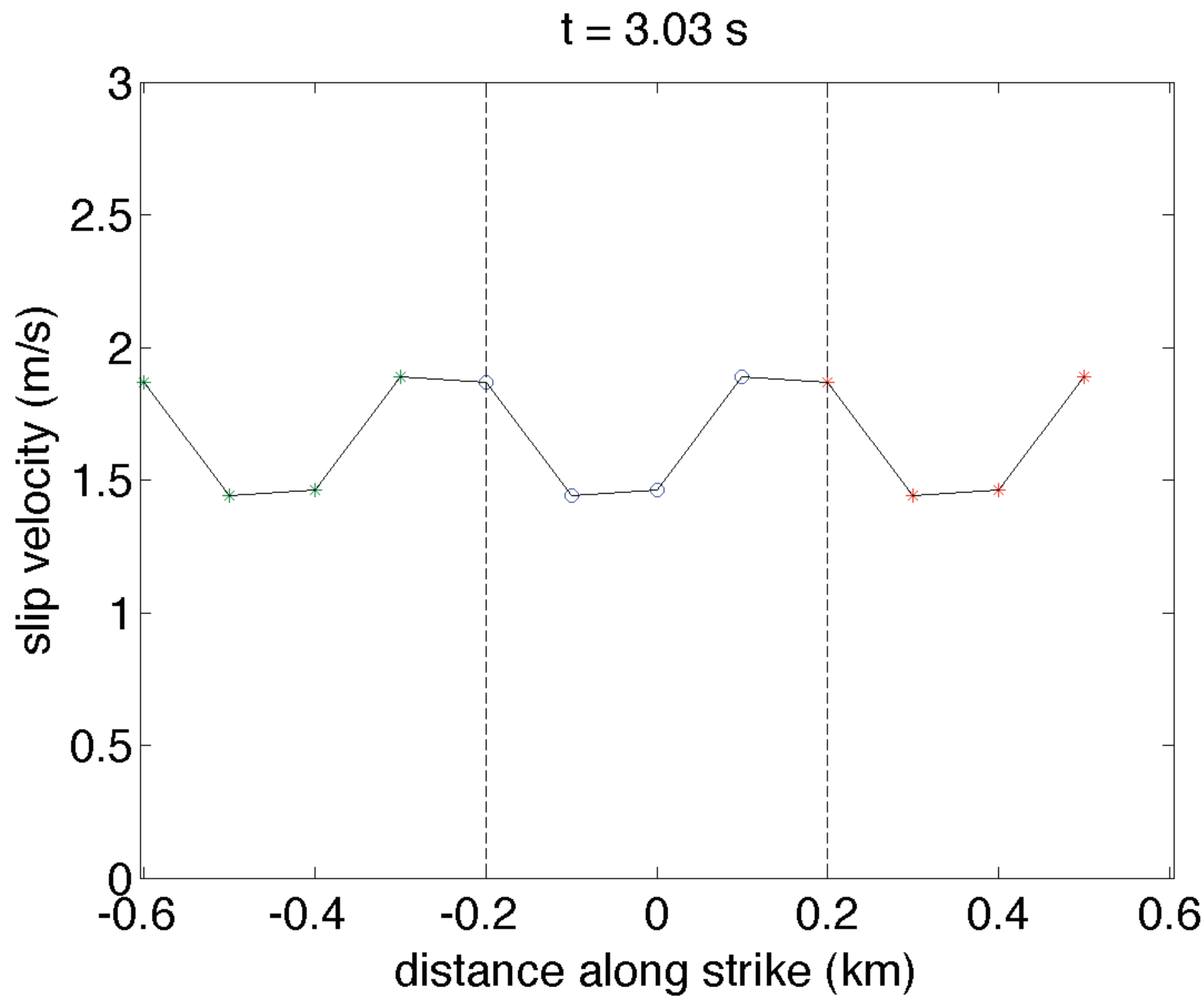


Ex. 2: Unregularized Ill-posed (TPV7)

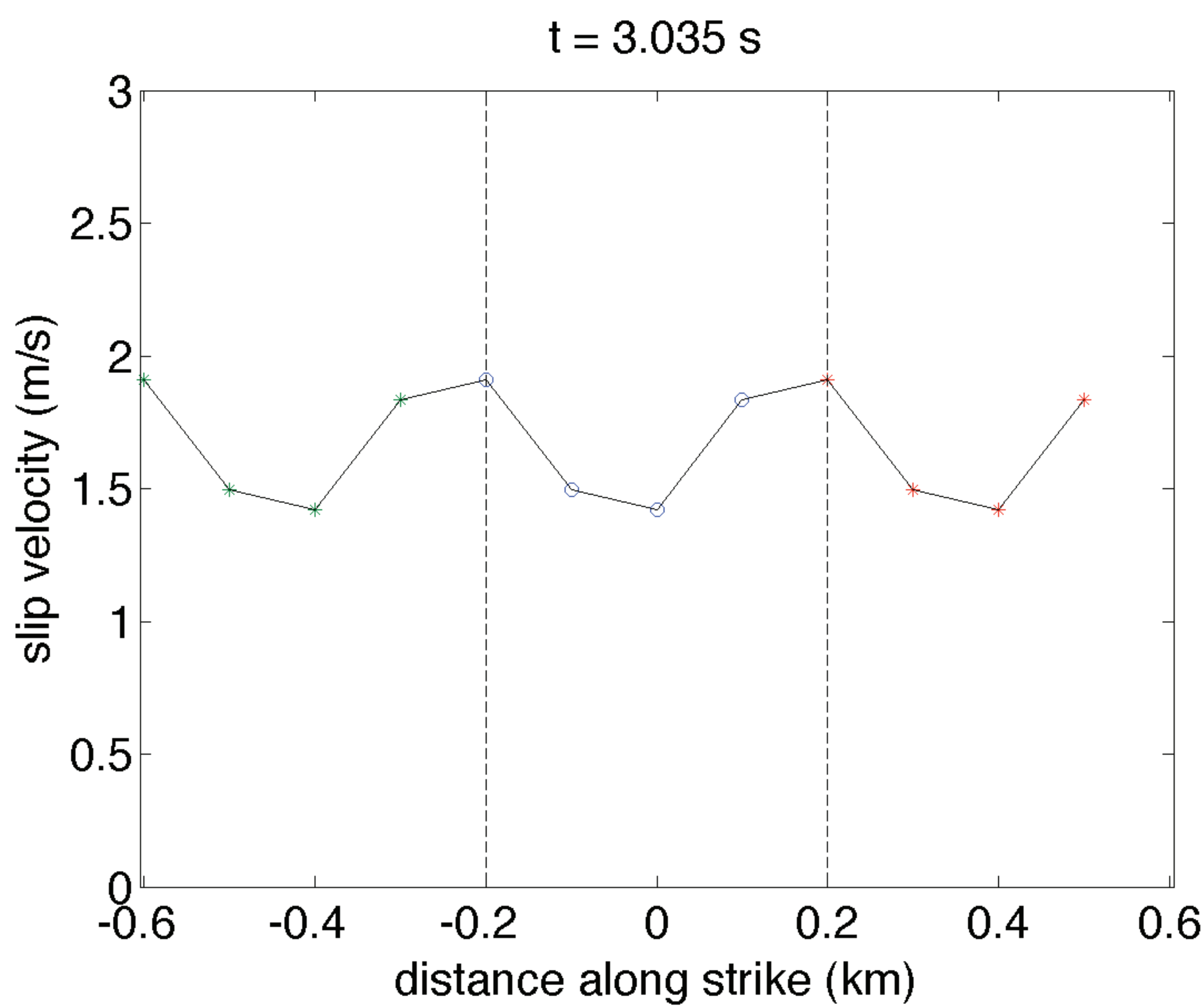


Example: λ=400 m Perturbation

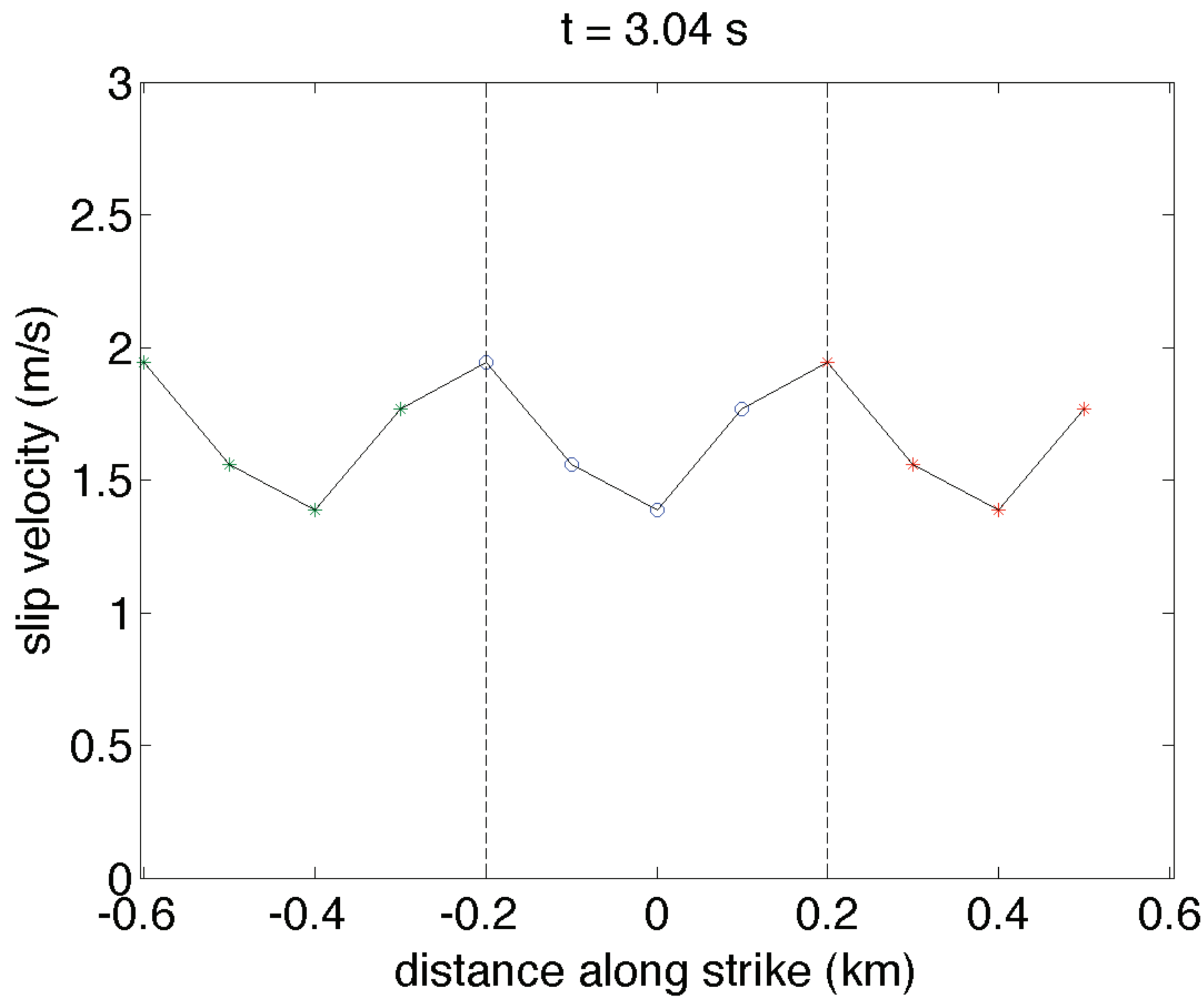


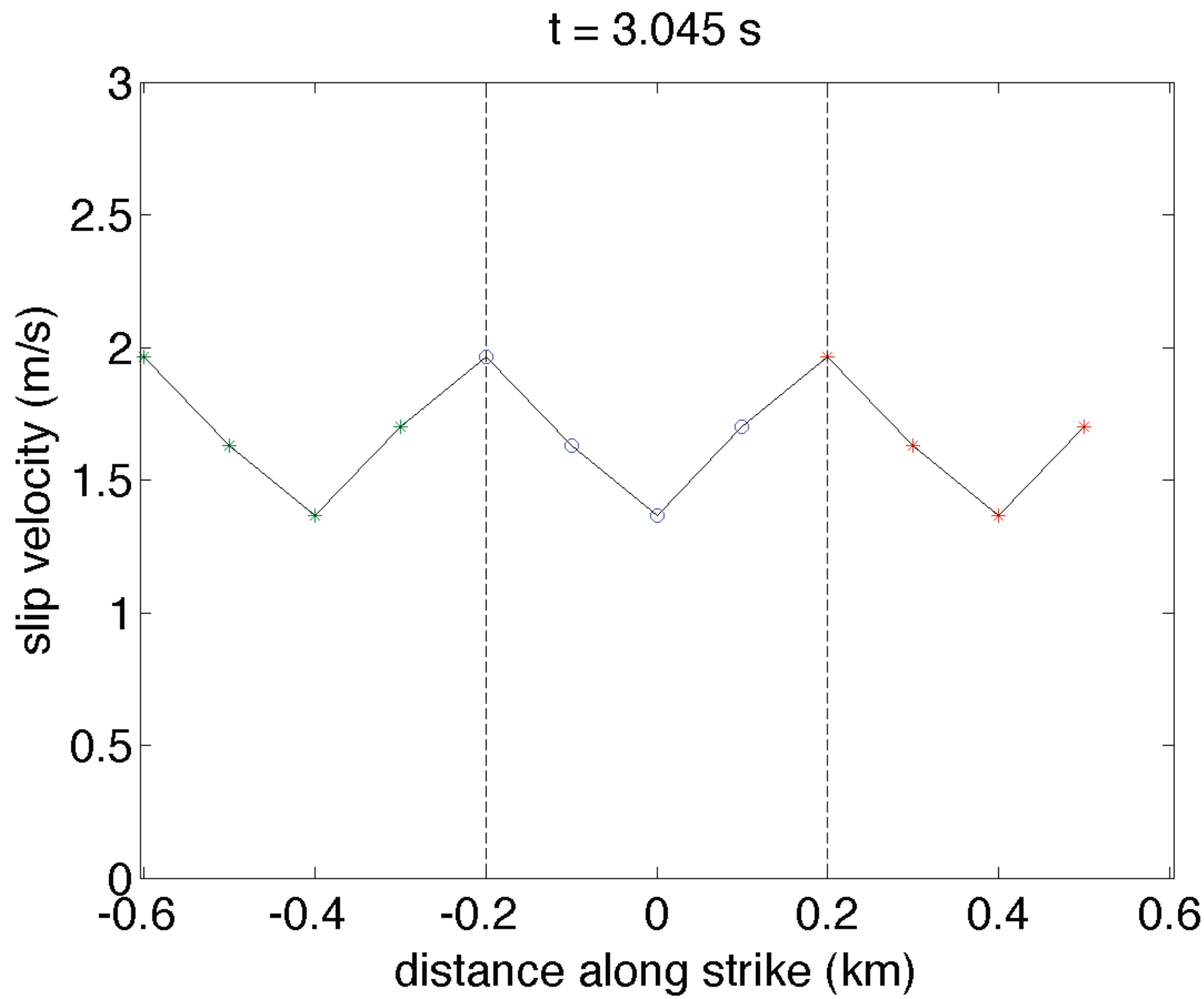


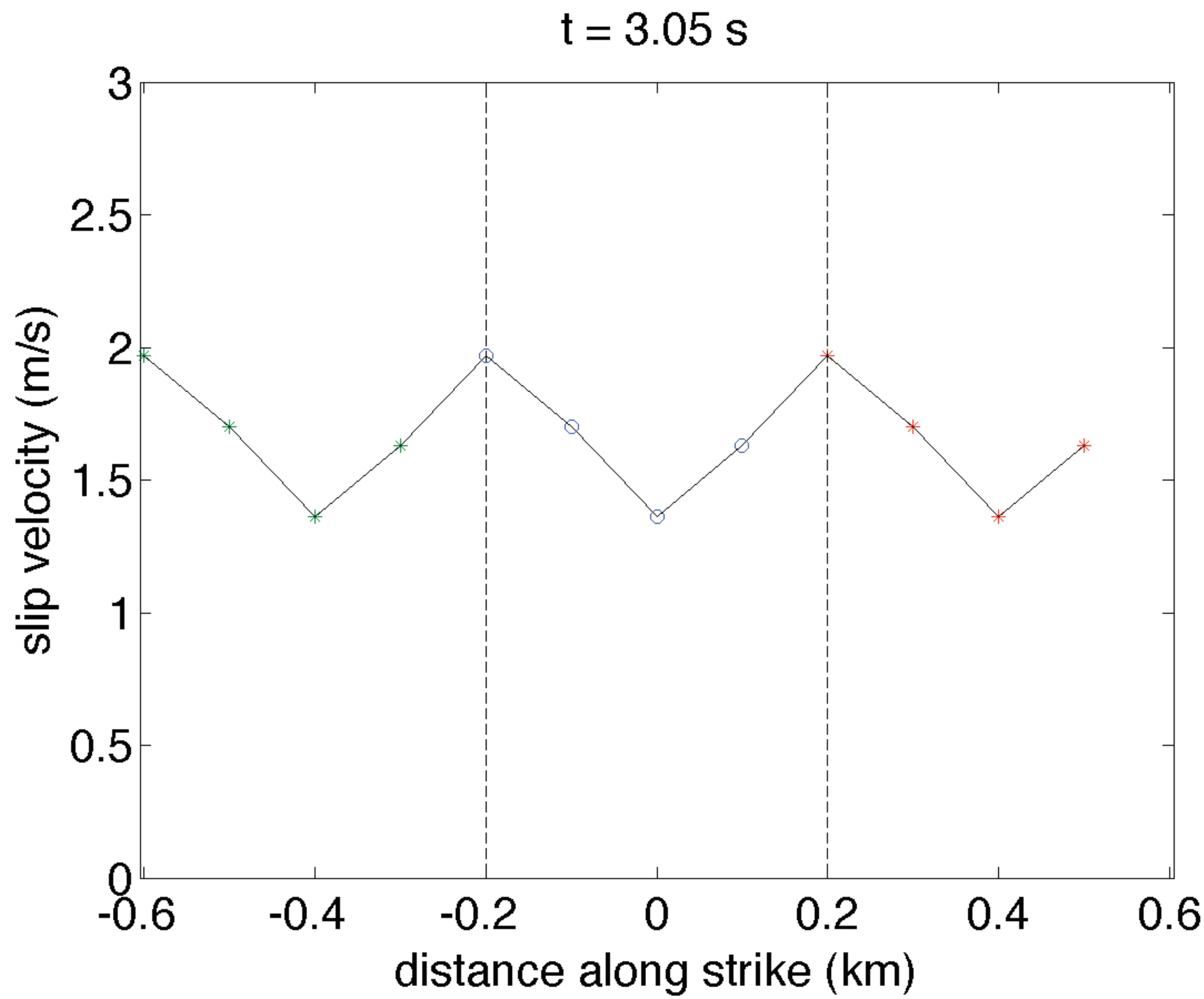
t = 3.03 s
slip velocity (m/s)
distance along strike (km)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

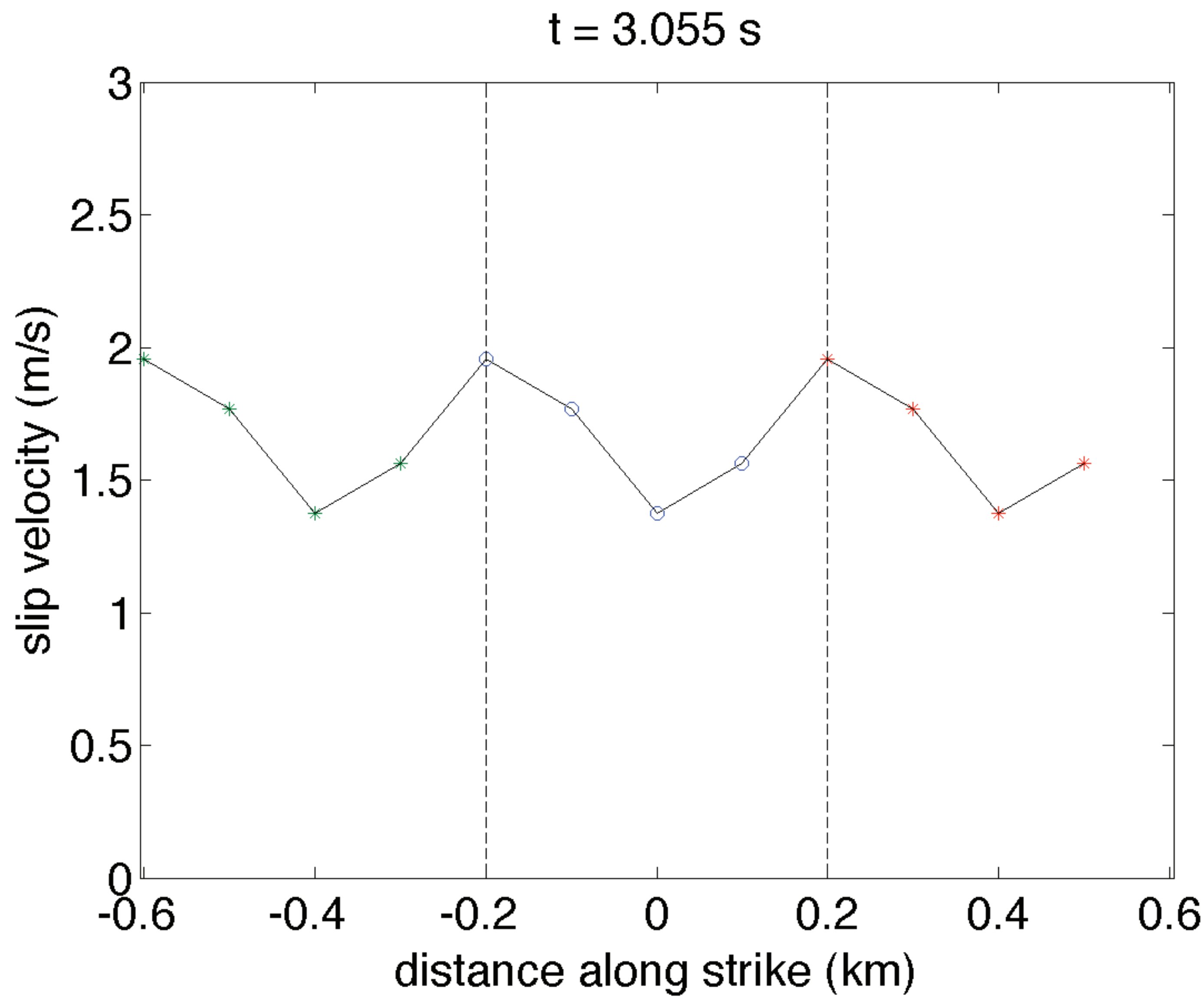


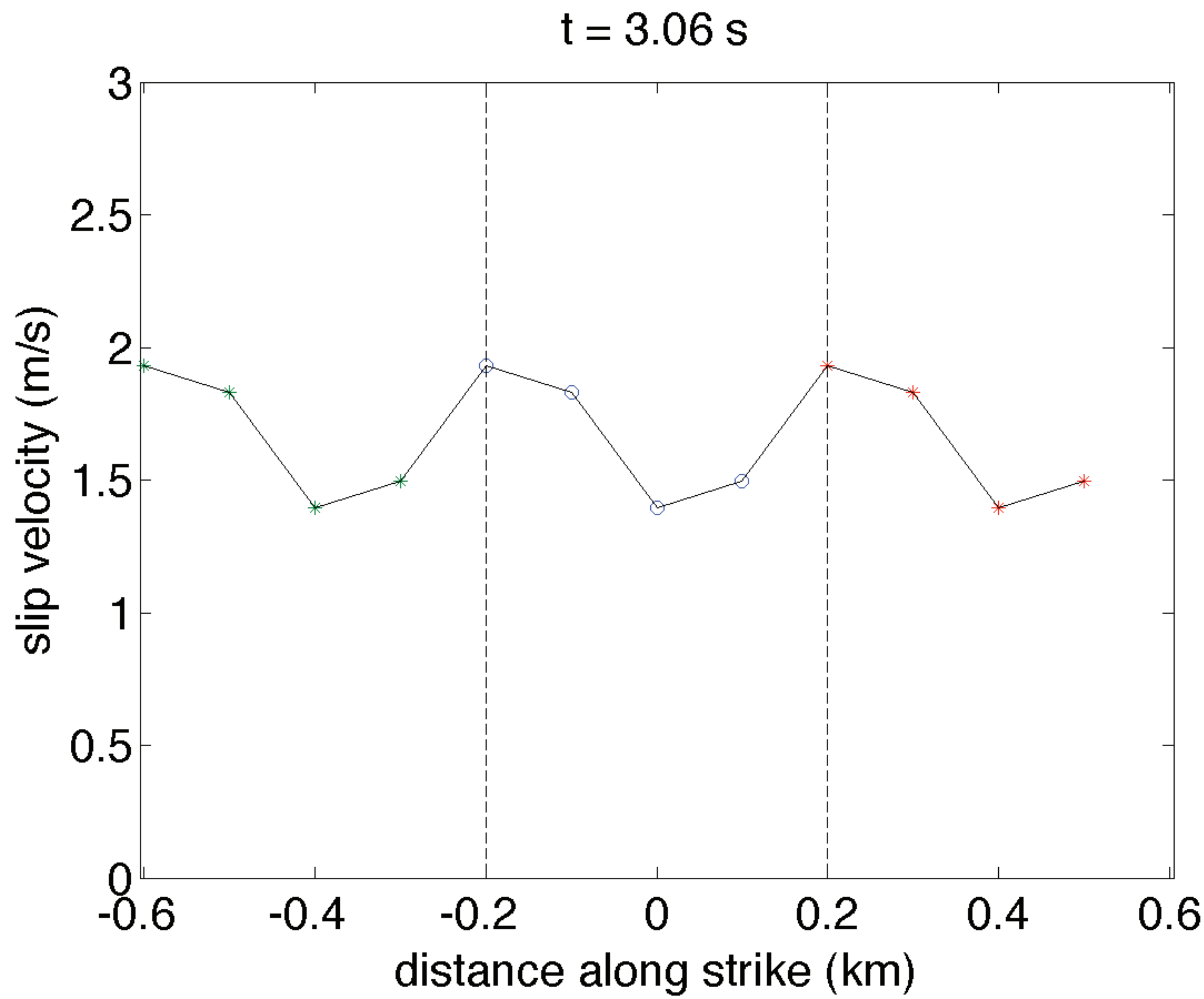
t = 3.035 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

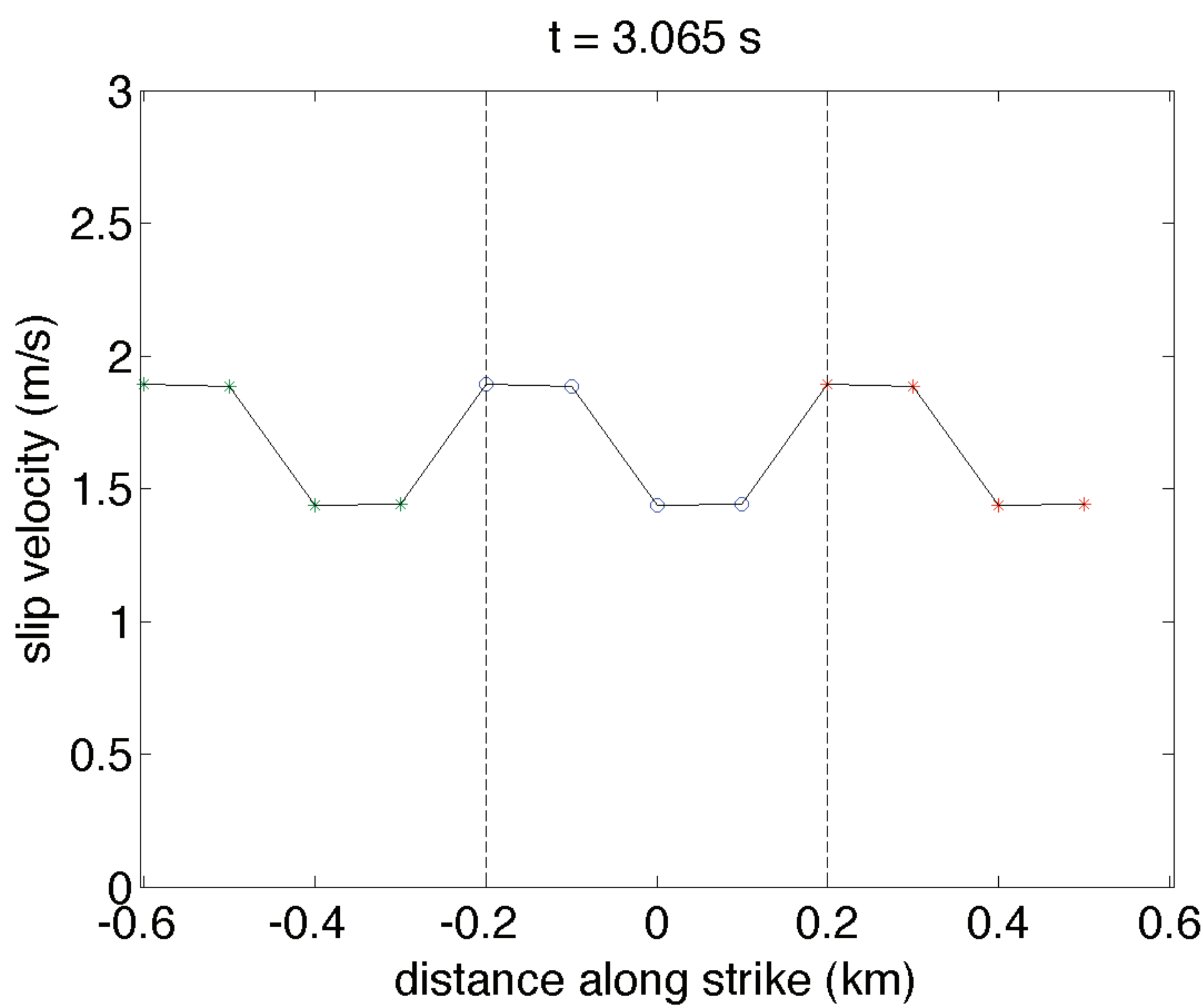

t = 3.04 s
slip velocity (m/s)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

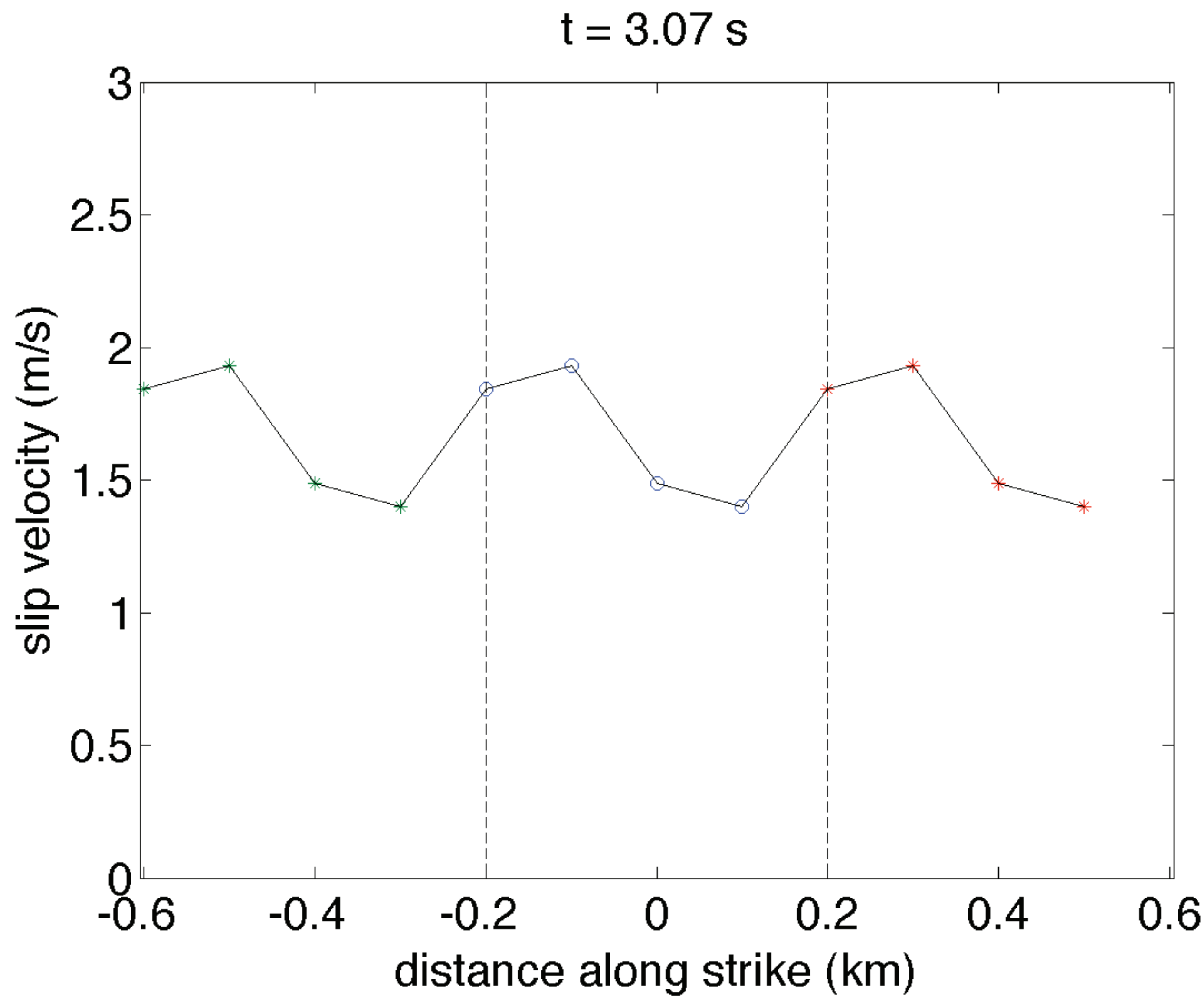

t = 3.045 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

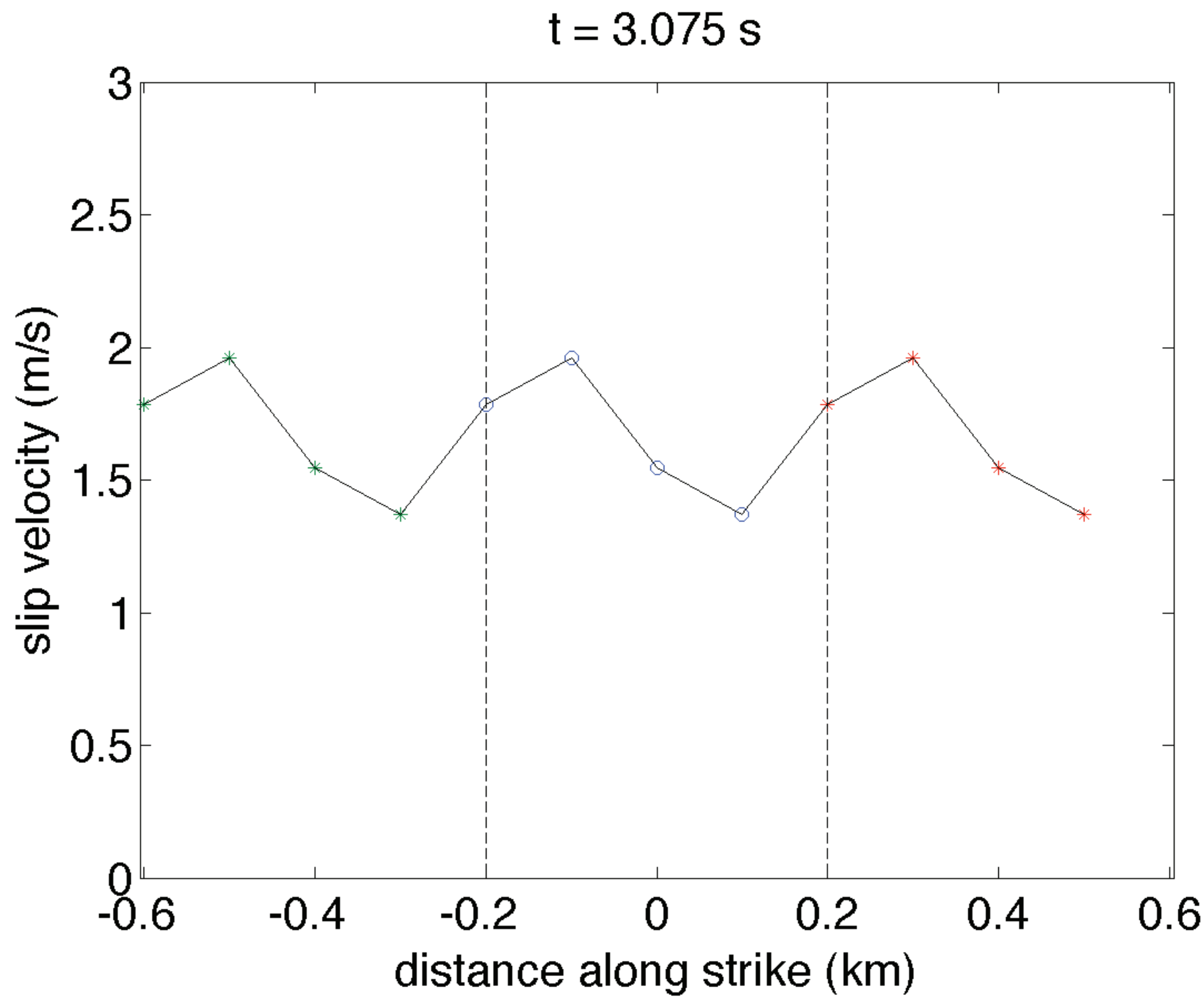

t = 3.05 s
slip velocity (m/s)
distance along strike (km)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


t = 3.055 s
slip velocity (m/s)
distance along strike (km)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

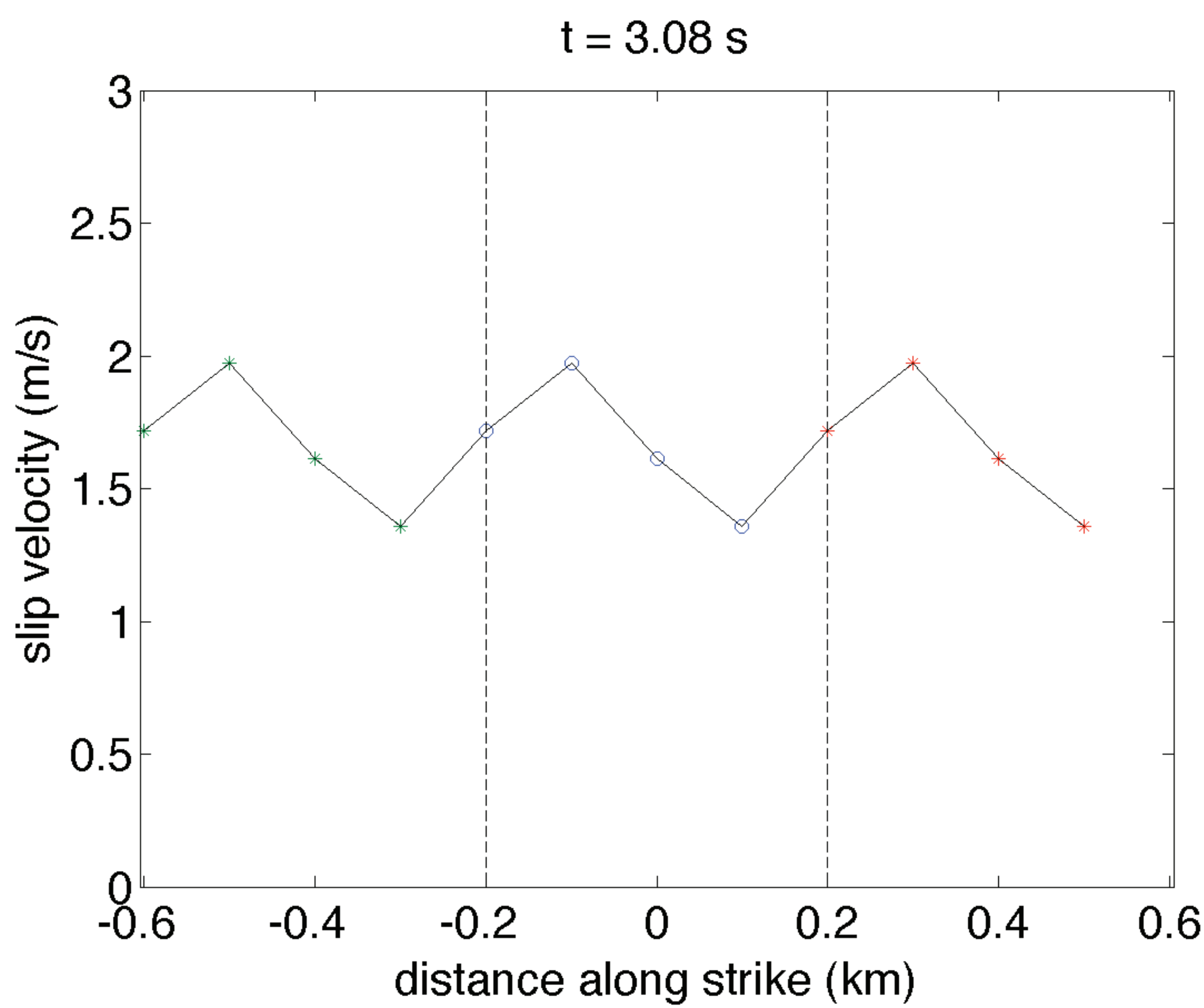

t = 3.06 s
slip velocity (m/s)
distance along strike (km)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

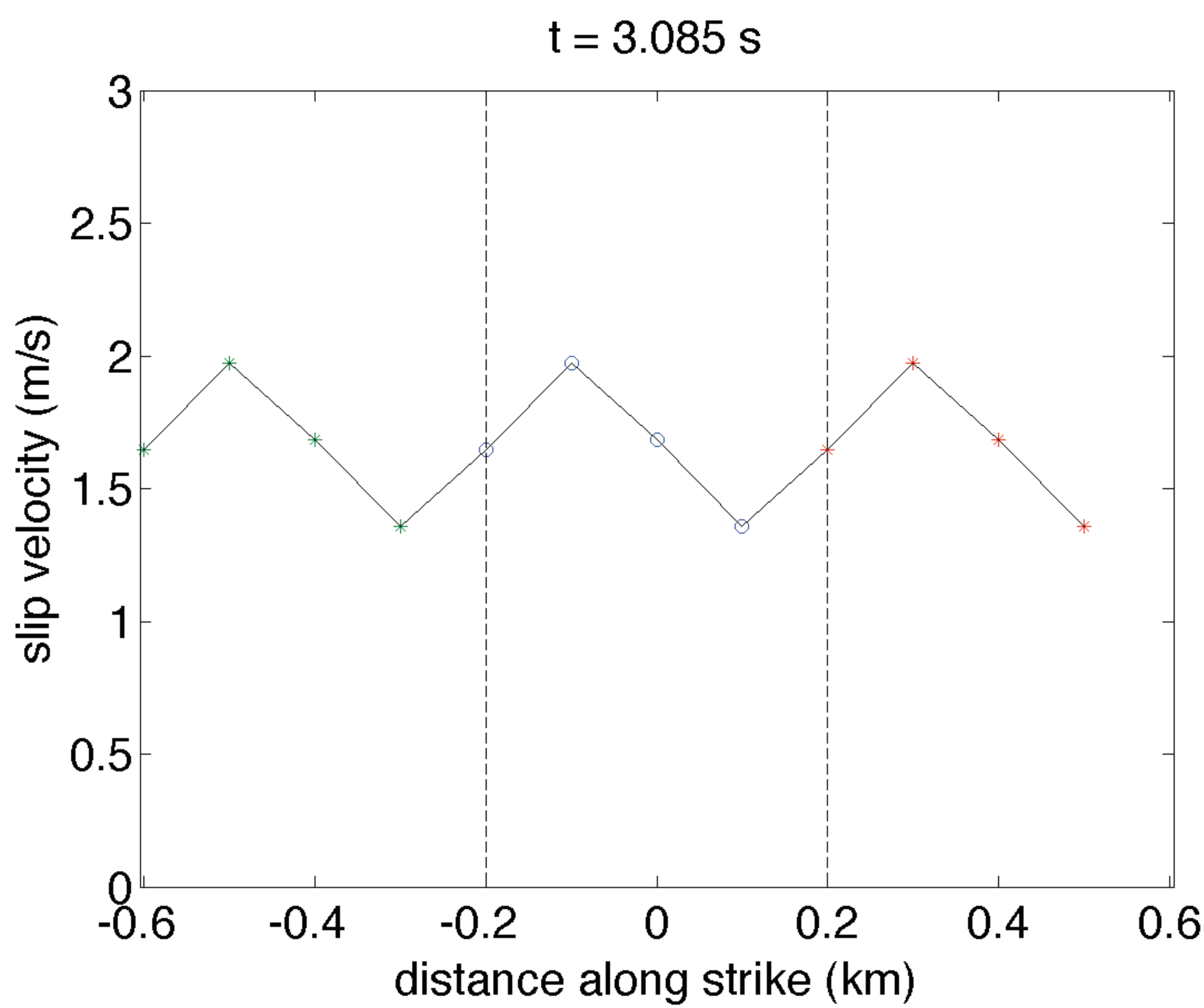

t = 3.065 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

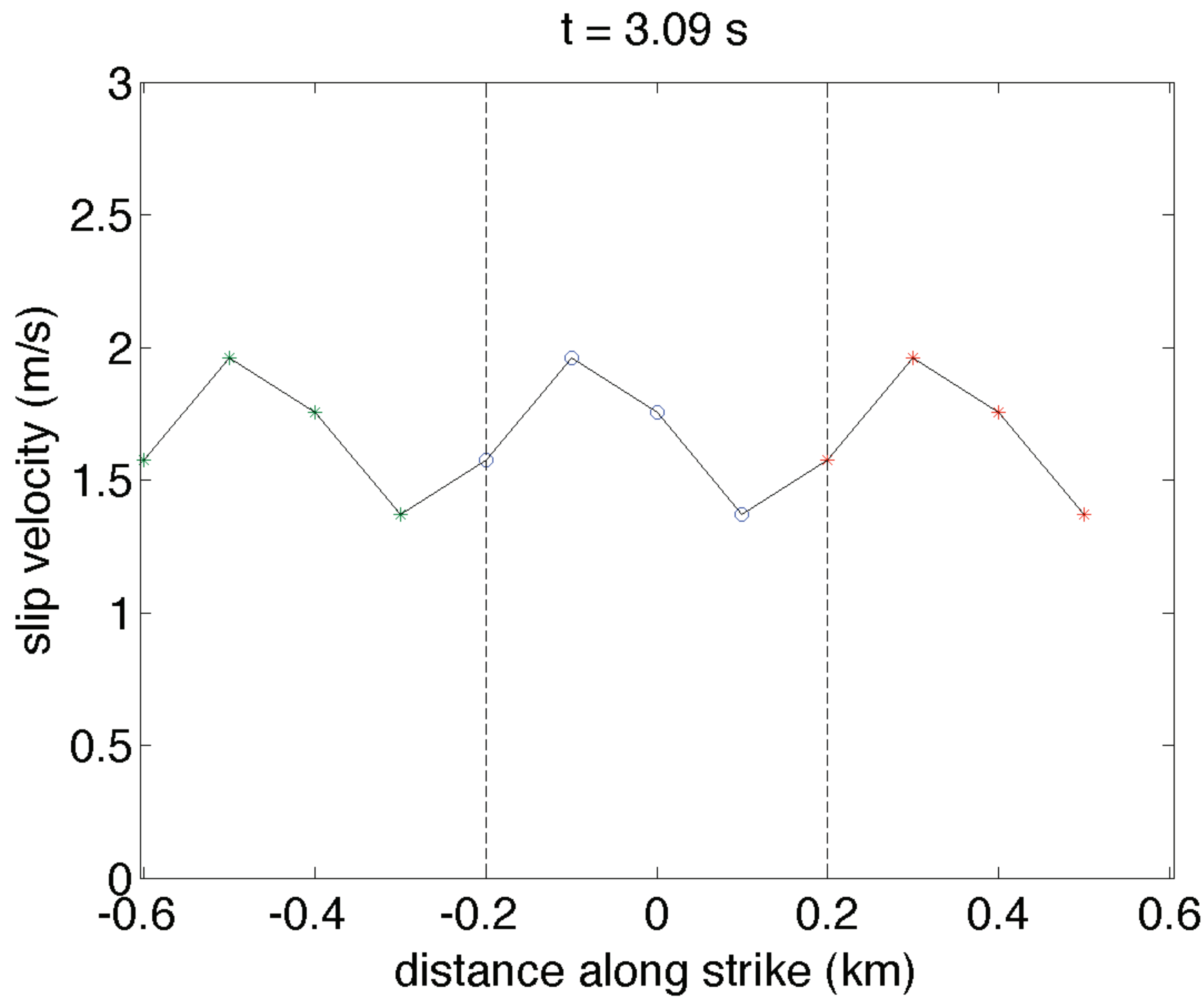

t = 3.07 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

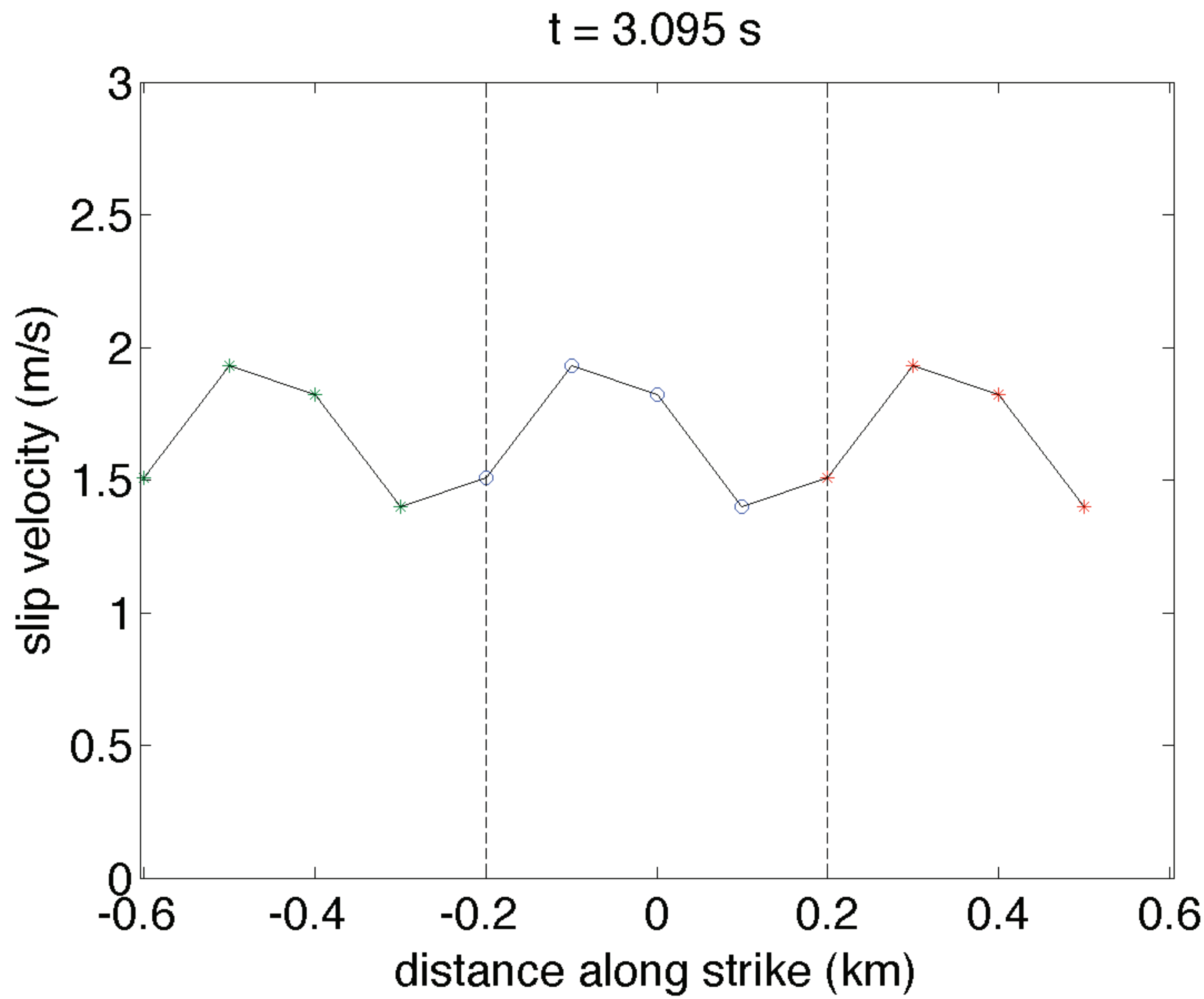


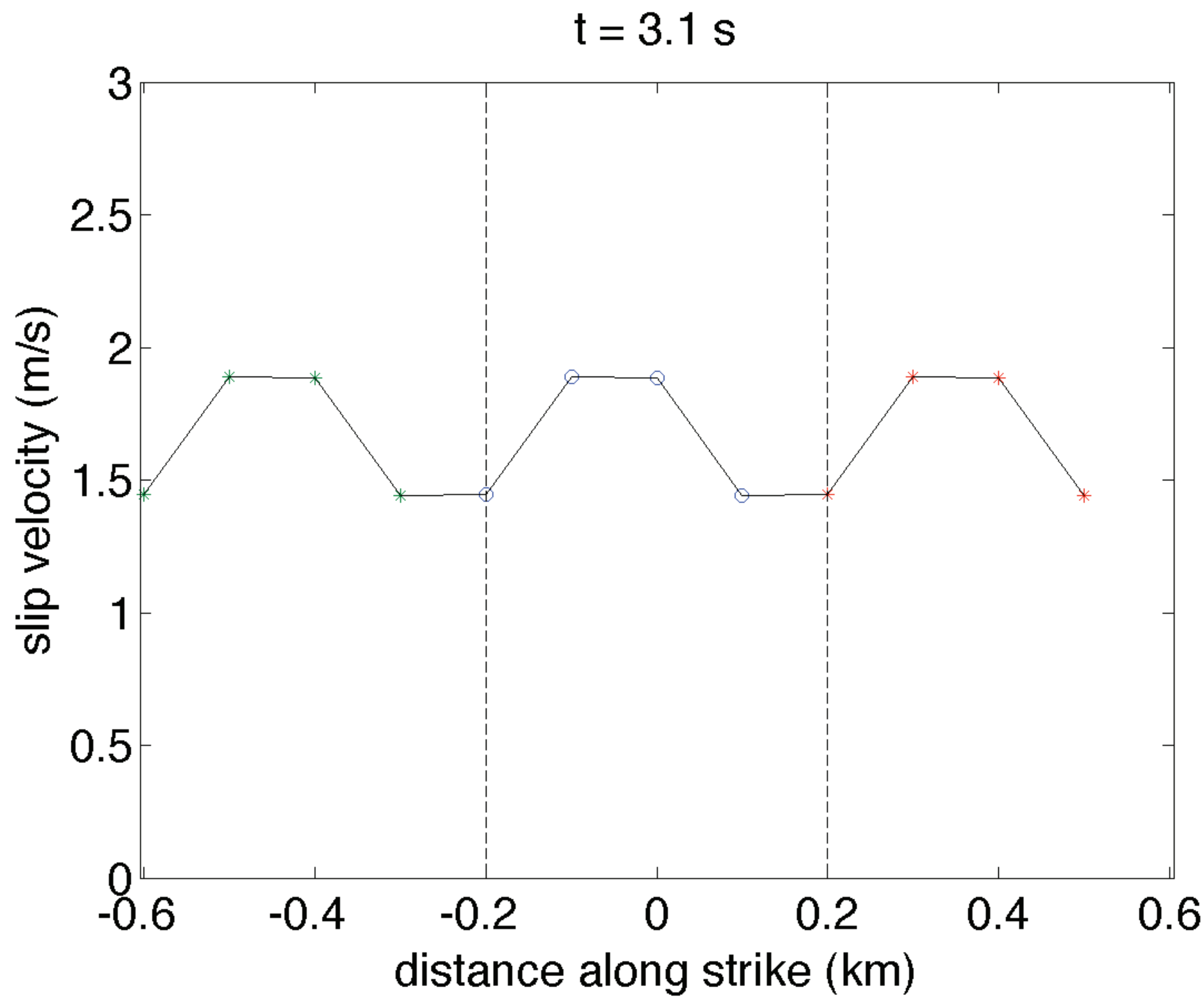
t = 3.075 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

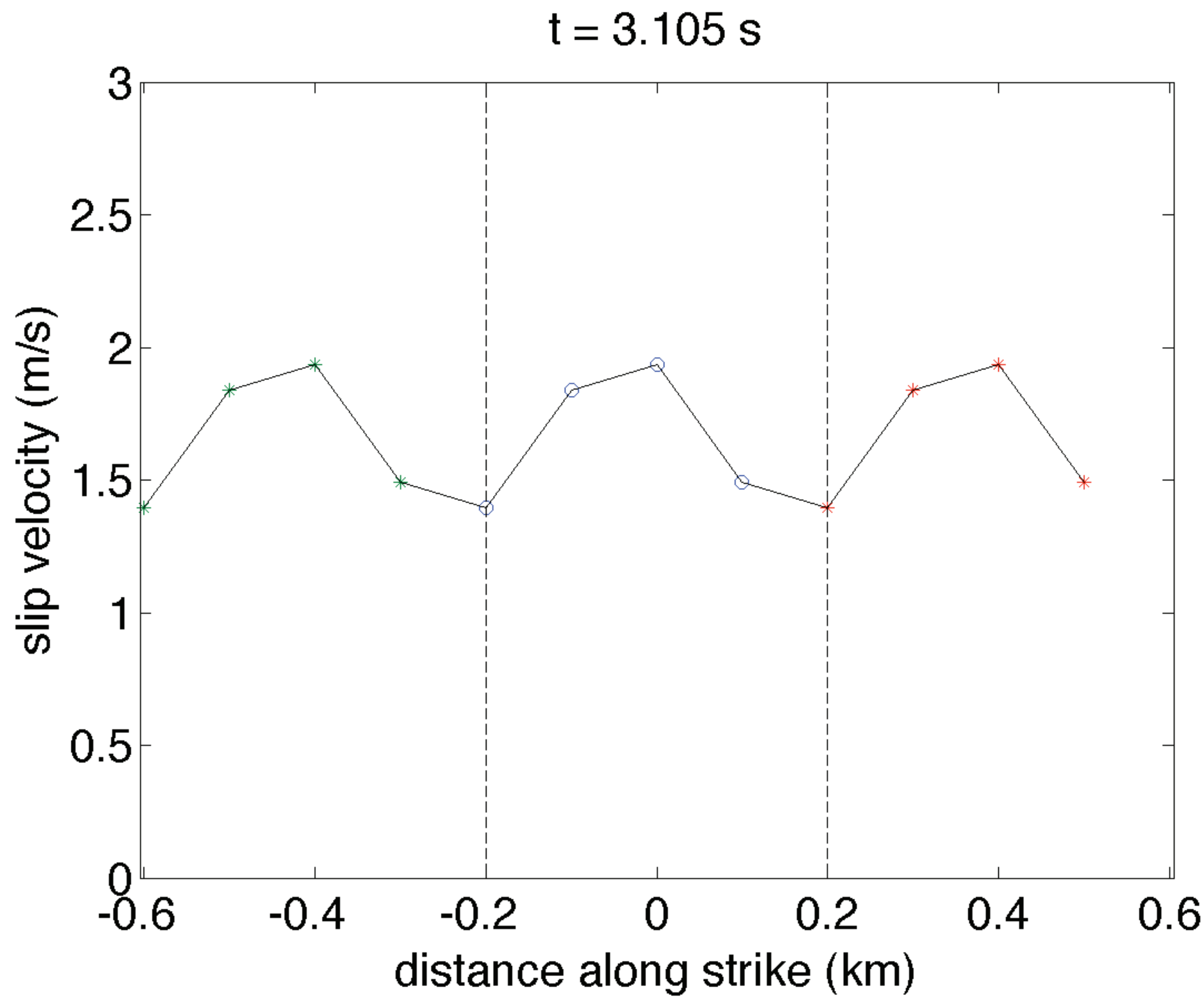

t = 3.08 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

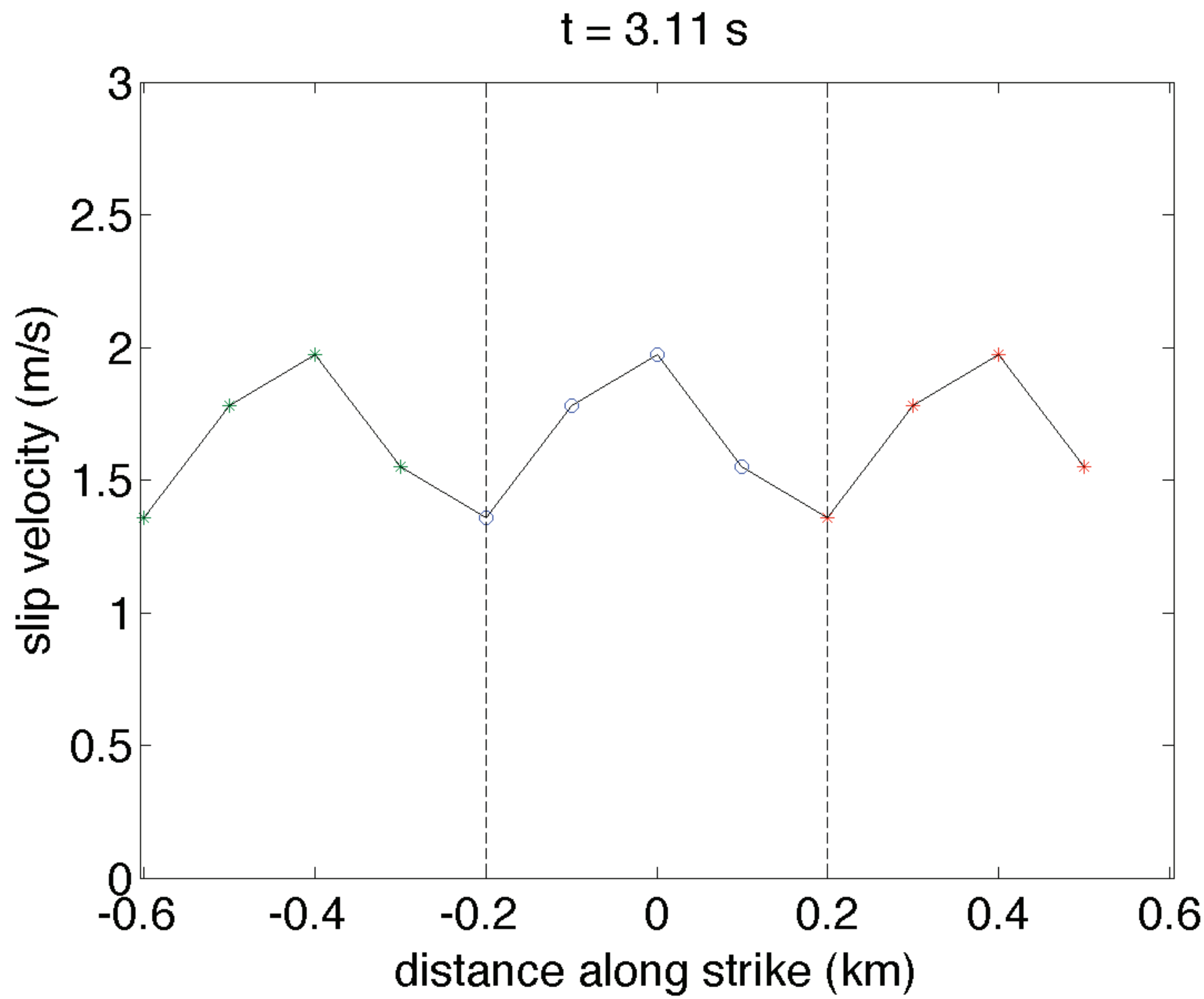

t = 3.085 s
slip velocity (m/s)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

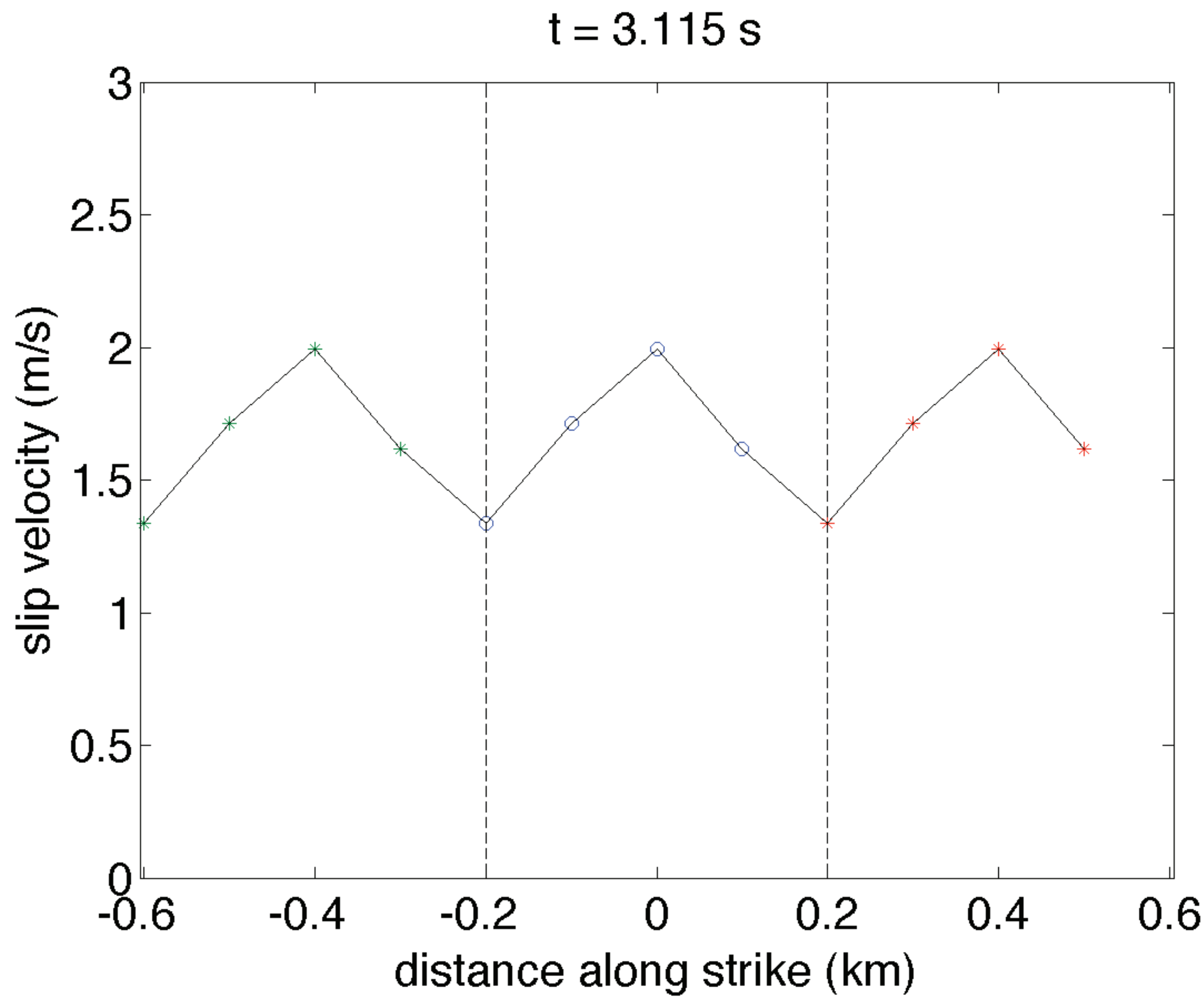

t = 3.09 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

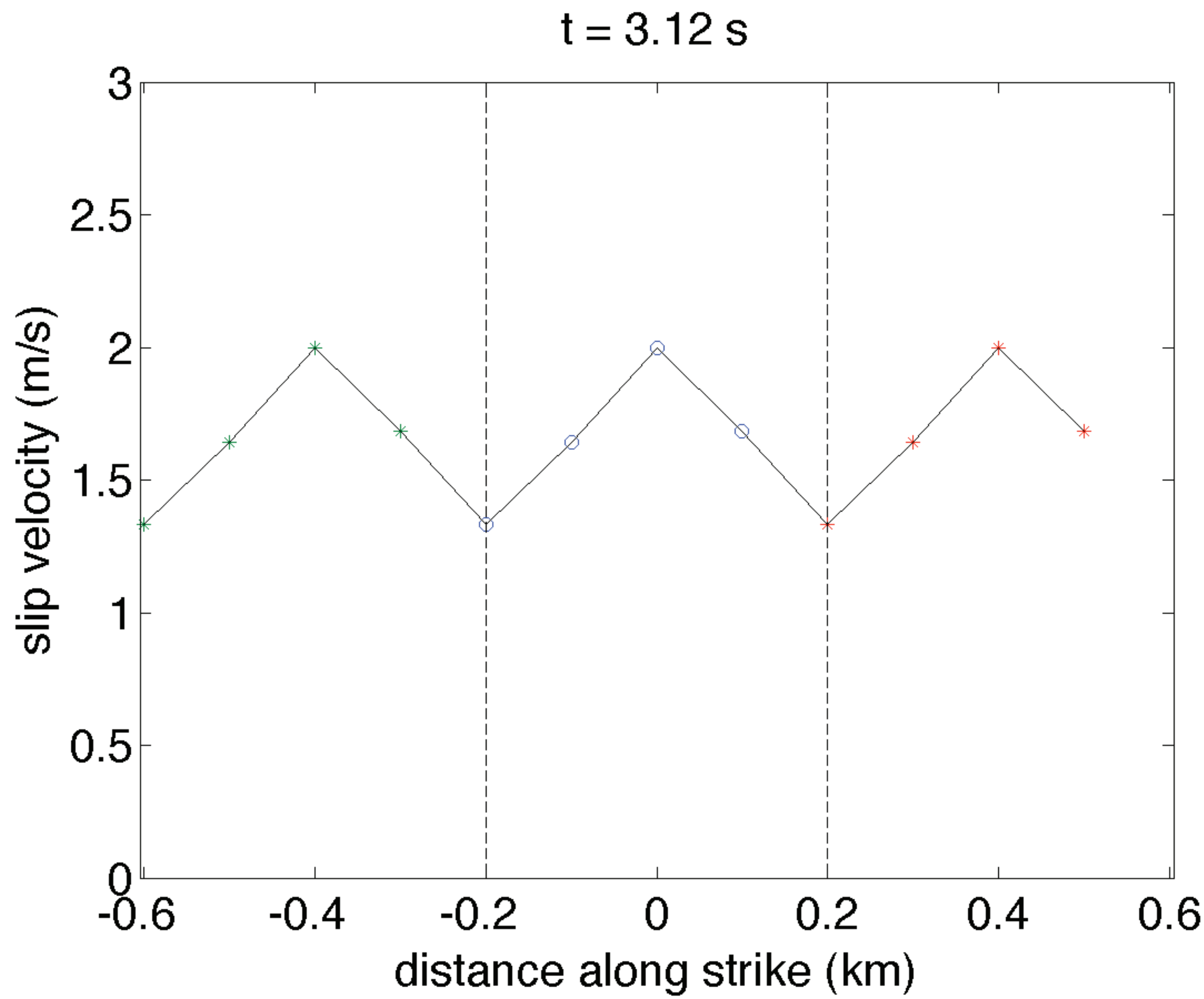

t = 3.095 s
slip velocity (m/s)
distance along strike (km)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

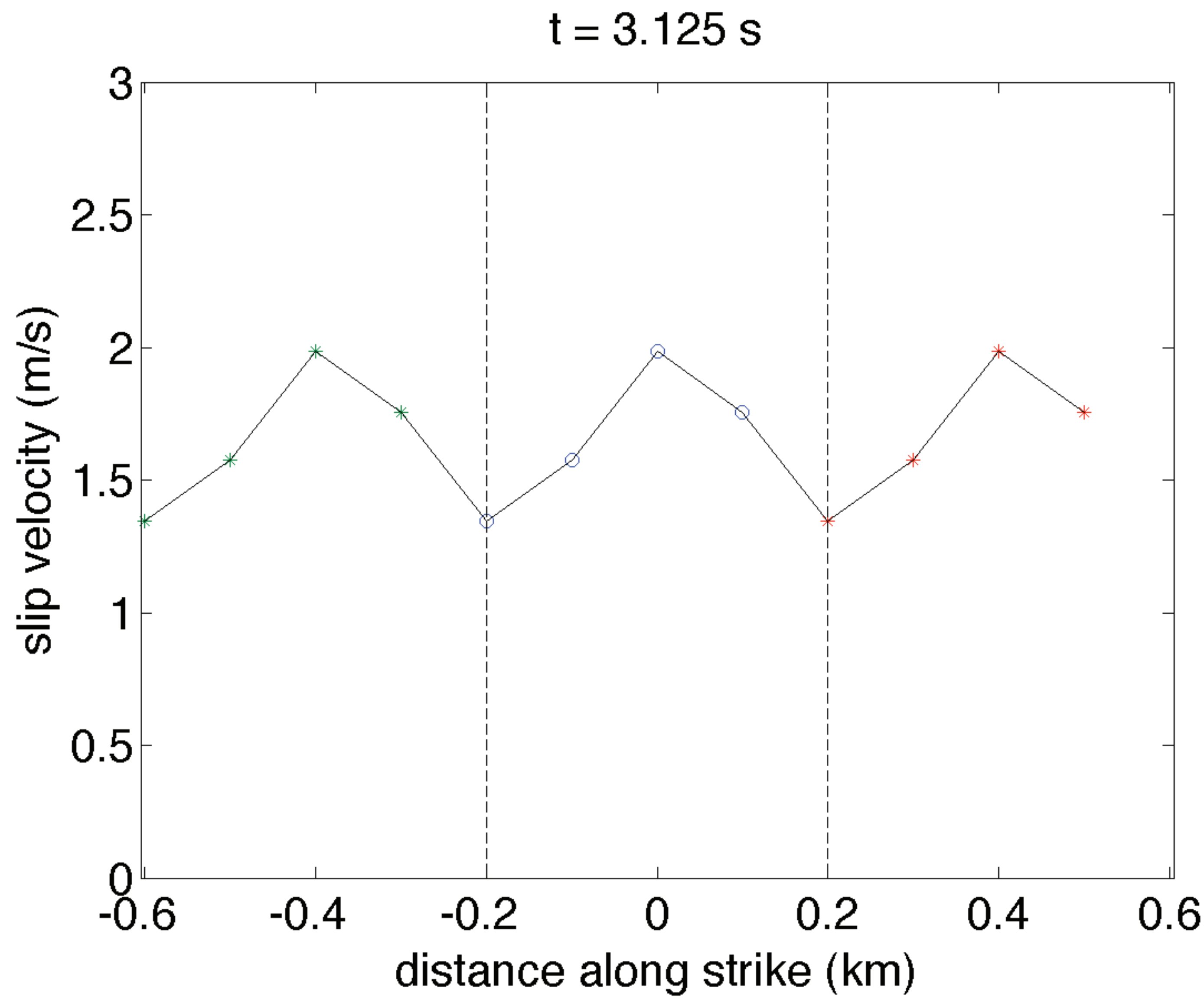

t = 3.1 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

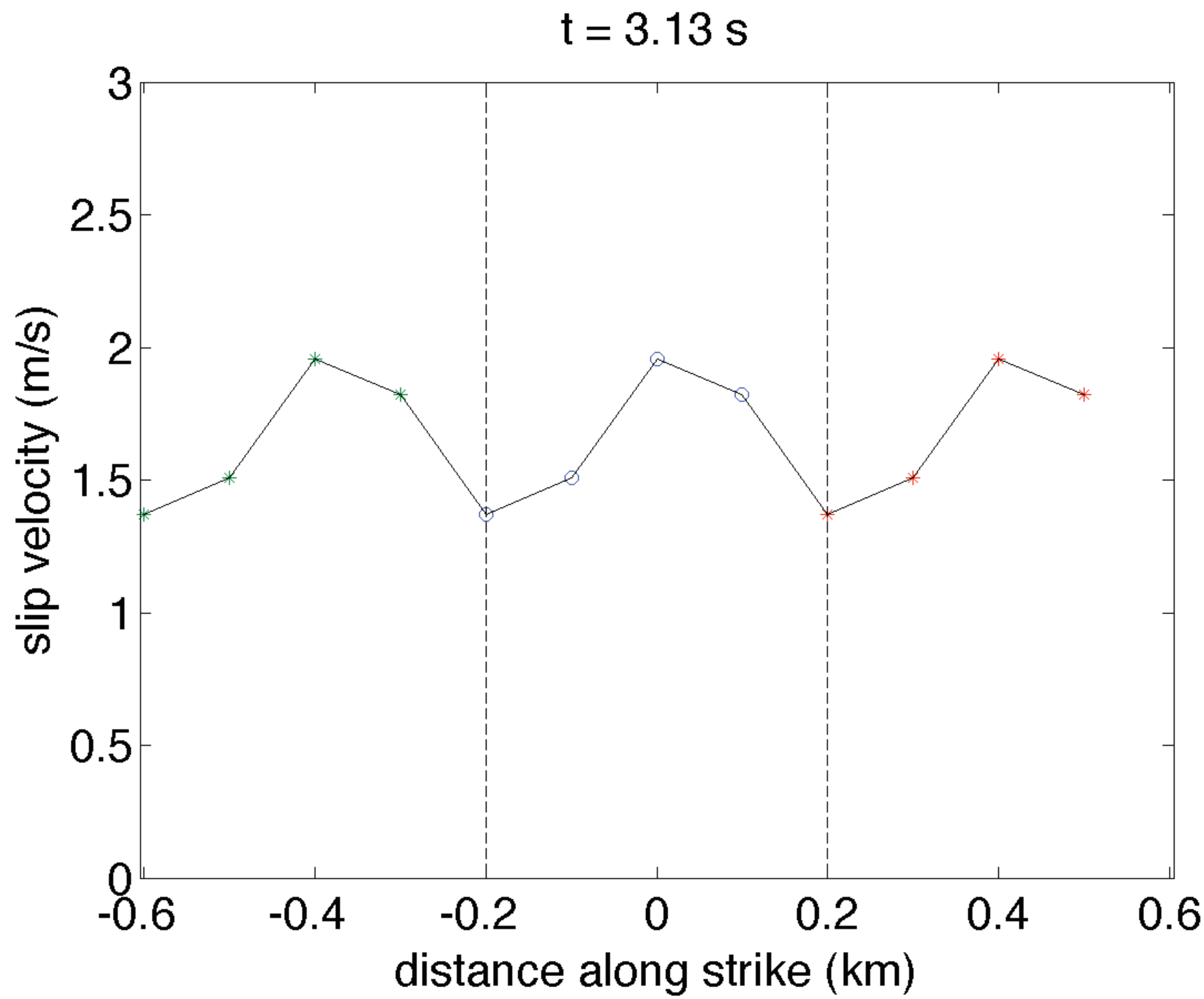

t = 3.105 s
slip velocity (m/s)
distance along strike (km)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


t = 3.11 s
slip velocity (m/s)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

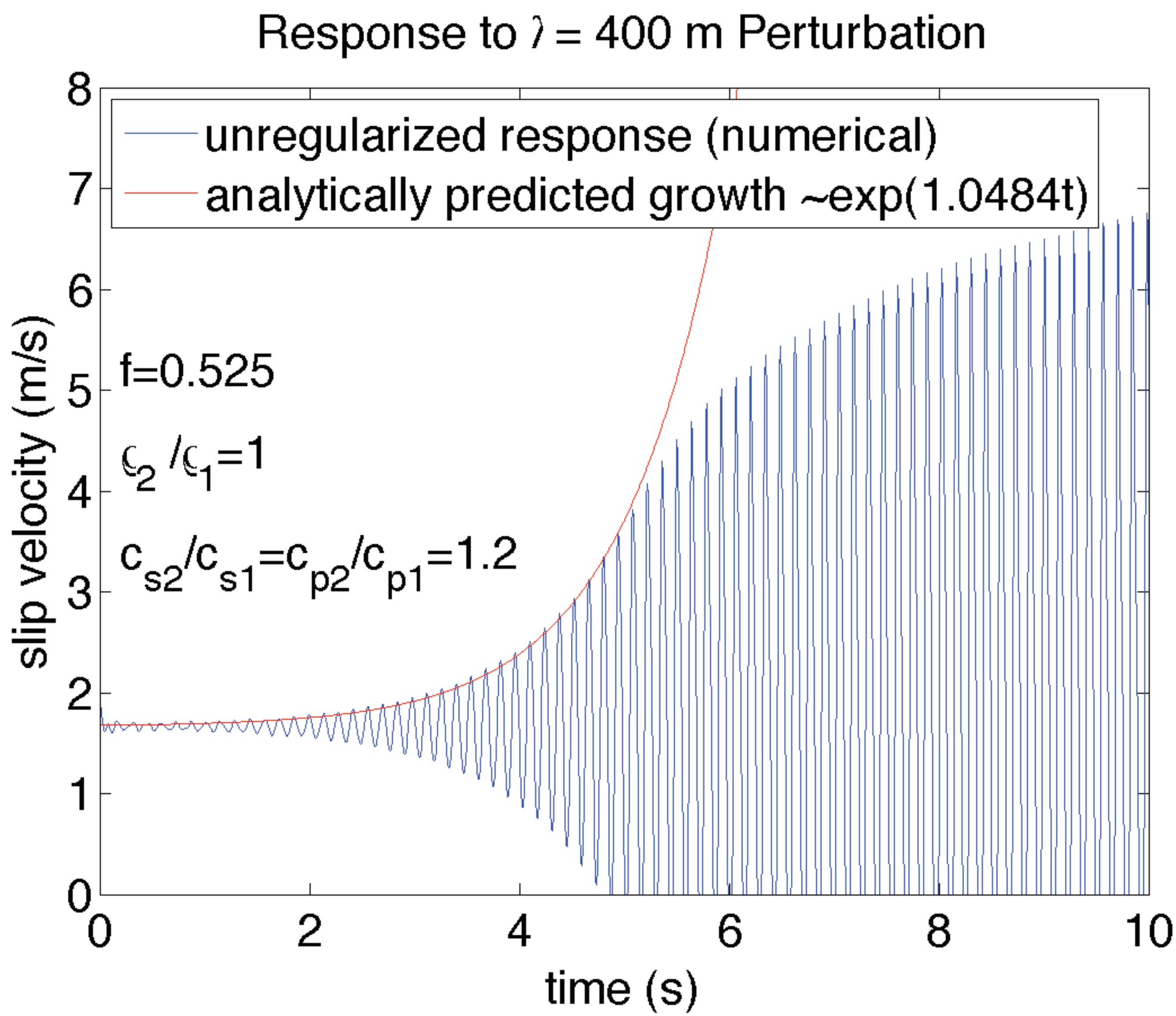

t = 3.115 s
slip velocity (m/s)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)


t = 3.12 s
slip velocity (m/s)
distance along strike (km)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

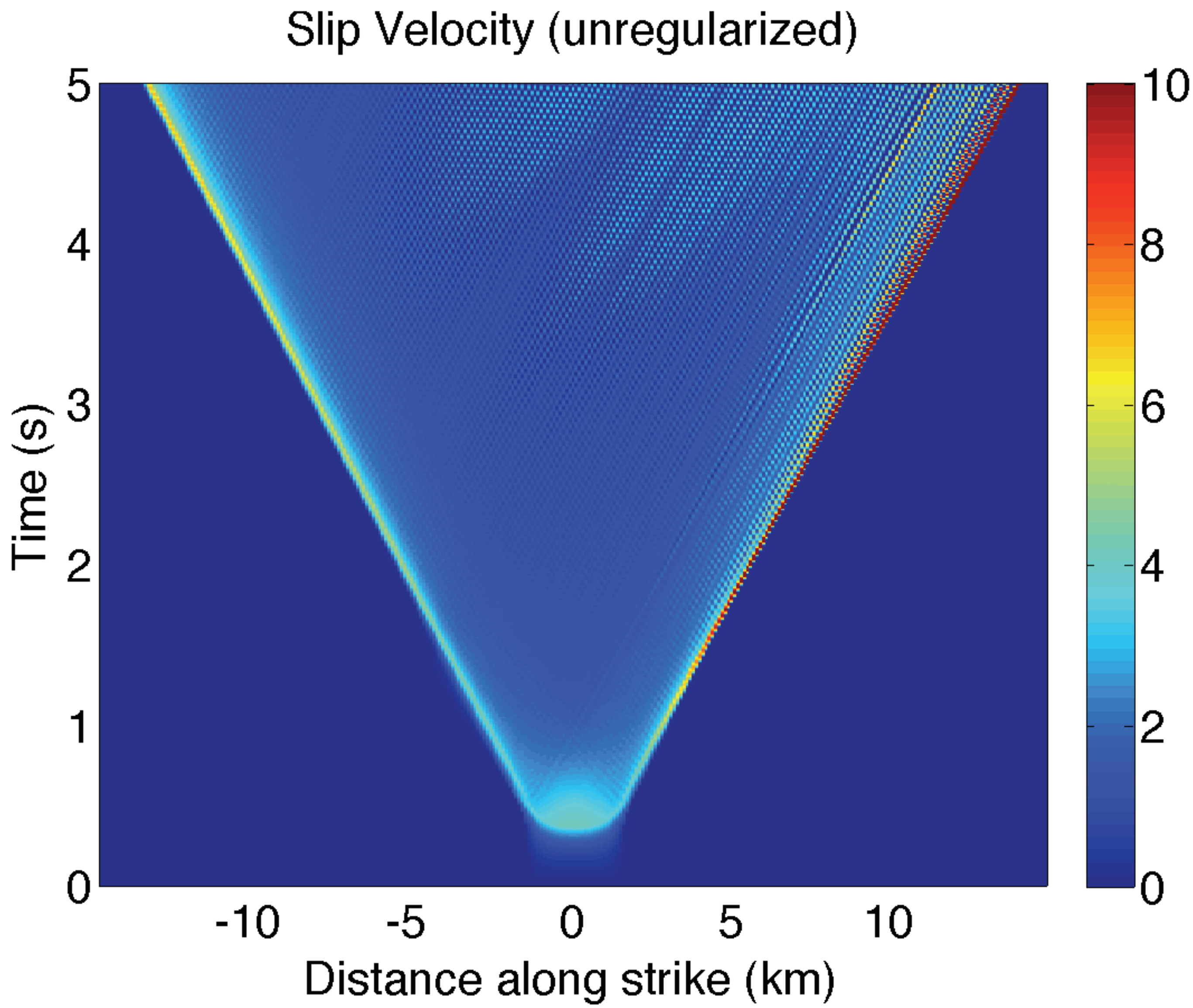

t = 3.125 s
slip velocity (m/s)
distance along strike (km)
0
0.5
1
1.5
2
2.5
3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


t = 3.13 s
slip velocity (m/s)
3
2.5
2
1.5
1
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
distance along strike (km)

Growth of Unstable Mode



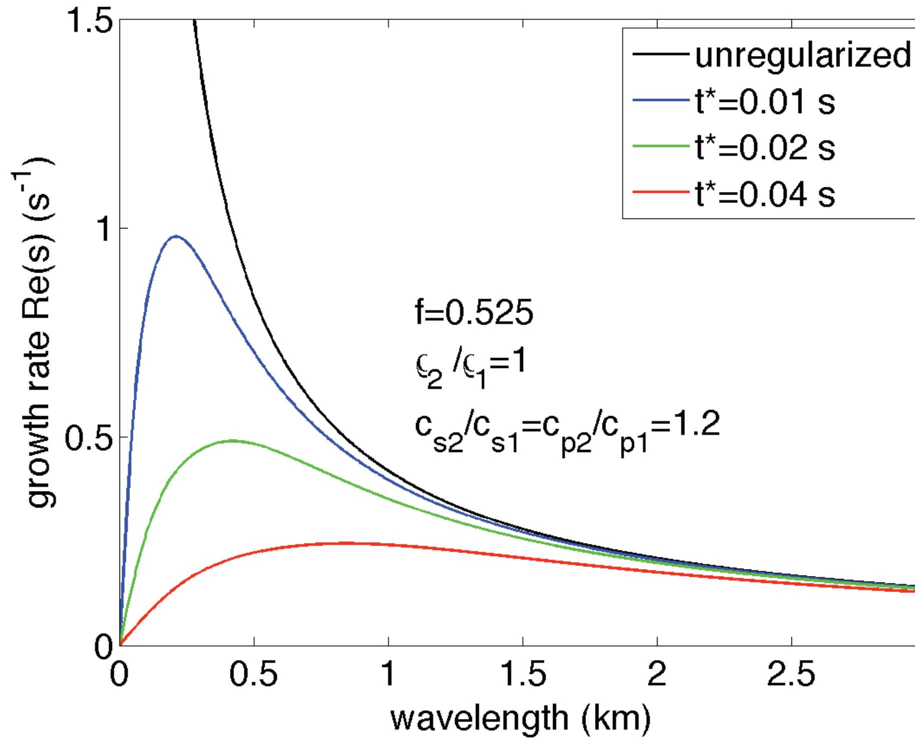
Generalized Rayleigh Waves Behind Rupture Front (TPV7 h=100 m)



Estimates of Growth Rate:
Why TPV7 (At Current Resolution) Might Look OK, But Not Be!

Growth of unstable GR wave (for TPV7 parameters, unregularized) is

$$e^{\left(\frac{200\ \mathrm{m}}{\lambda}\right)\left(\frac{t}{0.48\ \mathrm{s}}\right)}$$



so a perturbation at Nyquist wavelength $\lambda=2h$ for $h=100$ m will e-fold every 0.48 s.

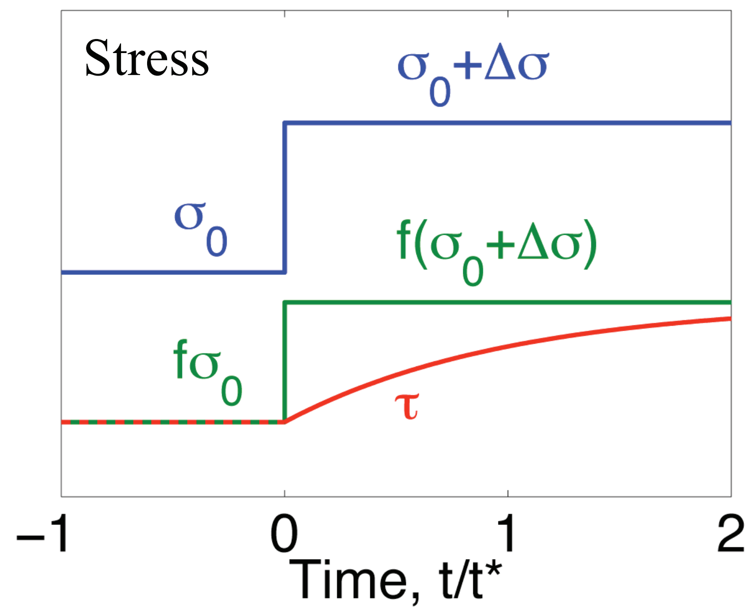
Growth limited by:

1. preventing reverse slip (by having initial stress>>stress drop)
2. damping in code (intrinsic to numerical method or explicitly added)

On-Fault Regularization Options

1. Evolution over fixed time scale

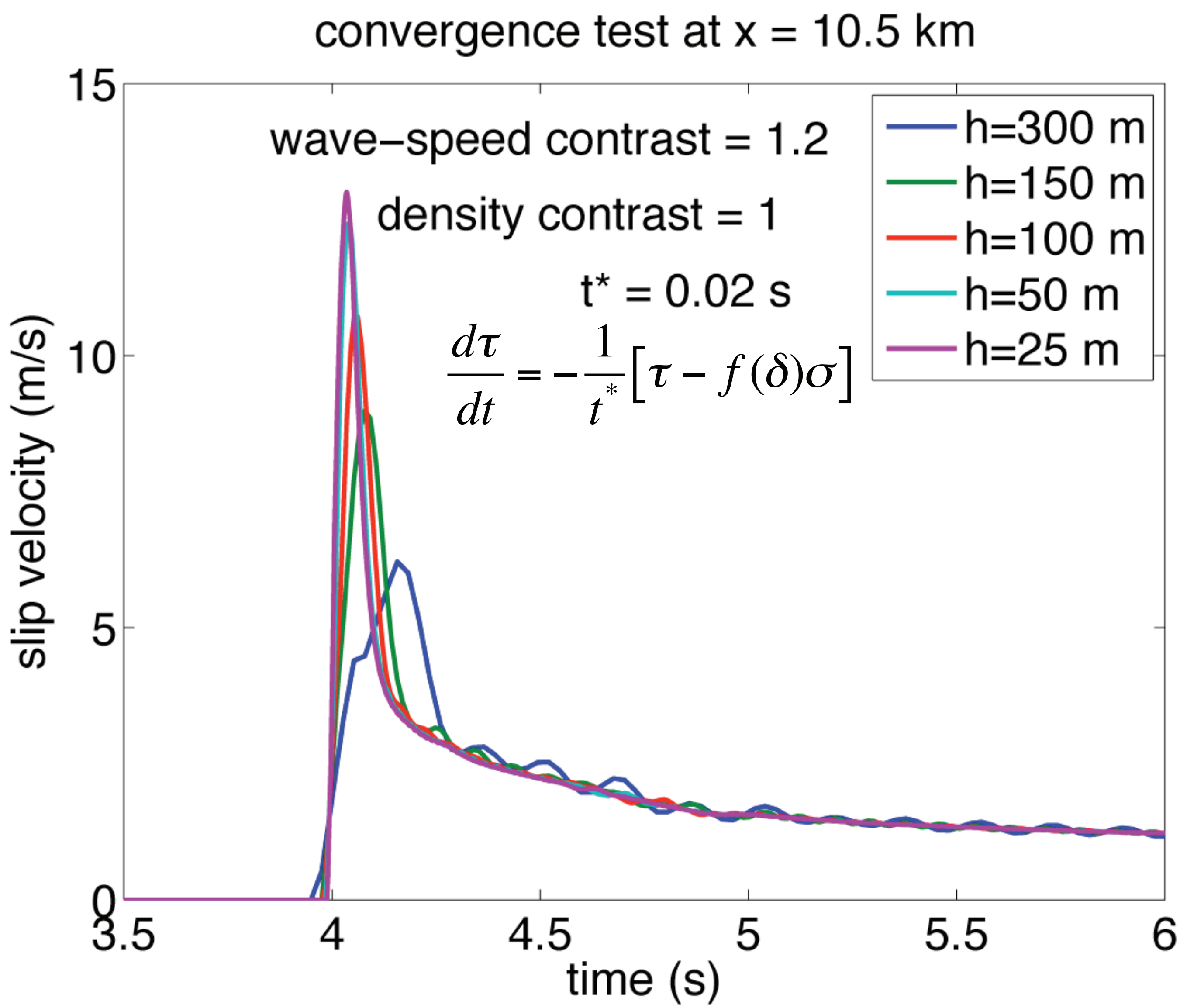
$$\frac{d\tau}{dt} = -\frac{1}{t^*}\left[\tau - f(\delta)\sigma\right]$$

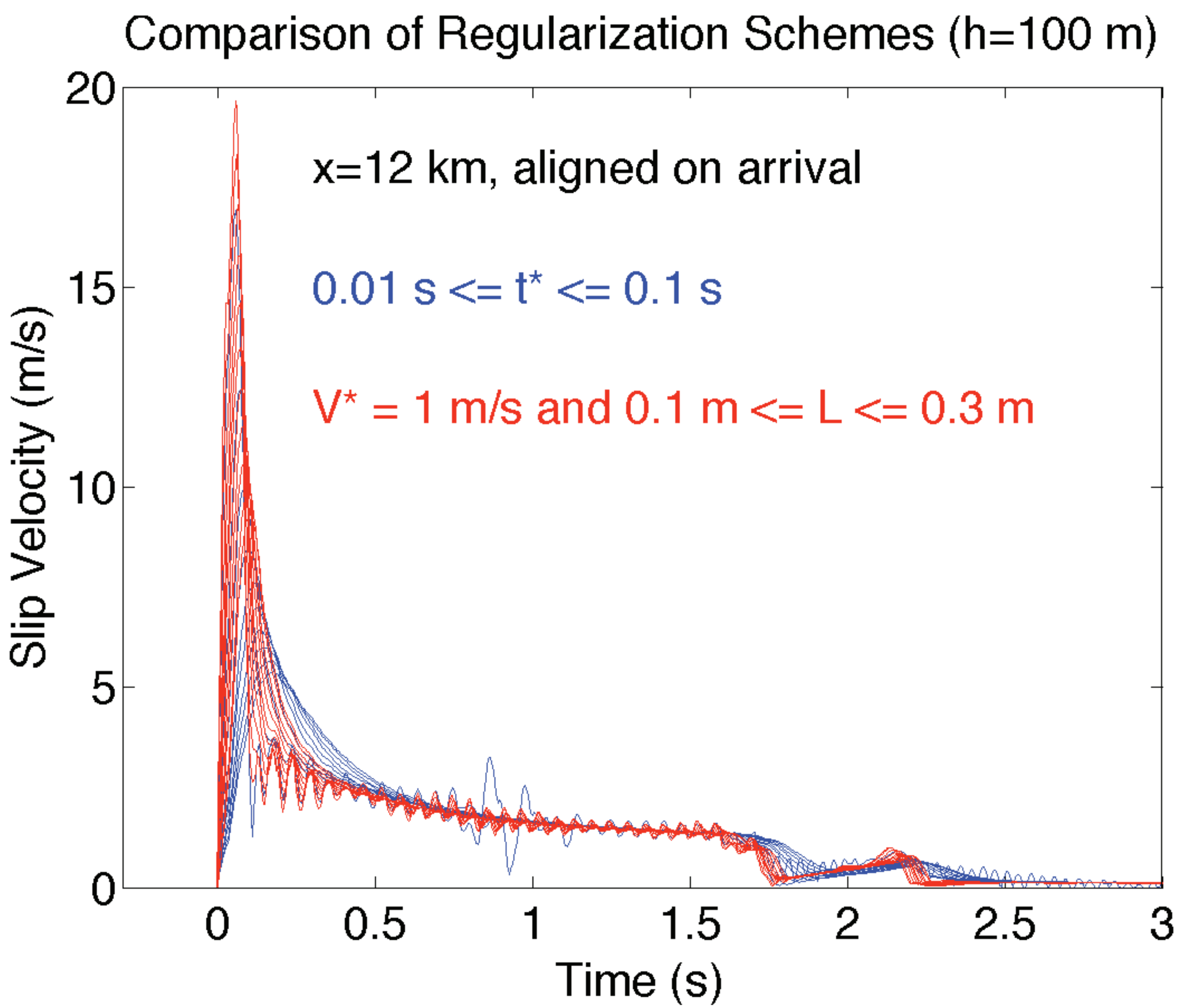


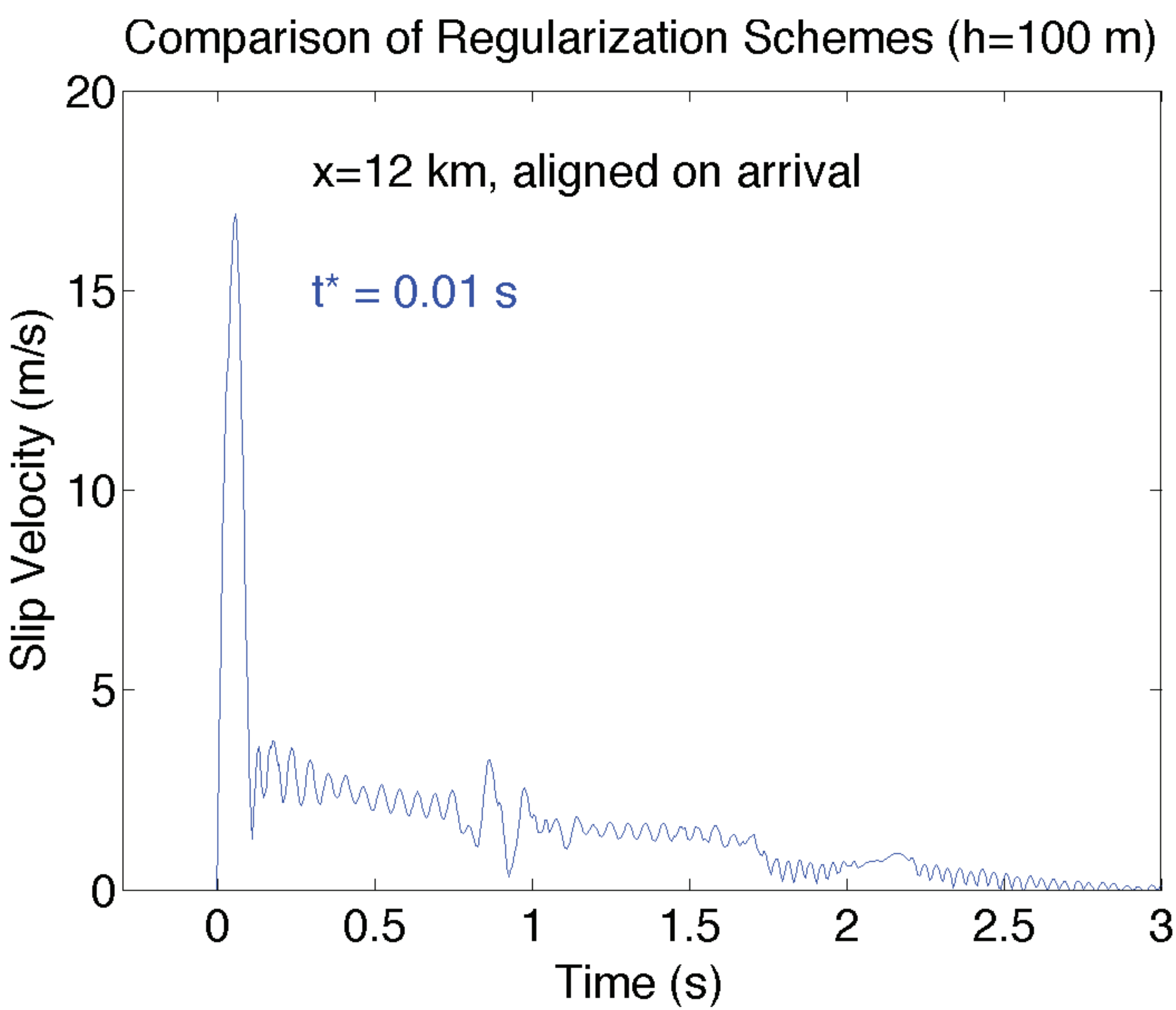
2. Prakash-Clifton law (evolution over fixed slip scale), with Cochard-Rice modification (V^*) to allow evolution when V=0

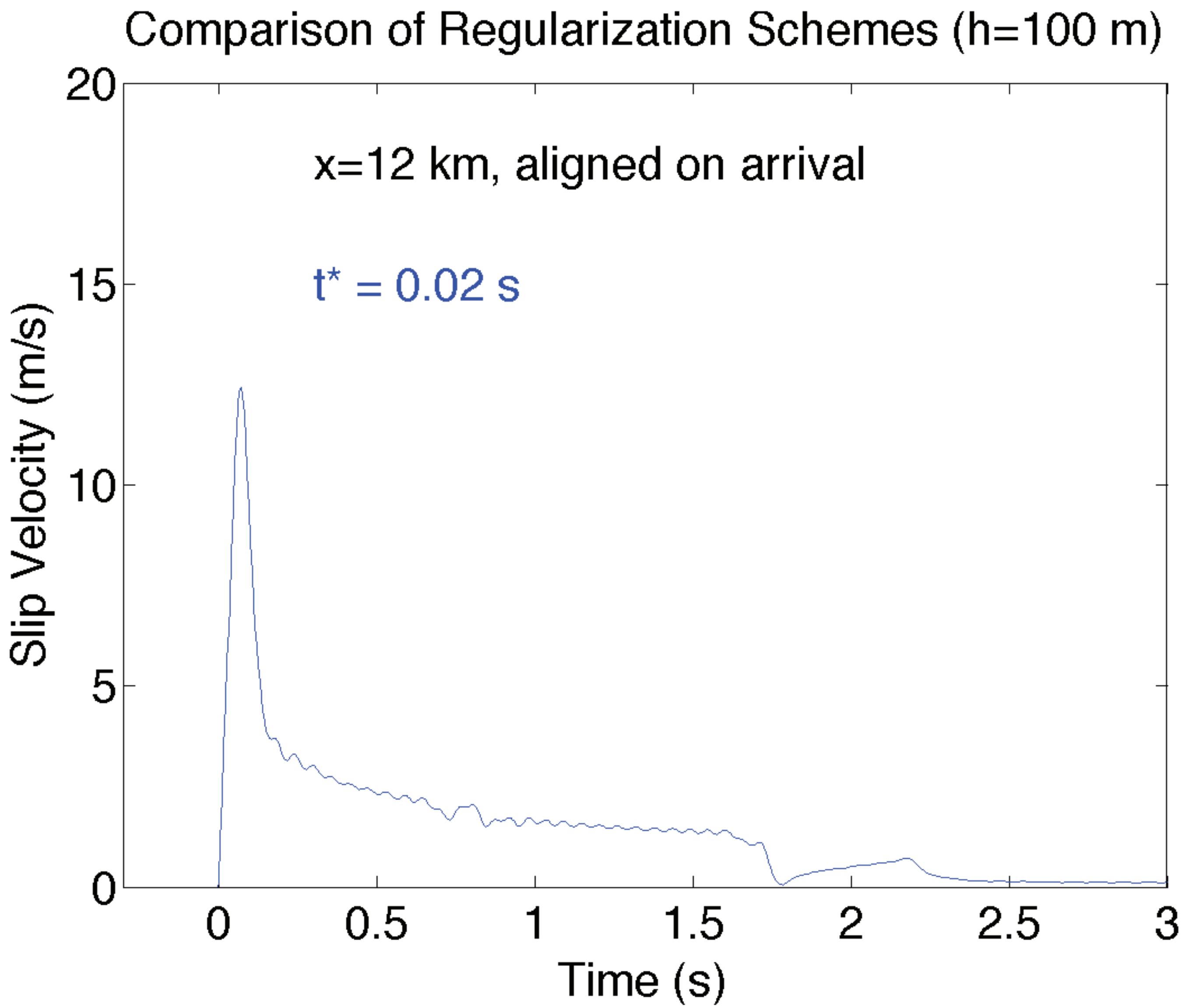
$$\frac{d\tau}{dt} = -\frac{V + V^*}{L}\left[\tau - f(\delta)\sigma\right]$$

Ex. 2: Regularized Ill-posed

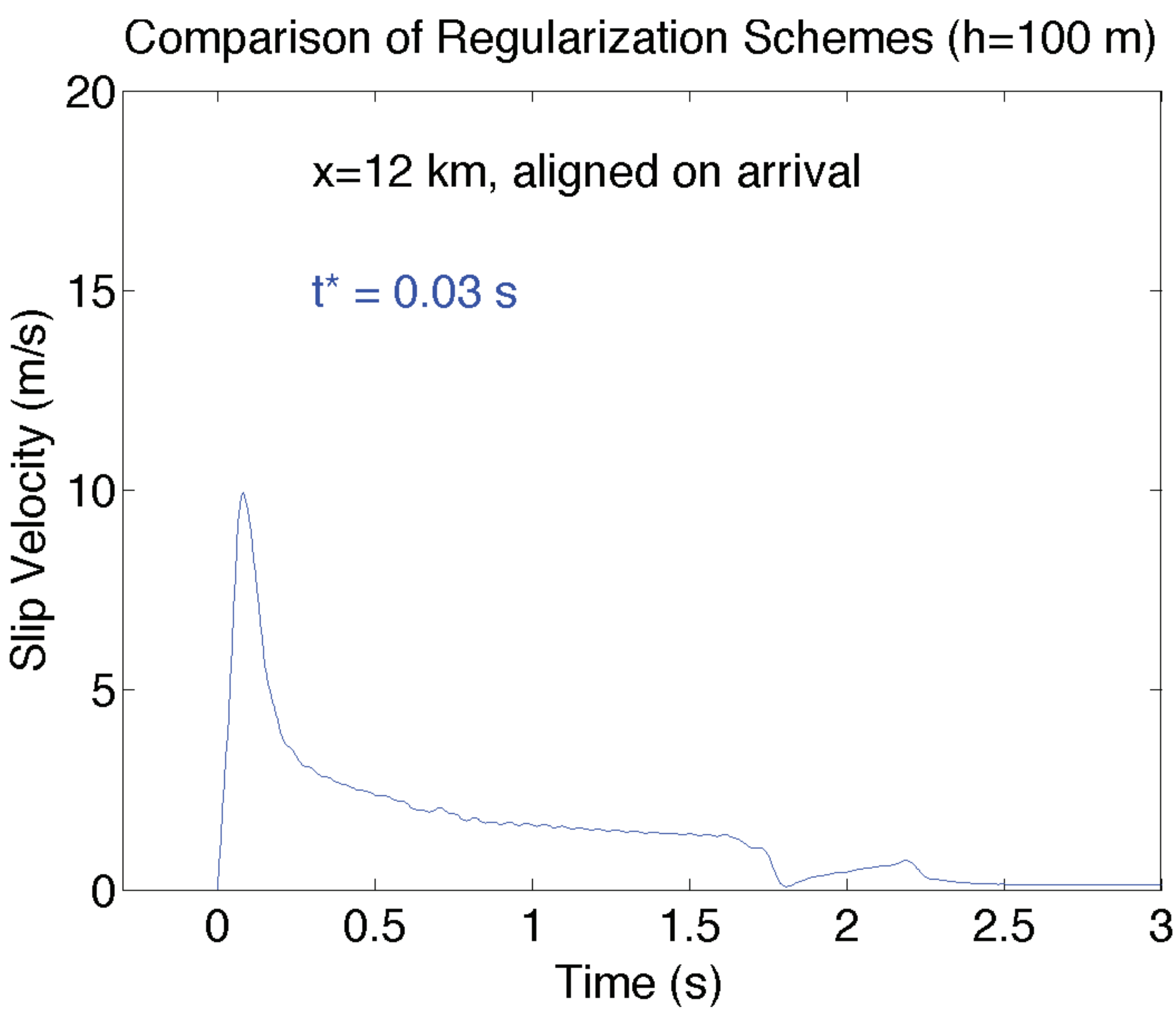


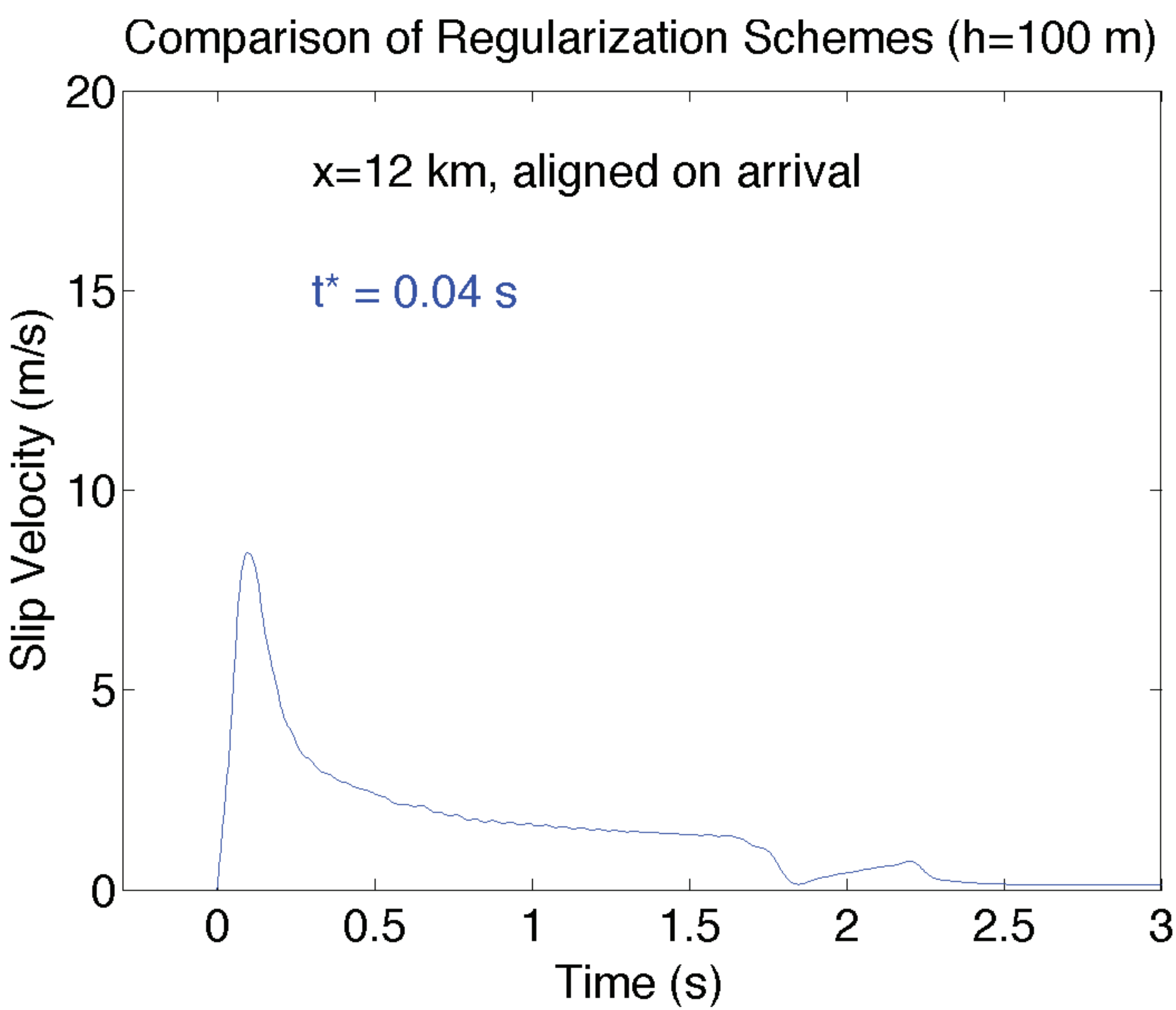

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
0.01 s <= t* <= 0.1 s
V* = 1 m/s and 0.1 m <= L <= 0.3 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.01 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

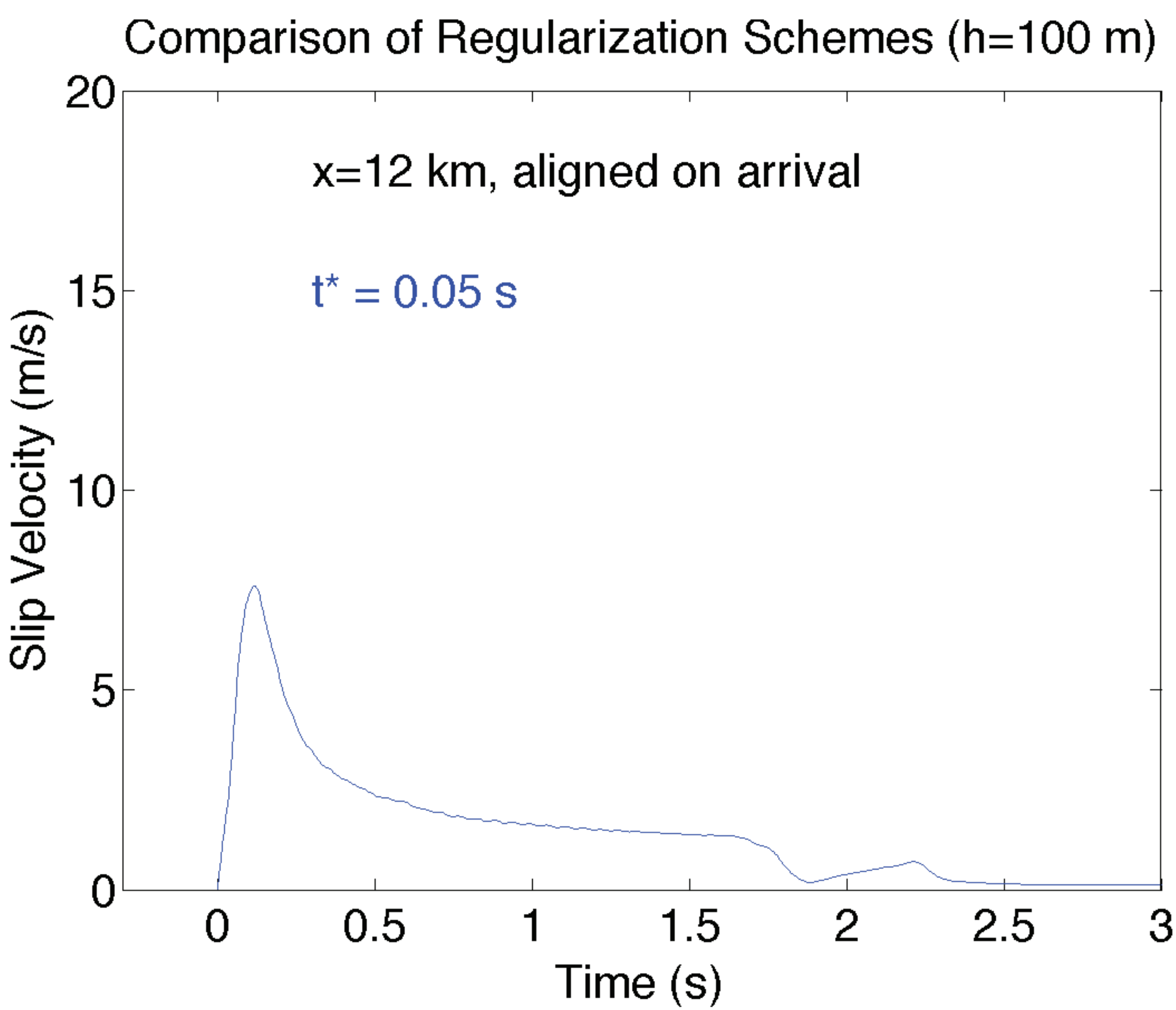


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.02 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

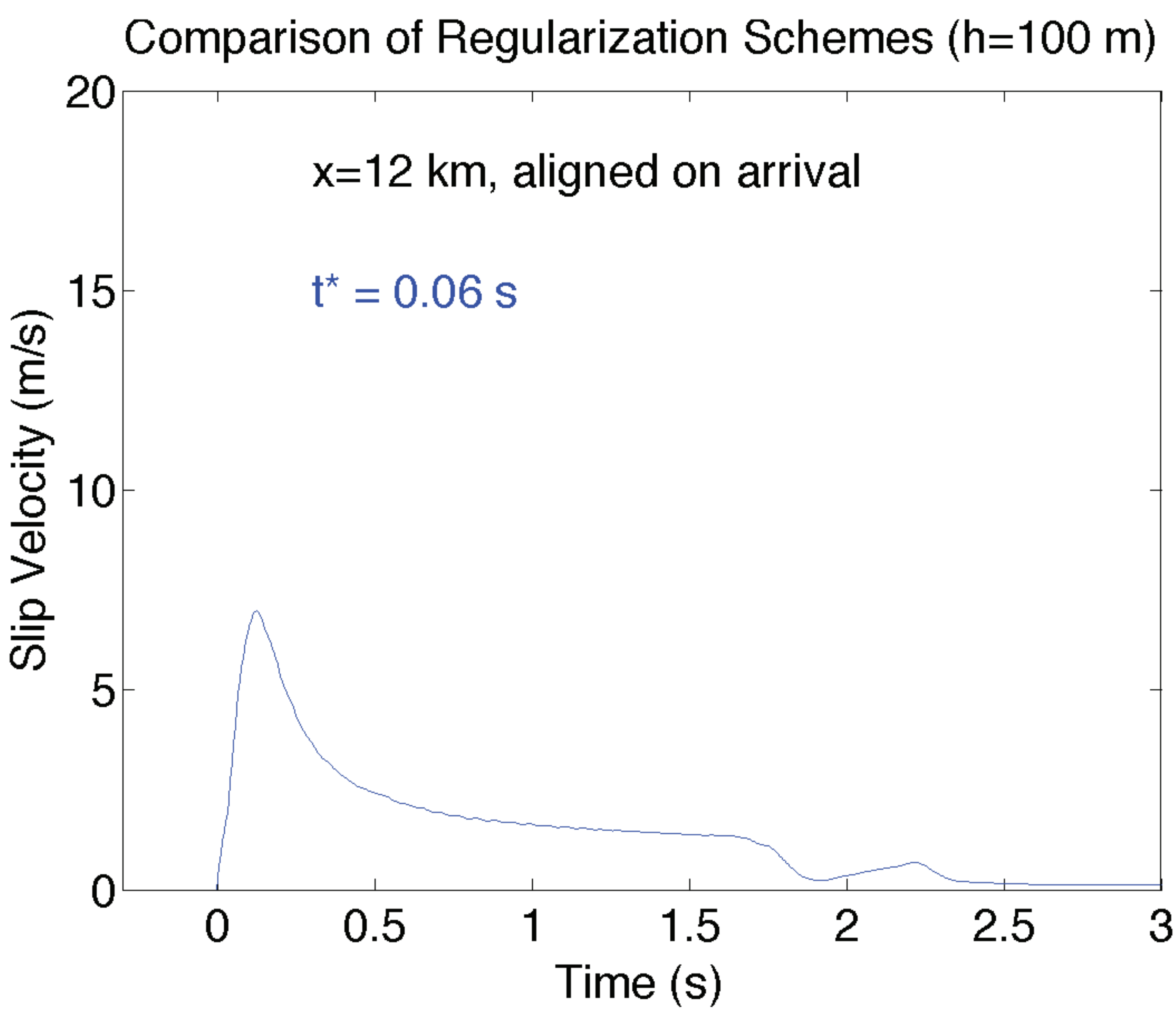

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.03 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3



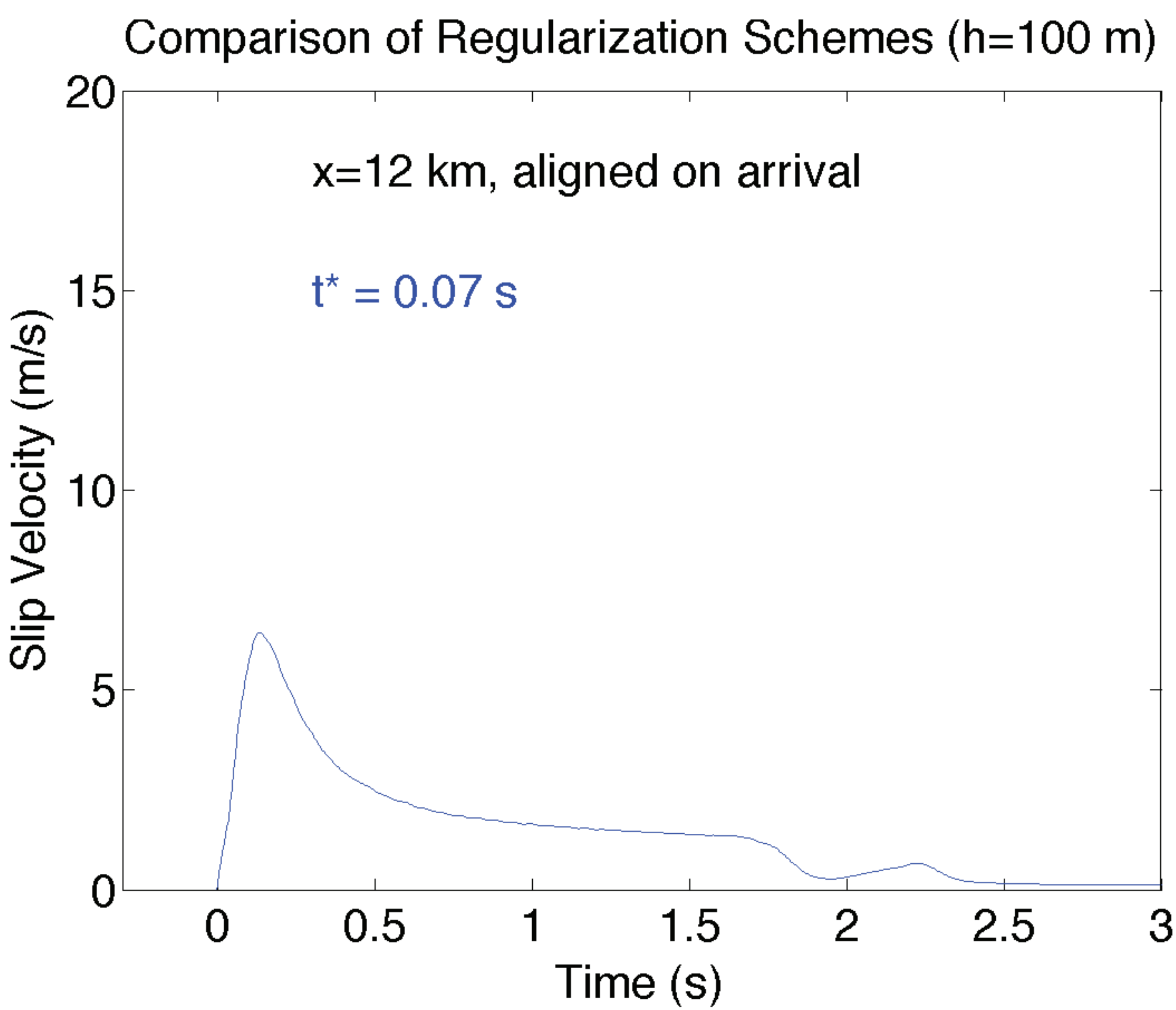
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.04 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

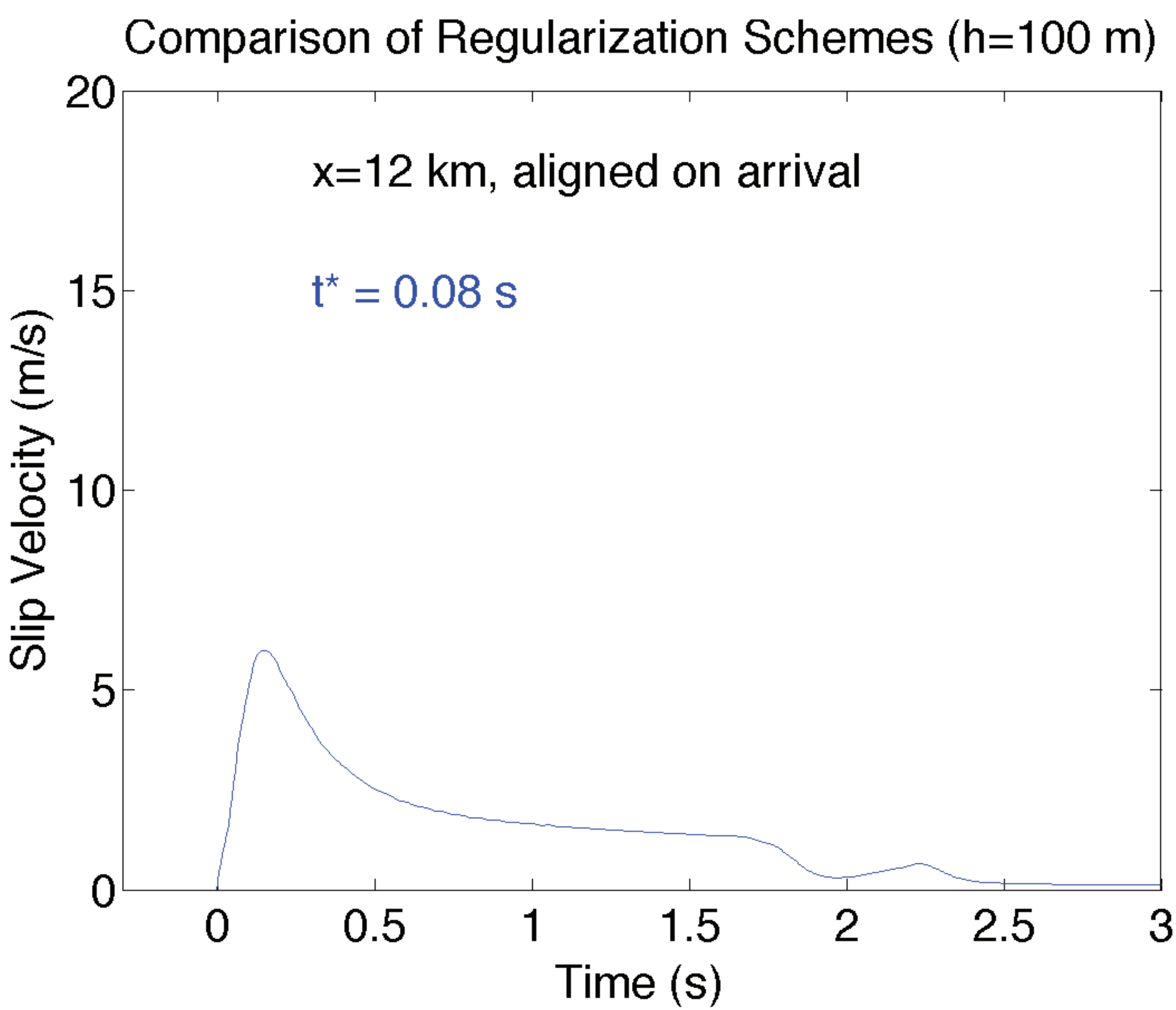


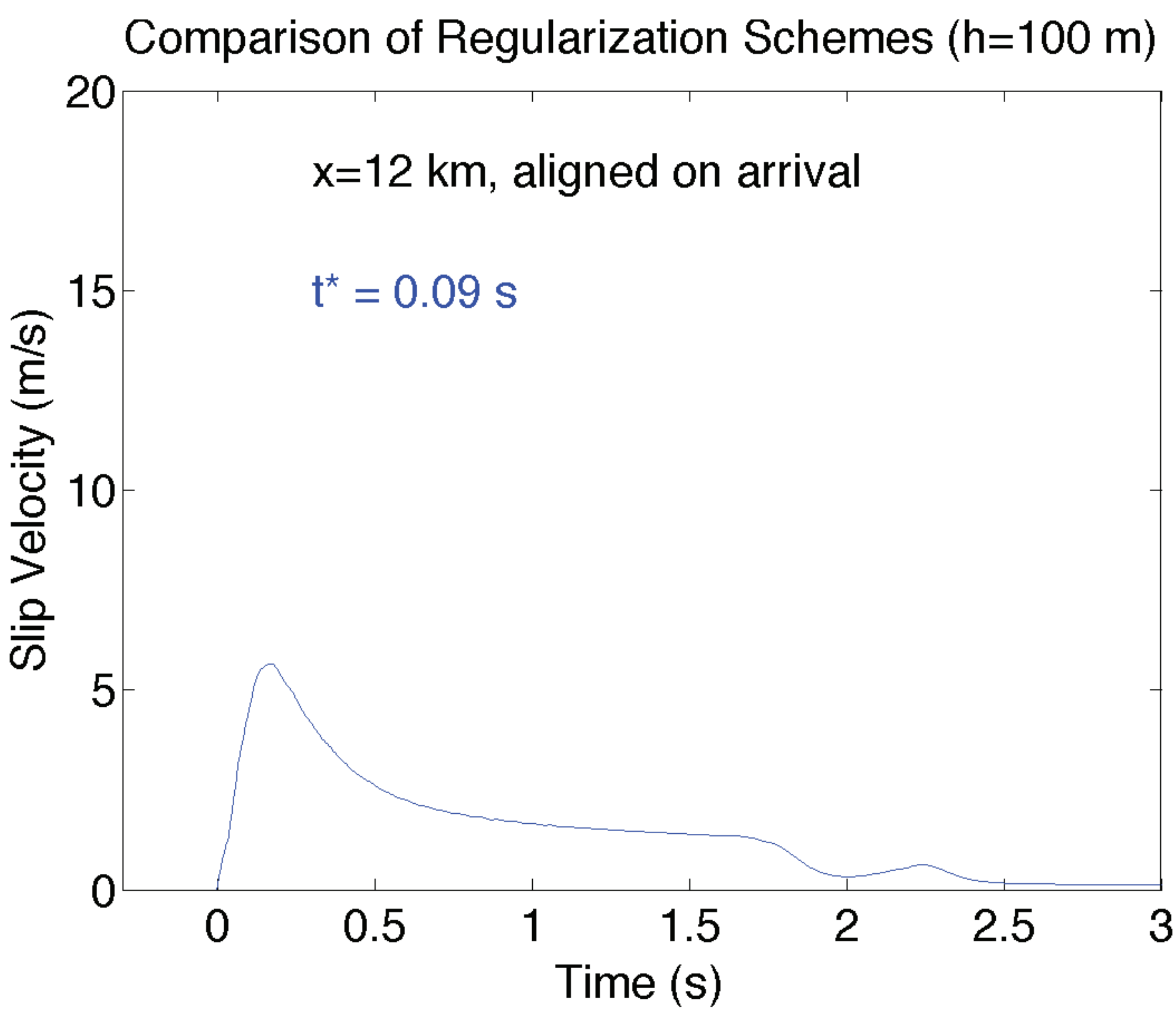
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.05 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

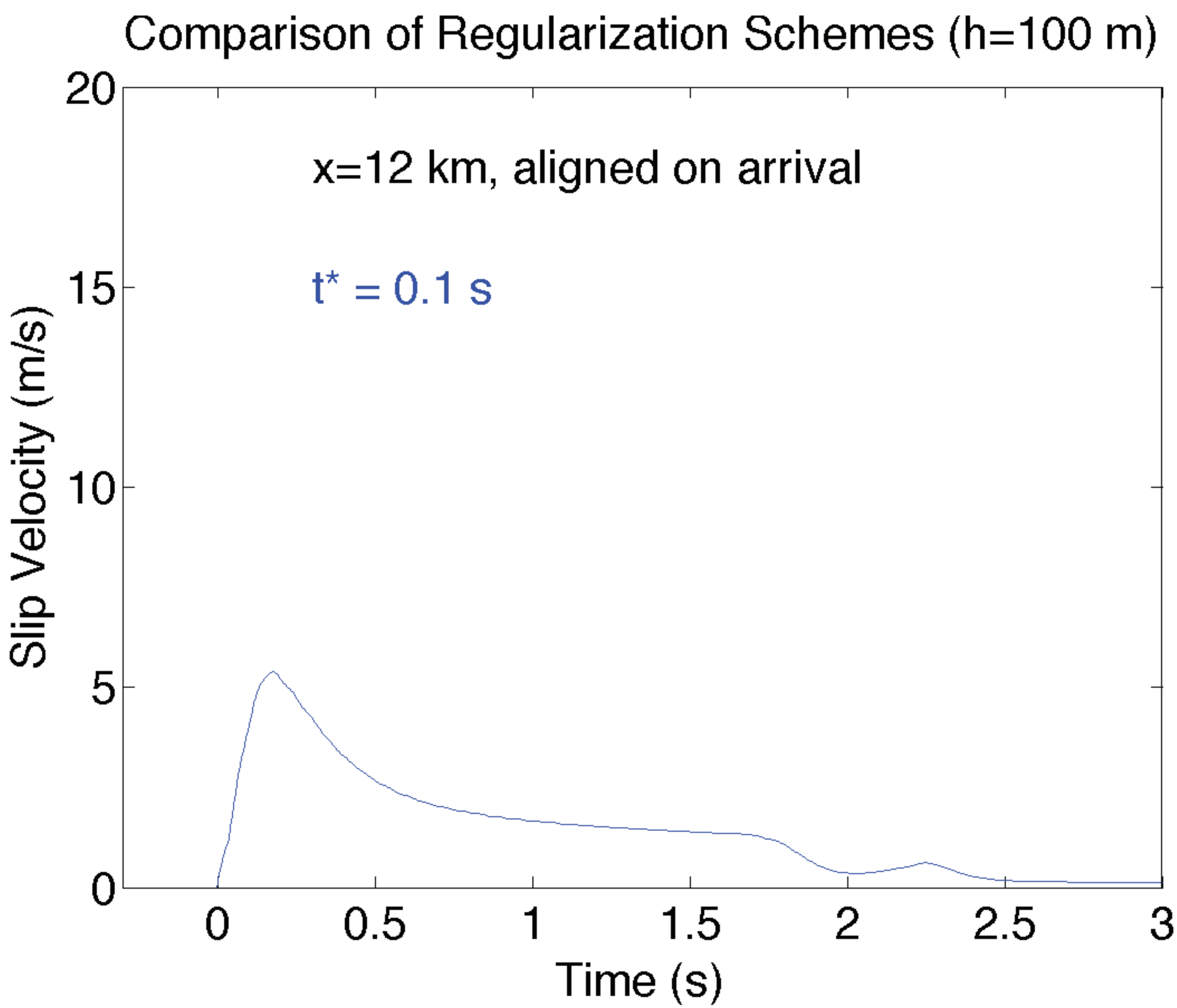


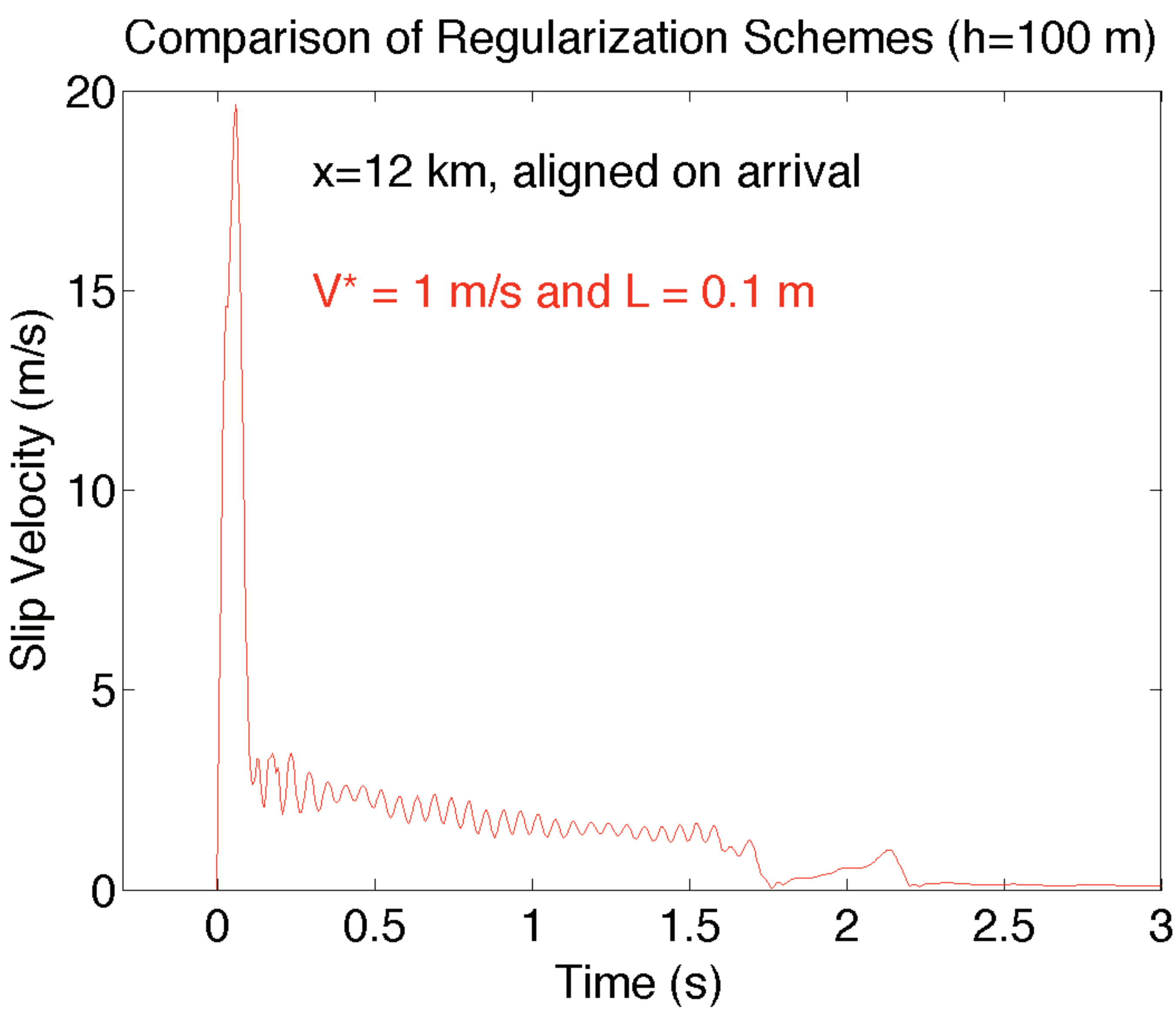
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.06 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

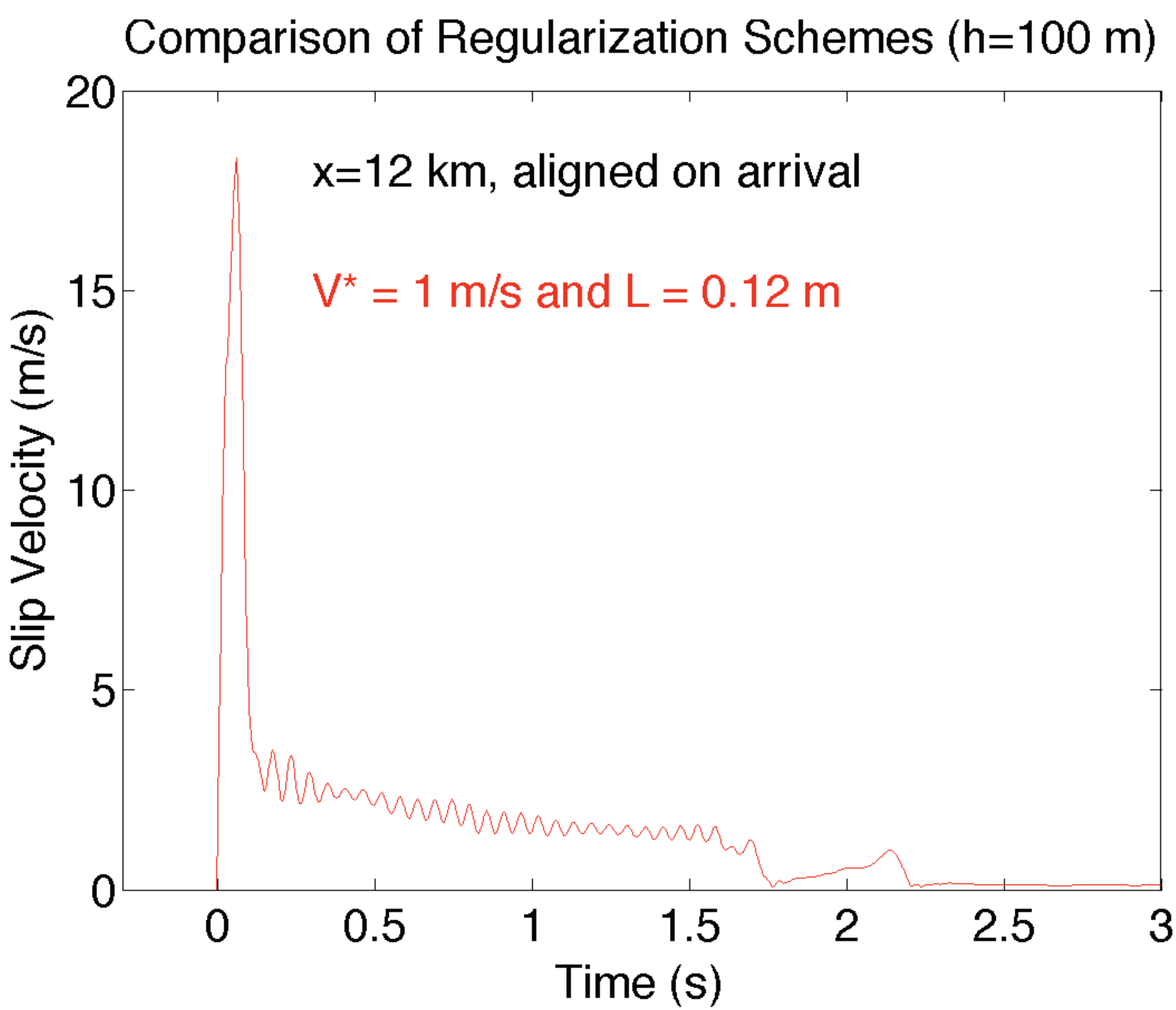

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.07 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

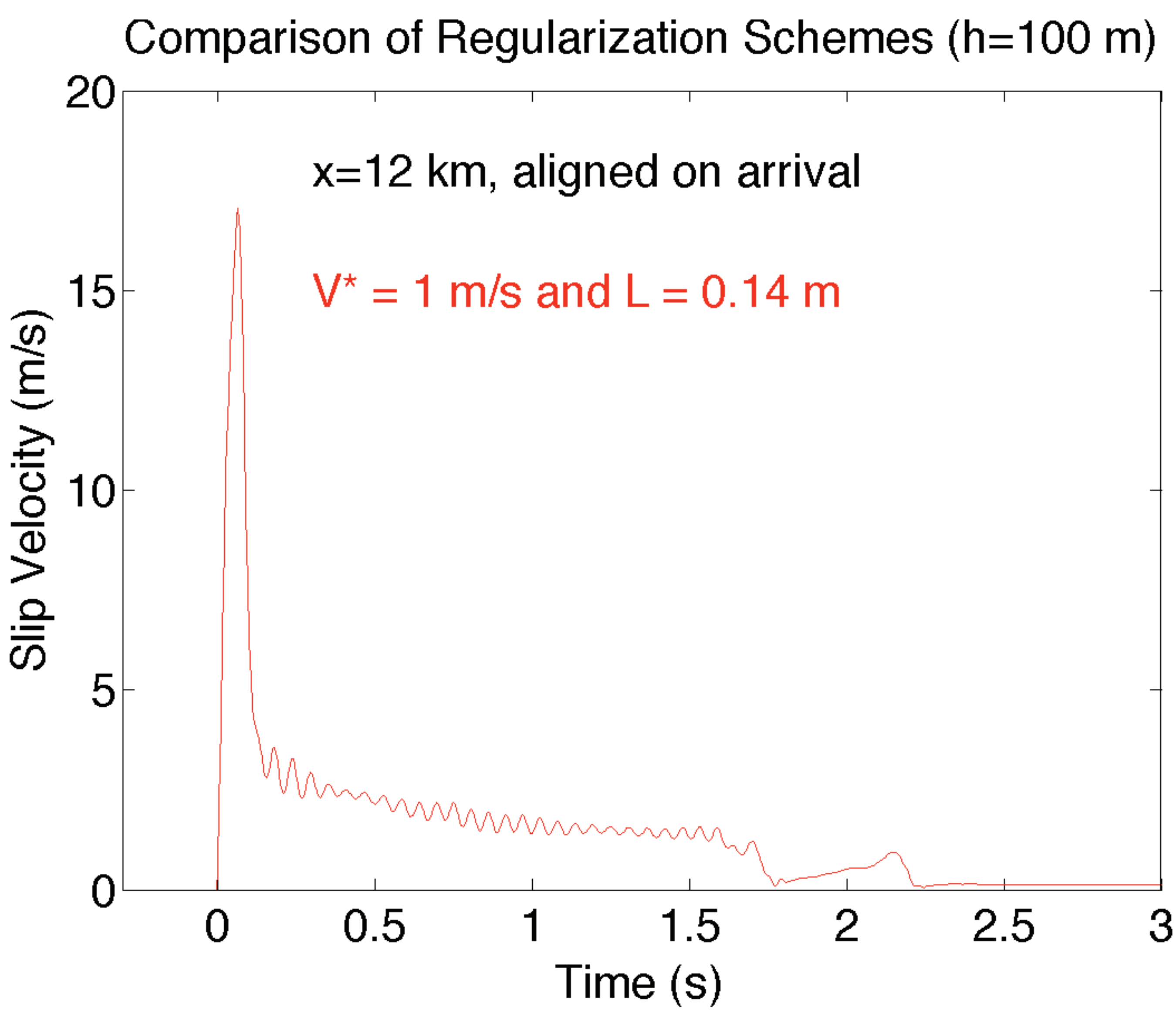

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.08 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.09 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

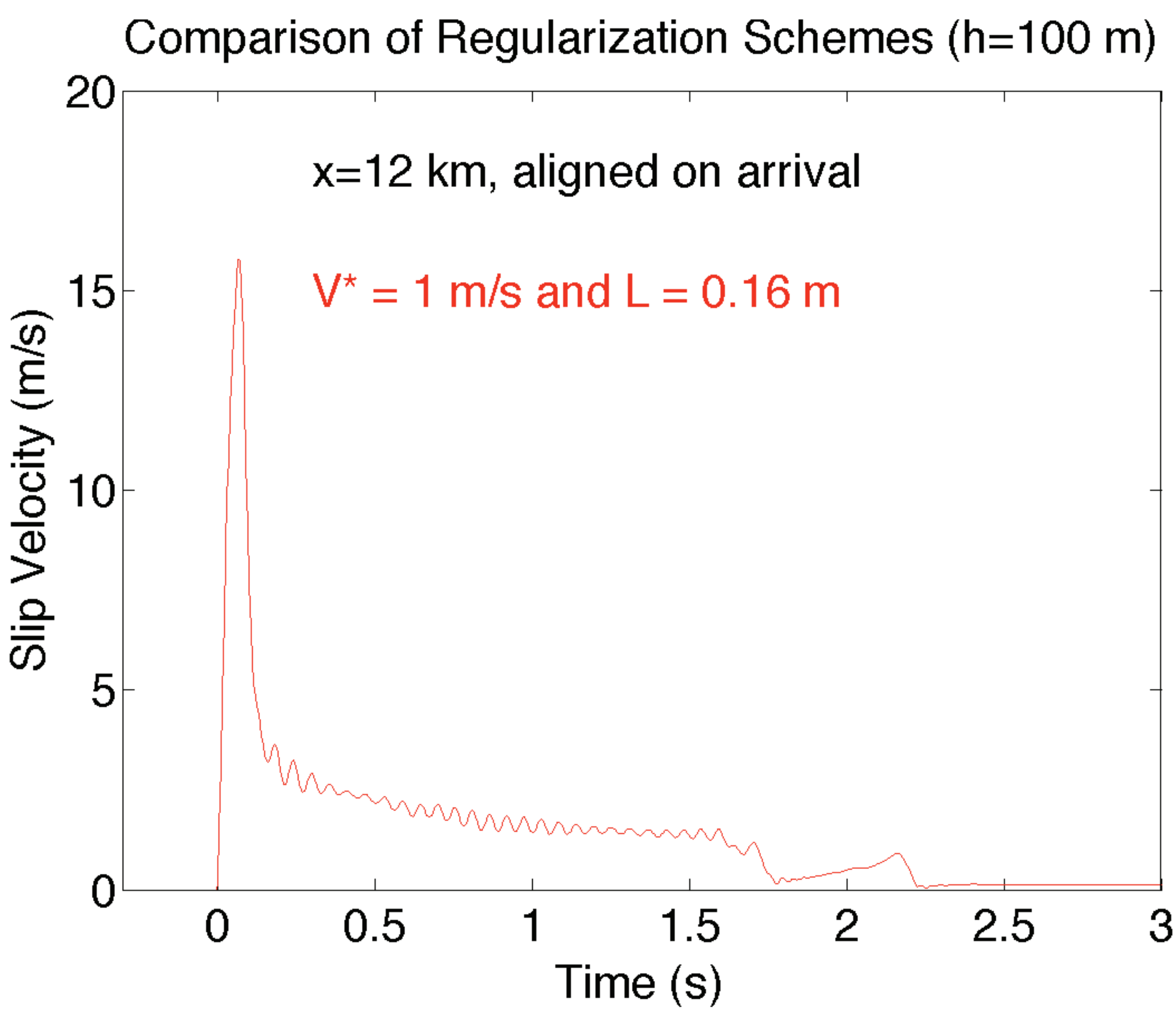

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.1 s
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

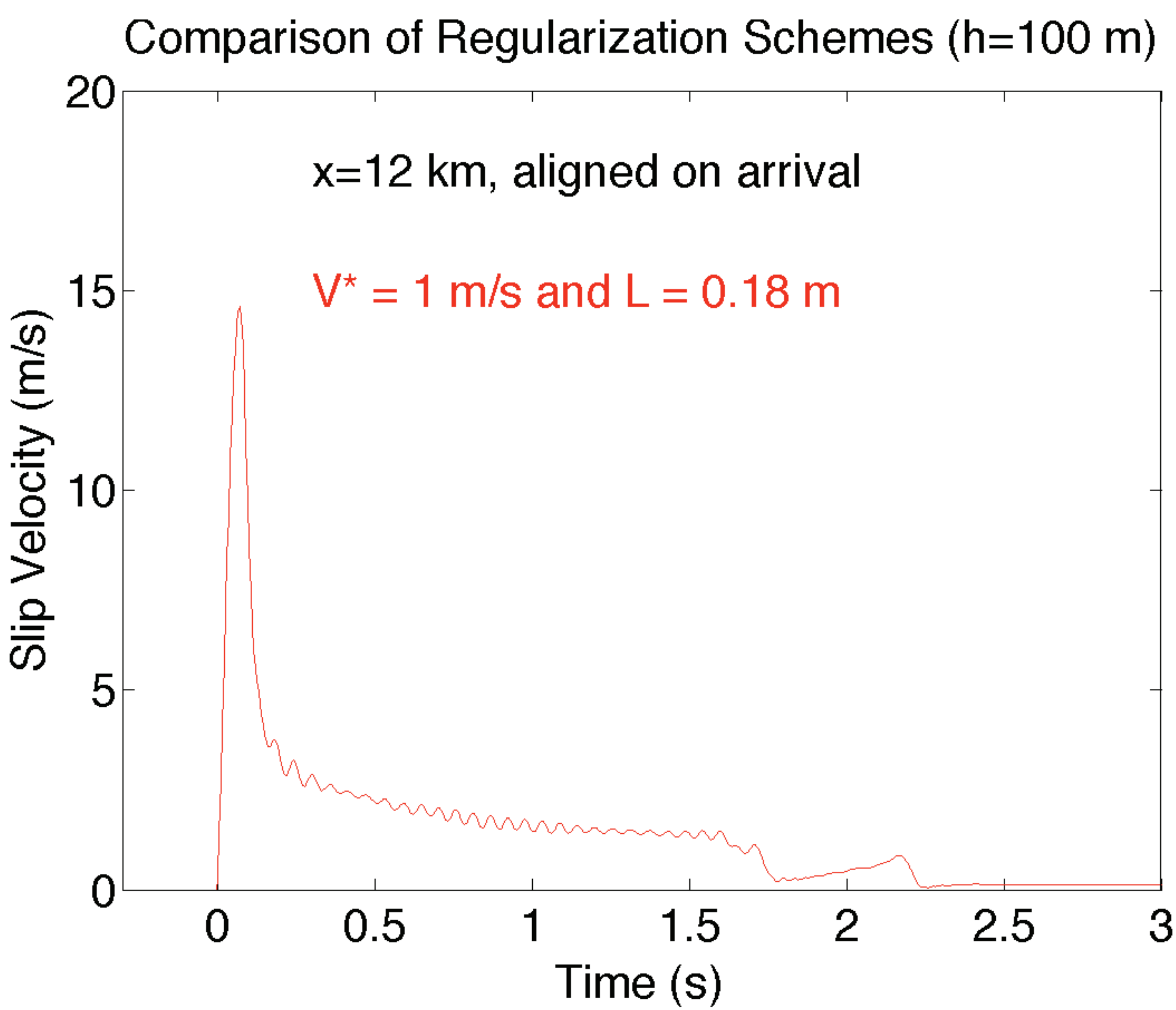

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.1 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

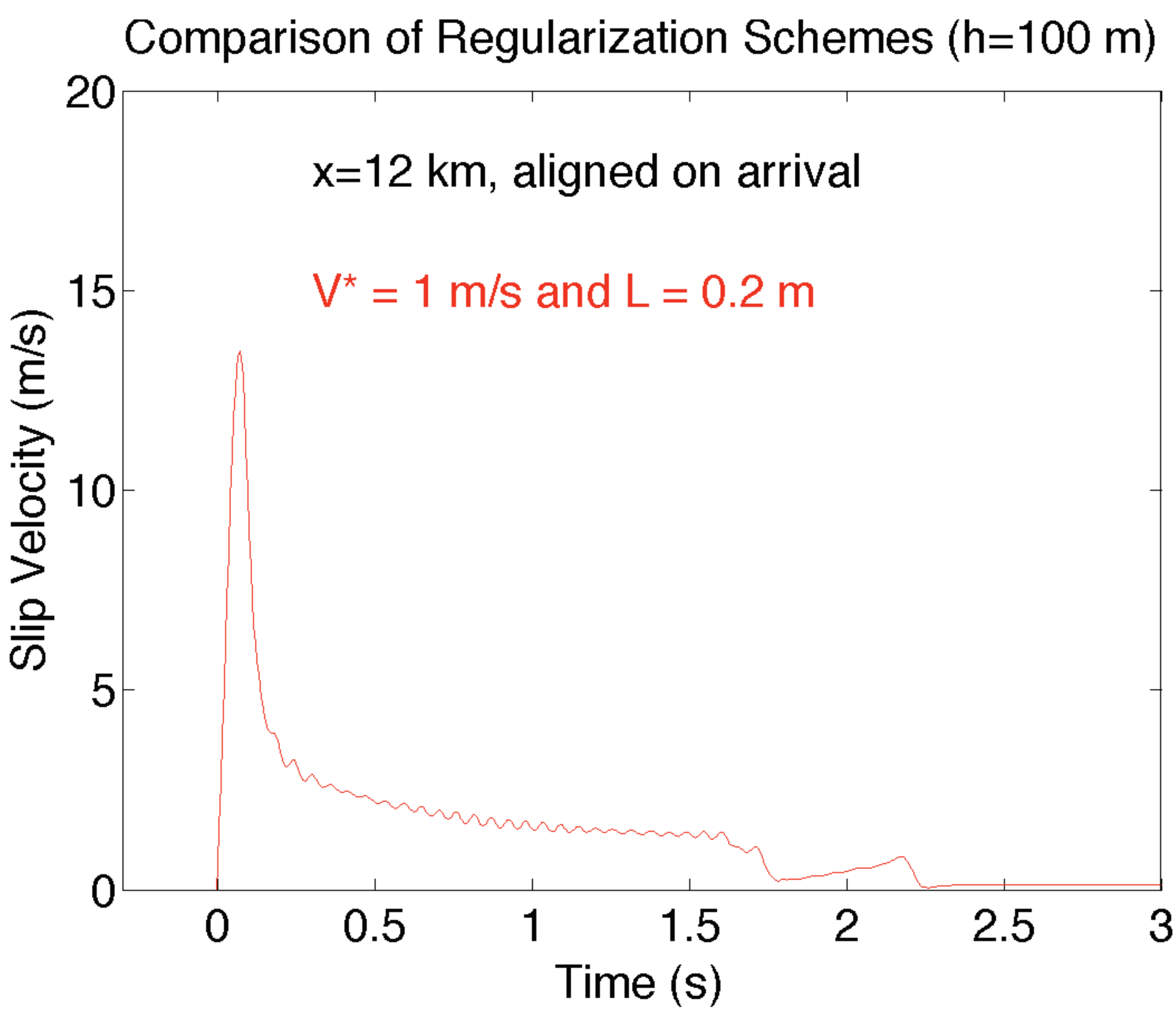

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.12 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

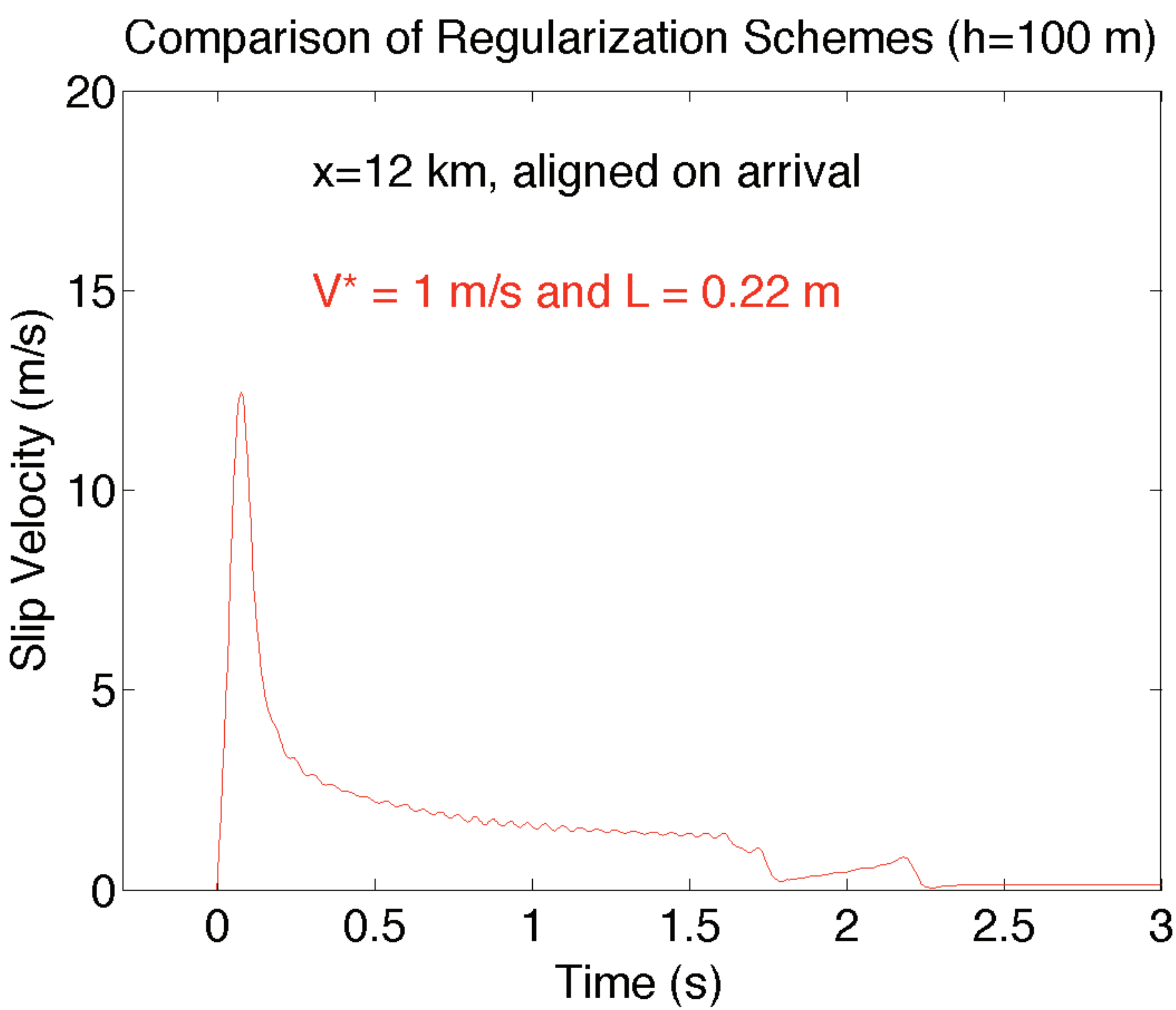


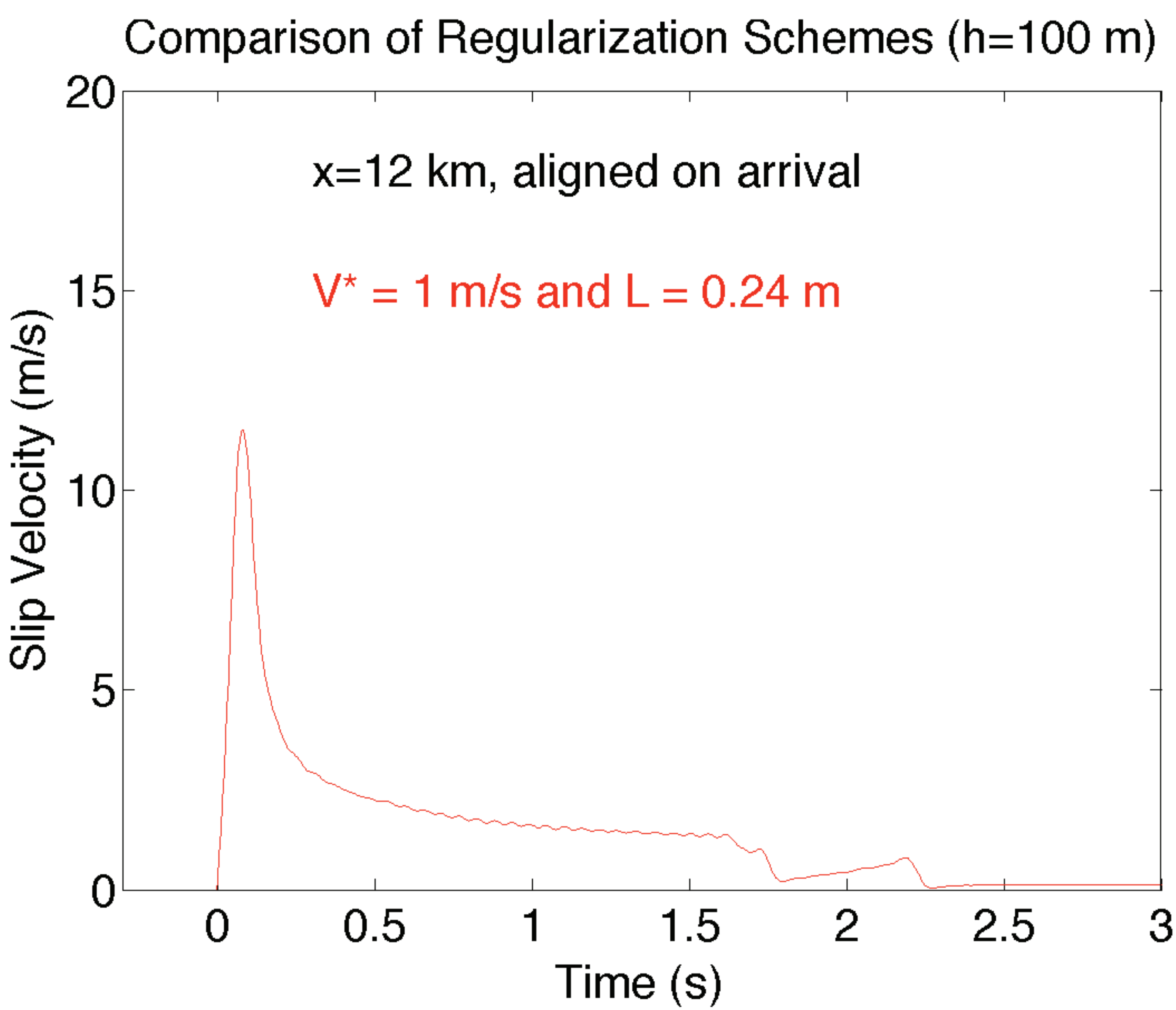
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.14 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

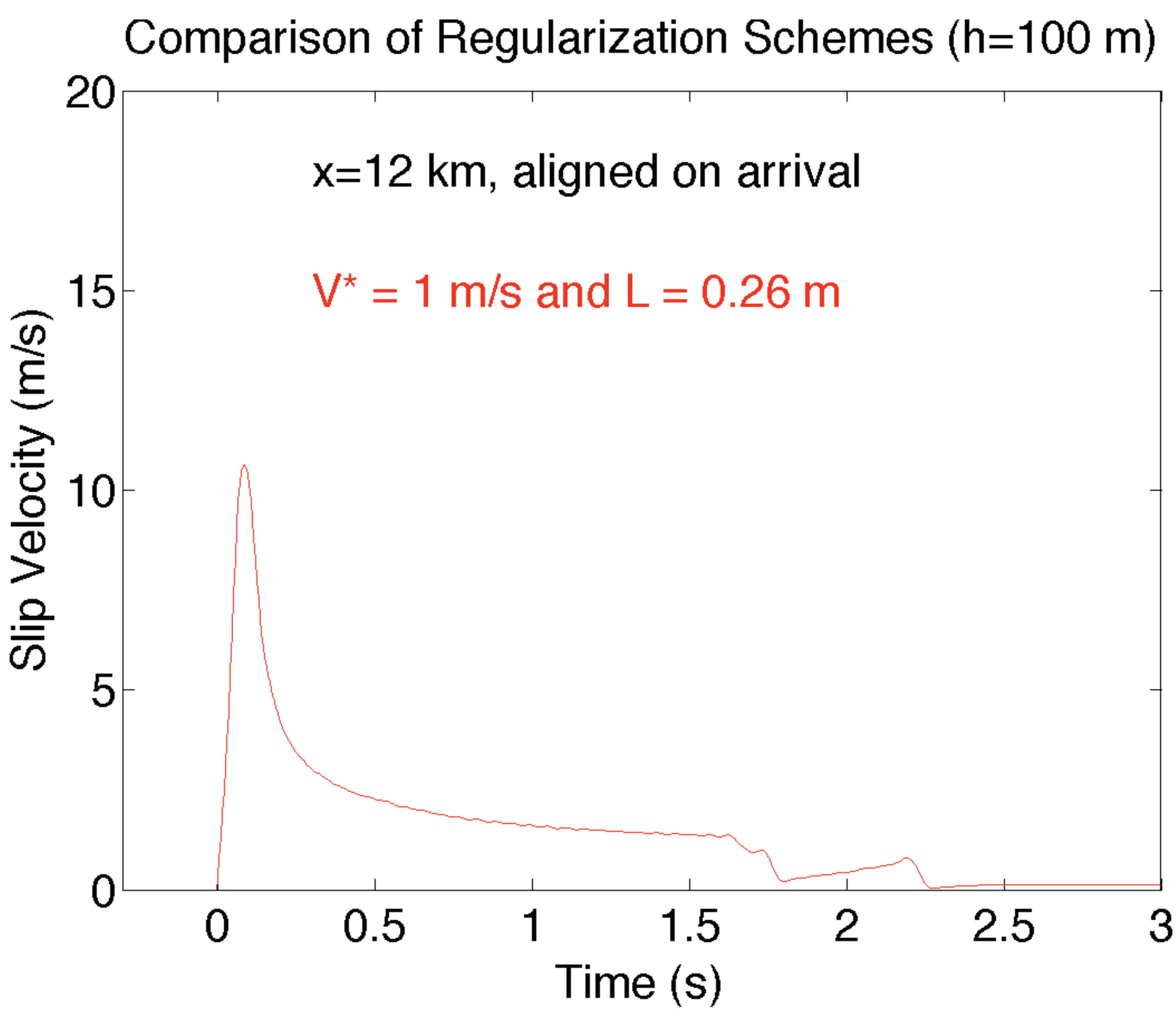

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.16 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

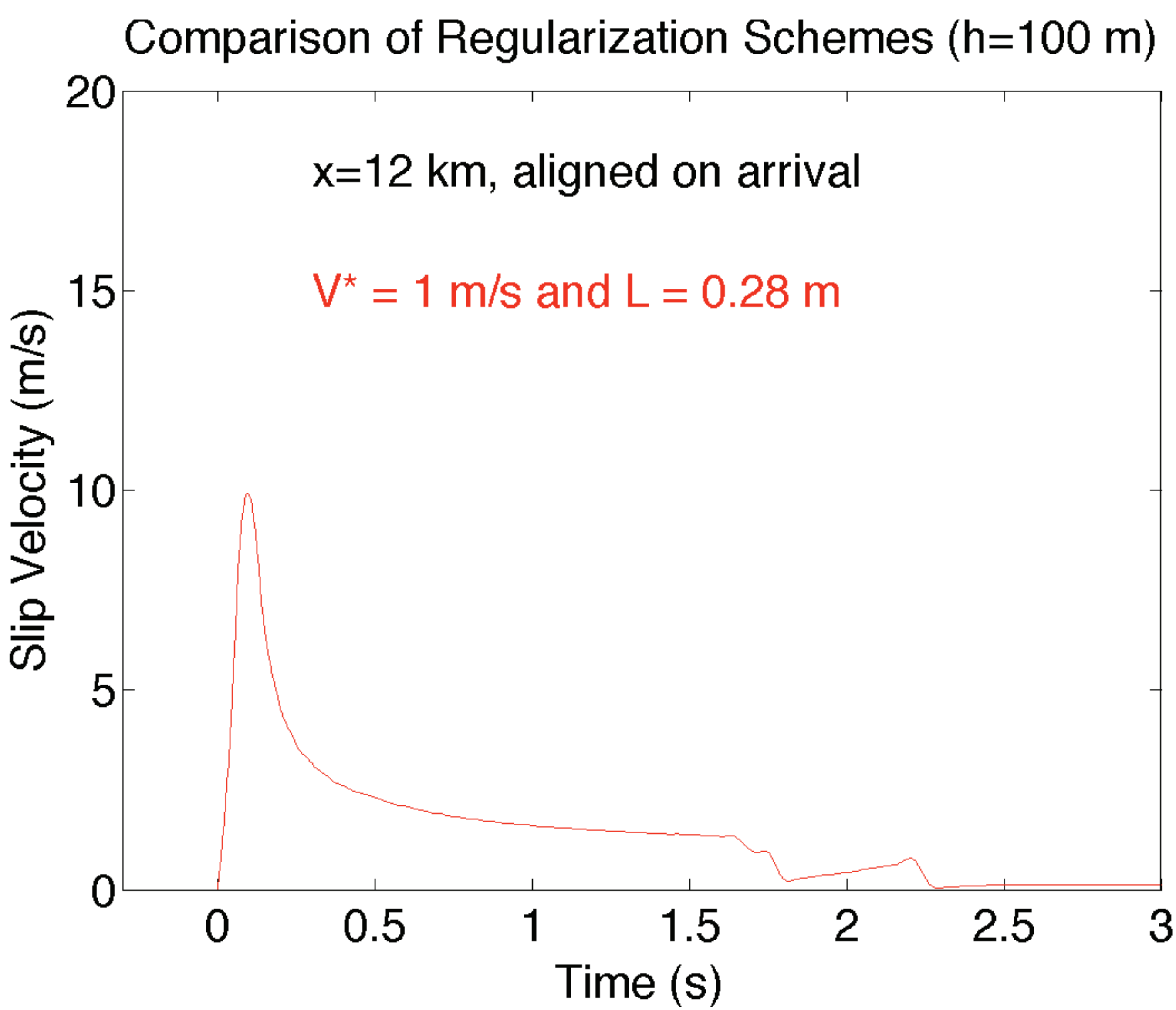

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.18 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

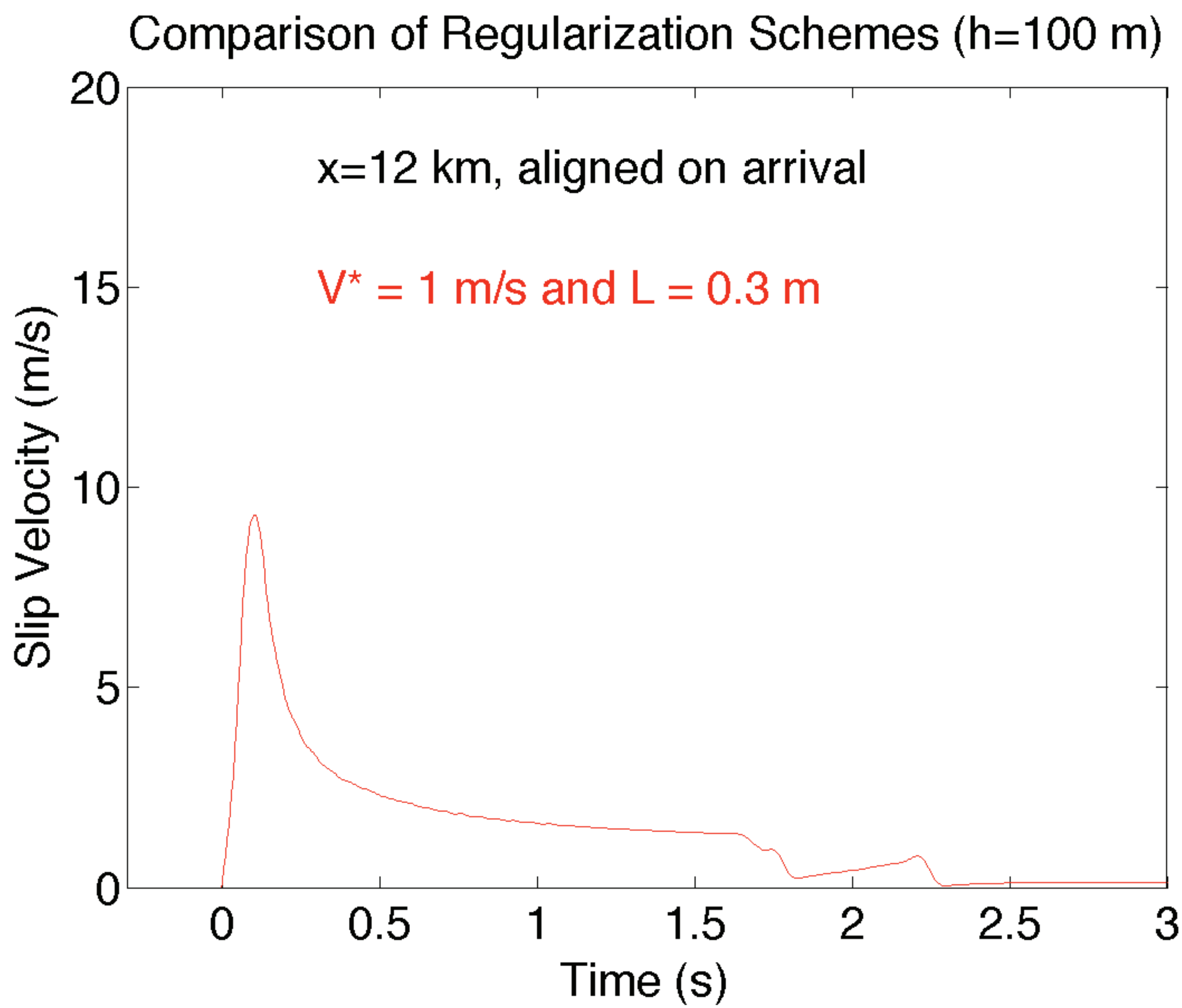

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.2 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.22 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

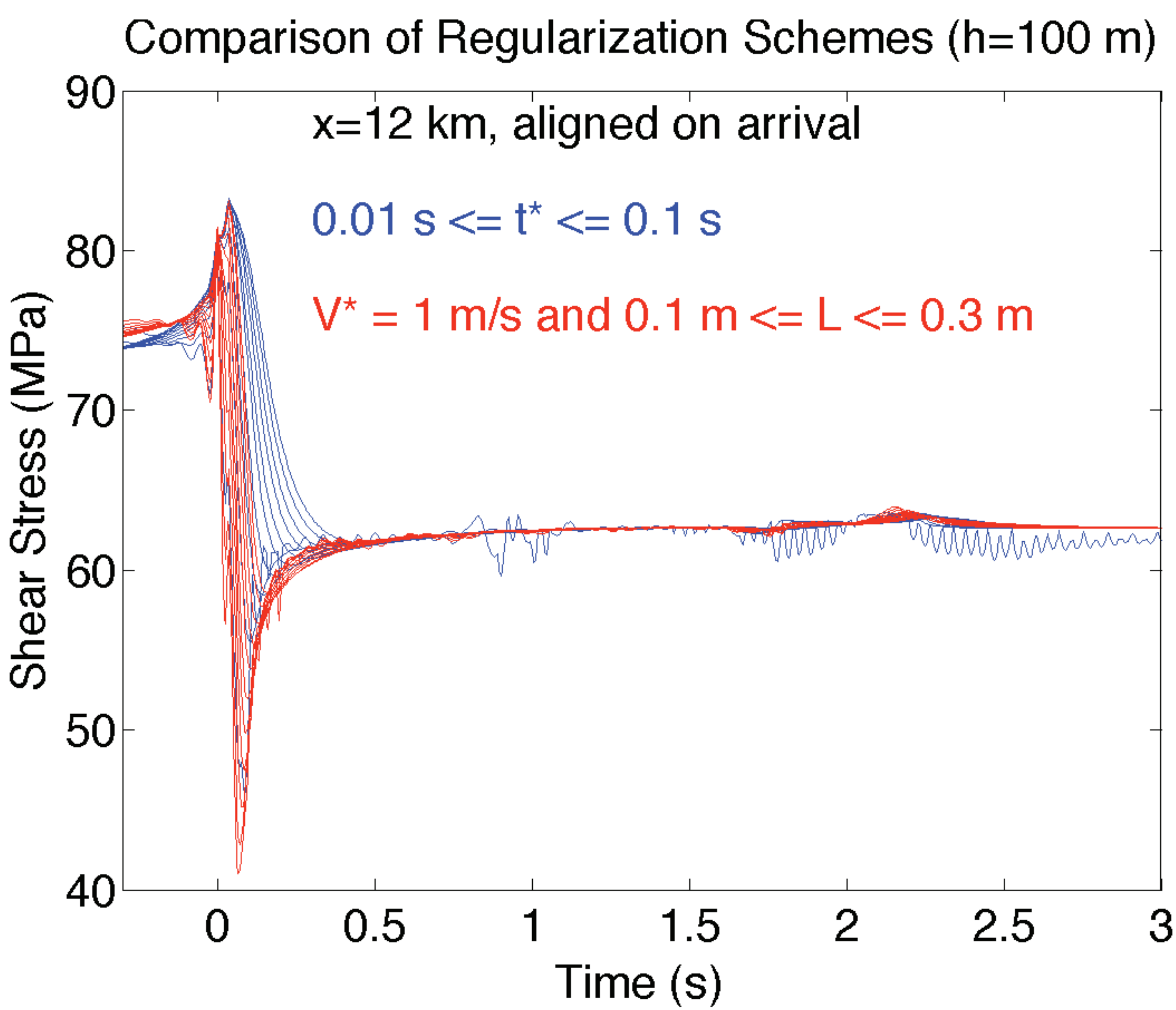

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.24 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

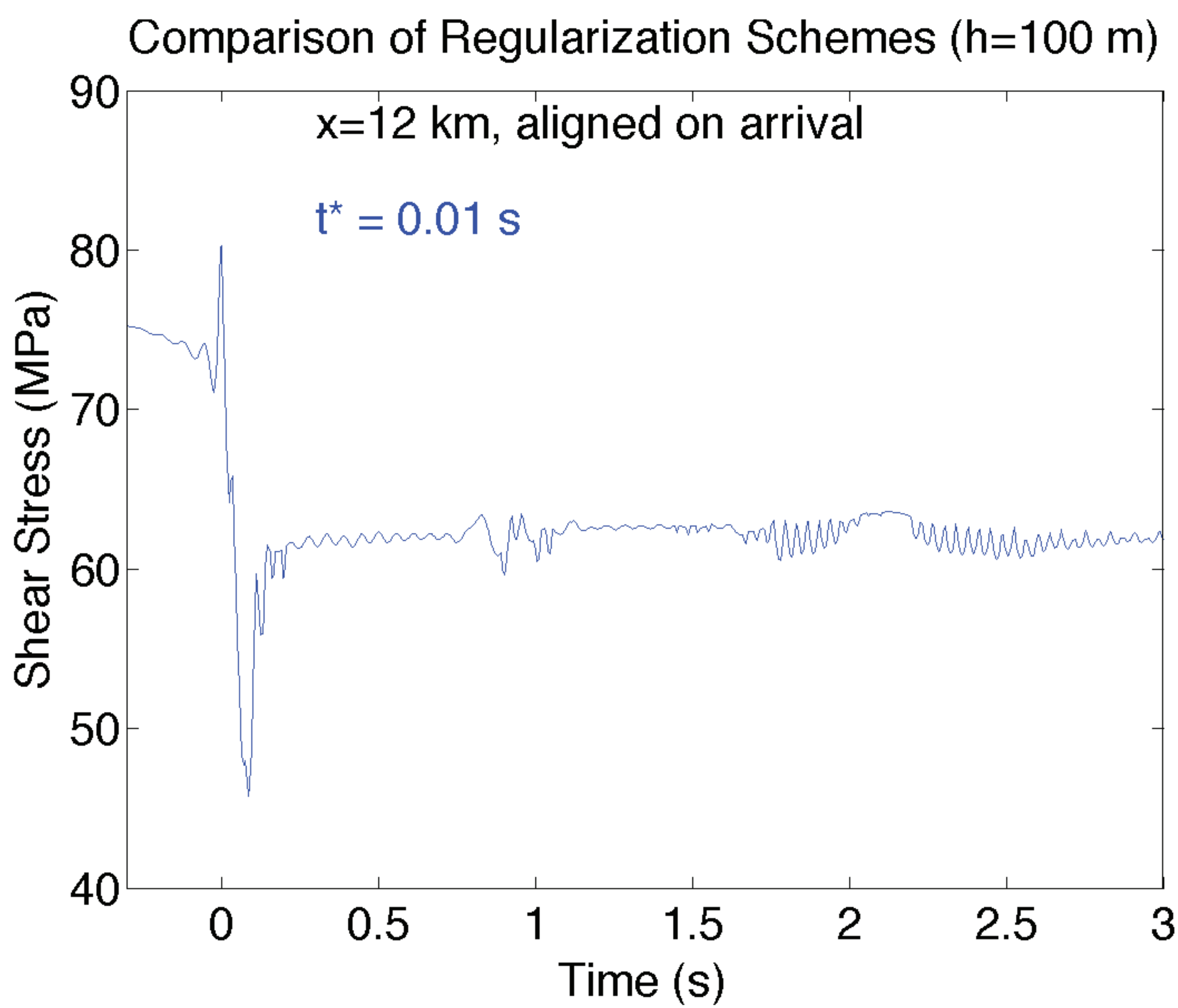

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.26 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

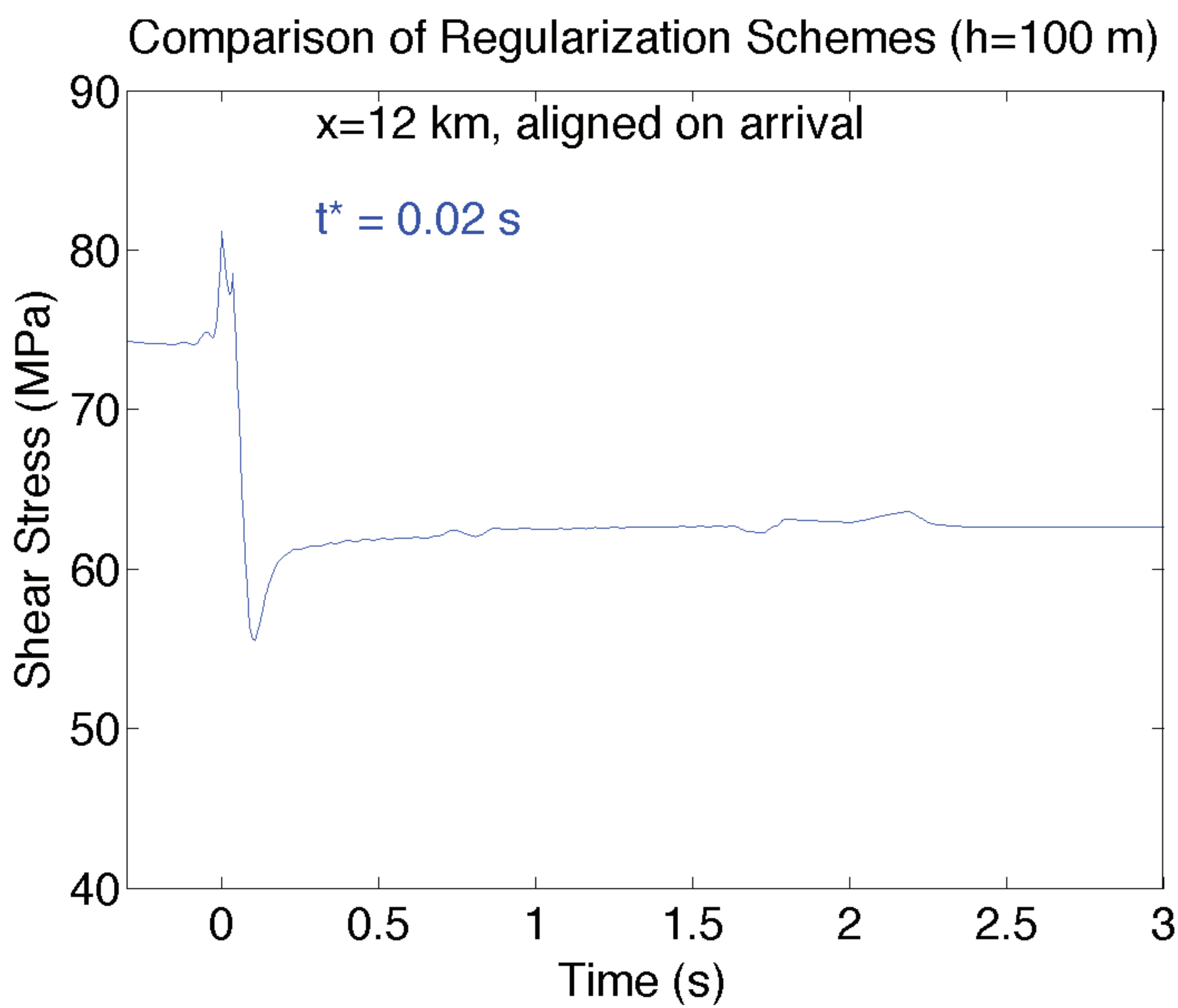

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.28 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

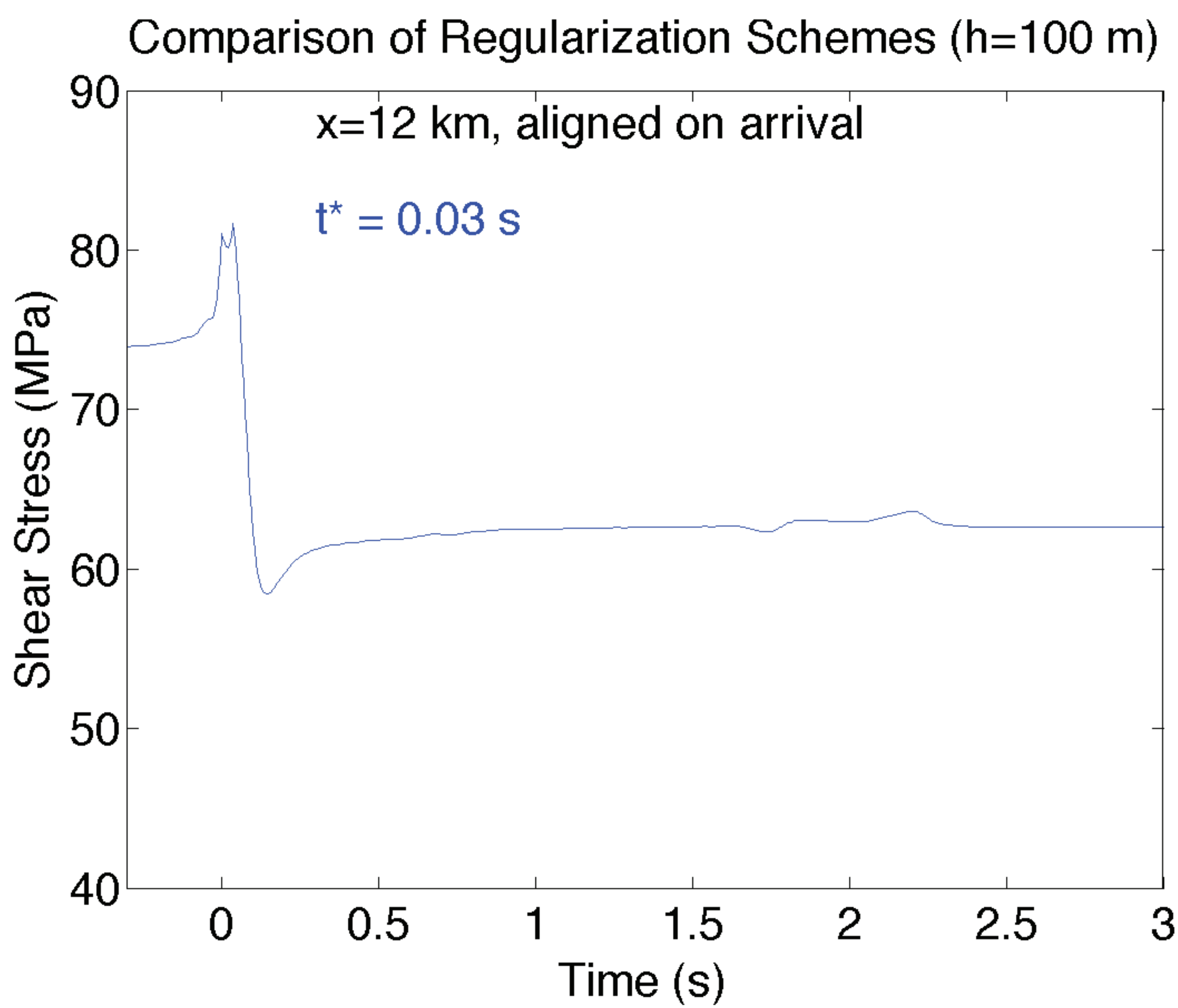


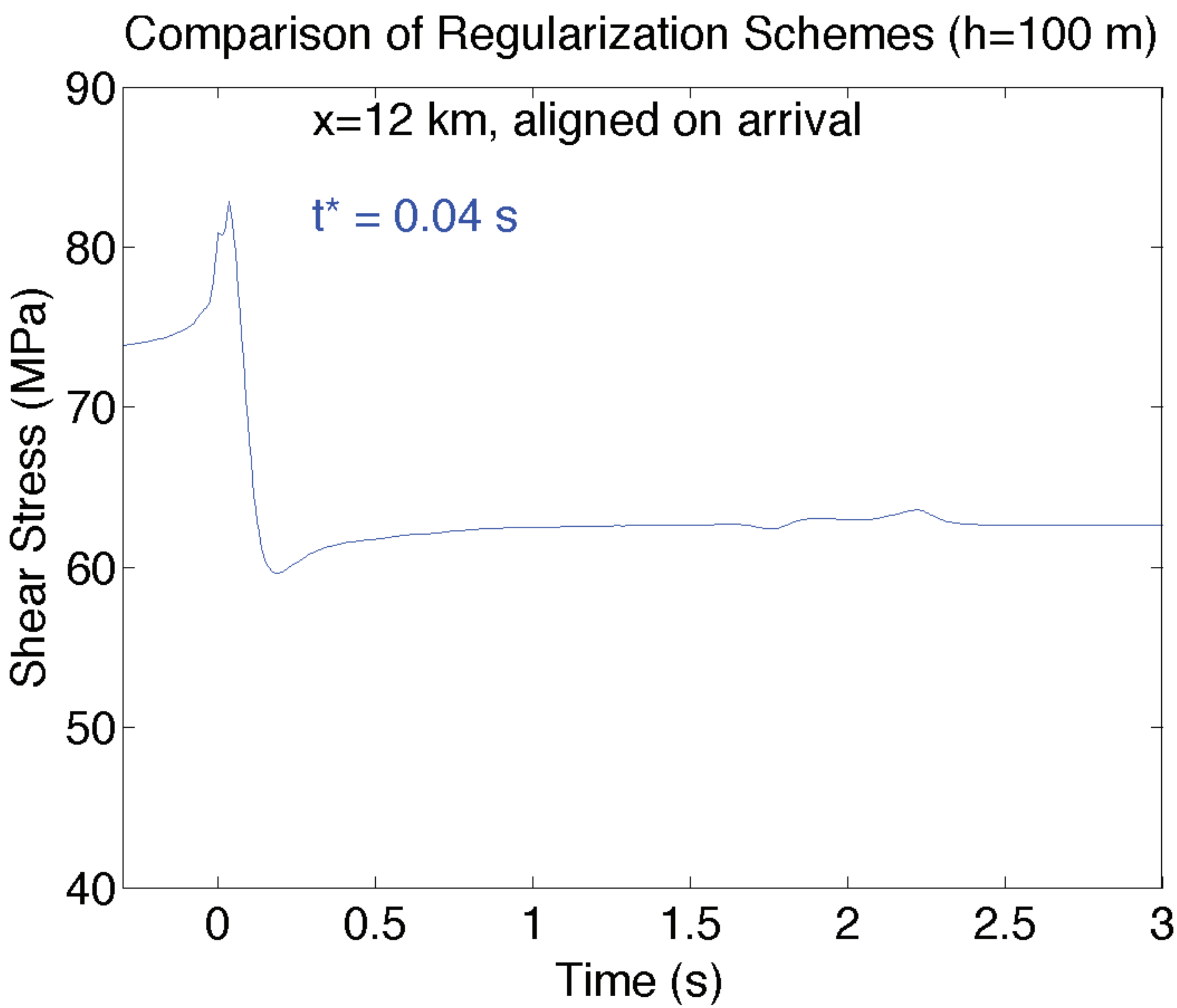
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.3 m
Slip Velocity (m/s)
Time (s)
0
5
10
15
20
0
0.5
1
1.5
2
2.5
3

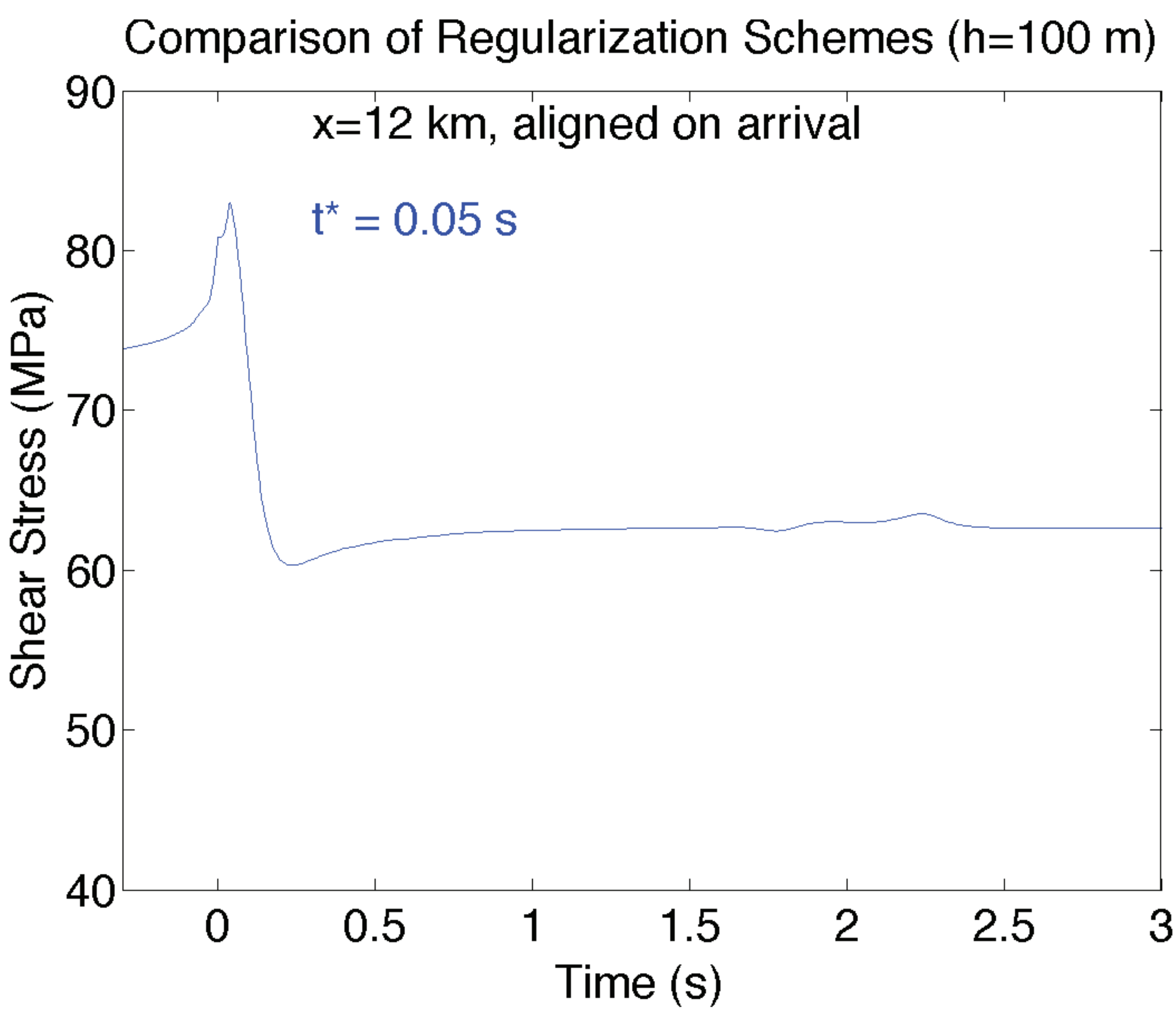

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
0.01 s <= t* <= 0.1 s
V* = 1 m/s and 0.1 m <= L <= 0.3 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

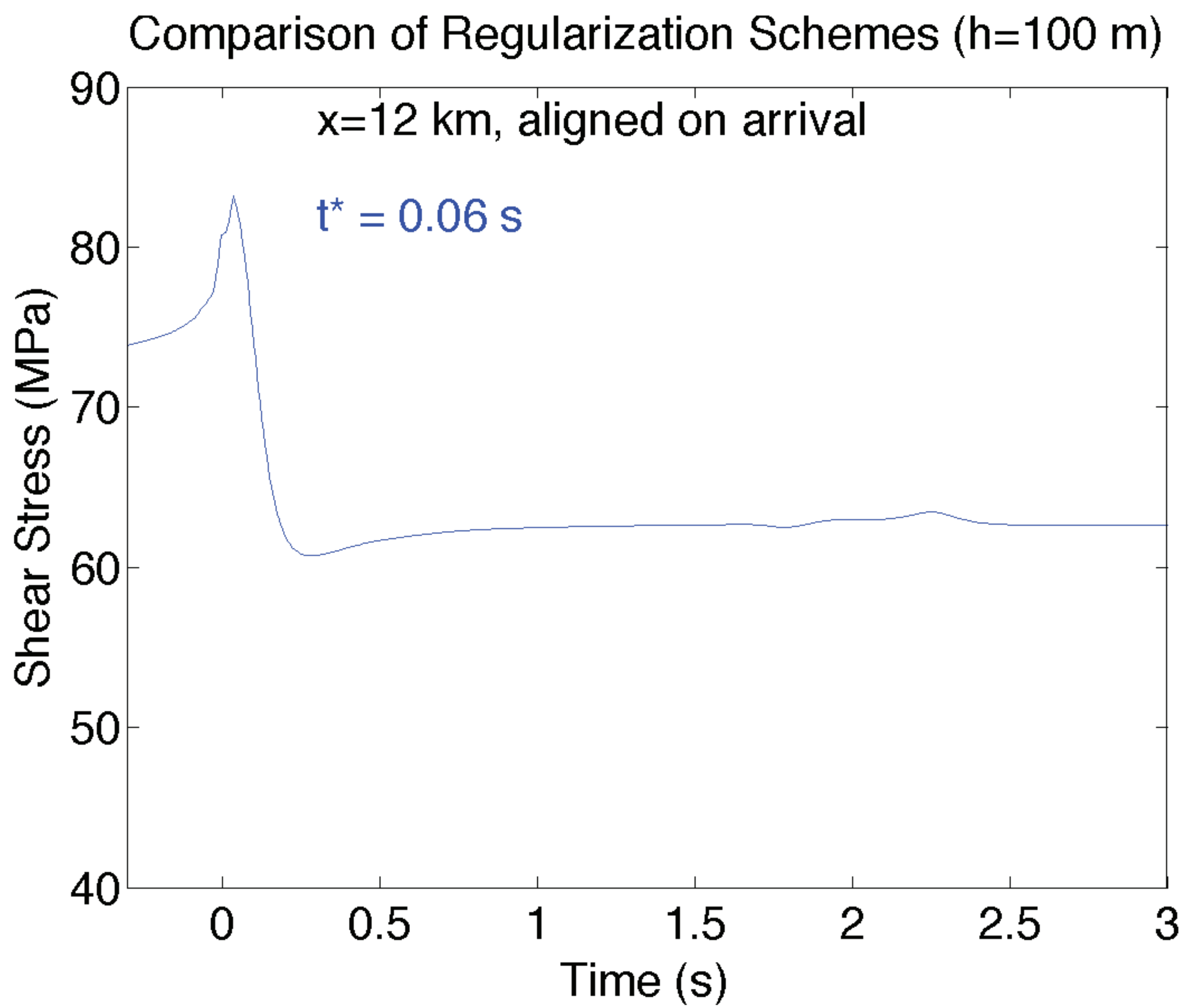

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.01 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

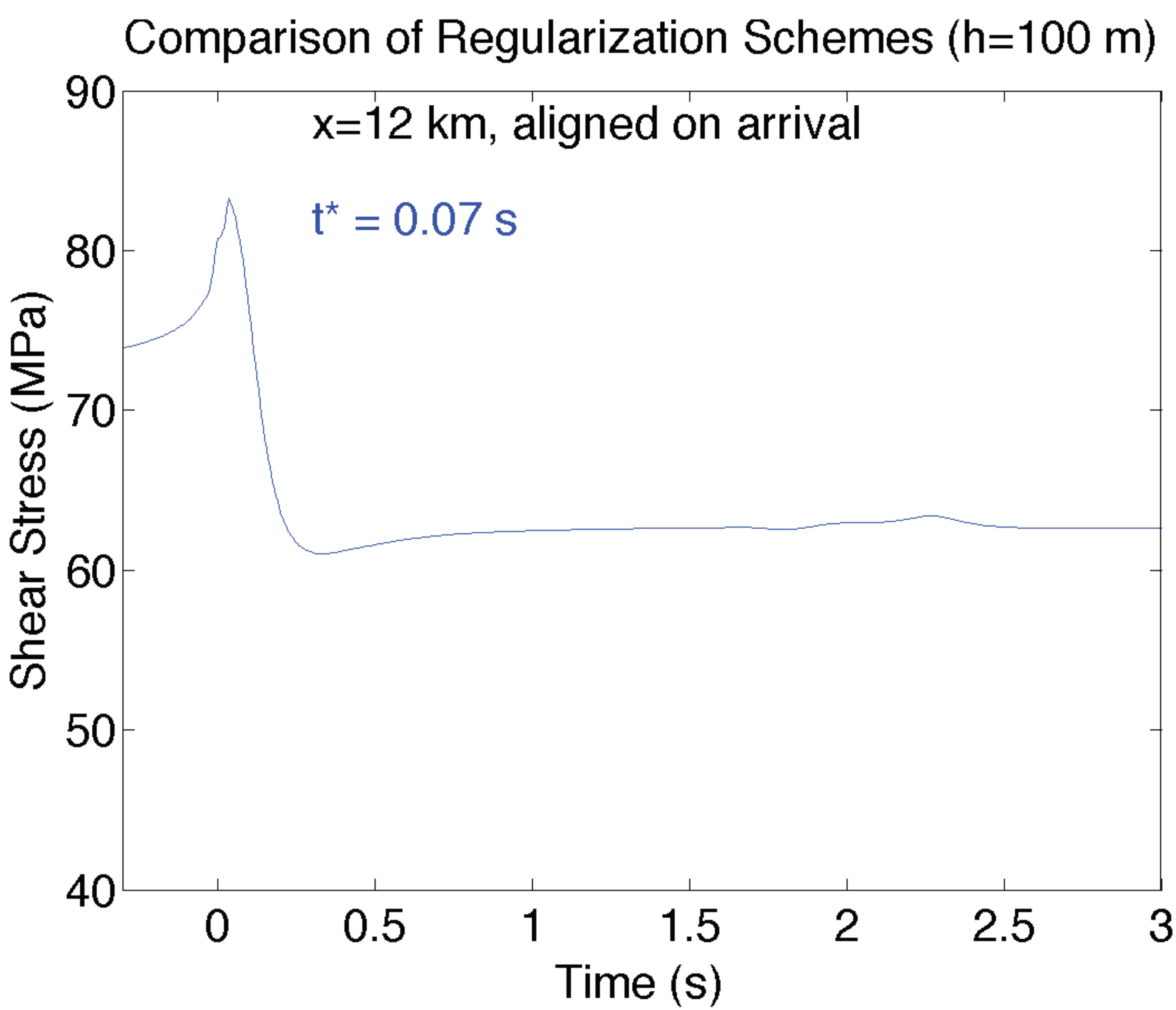

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.02 s
Shear Stress (MPa)
Time (s)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3

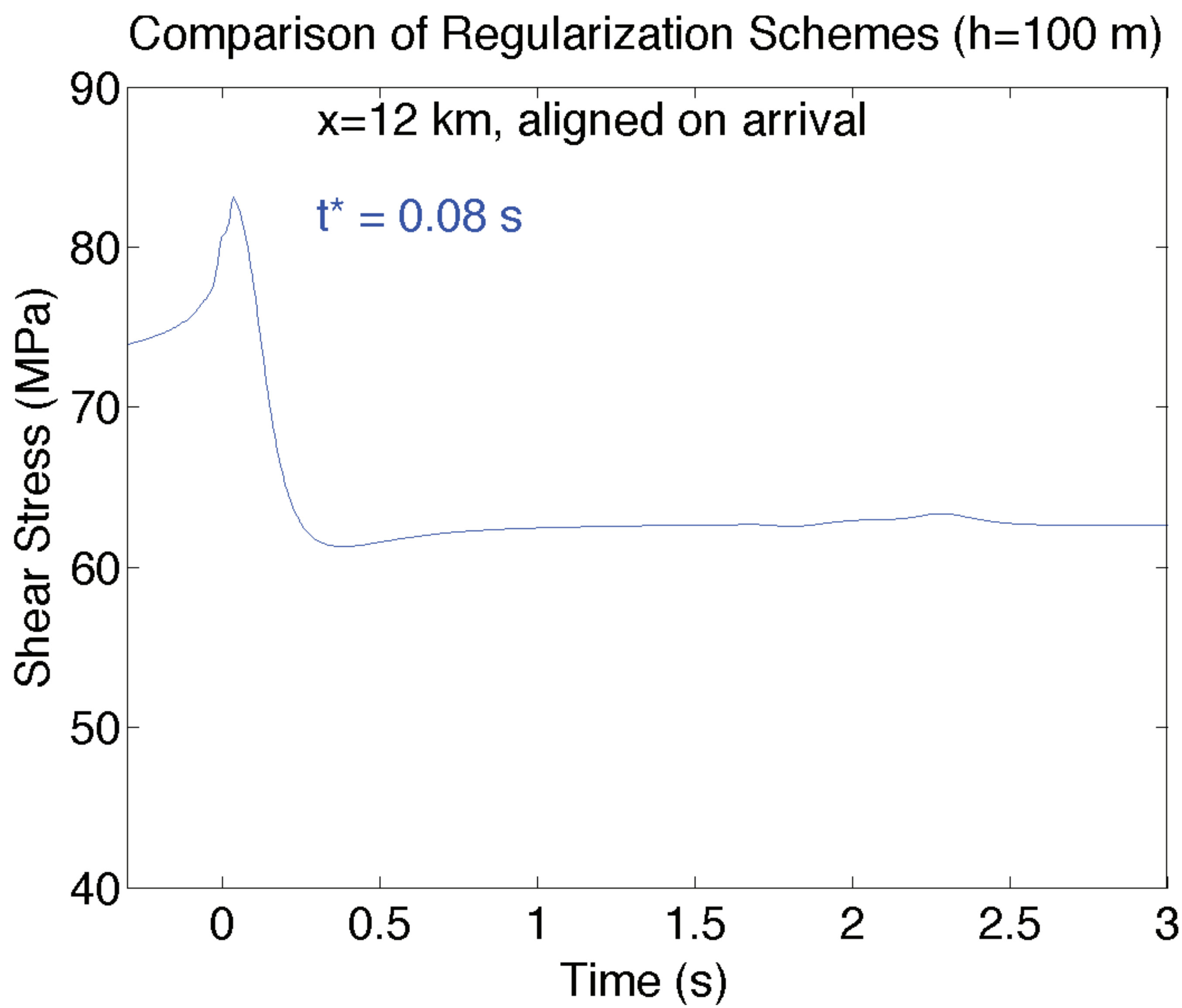

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.03 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

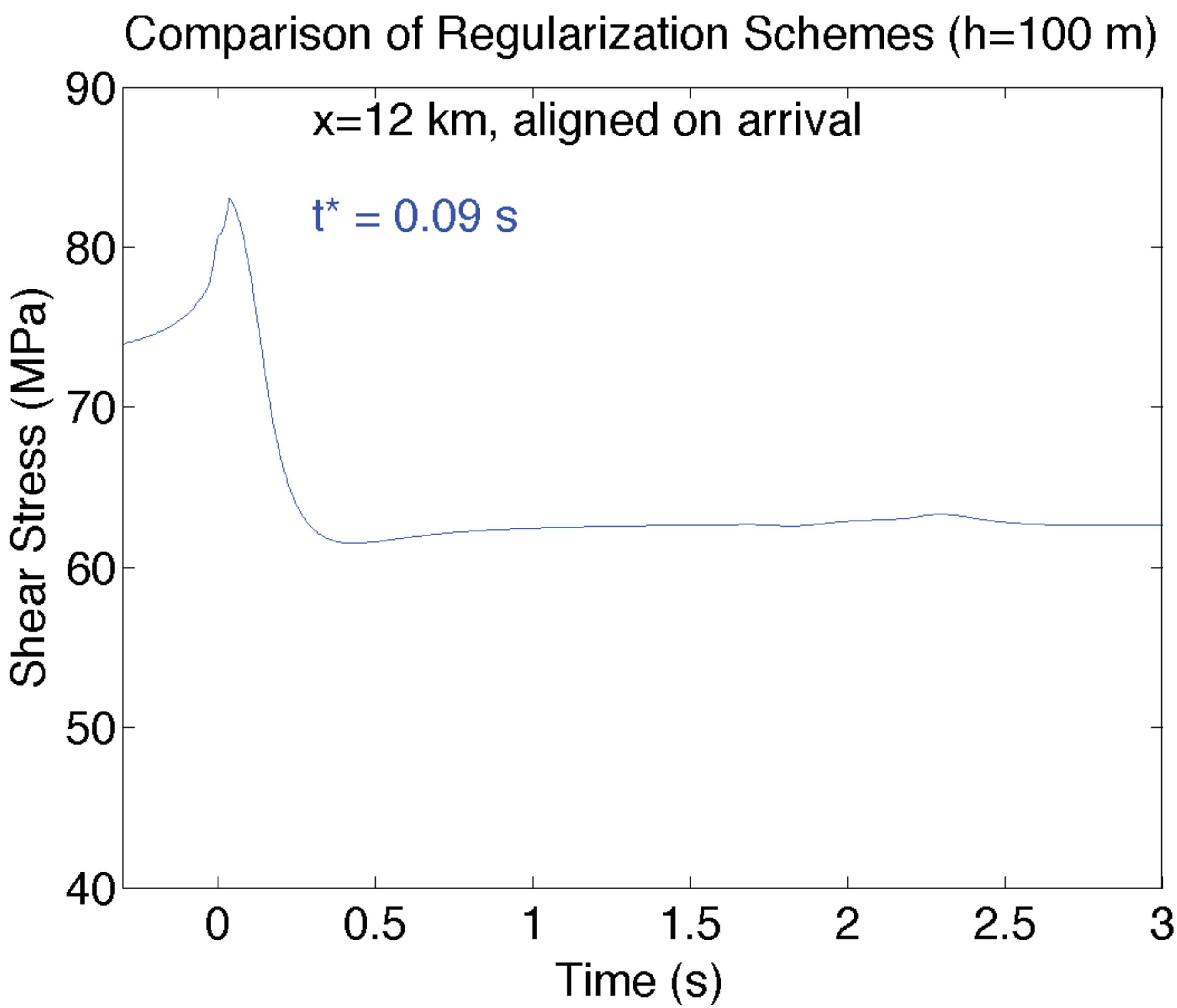

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.04 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

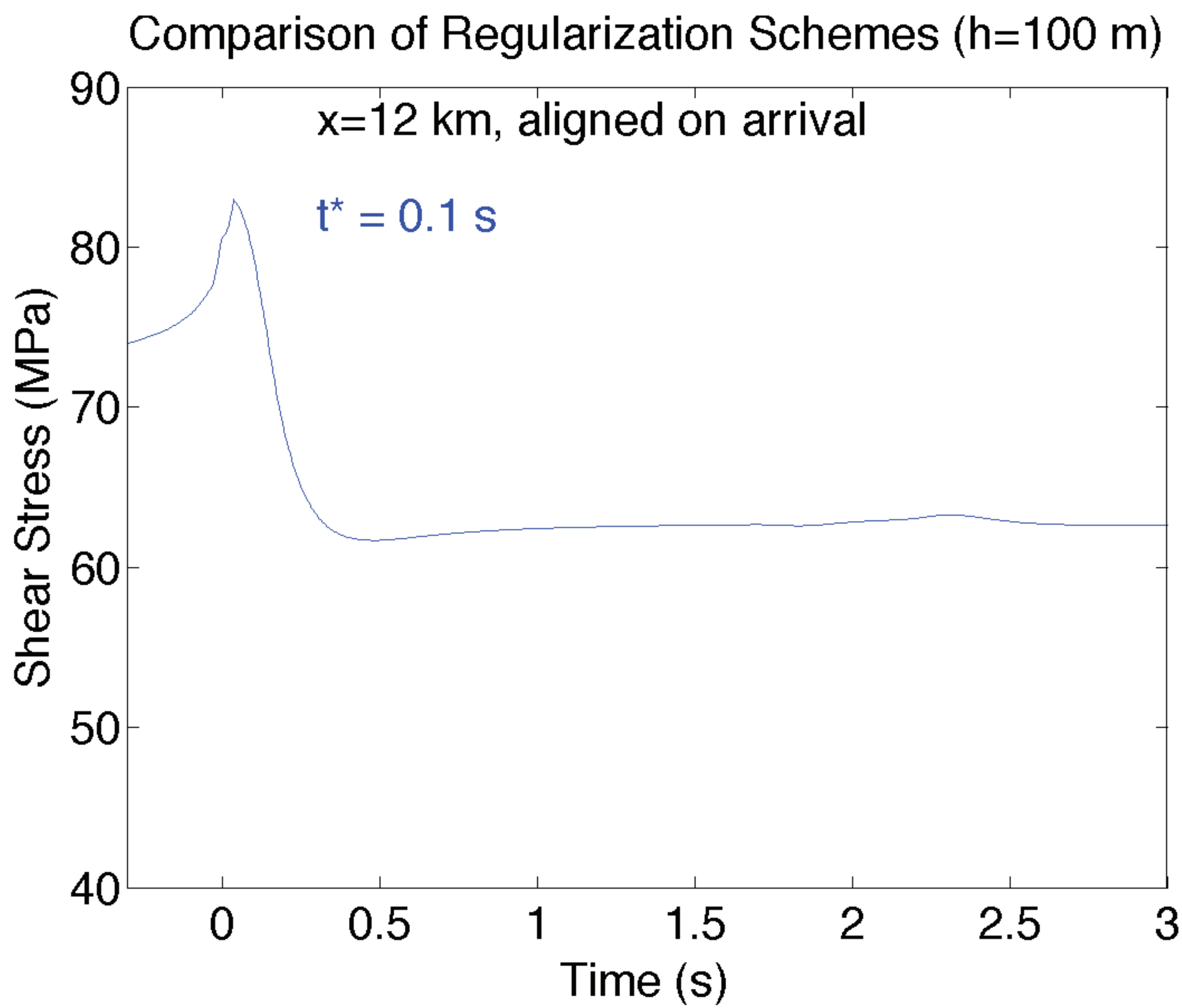

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.05 s
Shear Stress (MPa)
Time (s)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3

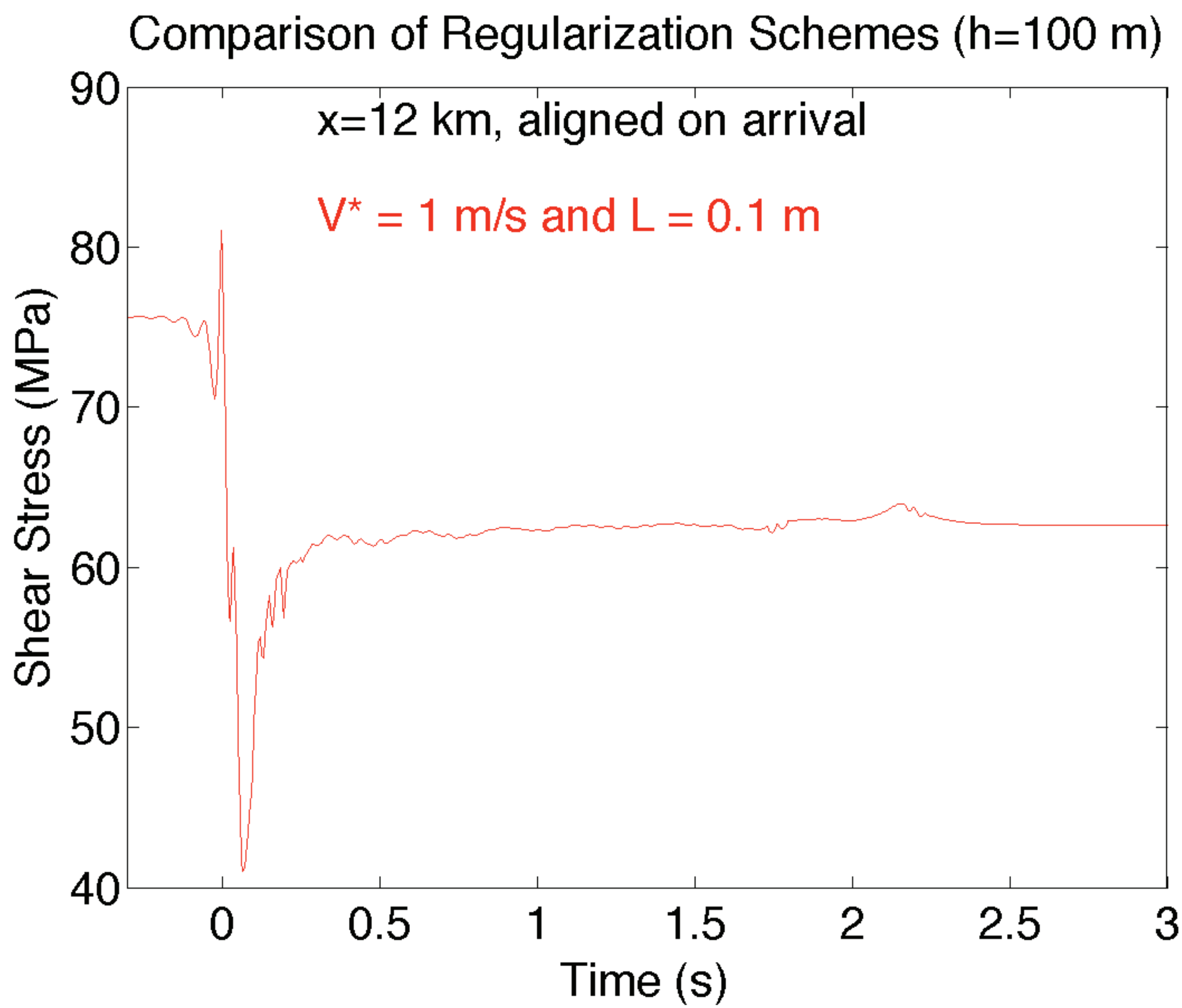

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.06 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

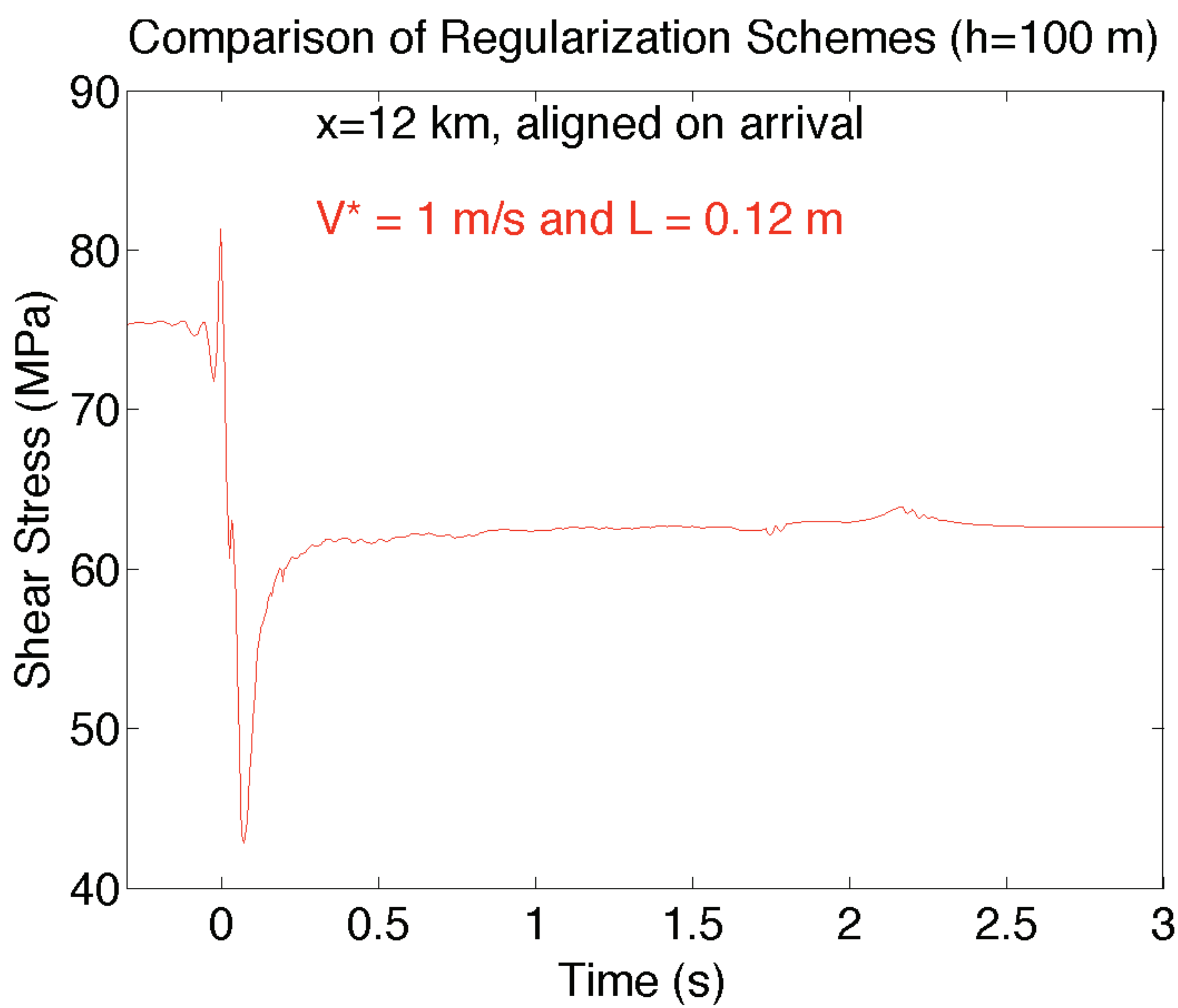

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.07 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

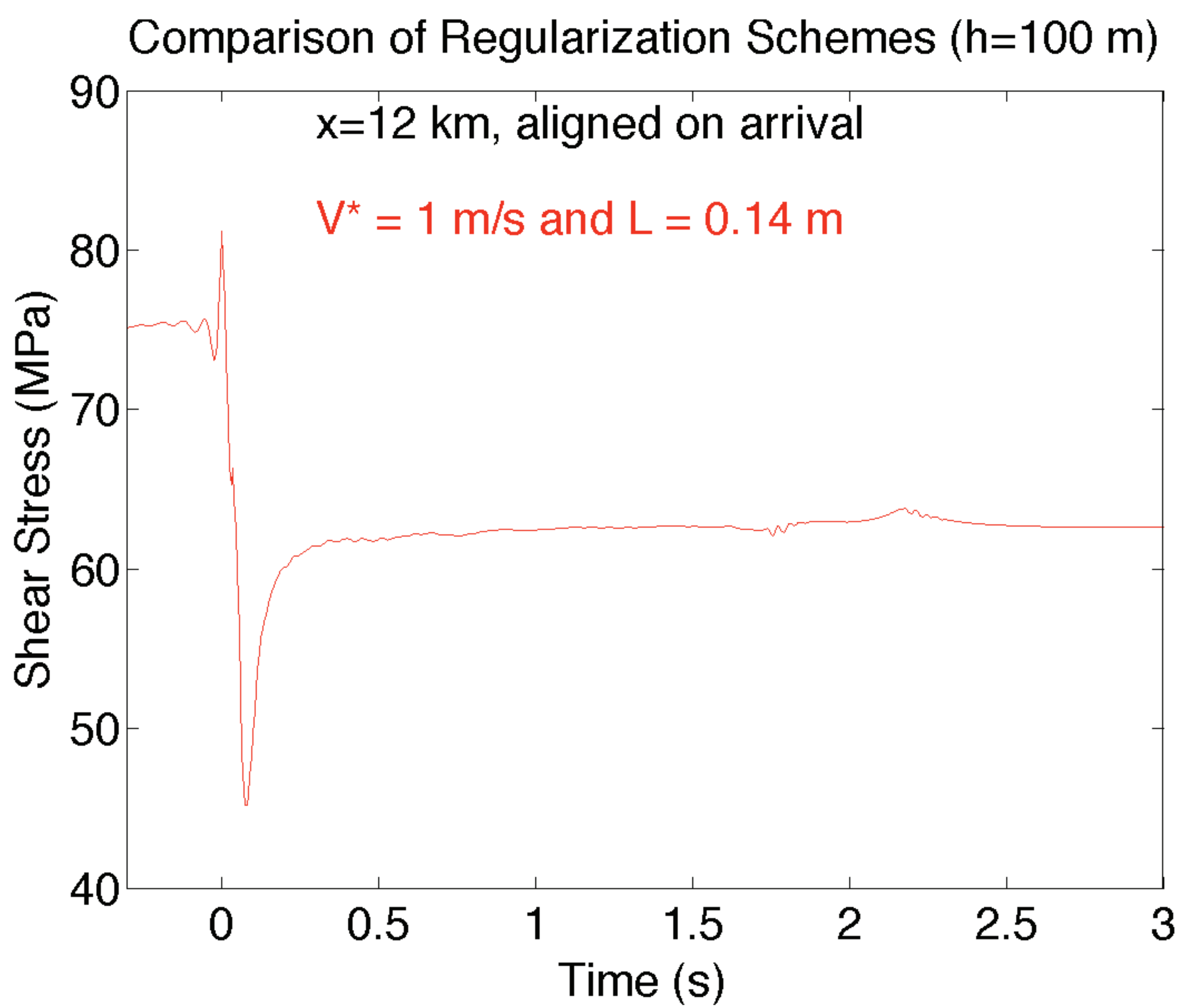

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.08 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

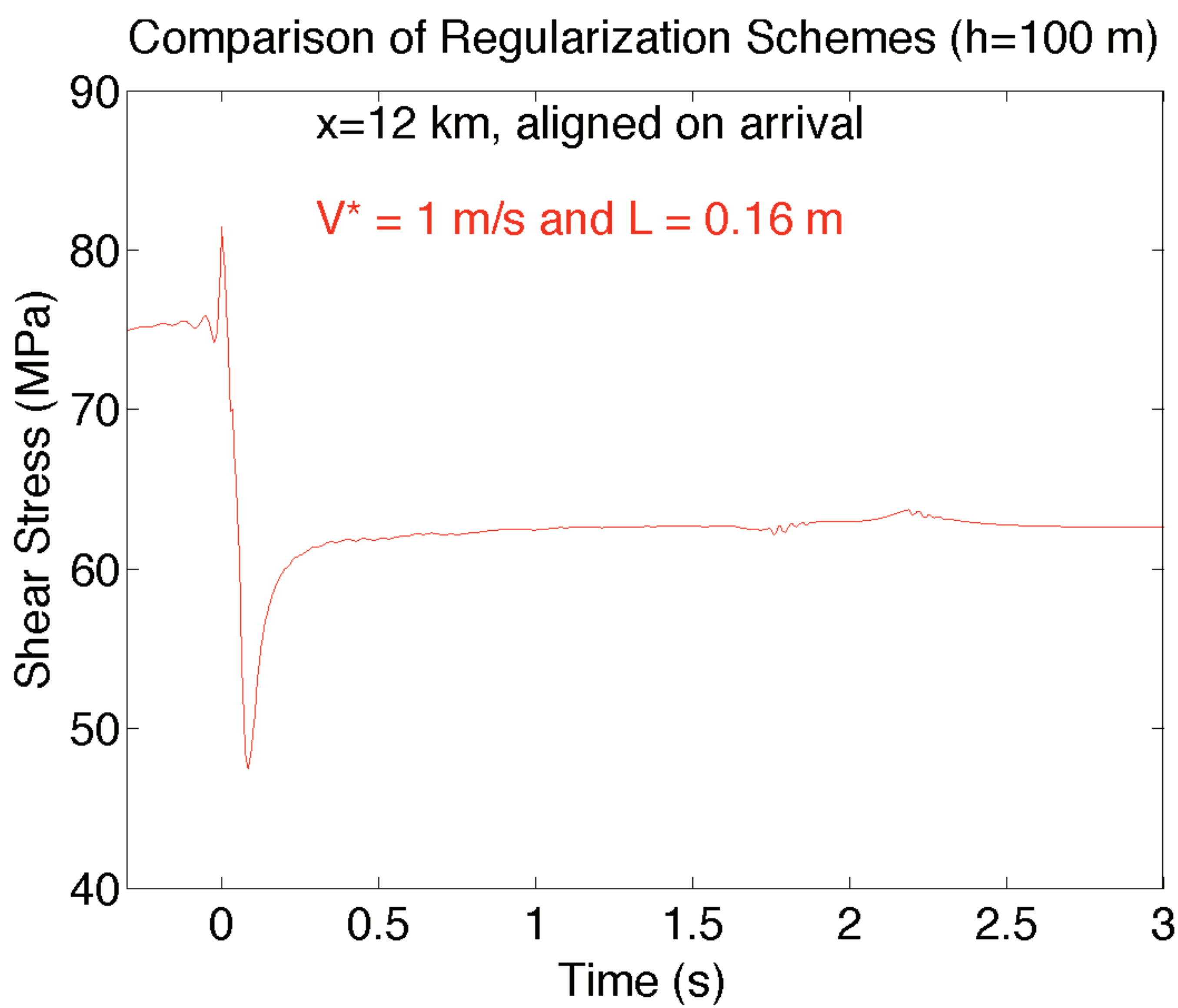

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.09 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

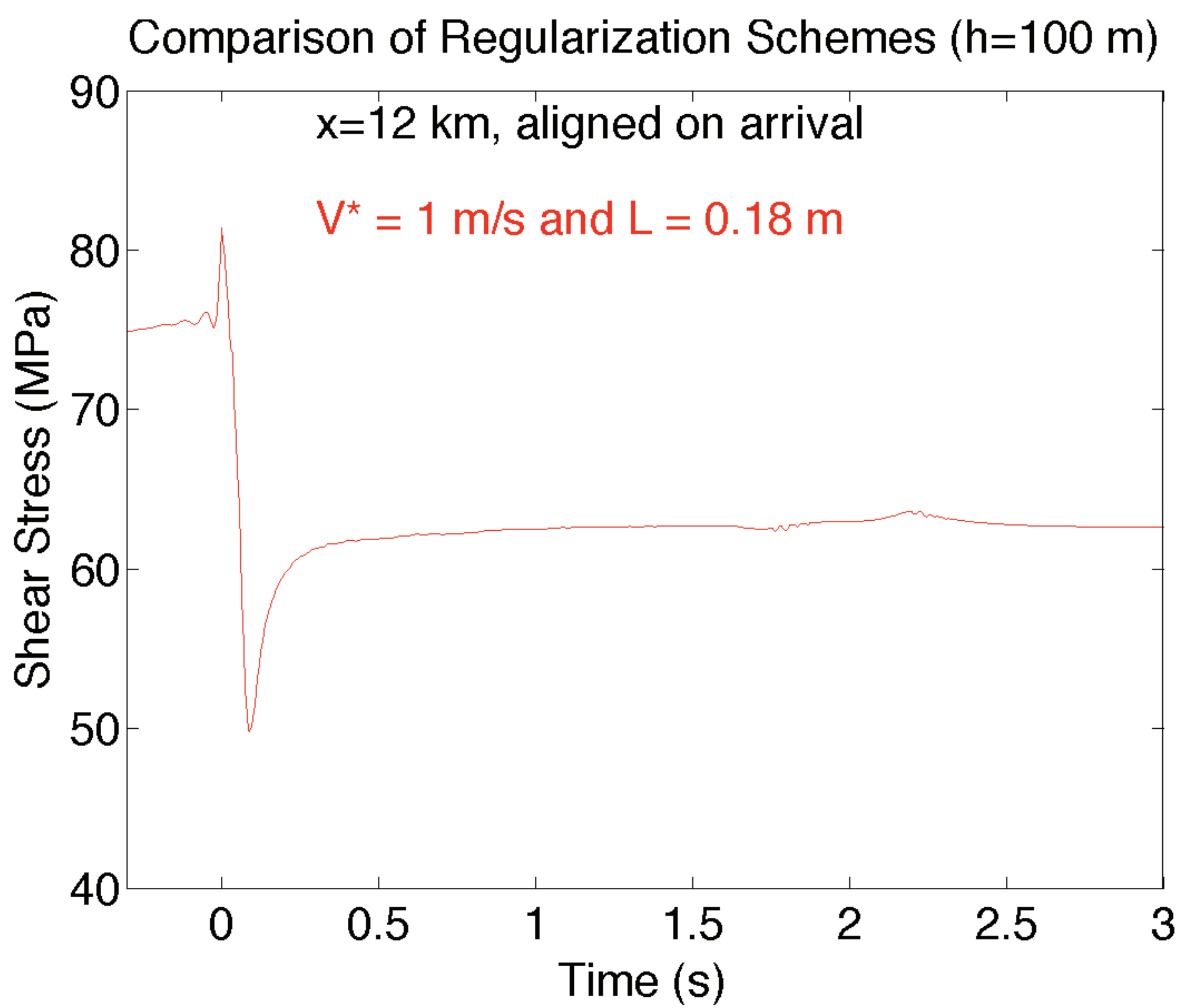

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.1 s
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

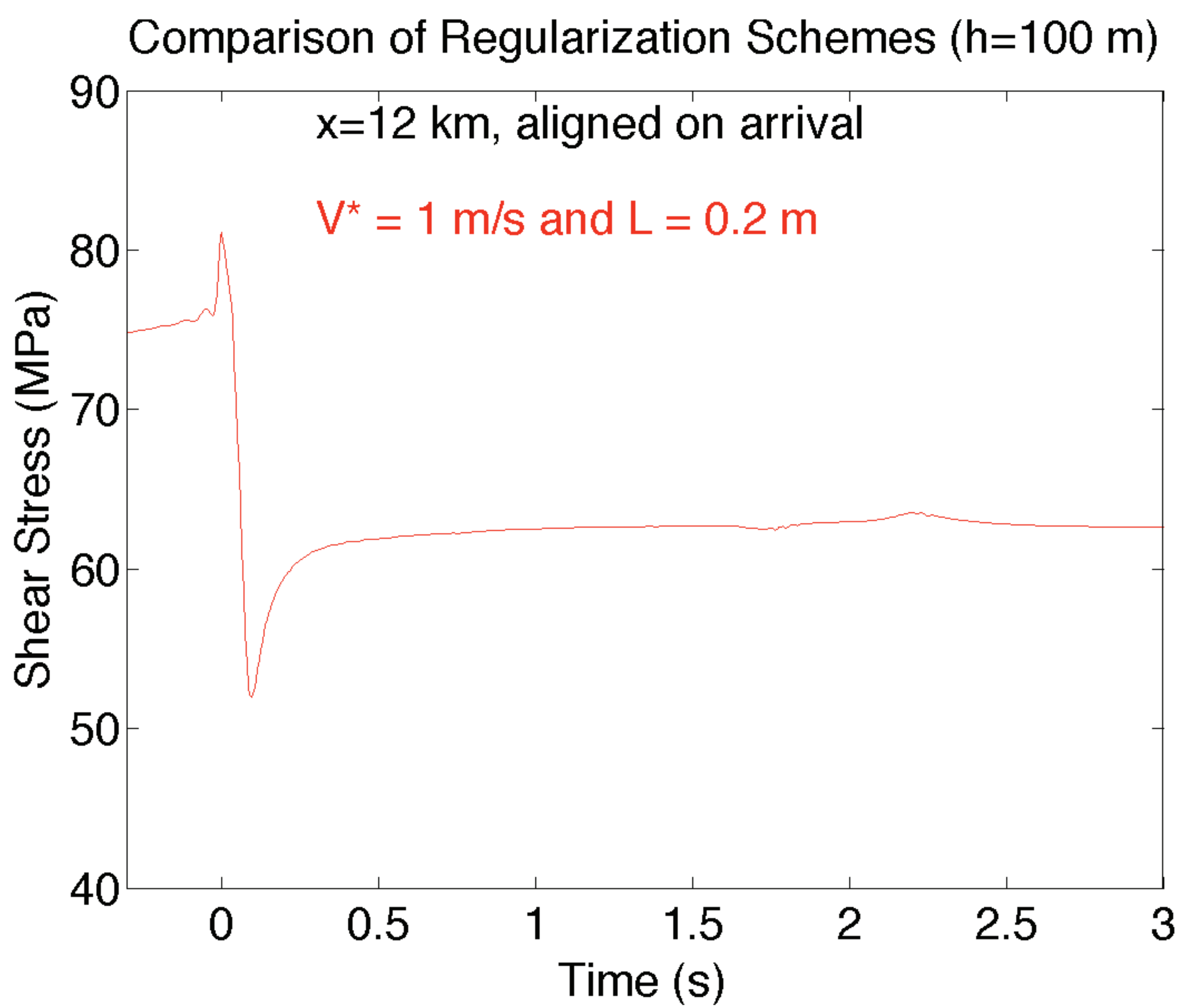

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.1 m
Shear Stress (MPa)
Time (s)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.12 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

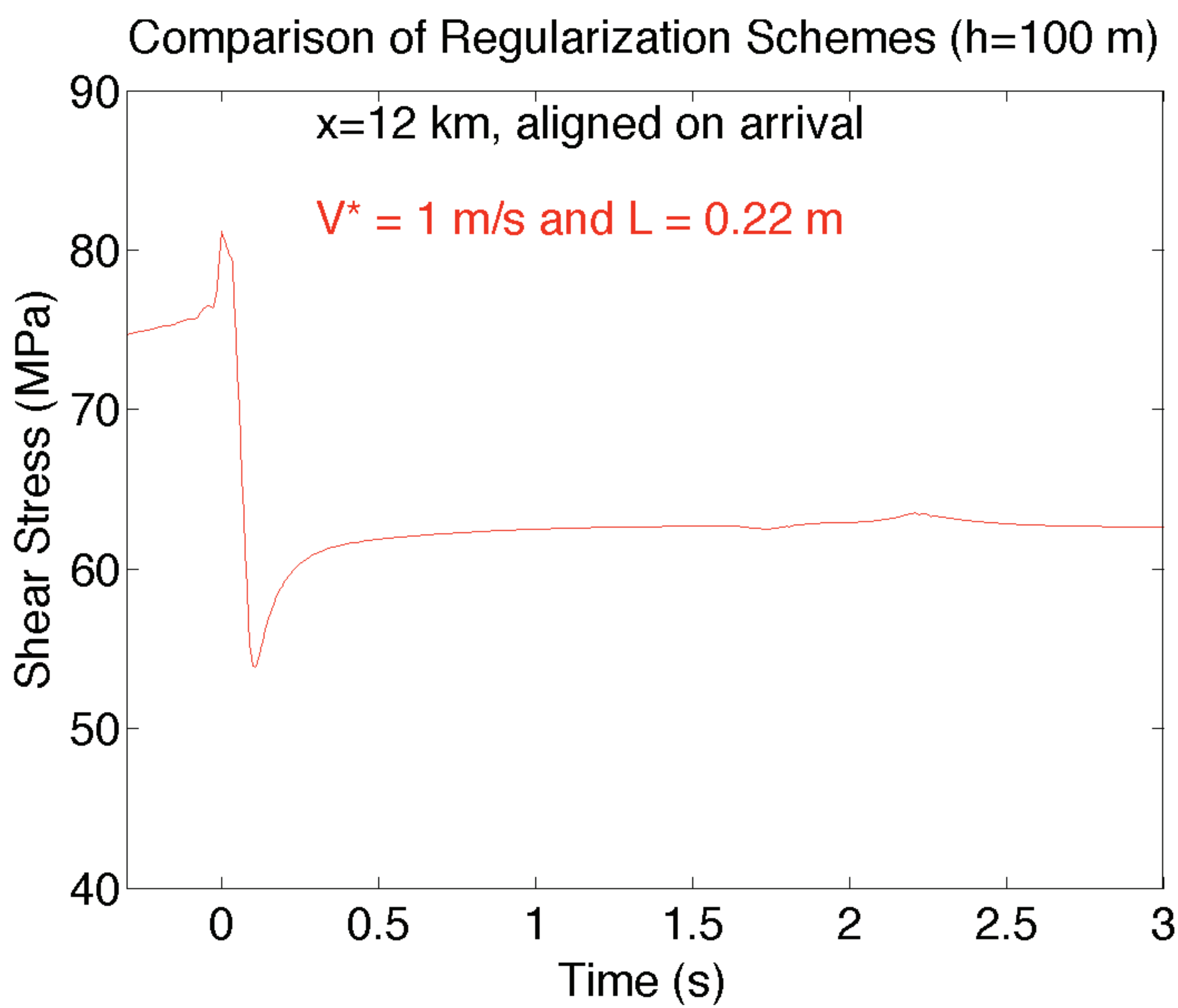

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.14 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

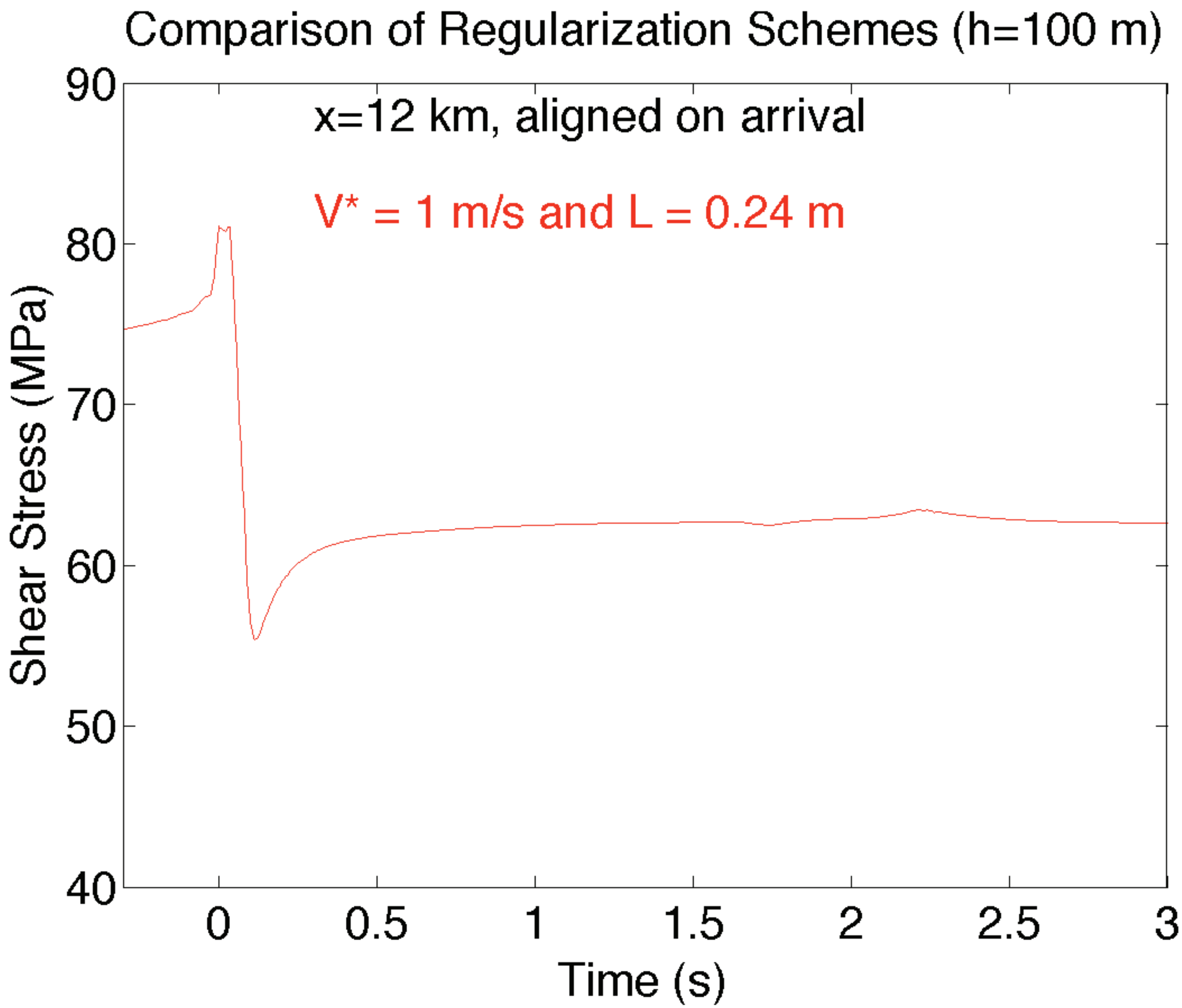

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.16 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

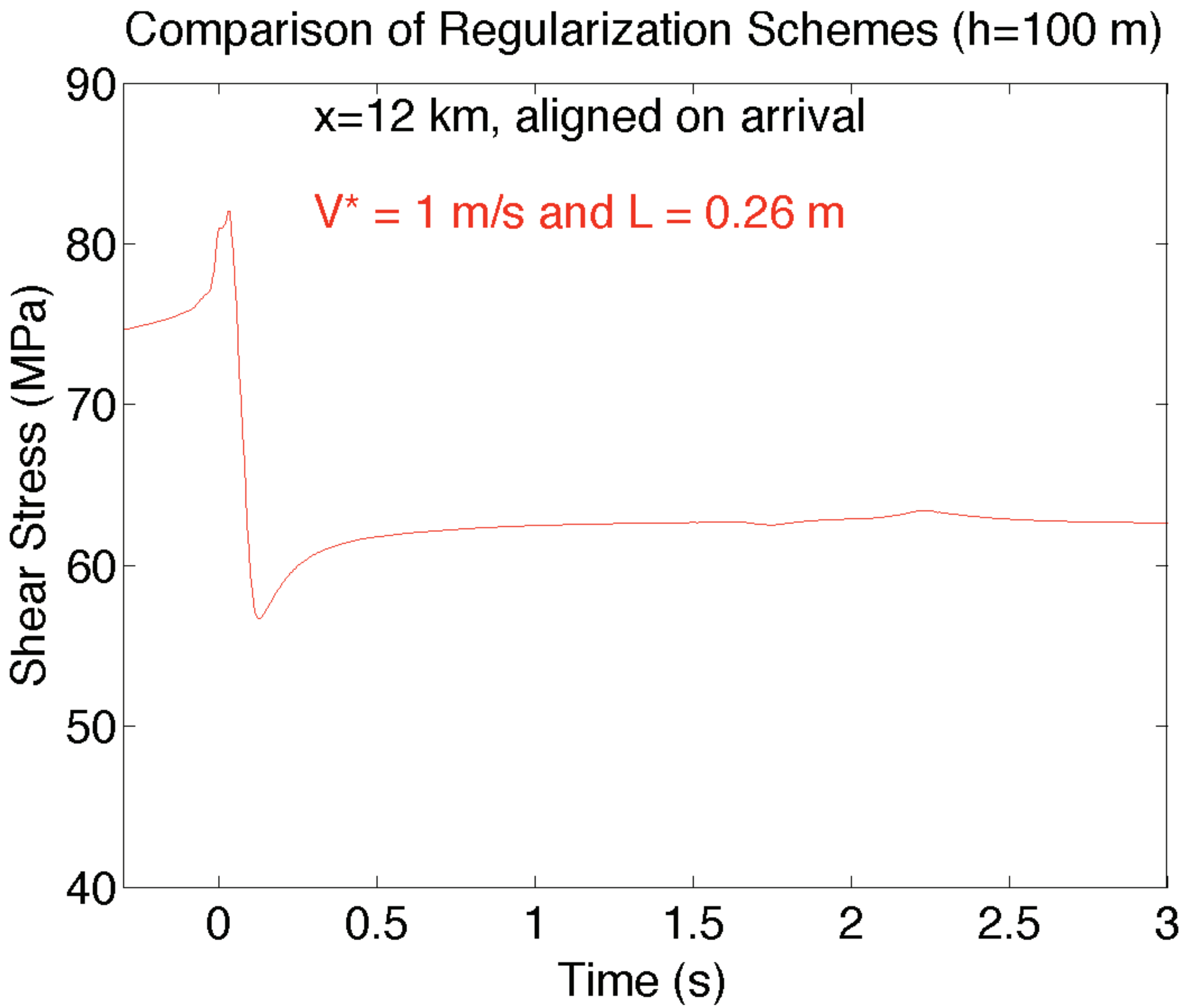

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.18 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)



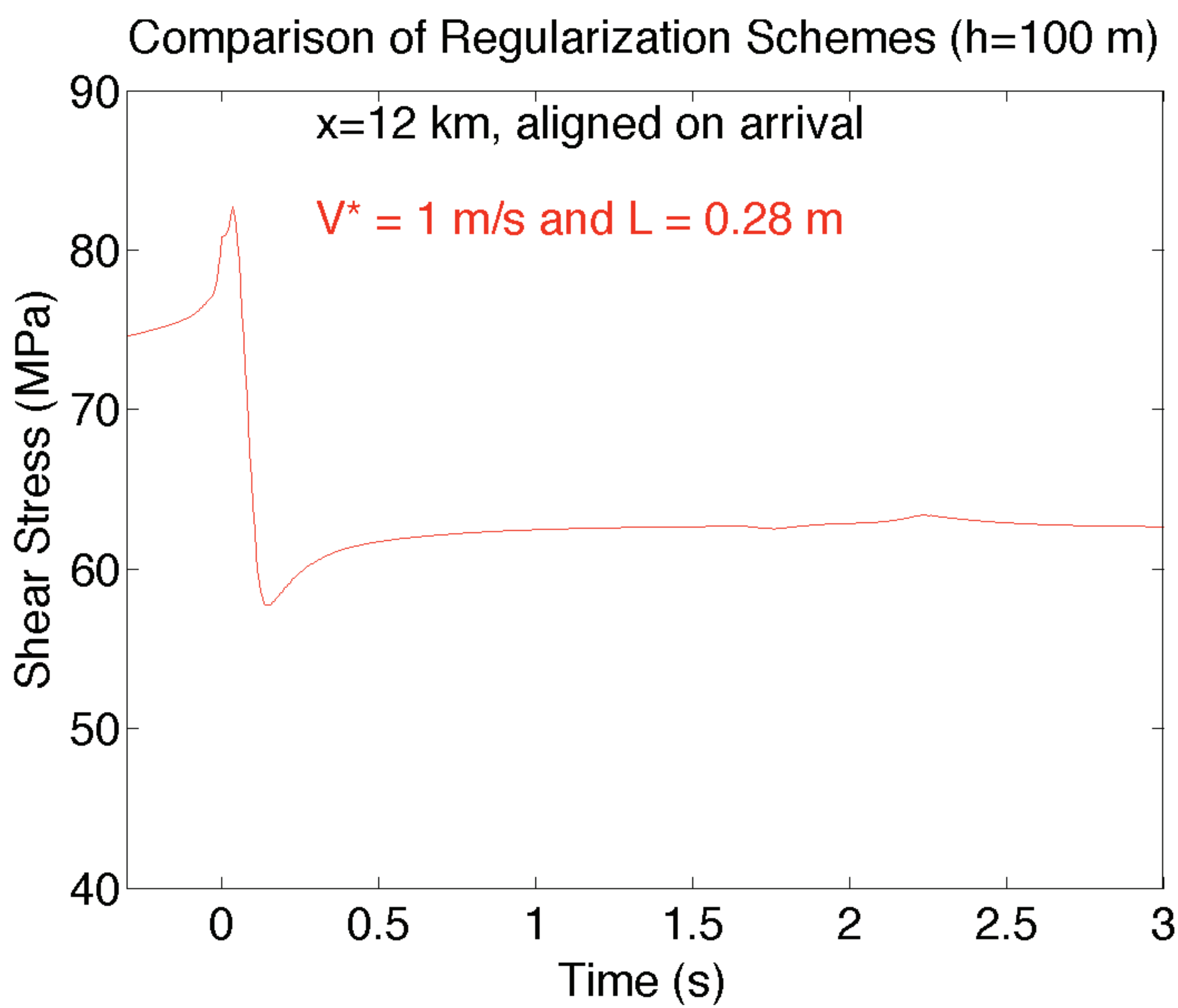
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.2 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.22 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

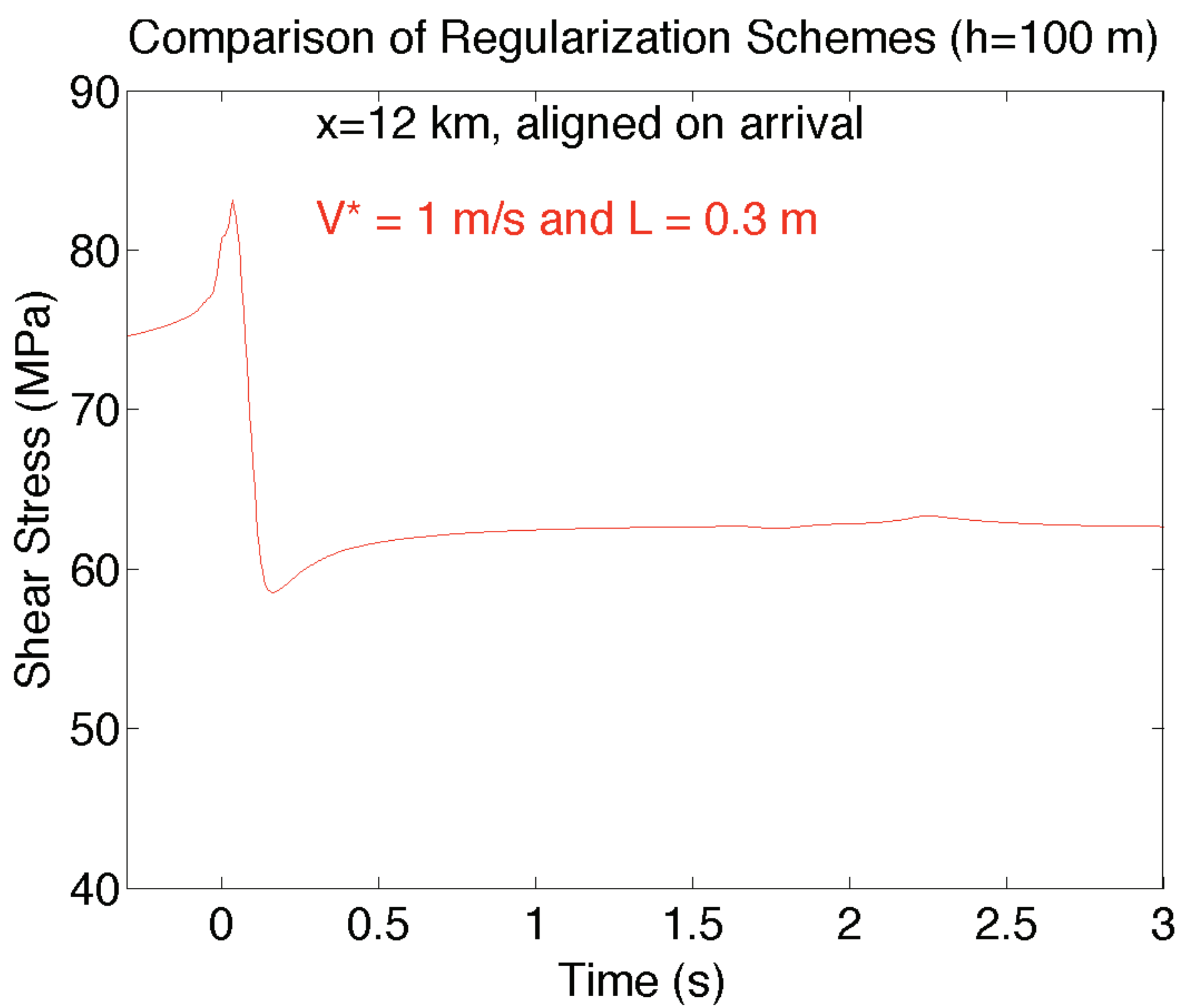

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.24 m
Shear Stress (MPa)
Time (s)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3

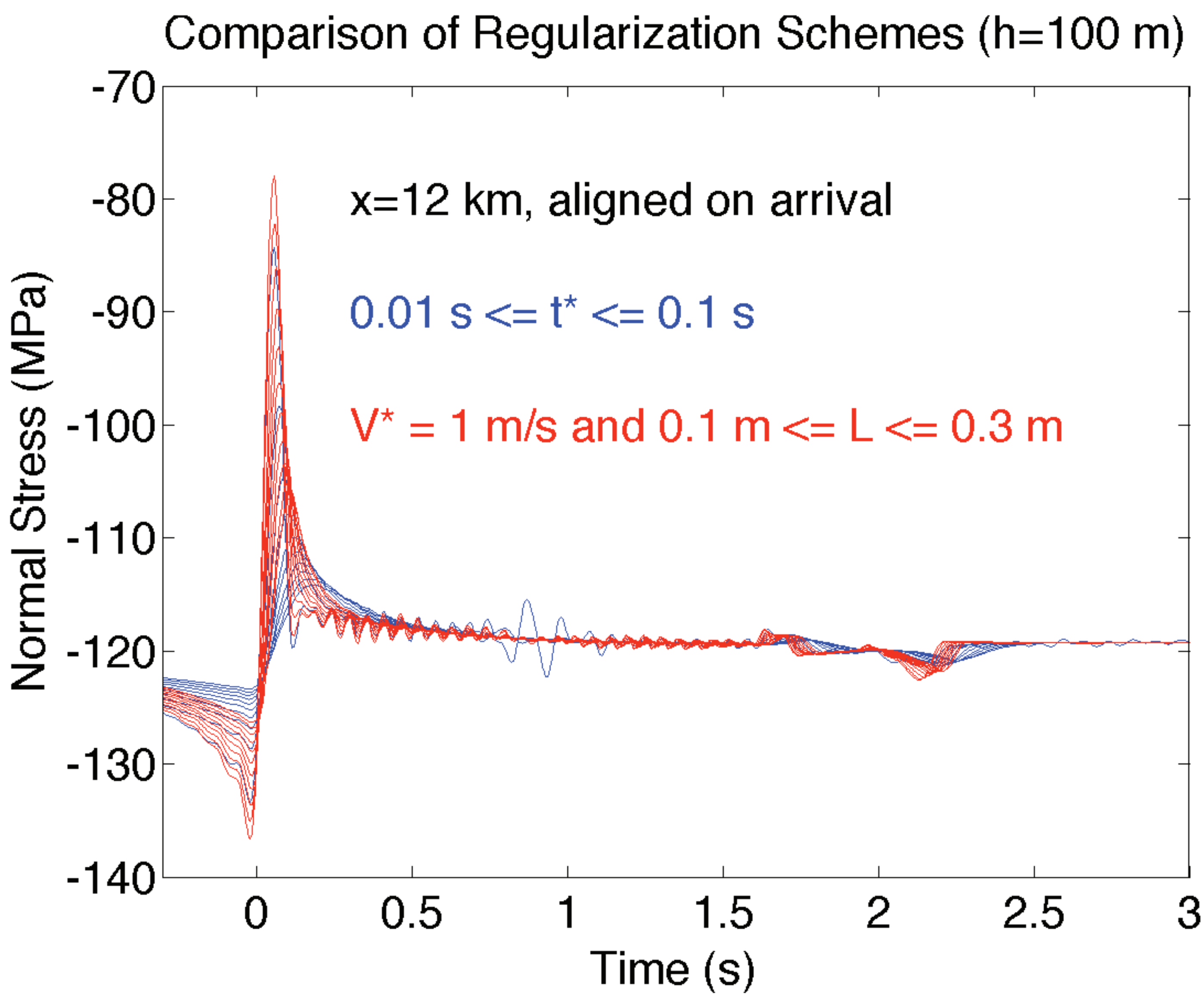


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.26 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

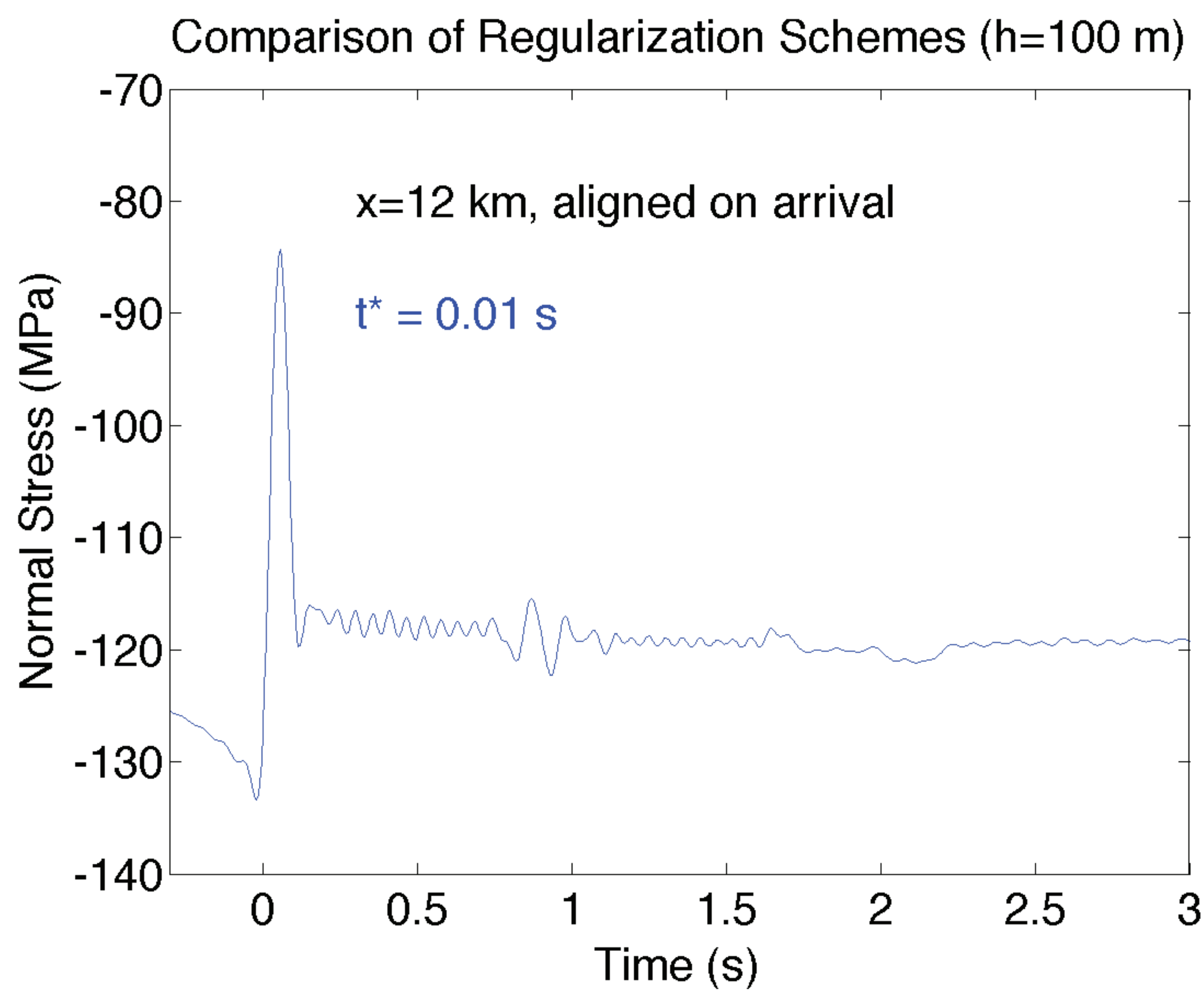


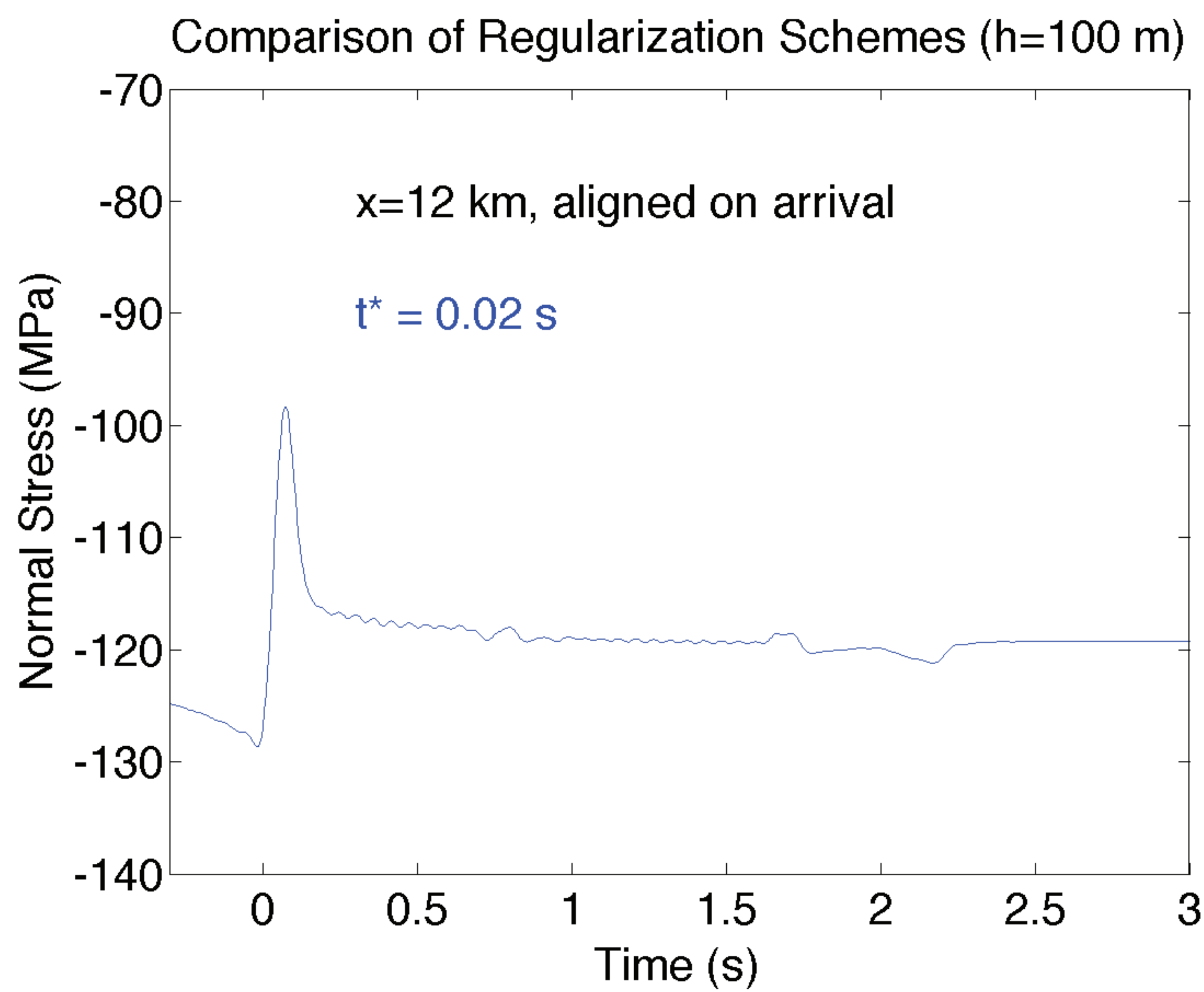
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.28 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

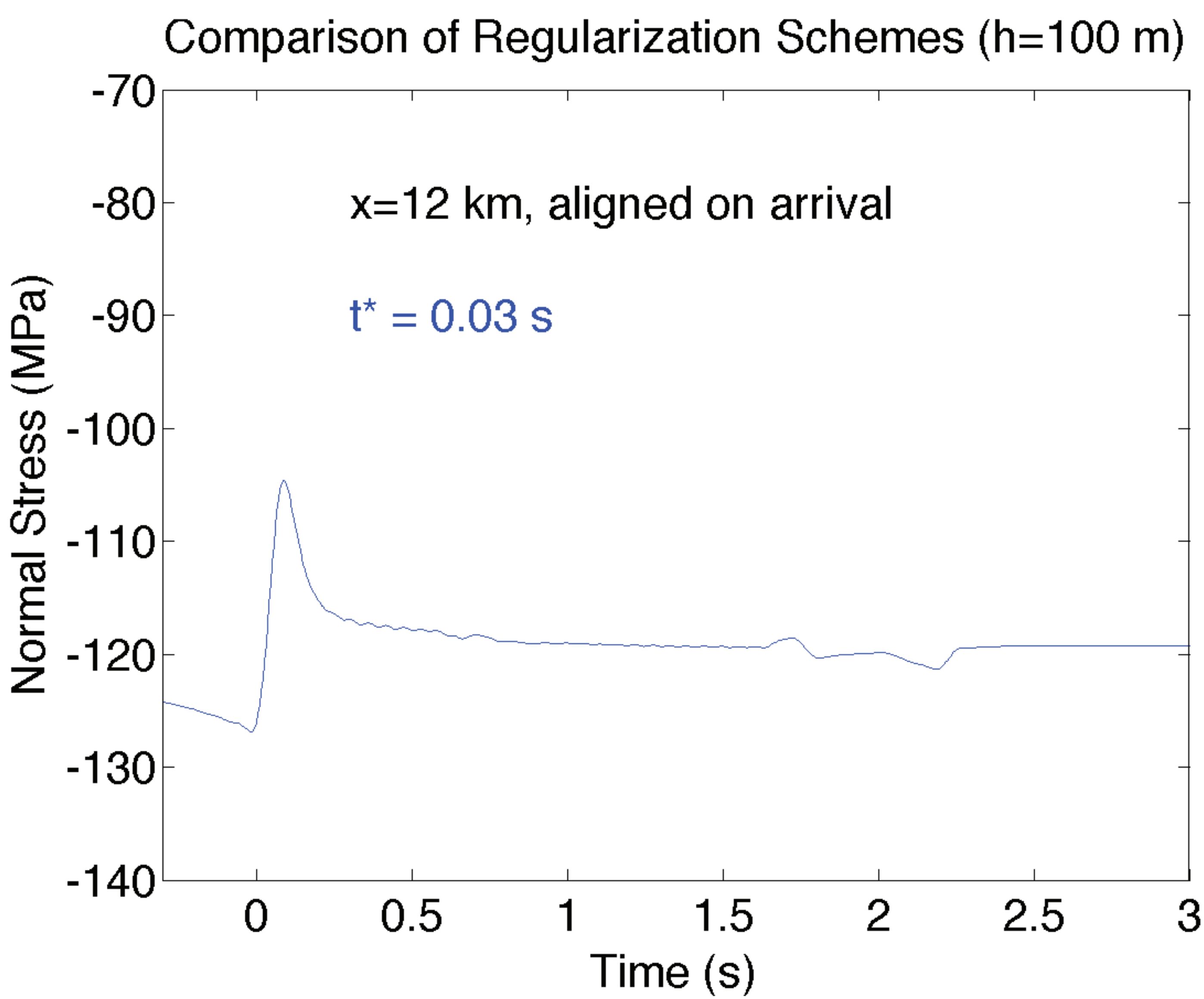

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.3 m
Shear Stress (MPa)
90
80
70
60
50
40
0
0.5
1
1.5
2
2.5
3
Time (s)

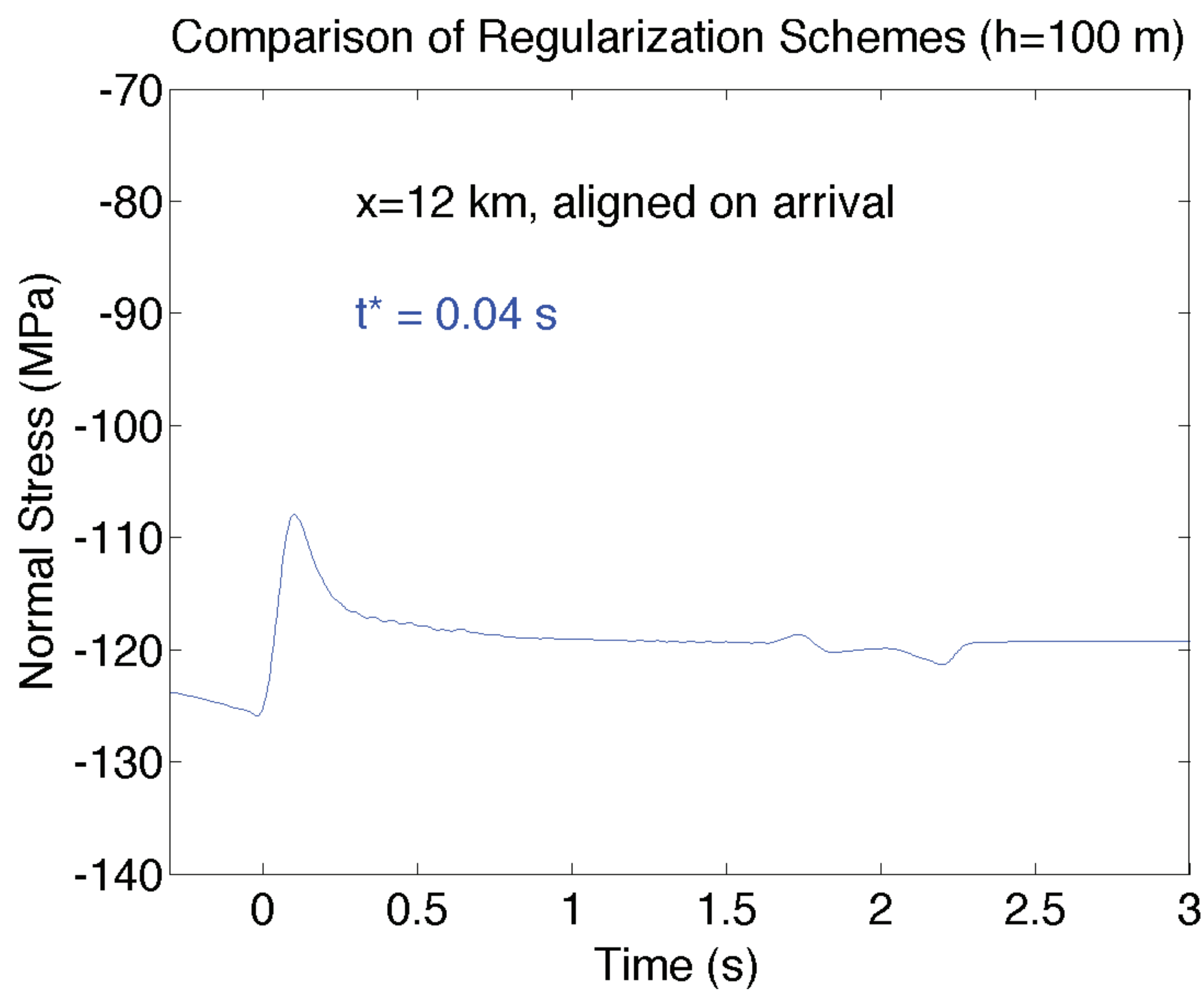


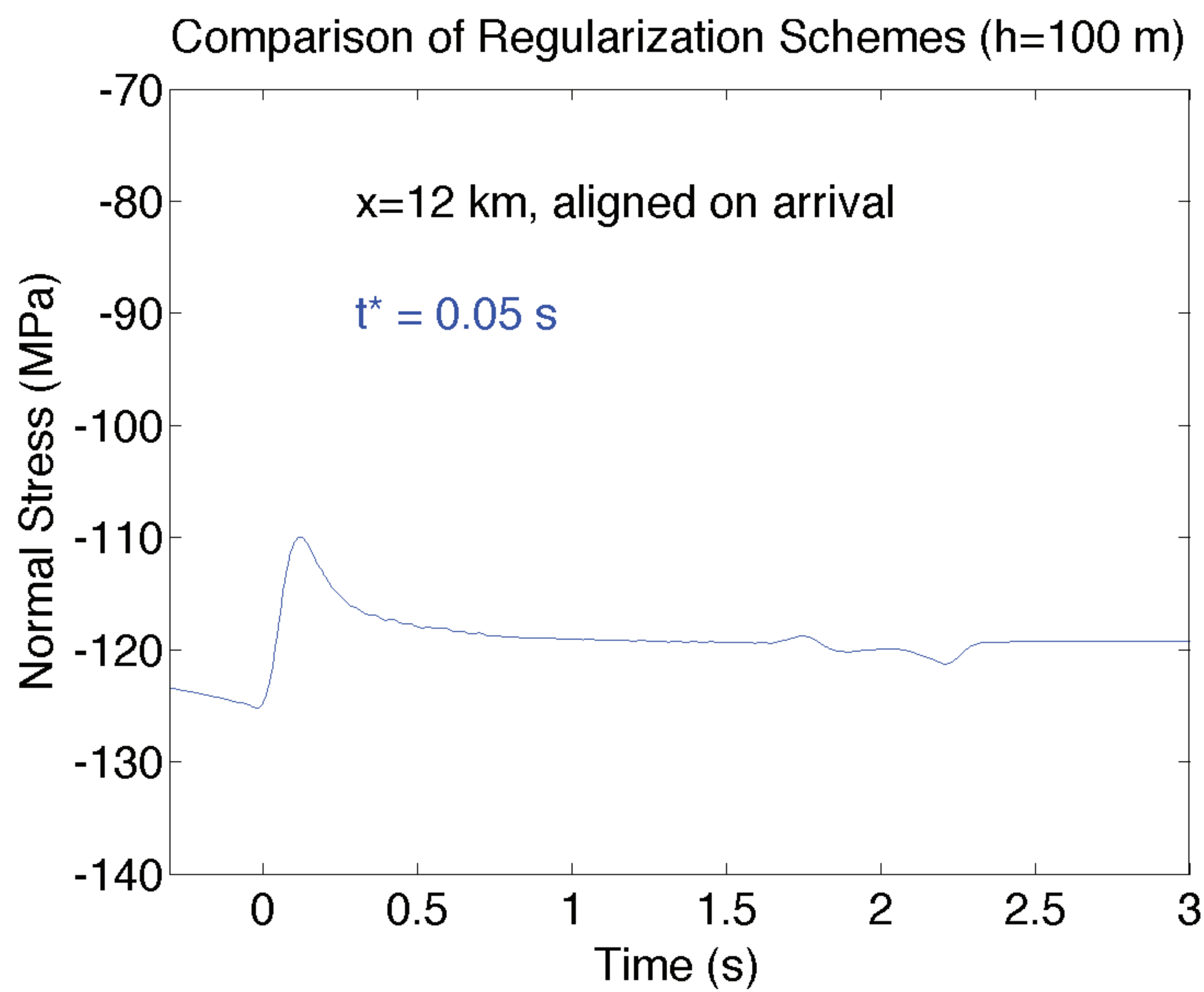
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
0.01 s <= t* <= 0.1 s
V* = 1 m/s and 0.1 m <= L <= 0.3 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

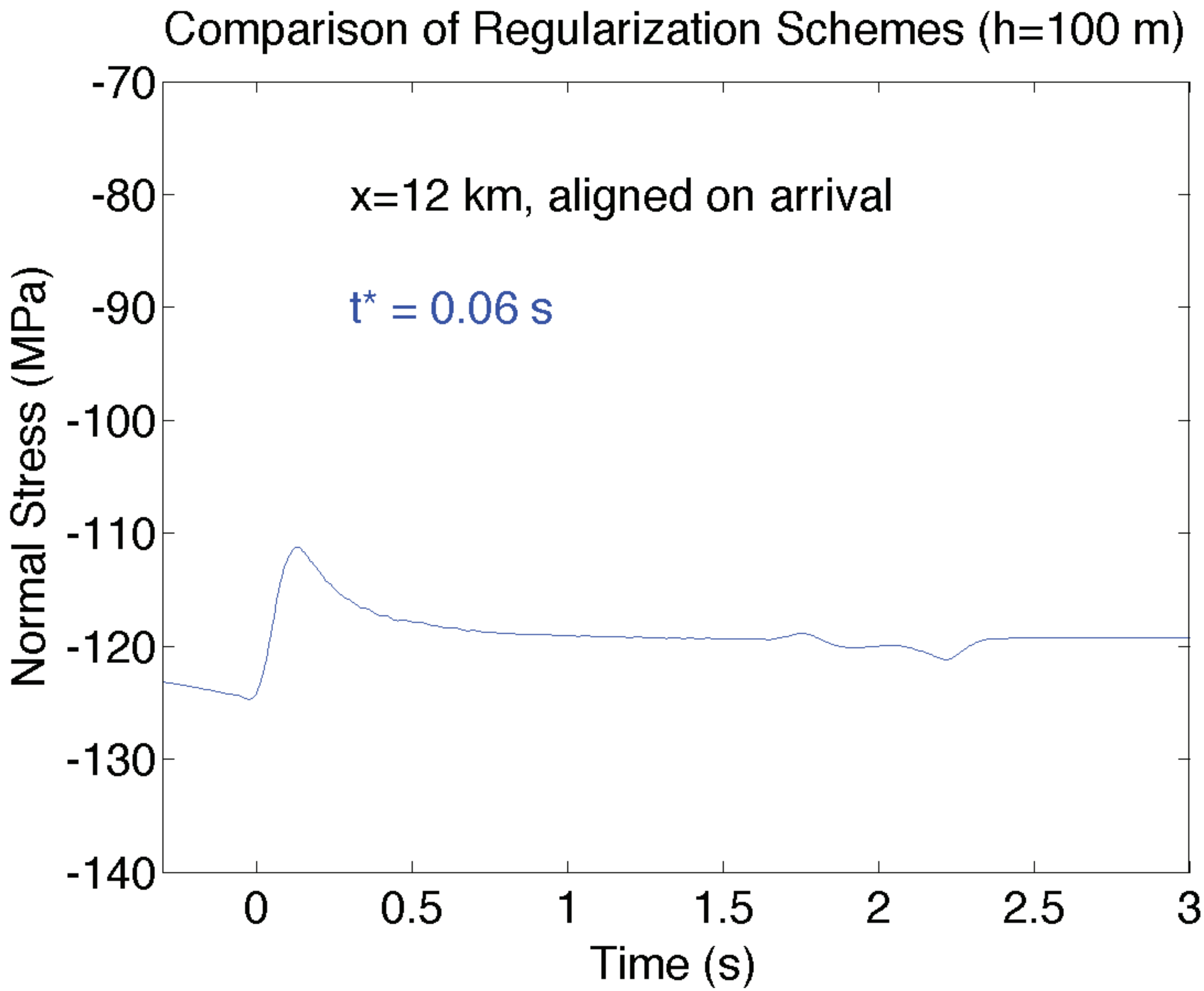

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.01 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

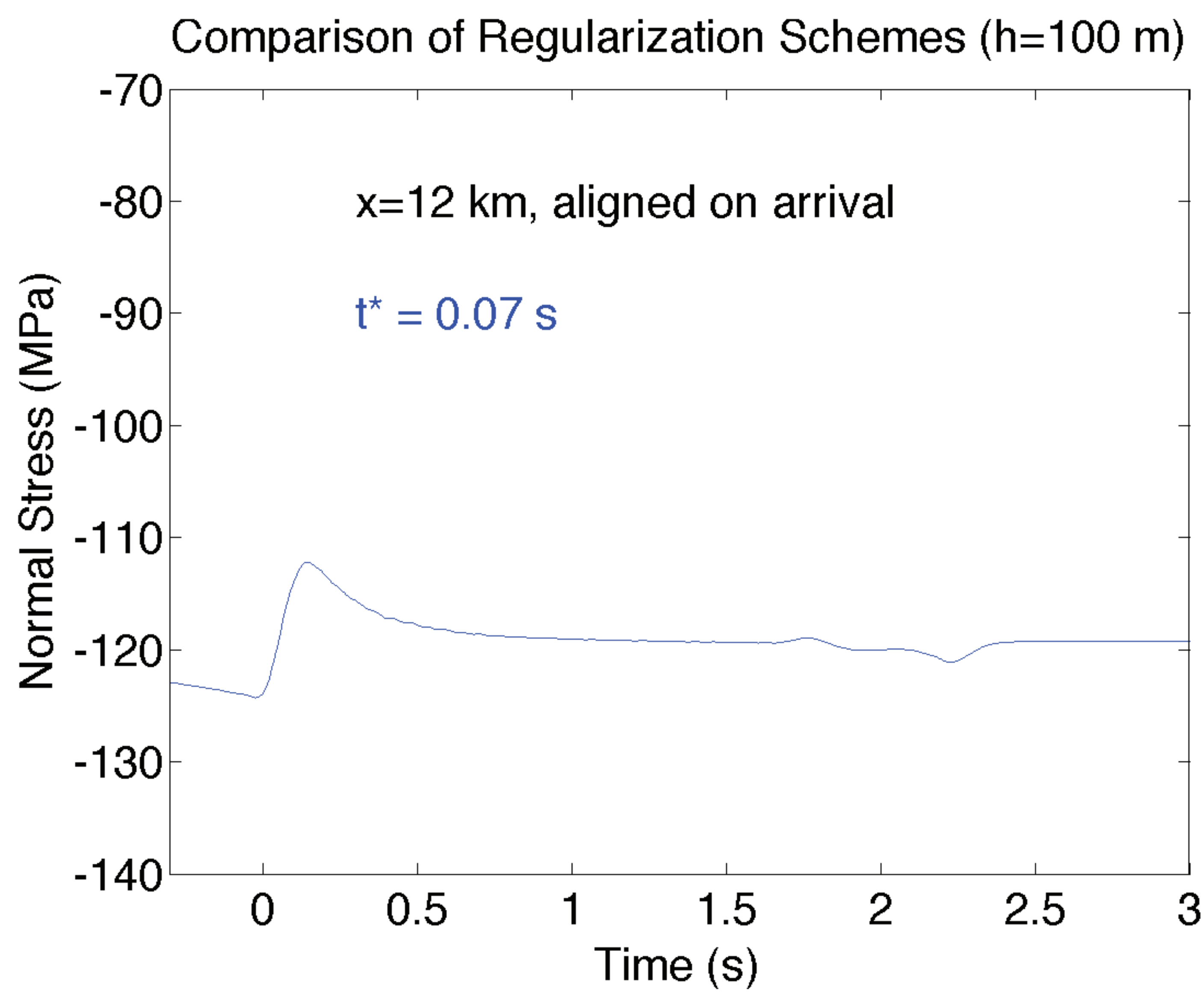

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.02 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

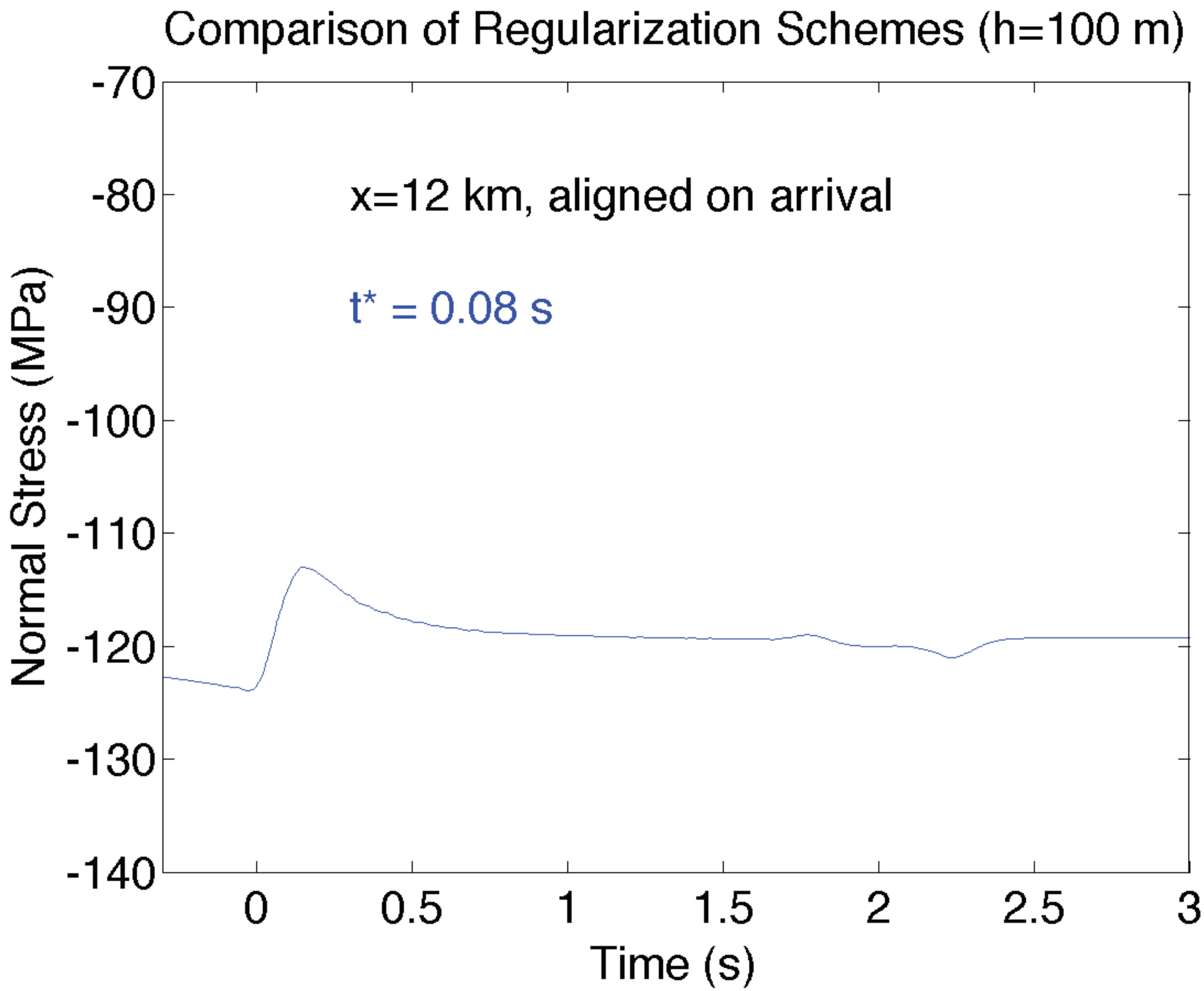

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.03 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

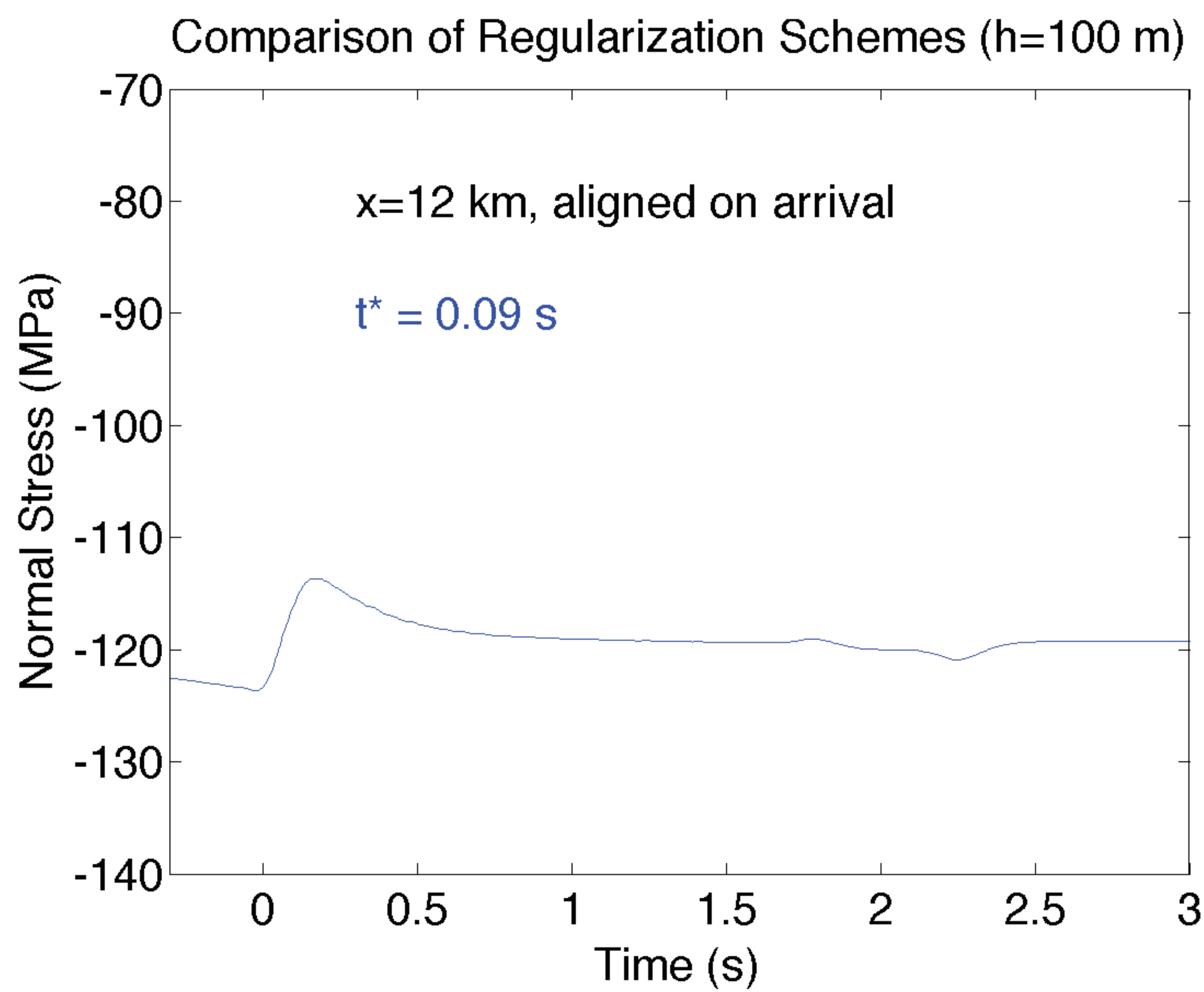

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.04 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

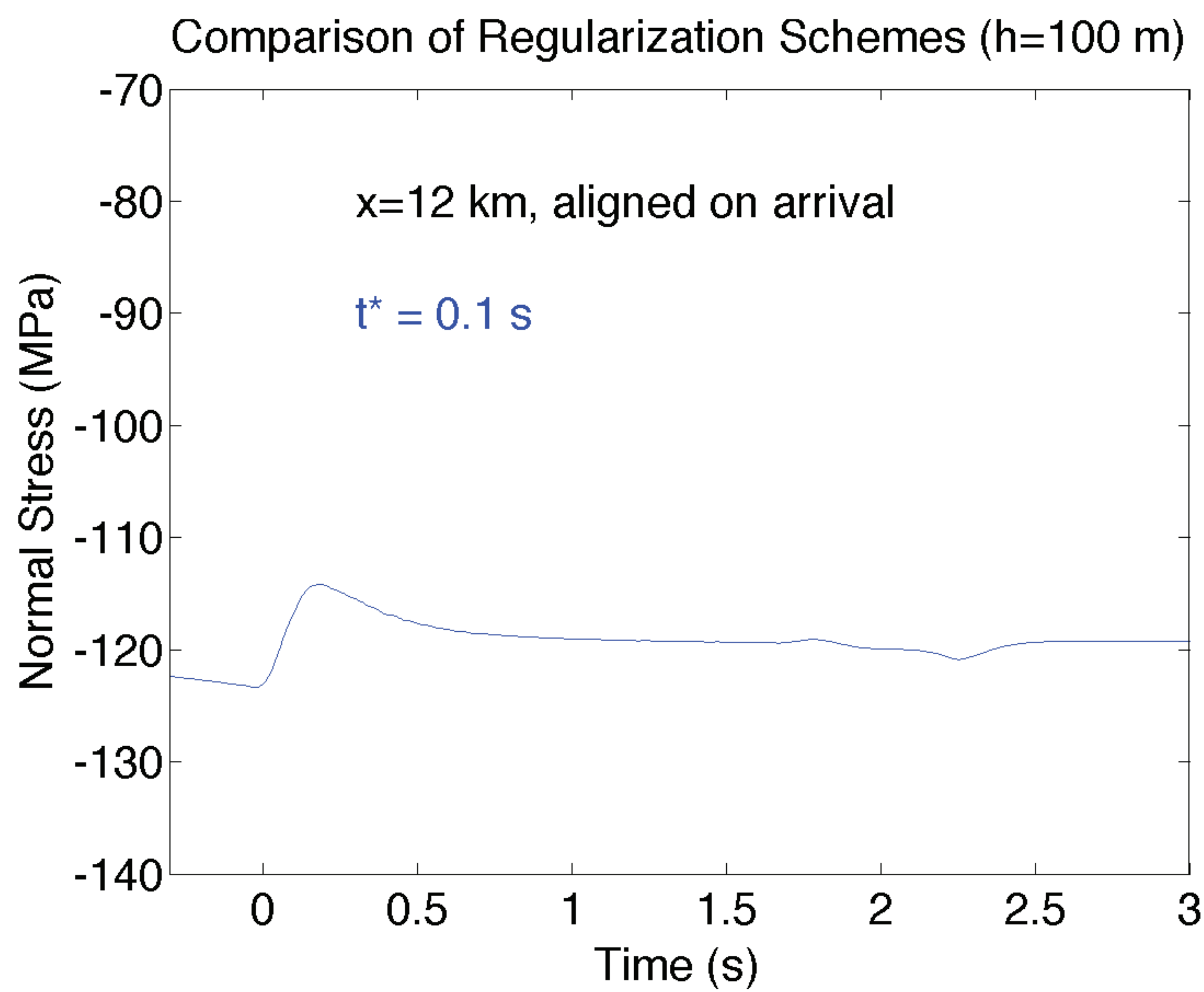

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.05 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

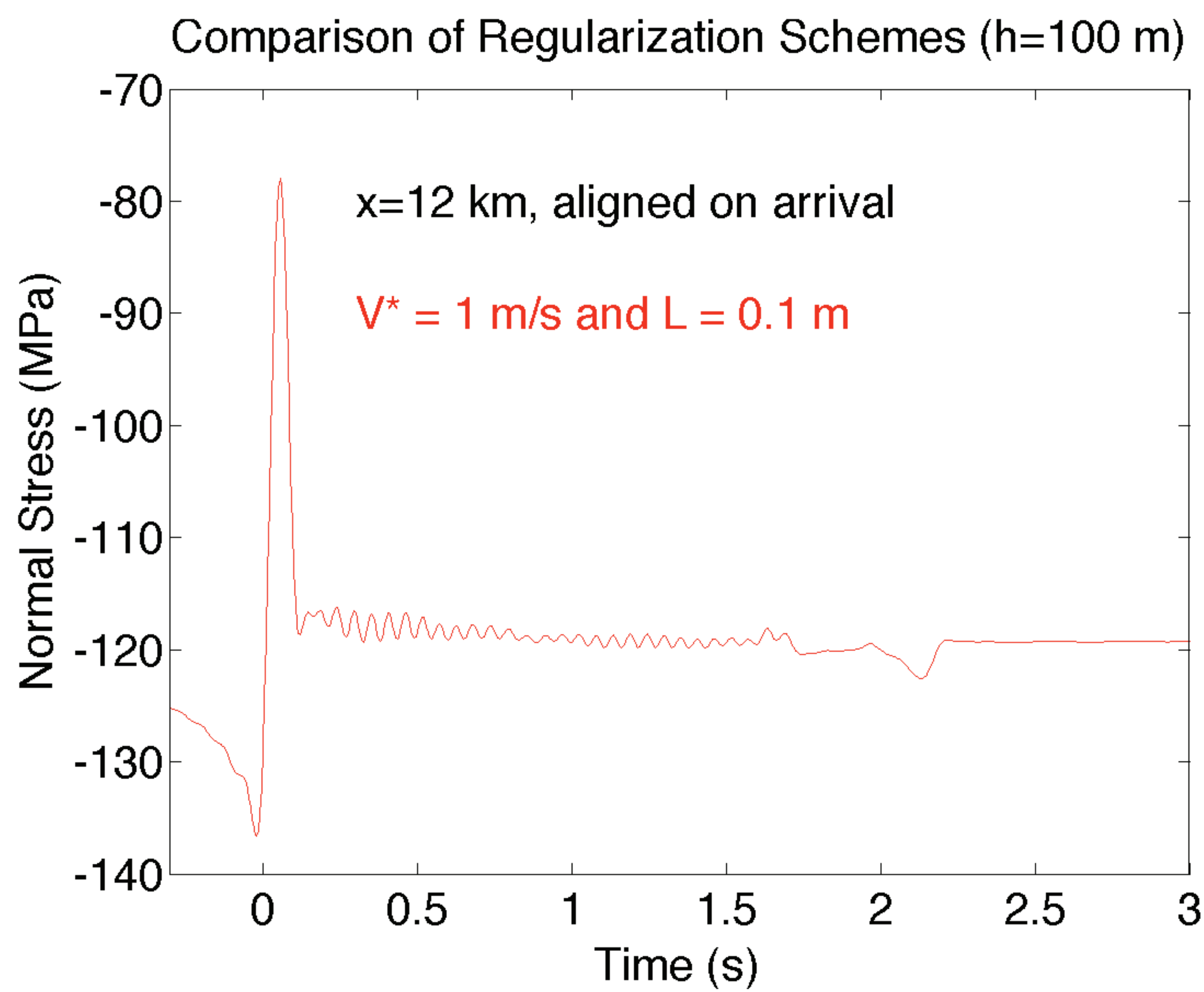

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.06 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.07 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

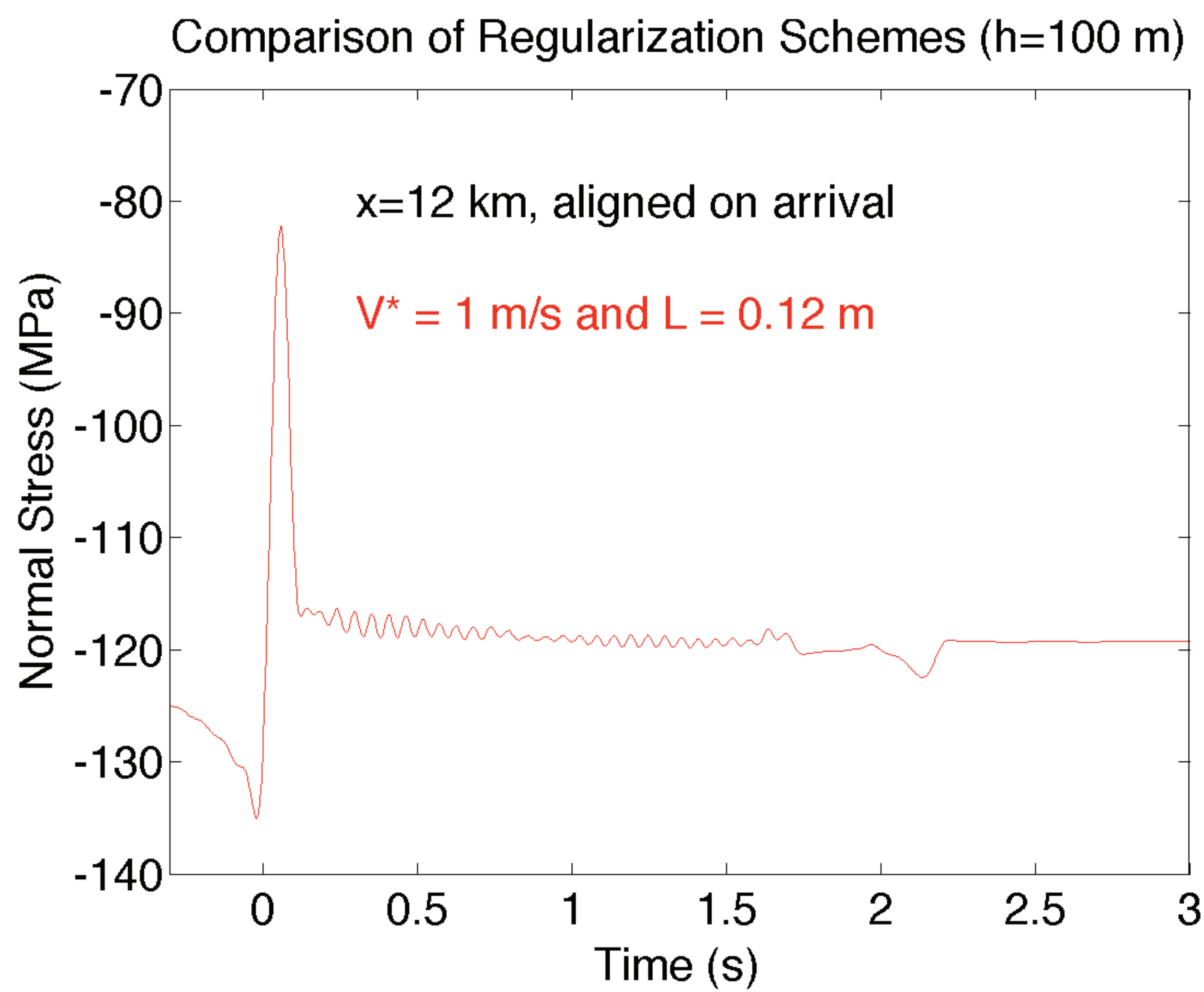

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.08 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

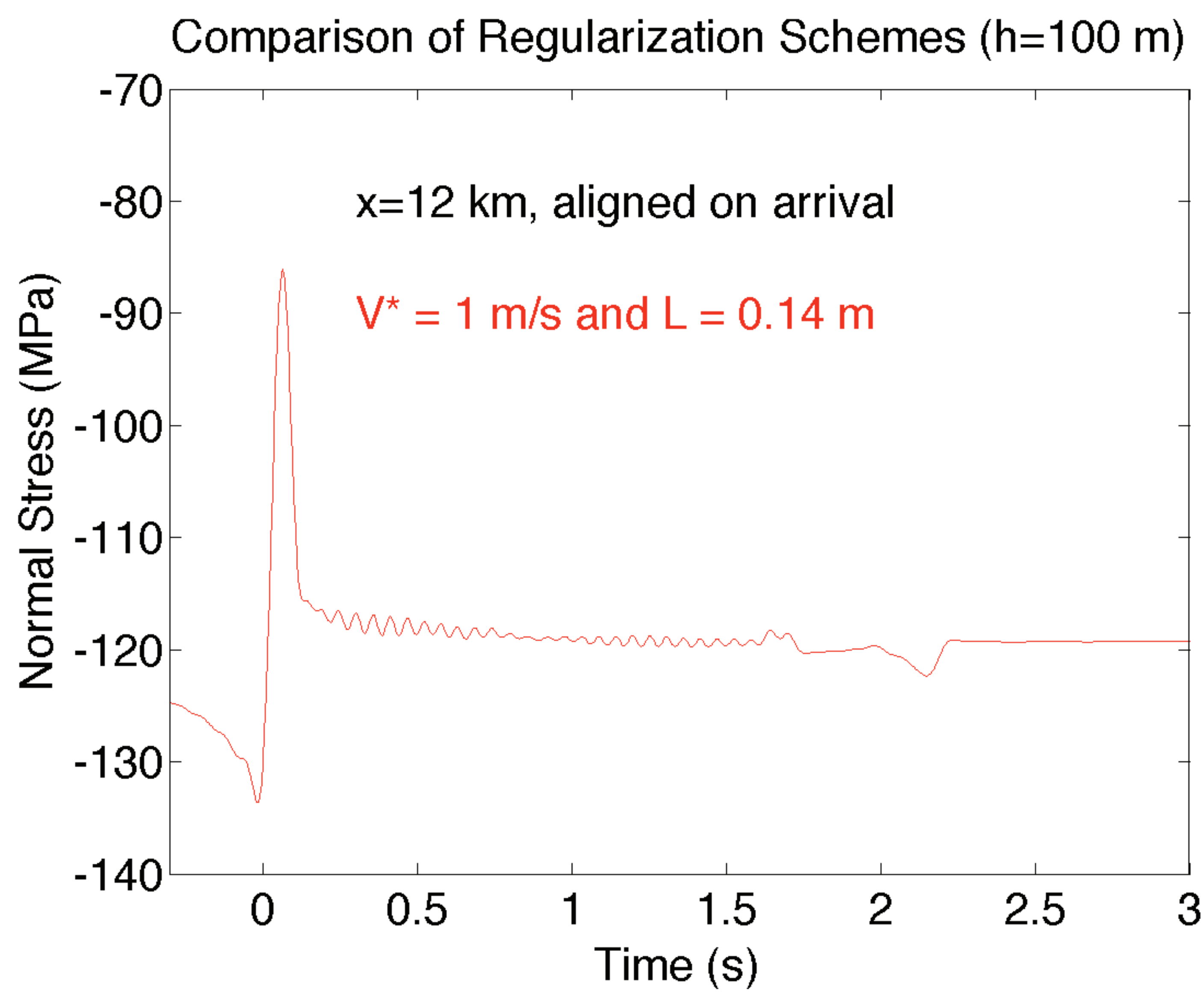

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.09 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

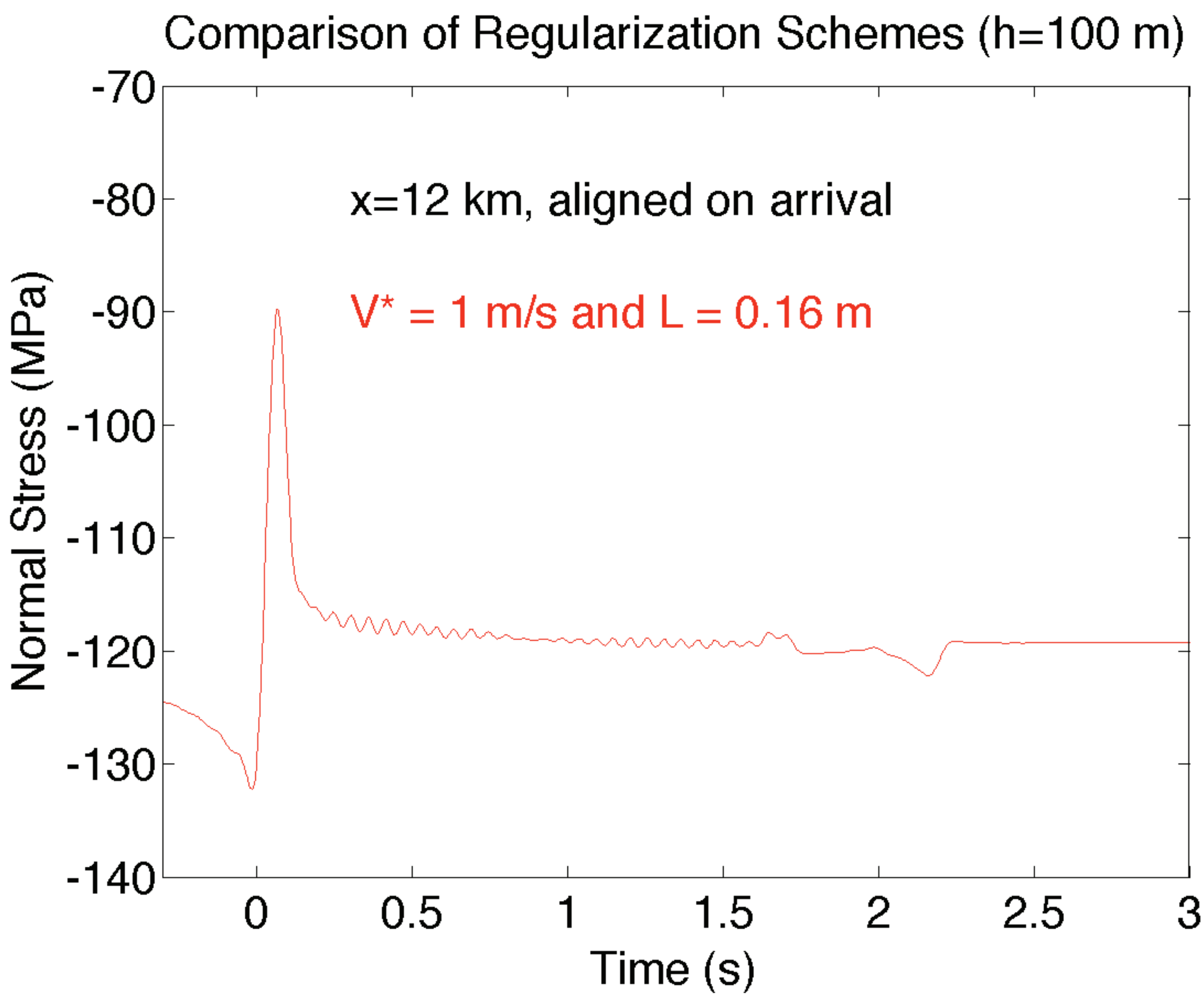

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
t* = 0.1 s
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)



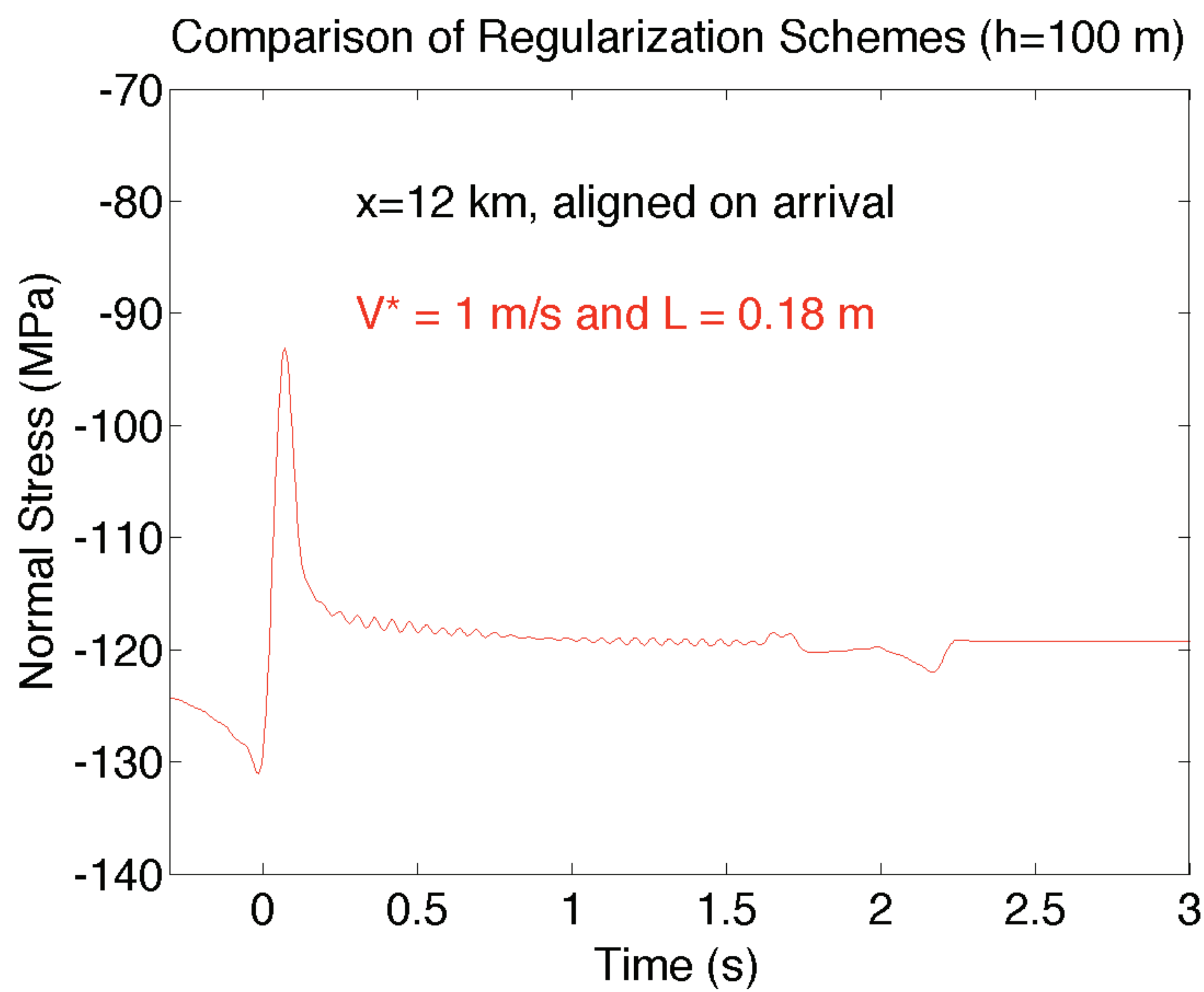
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.1 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

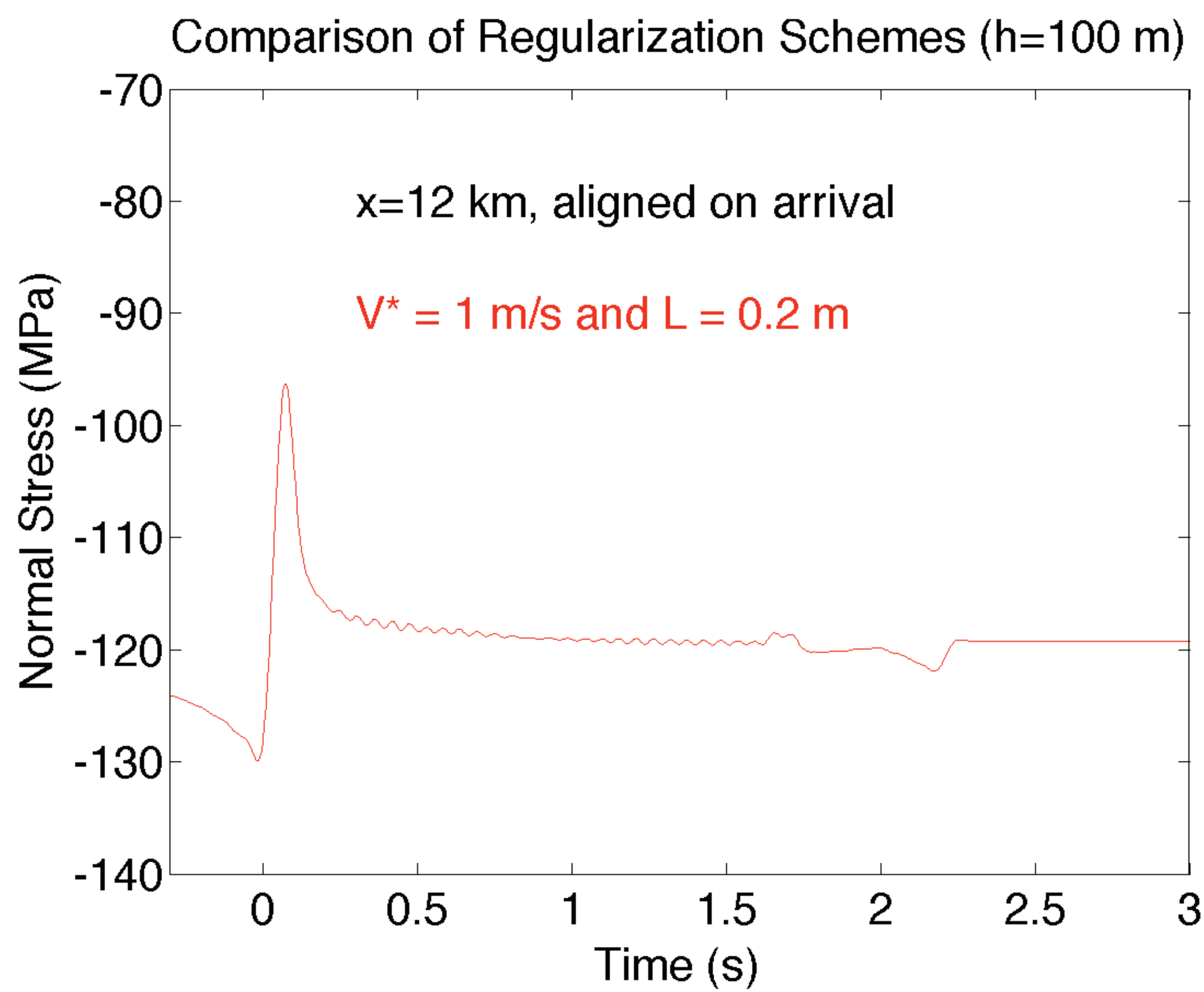

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.12 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

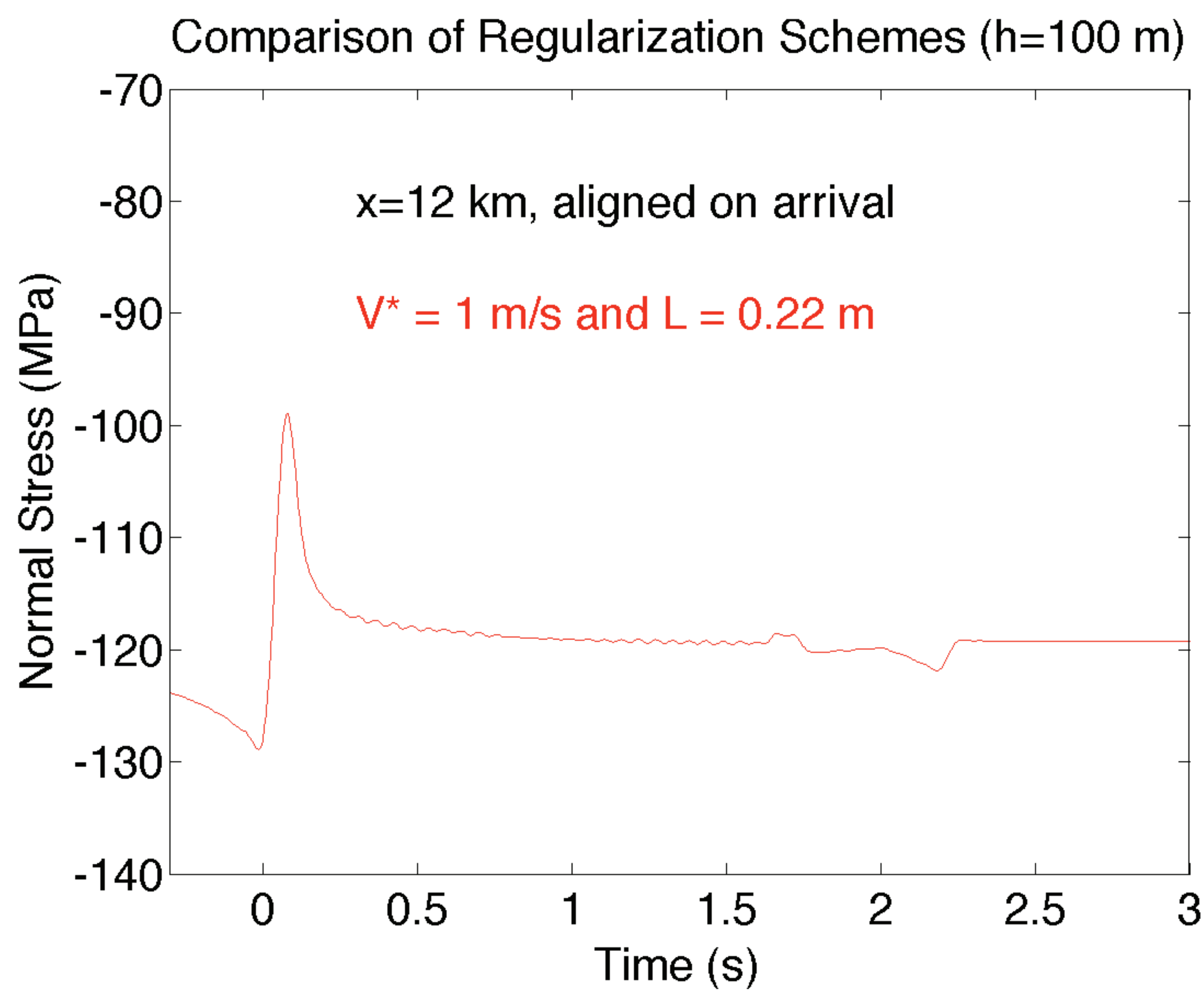

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.14 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

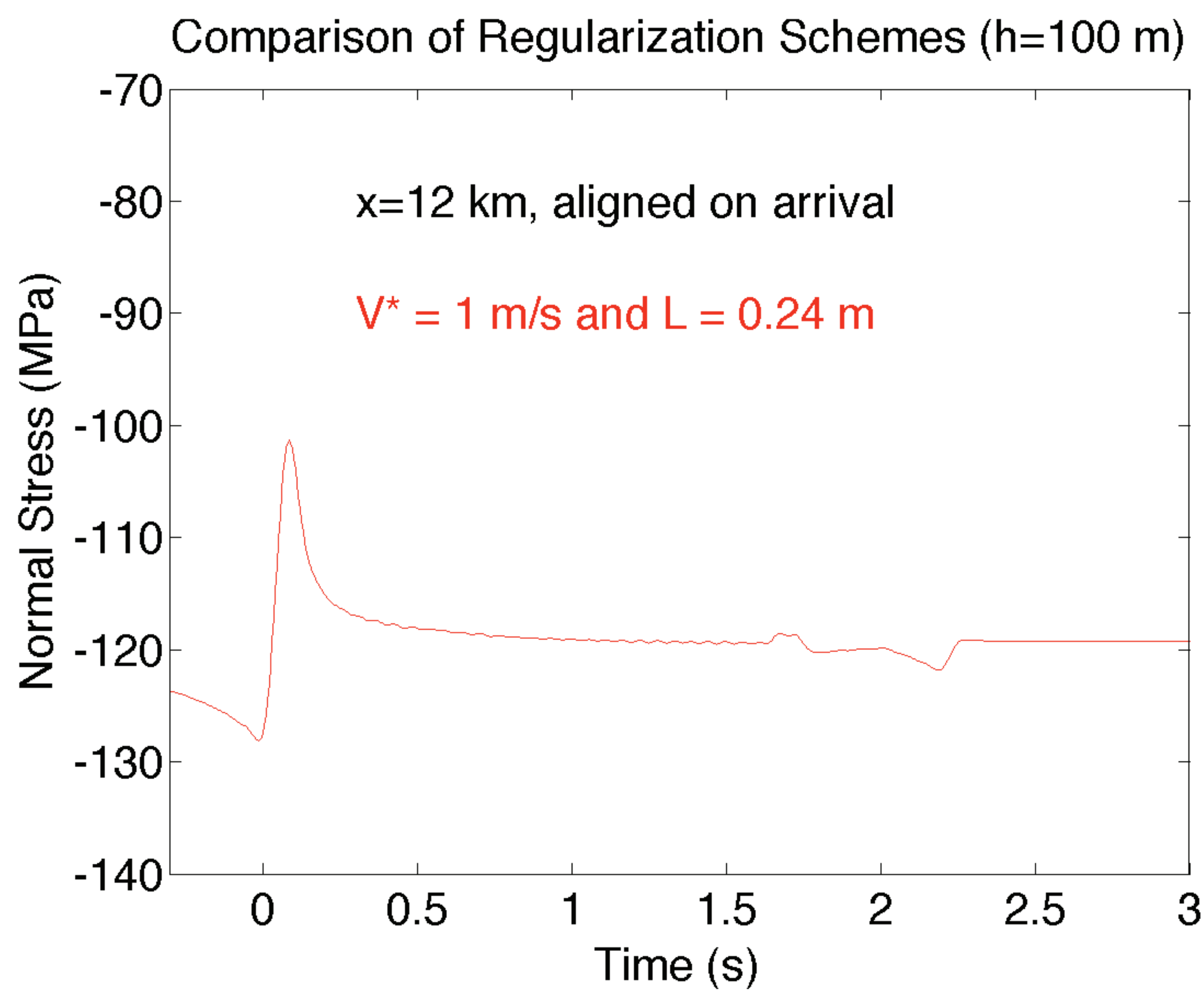


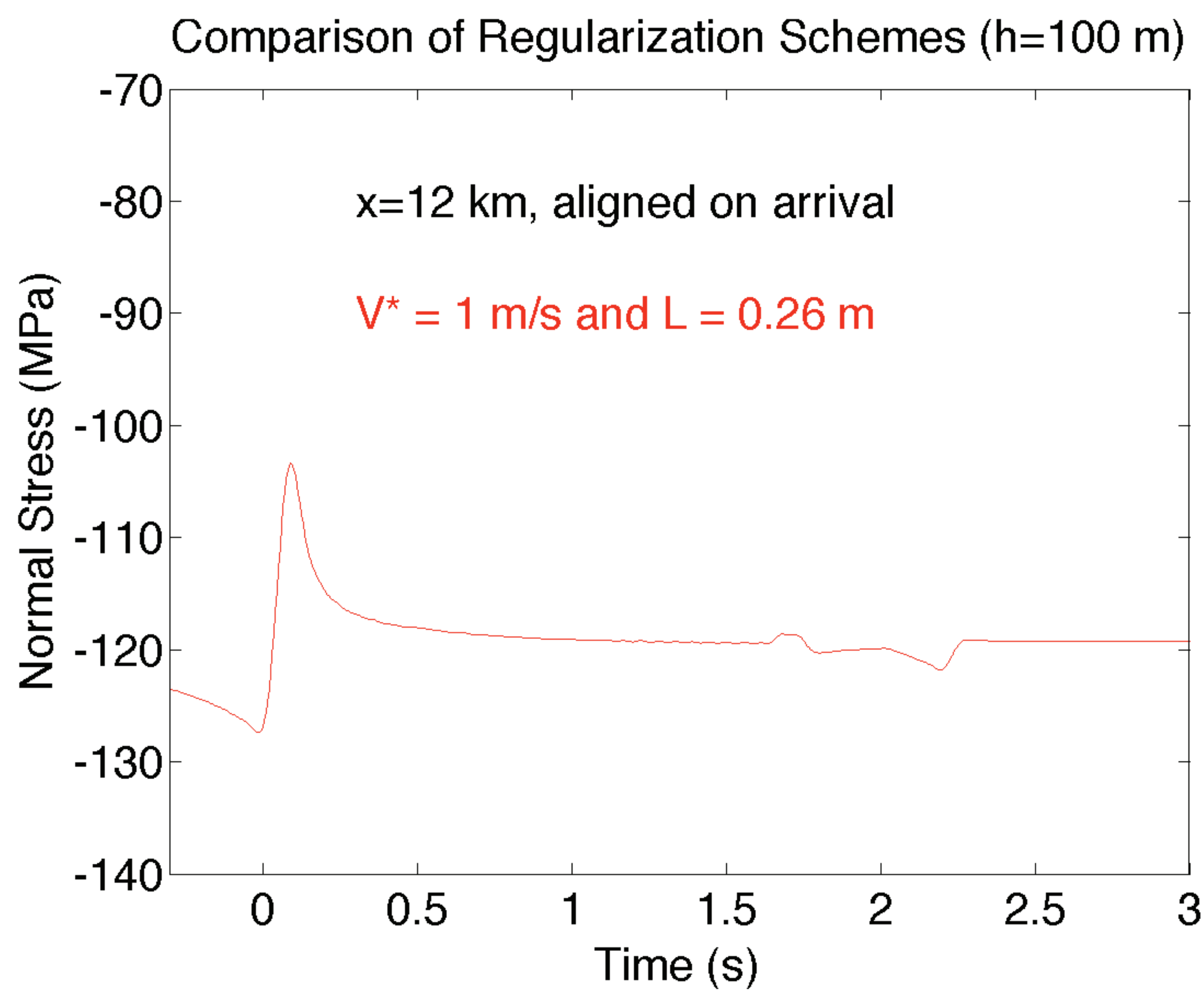
Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.16 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

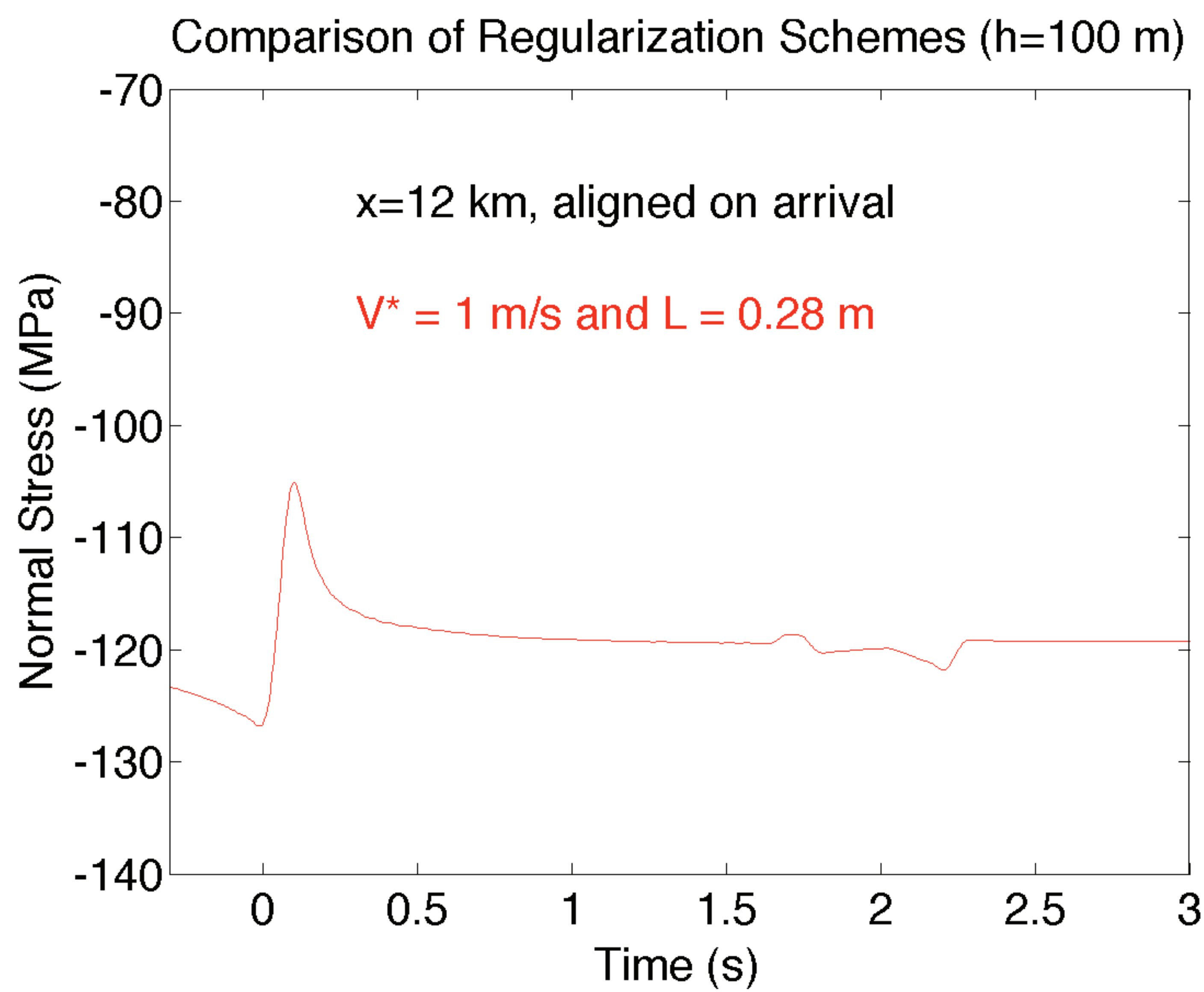

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.18 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

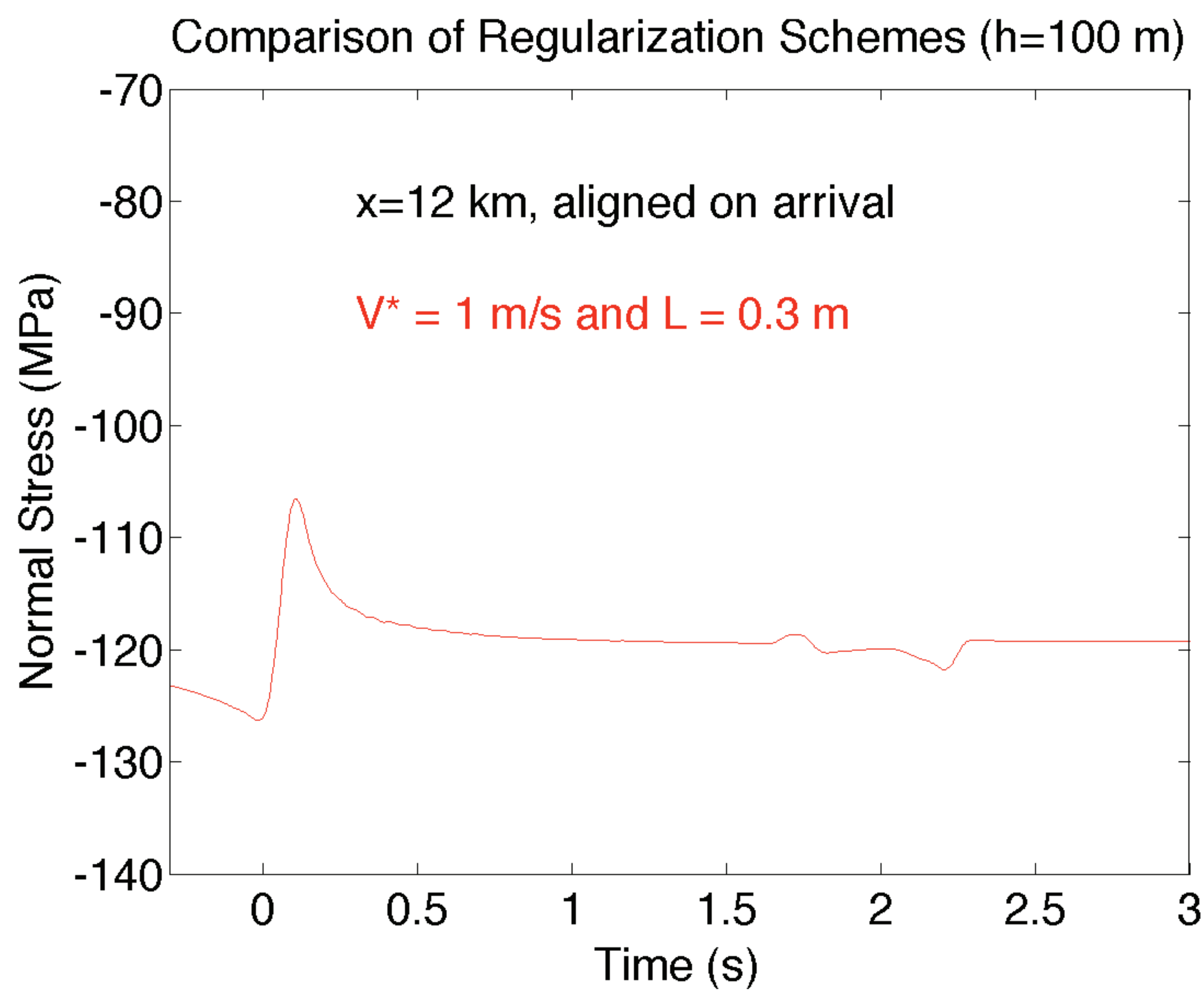

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.2 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.22 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

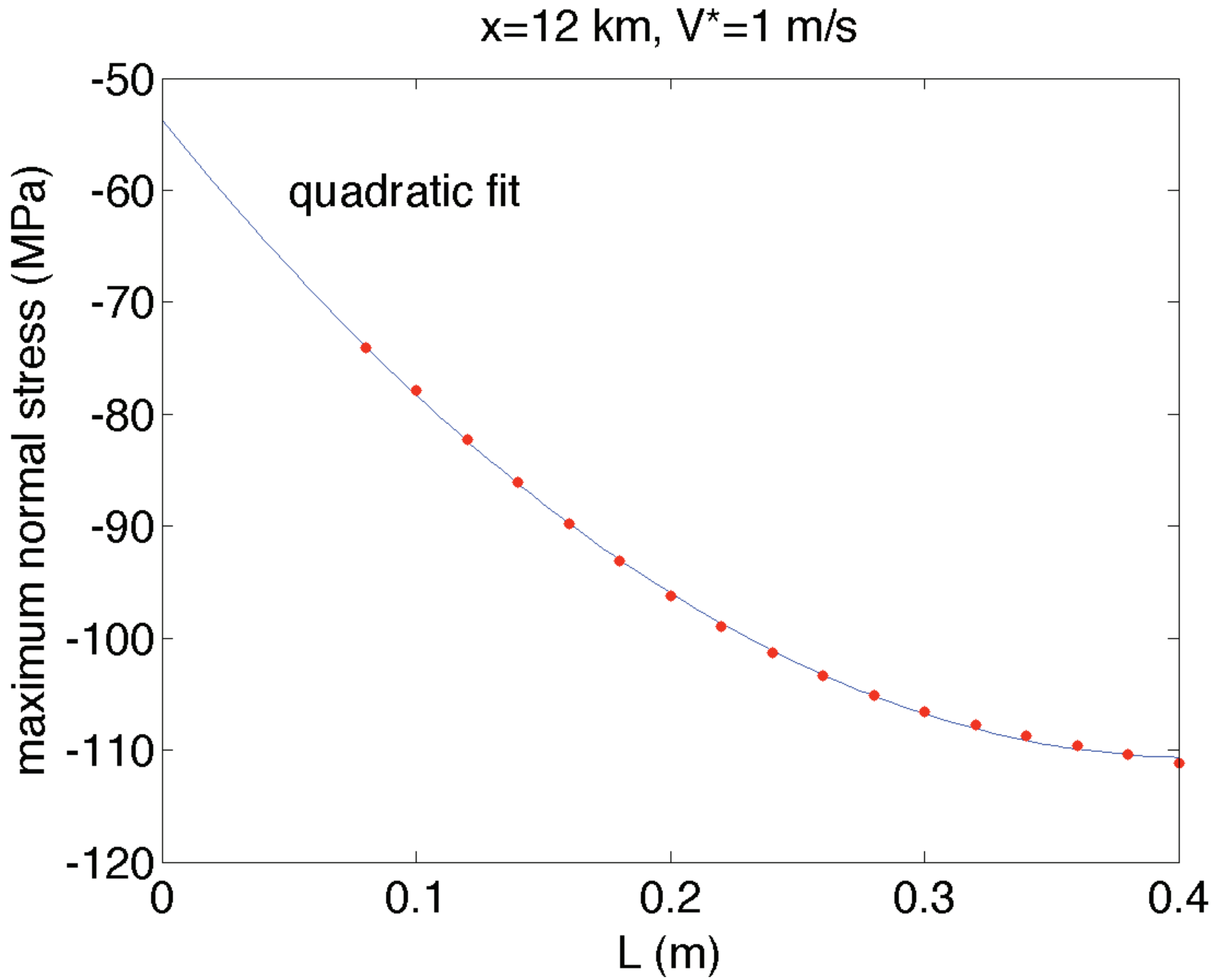

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.24 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.26 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

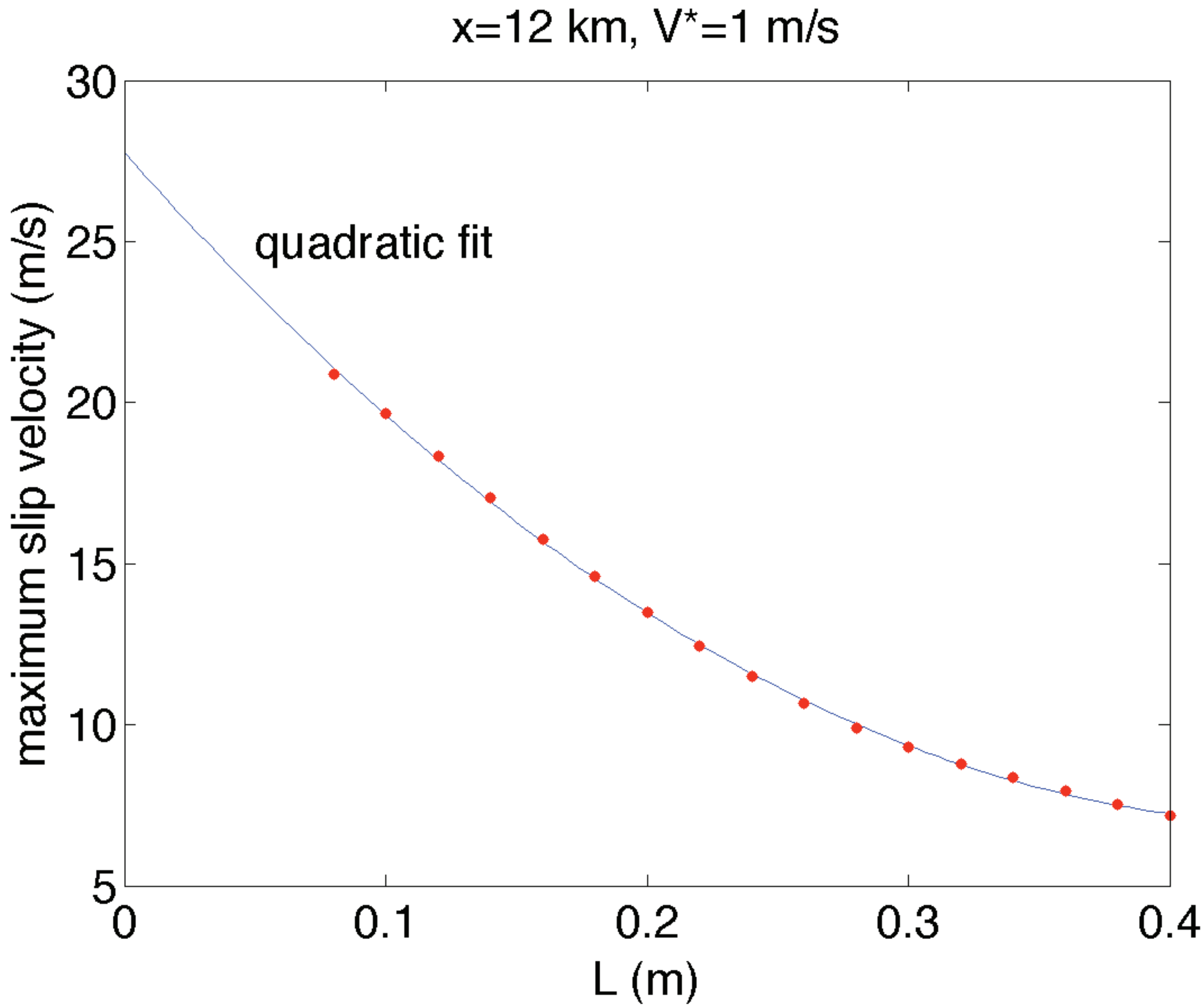

Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.28 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)


Comparison of Regularization Schemes (h=100 m)
x=12 km, aligned on arrival
V* = 1 m/s and L = 0.3 m
Normal Stress (MPa)
-70
-80
-90
-100
-110
-120
-130
-140
0
0.5
1
1.5
2
2.5
3
Time (s)

Limit as *L* Vanishes: No Opening?



Limit as *L* Vanishes: Peak Velocity



Implications for Numerical Modeling of Bimaterial Problems

TPV6 (well-posed bimaterial problem):

- probably not converged at chosen resolution

TPV7 (unregularized ill-posed bimaterial problem):

- many codes show “clean” results, probably due to damping (either explicitly added or intrinsic to numerical method)
- damping likely regularizes response, but causes results (e.g., peak velocities) to be sensitive to damping method (and probably grid spacing h, since artifical viscosities depend on h)

TPV8 (regularized ill-posed bimaterial problem):

- for consistent results among different groups, we must agree upon regularization method and check convergence

Implementation of Time Regularization

implicit discretization easy with TSN (for FE and FD codes)

$$\frac{d\tau}{dt} = -\frac{1}{t^*}\left[\tau - f(\delta)\sigma\right] \qquad \text{with initial condition } \tau^0 = f_s\sigma^{\text{init}}$$

$$\frac{\tau^{n+1} - \tau^n}{\Delta t} = -\frac{1}{t^*}\left[\tau^{n+1} - f(\delta^{n+1})\sigma^{n+1}\right]$$

$$\tau^{n+1} = \frac{f(\delta^{n+1})\sigma^{n+1} - \left(t^*/\Delta t\right)\tau^n}{1 + t^*/\Delta t}$$

recover unregularized slip-weakening when $t^*=0$