TPV105-2D

Thermal pressurization with strongly rate-weakening friction

$$\tau = f(V,\Theta)(\sigma - p_0)$$

fault strength depends on both friction coefficient and effective stress

Eric M. Dunham (Stanford University)

Thermal Pressurization

• pore pressure changes due to thermal expansion (and confinement of fluids within matrix with vastly smaller thermal expansivity)
• without loss of heat (*adiabatic*) or fluid (*undrained*), pressurization is Λ~1 MPa/K
• along with extreme reduction of friction coefficient, TP is dynamic weakening mechanism that prevents fault melting (if shear zones are narrow)

$$\left.\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\tau\dot{\gamma}}{\rho c} \\ \frac{\partial p}{\partial t} &= \Lambda\frac{\partial T}{\partial t} \end{aligned}\right\} \Rightarrow \frac{\partial p}{\partial t} = \Lambda\frac{\tau\dot{\gamma}}{\rho c}$$

$$\int \dot{\gamma}(z,t)dz = V(t)$$

$$\dot{\gamma}(z,t) = V(t)\delta(z)$$

$$\frac{\partial p_0}{\partial t} = \frac{\Lambda f\dot{\gamma}}{\rho c}(\sigma - p_0) \Rightarrow p_0(t) = p_0(0)\exp(-t/t_{au})$$

$$\dot{\gamma}(z,t) = \begin{cases} V(t)/(2h), & |z| < h \\ 0, & \text{otherwise} \end{cases}$$

$$L_{au} = \frac{\rho c\sqrt{2\pi}h}{\Lambda f} = 3.4\ \text{m (TPV105)}$$

$$\dot{\gamma}(z,t) = \frac{V(t)}{h\sqrt{2\pi}}\exp\left(-\frac{z^2}{2h^2}\right)$$

effective exponential slip-weakening distance for adiabatic, undrained TP

Thermal Pressurization

(conservation of fluid mass and energy, Fourier's law and Darcy's law)

$$\frac{\partial T}{\partial t} = \alpha_{th} \frac{\partial^2 T}{\partial z^2} + \frac{\tau \dot{\gamma}}{\rho c}$$

$$\frac{\partial p}{\partial t} = \alpha_{hy} \frac{\partial^2 p}{\partial z^2} + \Lambda \frac{\partial T}{\partial t}$$

$$T(z,0) = T_{ini}, \quad p(z,0) = p_{ini}$$

$$\left.\frac{dT}{dz}\right|_{z=0} = 0, \quad \left.\frac{dp}{dz}\right|_{z=0} = 0 \ (\text{symmetry})$$

$$T \to T_{ini}, \ p \to p_{ini} \text{ as } z \to \infty$$

$$\dot{\gamma}(z,t) = \frac{V(t)}{h\sqrt{2\pi}} \exp\left(-\frac{z^2}{2h^2}\right)$$

(Gaussian shear zone in TPV105)

$$\text{diffusion length} \sim \sqrt{4\alpha_{\max} t}, \quad \alpha_{\max} = \max\{\alpha_{th}, \alpha_{hy}\}$$

(provides estimate of where domain can be truncated without influencing solution)

Thermal Pressurization

two limiting cases:

$$t << \frac{h^2}{4\alpha_{\max}} \qquad\qquad t >> \frac{h^2}{4\alpha_{\max}}$$

(adiabatic and undrained) (slip-on-a-plane)

slip-on-a-plane has analytical solution for constant slip velocity, weakening over slip distance

$$L^* = \frac{4}{f^2}\left(\frac{\rho c}{\Lambda}\right)^2 \frac{\left(\sqrt{\alpha_{th}} + \sqrt{\alpha_{hy}}\right)^2}{V} = 8.8 \text{ m (TPV105)}$$

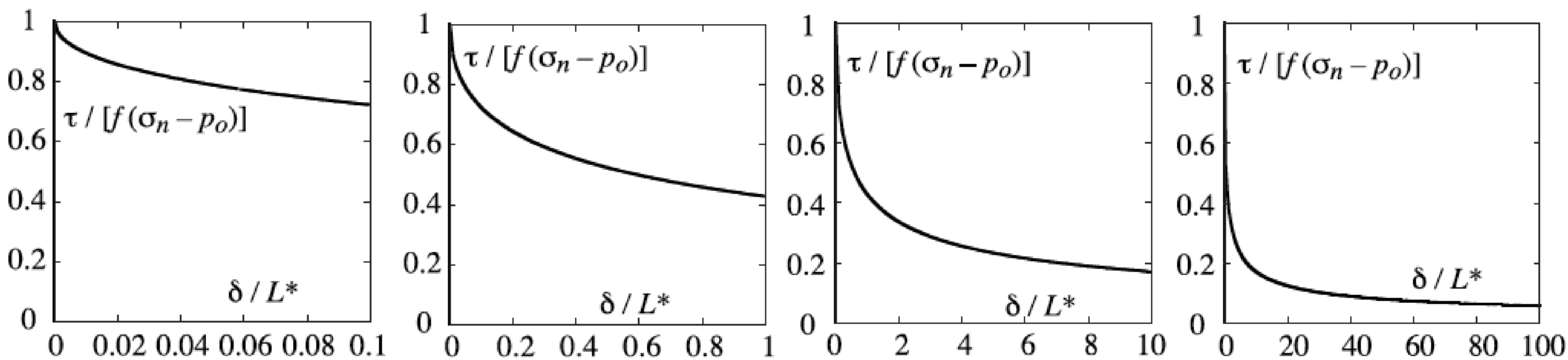


Figure 3. Prediction of shear strength τ versus slip δ due to thermal pressurization of pore fluid during slip on a plane, at constant rate V and with constant friction coefficient f, in a fluid-saturated solid. Note the multiscale nature of the weakening. Here σ_n is fault-normal stress and p_o is the pore pressure just after its reduction from ambient pressure by any dilatancy at onset of shear.

[Rice, 2006]

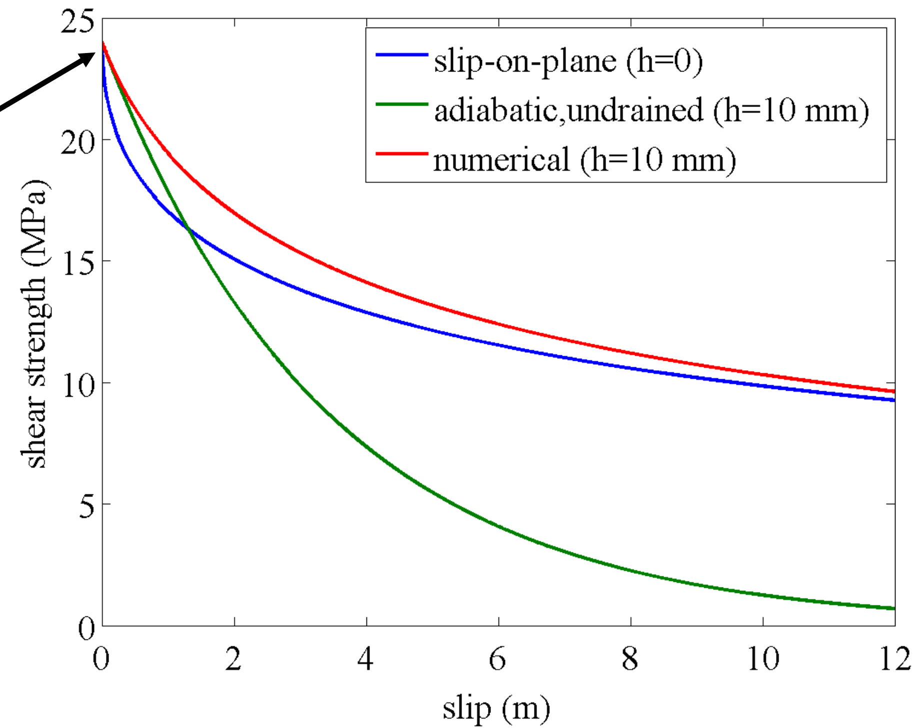
Stages of Dynamic Weakening

$L = 0.4\ \text{m}$ (rate-and-state evolution to fully weakened friction coefficient f_w=0.2)

$L_{au} = \dfrac{\rho c\sqrt{2\pi}h}{\Lambda f} = 3.4\ \text{m}$ (TP, adiabatic and undrained)

$L^* = \dfrac{4}{f^2}\left(\dfrac{\rho c}{\Lambda}\right)^2 \dfrac{\left(\sqrt{\alpha_{th}} + \sqrt{\alpha_{hy}}\right)^2}{V} = 8.8\ \text{m}$ (TP, slip-on-a-plane)

τ = 76 MPa for f_0=0.6 and initial effective stress



Numerical Methods

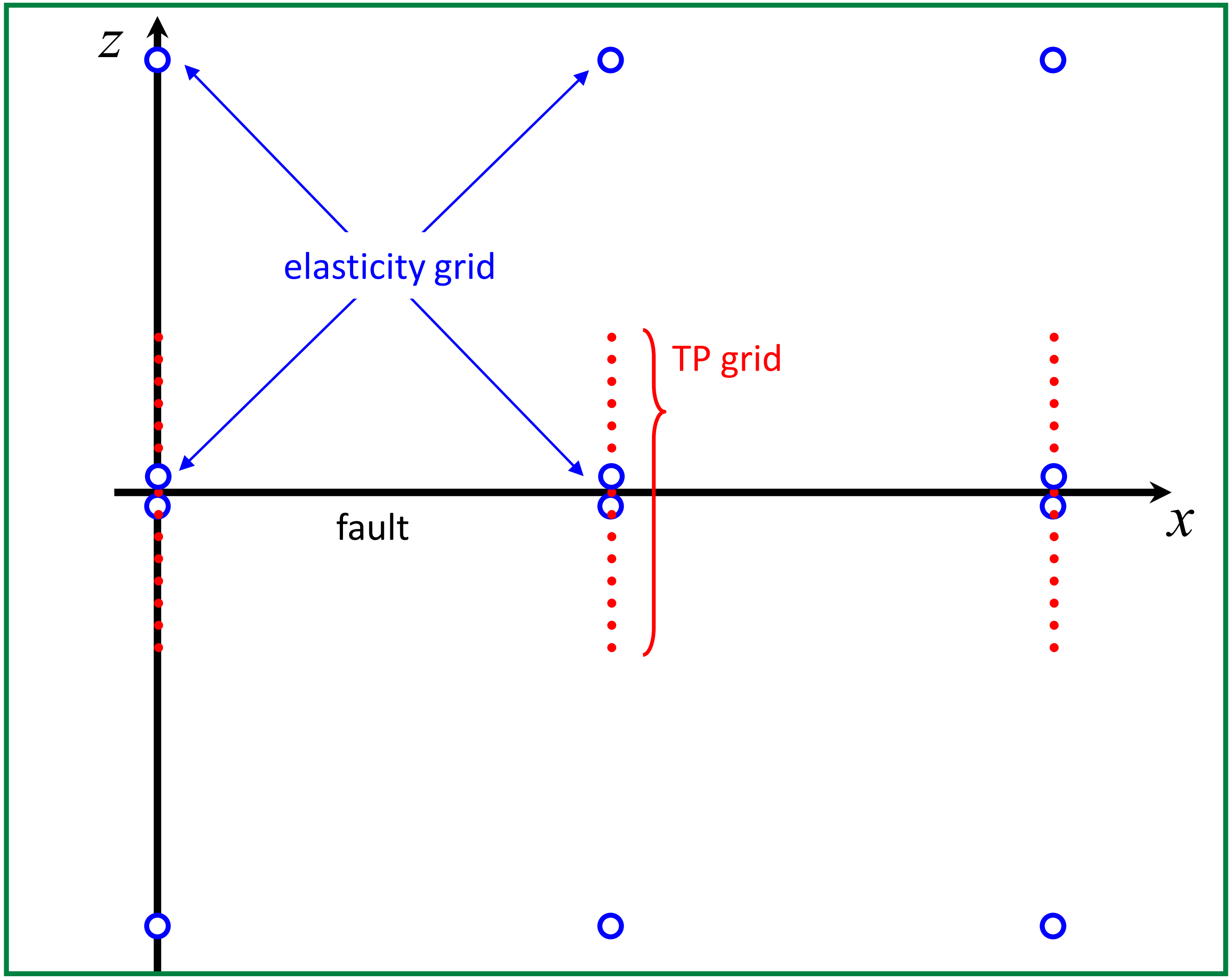
Many methods for solving TP equations:

- finite differences
- convolution of shear heating history with Green's function
- spectral representation of *T* and *p* fields

and coupling TP solver with elastodynamics

- operator splitting
- simultaneous time integration

Finite Differences



Finite Differences

$$\frac{dT_i}{dt} = \alpha_{th} \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta z^2} + \frac{\tau \dot{\gamma}_i}{\rho c}, \quad \frac{dp_i}{dt} = \alpha_{hy} \frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta z^2} + \Lambda \frac{dT_i}{dt}$$

Explicit time integration limited by stability constraints to time steps

$$\Delta t \le C \frac{\Delta z^2}{\alpha_{\max}}, \quad C = O(1)$$

Implicit time integration limited only by desired accuracy of solution (so time steps do not need to be smaller than elastic step)

<u>Operator Splitting for Time Integration</u>

1. Given fields at time t^n: v^n, σ^n, p^n, T^n
2. Integrate elasticity equations over full time step (using *any* method), holding $p=p^n$ fixed over time step (coupling via effective stress) to get v^{n+1}, σ^{n+1}
3. Return to t^n, integrate TP equations over full time step (using *any* method, including substeps), holding $v=v^{n+1}$, $\sigma=\sigma^{n+1}$ fixed over time step (coupling via shear heating) to get p^{n+1}, T^{n+1}

Green's Functions

$$p(0,t) = \int_0^t G(t-t')\tau(t')V(t')dt'$$

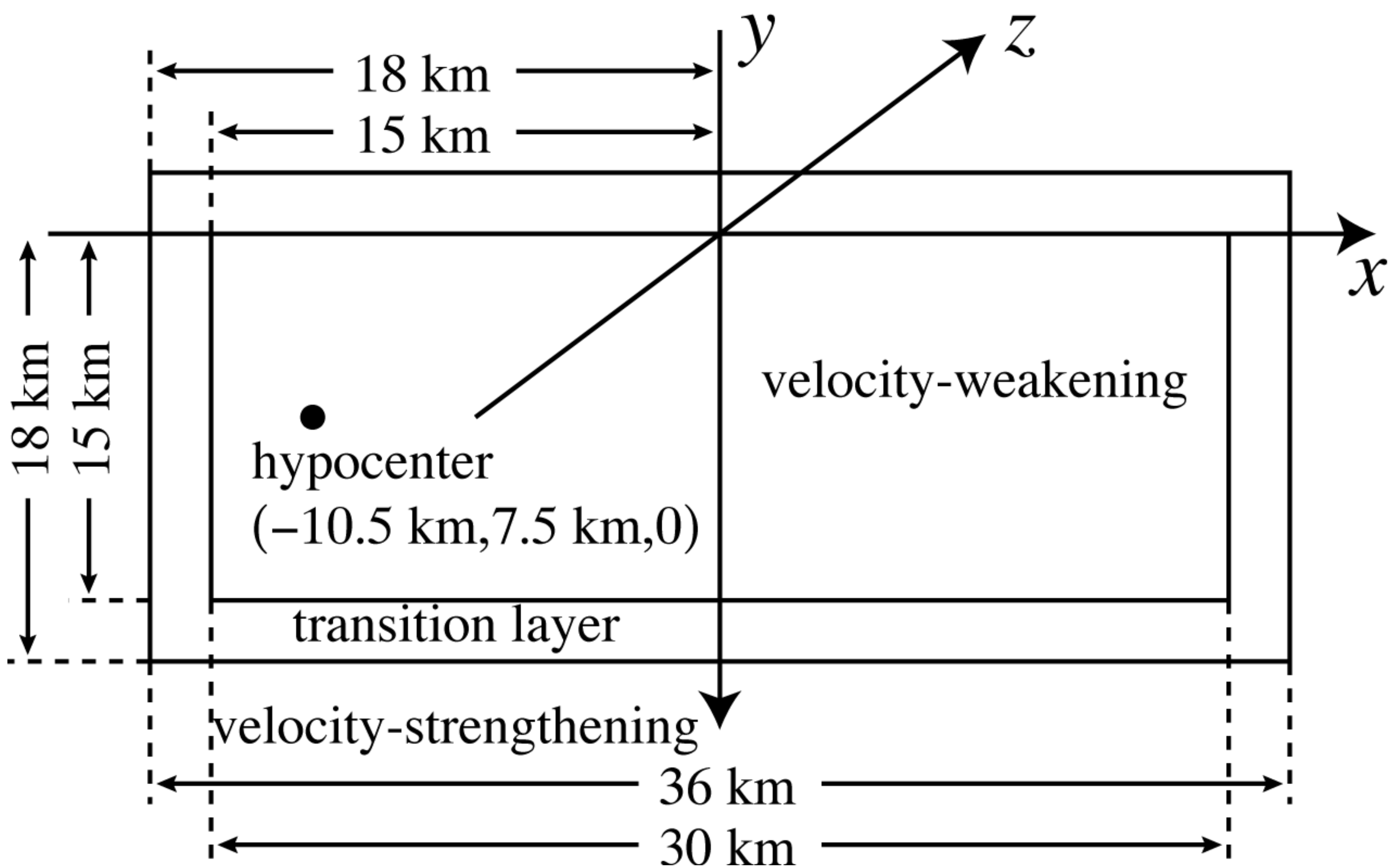
$$\approx \sum_{m=0}^{n-1} G\left(t^n - t^{m+1/2}\right)\tau\left(t^{m+1/2}\right)V\left(t^{m+1/2}\right)\Delta t$$

(using midpoint rule, but could also use trapezoidal rule or higher order quadrature rule)

$$G_{\mathrm{hy}}(0,t) = \frac{\Lambda}{\rho c(\alpha_{\mathrm{hy}} - \alpha_{\mathrm{th}})2\sqrt{\pi}}\left[\frac{\alpha_{\mathrm{hy}}}{\sqrt{\alpha_{\mathrm{hy}}t + \frac{1}{2}h^2}} - \frac{\alpha_{\mathrm{th}}}{\sqrt{\alpha_{\mathrm{th}}t + \frac{1}{2}h^2}}\right]$$

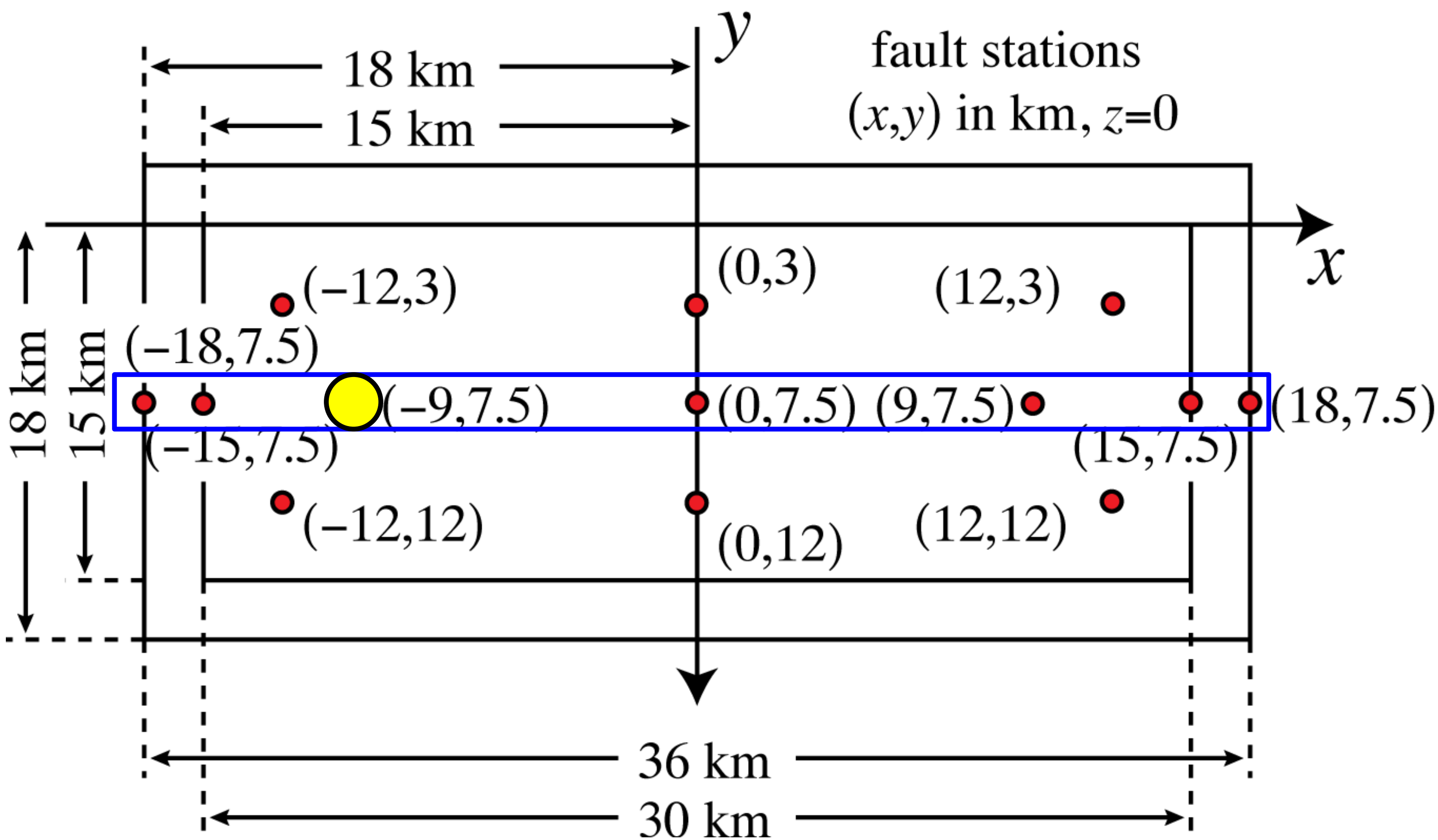
shear heating history stored for all elastic time steps

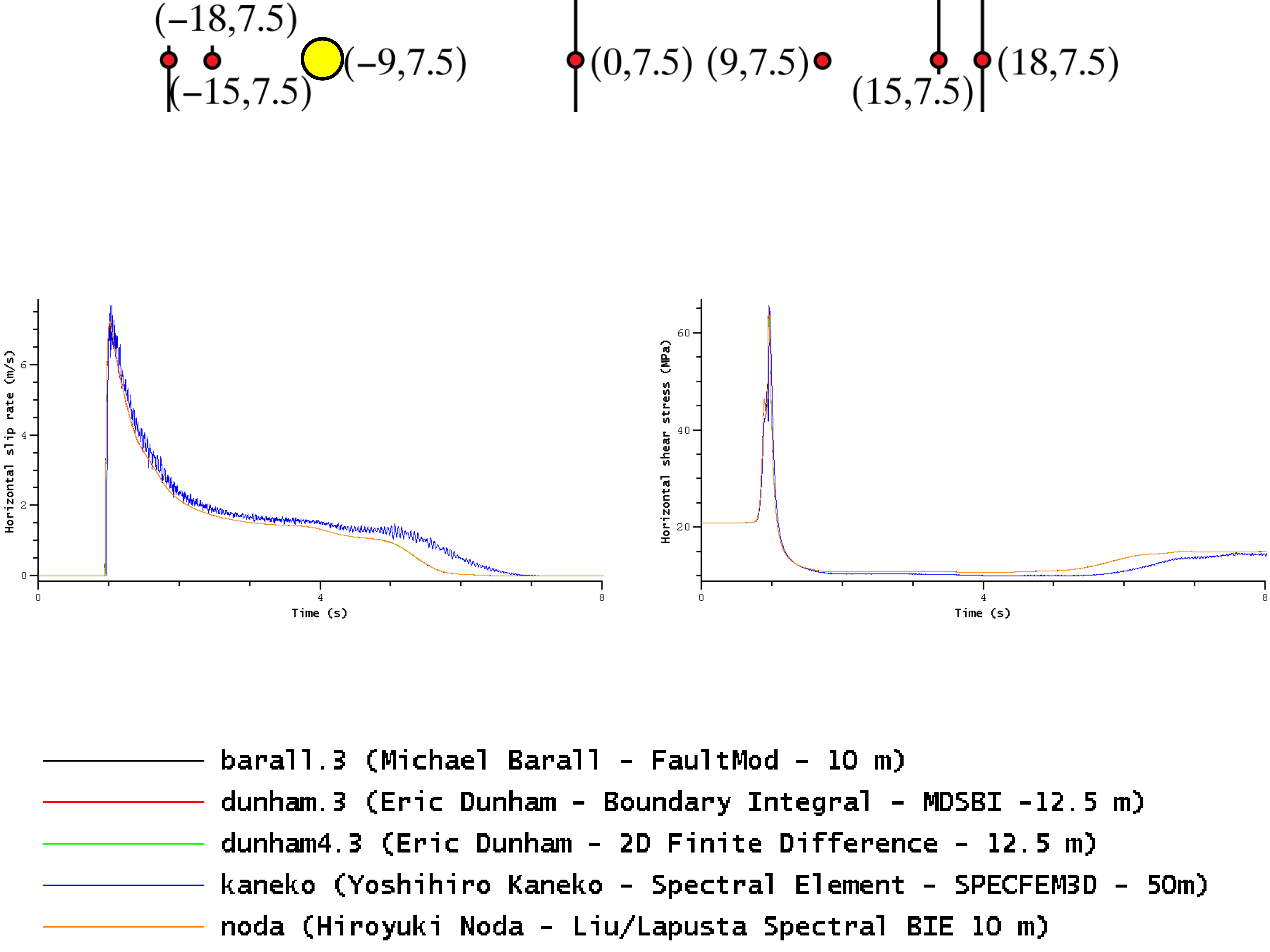
TPV105-2D



…but only in 2D for now (computational limitations)

Comparison of Results

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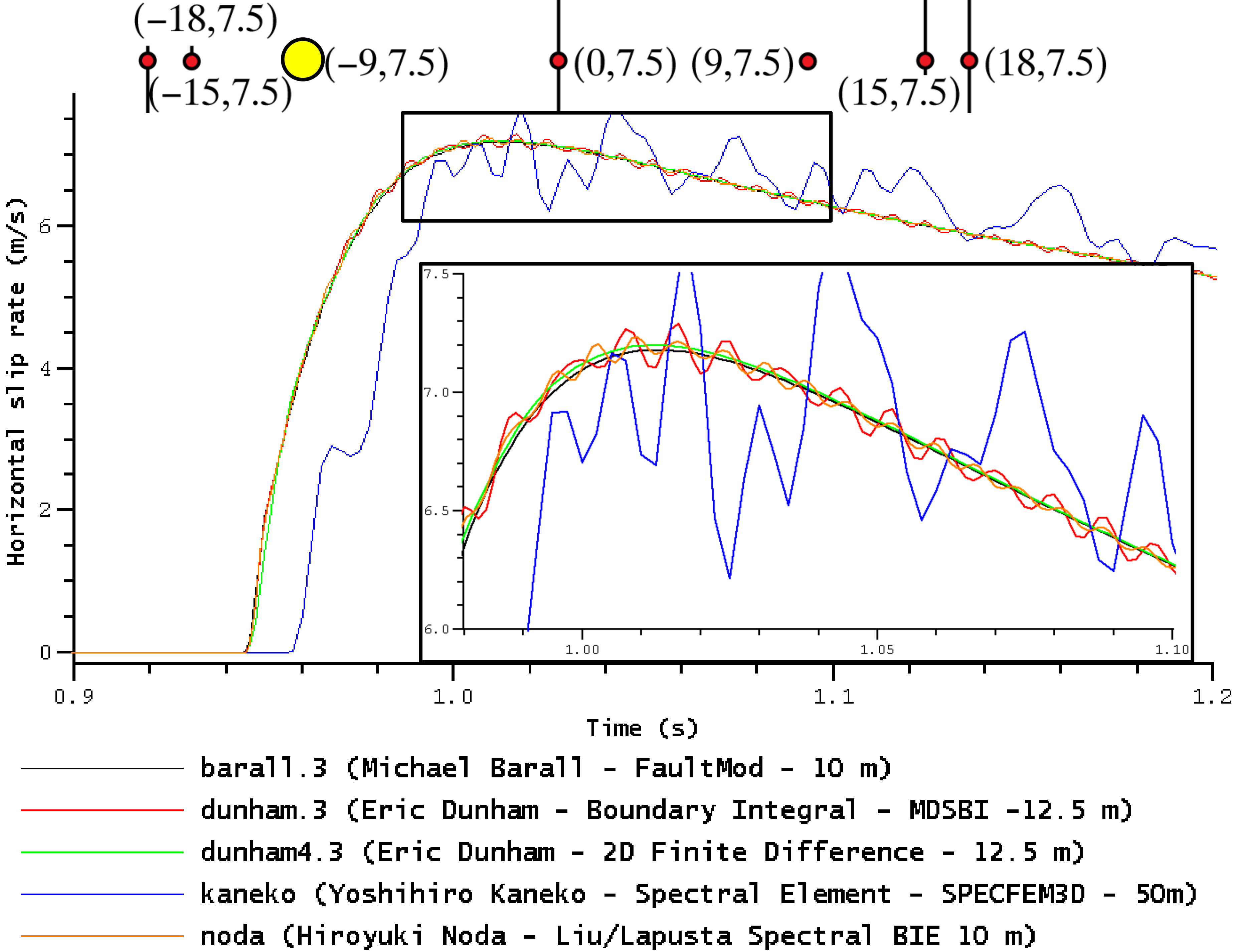
(−15,7.5)
(−9,7.5)
(0,7.5)
(9,7.5)
(15,7.5)
(18,7.5)
Horizontal slip rate (m/s)
Time (s)
Horizontal shear stress (MPa)
Time (s)
barall.3 (Michael Barall - FaultMod - 10 m)
dunham.3 (Eric Dunham - Boundary Integral - MDSBI -12.5 m)
dunham4.3 (Eric Dunham - 2D Finite Difference - 12.5 m)
kaneko (Yoshihiro Kaneko - Spectral Element - SPECFEM3D - 50m)
noda (Hiroyuki Noda - Liu/Lapusta Spectral BIE 10 m)


(−18,7.5)
(−15,7.5)
(−9,7.5)
(0,7.5)
(9,7.5)
(15,7.5)
(18,7.5)
Horizontal slip rate (m/s)
0
2
4
6
0.9
1.0
1.1
1.2
Time (s)
7.5
7.0
6.5
6.0
1.00
1.05
1.10
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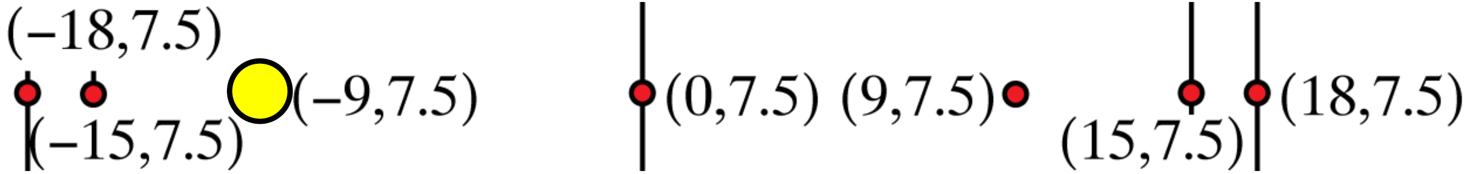
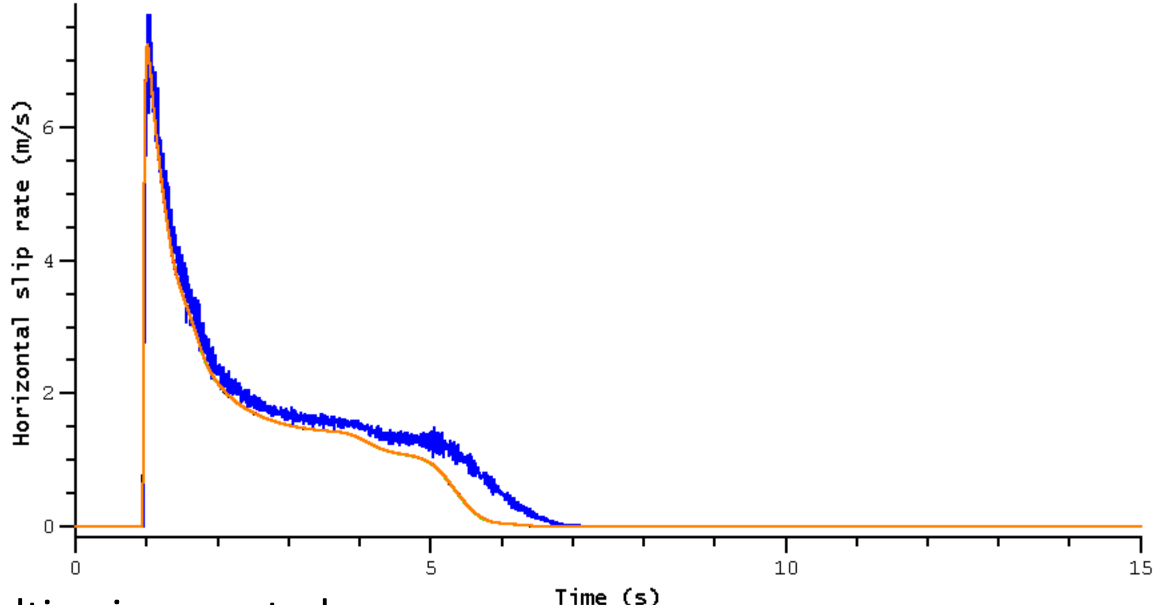
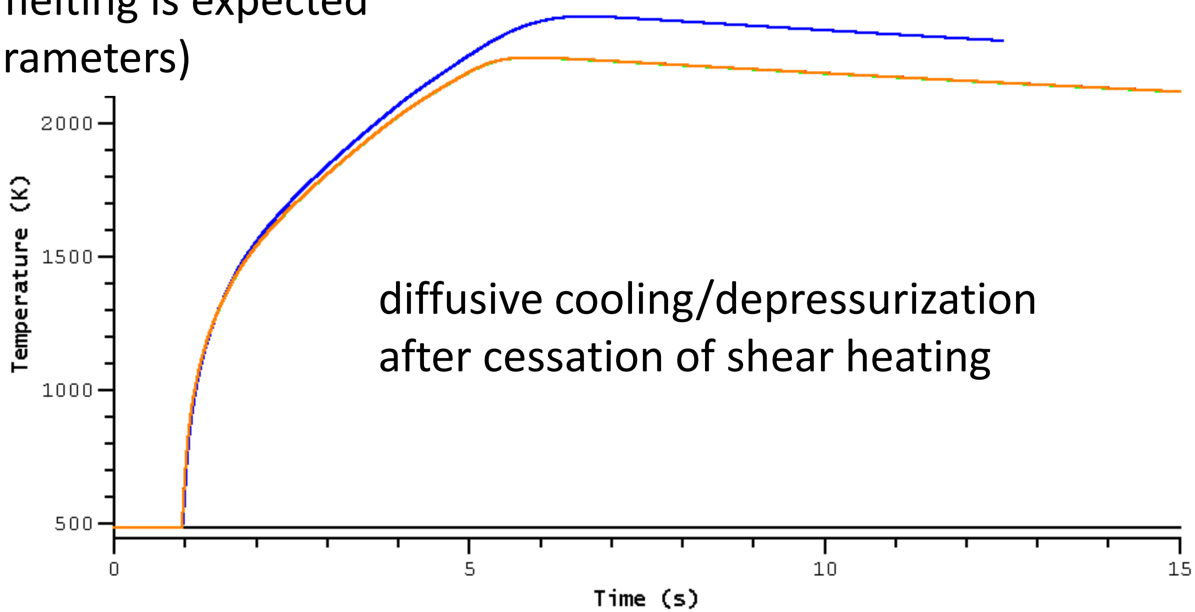
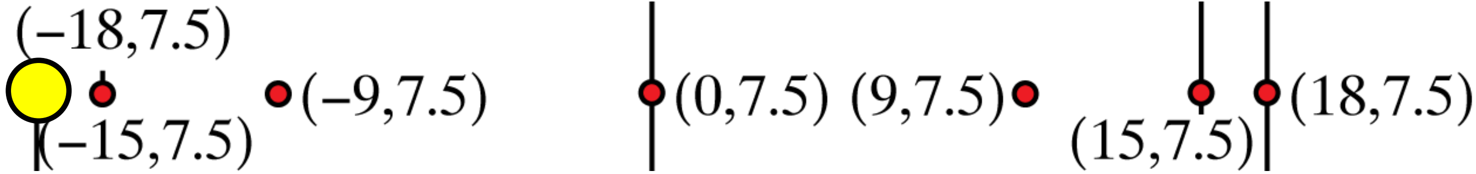
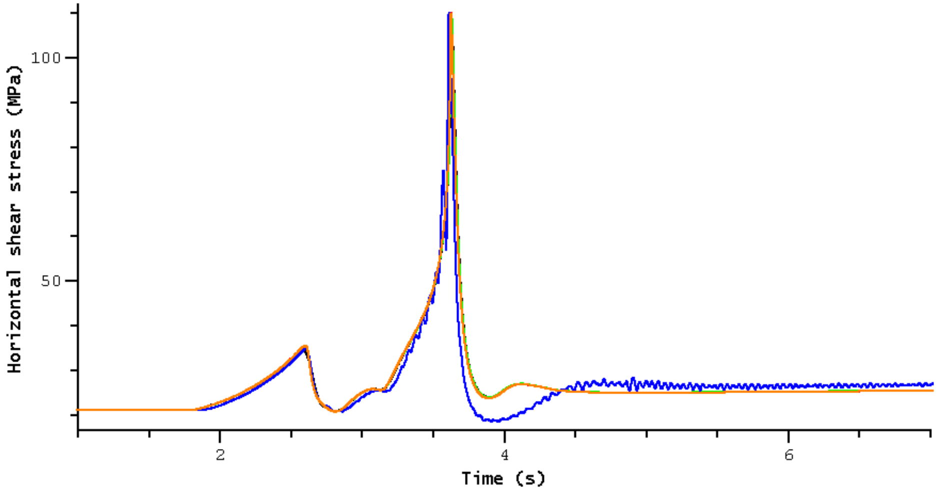




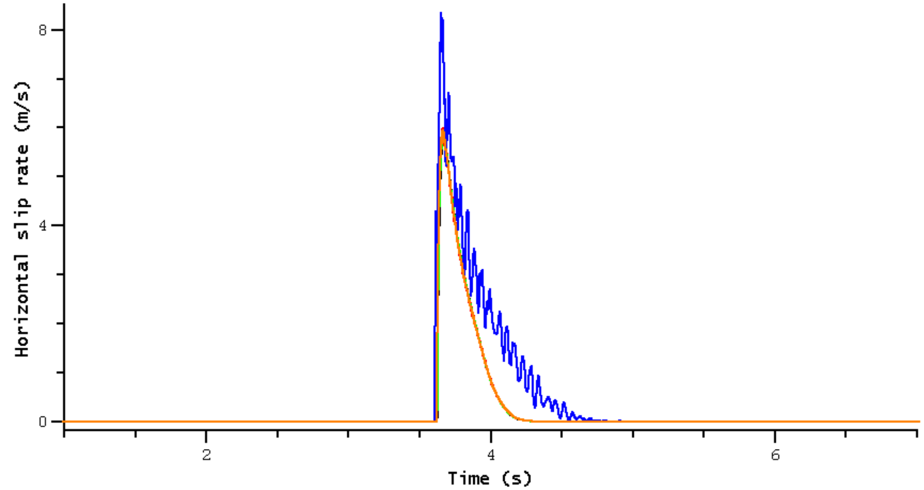
even with TP, melting is expected
(with these parameters)







1-7 s only



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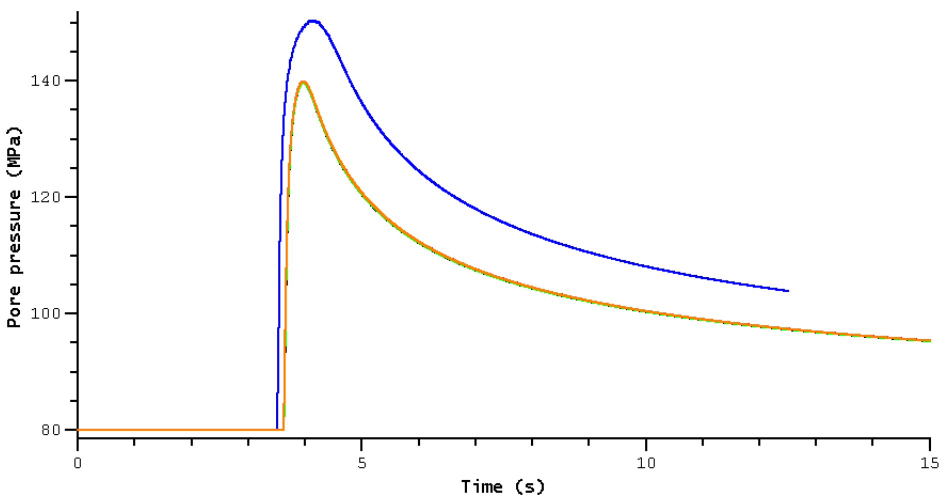
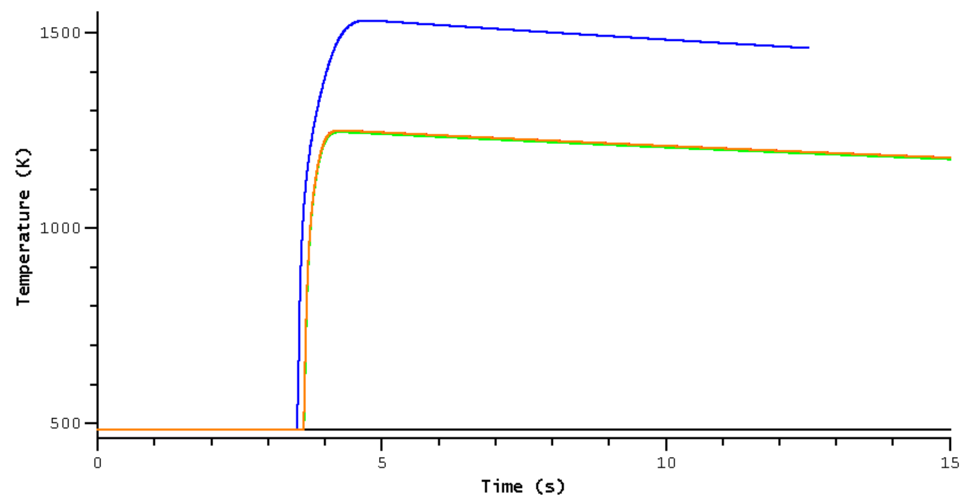
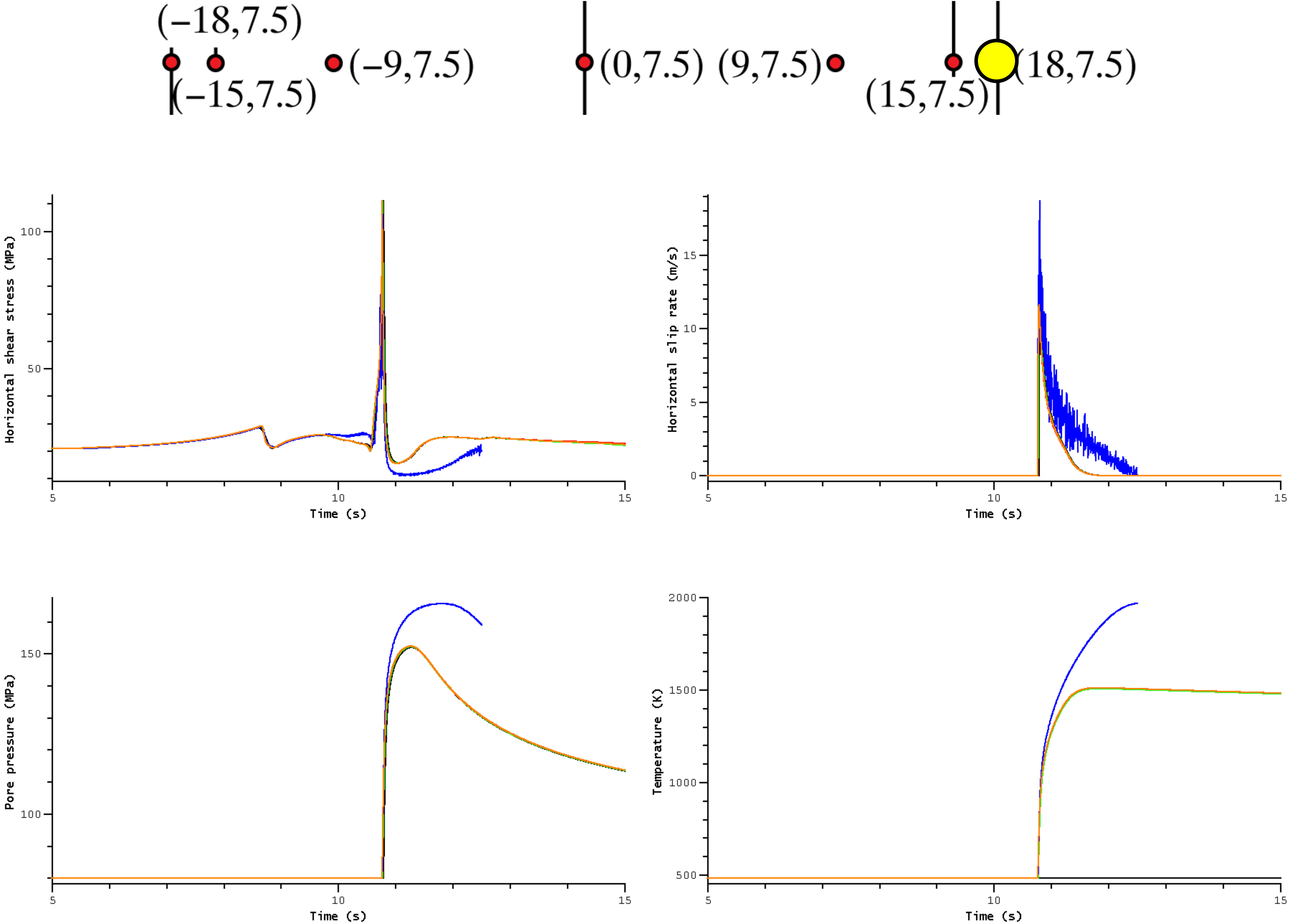


full 15 s



full 15 s



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(−9,7.5)
(0,7.5)
(9,7.5)
(15,7.5)
(18,7.5)
Horizontal shear stress (MPa)
Time (s)
Horizontal slip rate (m/s)
Time (s)
Pore pressure (MPa)
Time (s)
Temperature (K)
Time (s)

Conclusions

- All TP solvers (finite differences, Green’s functions, spectral representation) work
- Accuracy primarily limited by elastodynamics (for these parameters, at least)
- Much variability in efficiency and flexibility of TP solvers
  - FD: can handle nonlinearities and evolving properties, but inefficient (at least in current implementation)
  - Green’s functions: linearized equations only, efficiency decreases with number of time steps
  - spectral representation: linearized equations only, very efficient
- Moving forward:
  - More realistic parameter choices (no melting)
  - TPV105-3D?