SCEC-USGS Dynamic Rupture Code Validation Workshop
University of Southern California, Feb 2012

A Discontinuous Galerkin Approach for 3D Dynamic Rupture Modeling - Towards heterogeneous initial stress conditions

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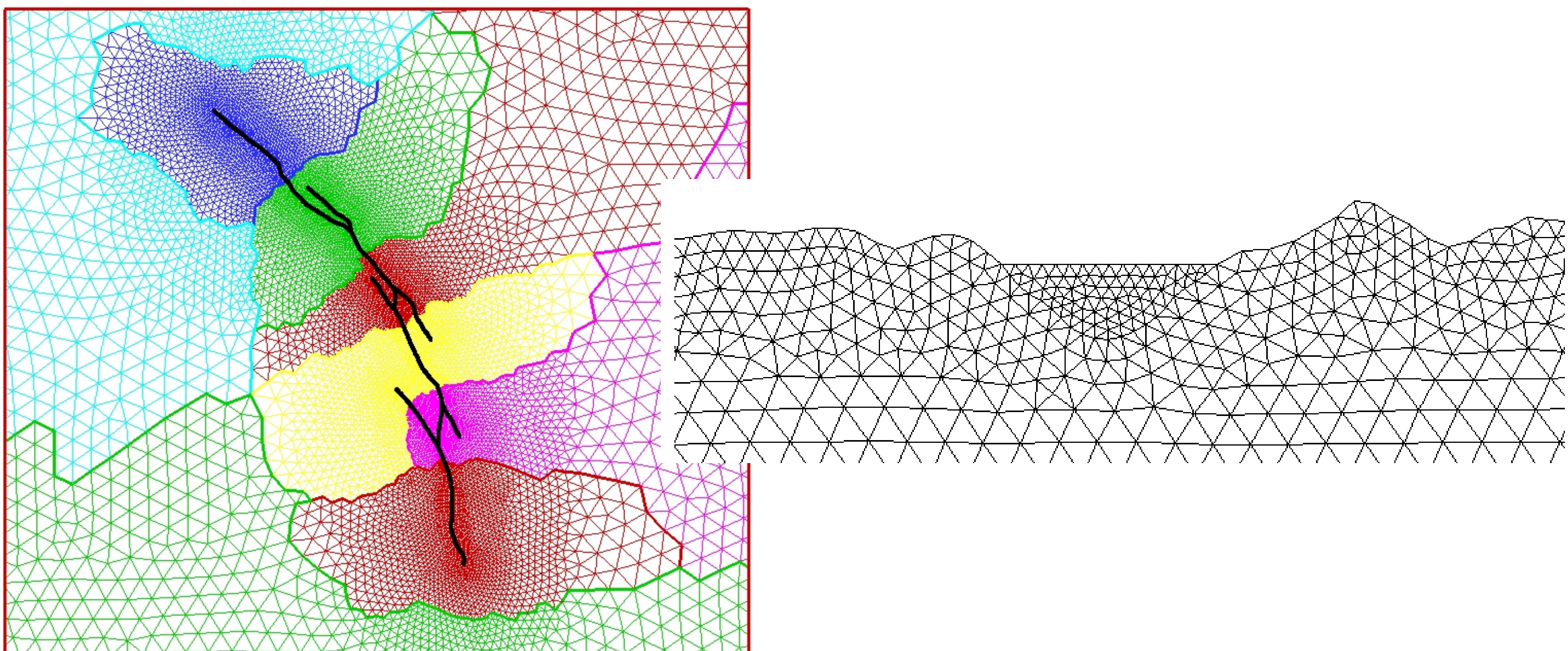
Why with the ADER-DG approach?

- High-accurate results of the rupture process – no spurious oscillations in the spectra
- Enables use of unstructured meshes – curved or kinked faults, branching, surface rupture, fault interaction
- High-accurate simulation of the wave propagation including heterogeneous media and topography (*hp*-adaptivity, ADER high-order time integration, local time stepping)
- Excellent scalability (32k+) - large scale strong ground motion simulations



2D wave equations in velocity-stress formulation

$$\frac{\partial}{\partial t}\sigma_{xx} - (\lambda + 2\mu)\frac{\partial}{\partial x}u - \lambda\frac{\partial}{\partial y}v = 0\,,$$

$$\frac{\partial}{\partial t}\sigma_{yy} - \lambda\frac{\partial}{\partial x}u - (\lambda + 2\mu)\frac{\partial}{\partial y}v = 0\,,$$

$$\frac{\partial}{\partial t}\sigma_{xy} - \mu(\frac{\partial}{\partial x}v + \frac{\partial}{\partial y}u) = 0\,,$$

$$\rho\frac{\partial}{\partial t}u - \frac{\partial}{\partial x}\sigma_{xx} - \frac{\partial}{\partial y}\sigma_{xy} = 0\,,$$

$$\rho\frac{\partial}{\partial t}v - \frac{\partial}{\partial x}\sigma_{xy} - \frac{\partial}{\partial y}\sigma_{yy} = 0\,,$$

more compact form:

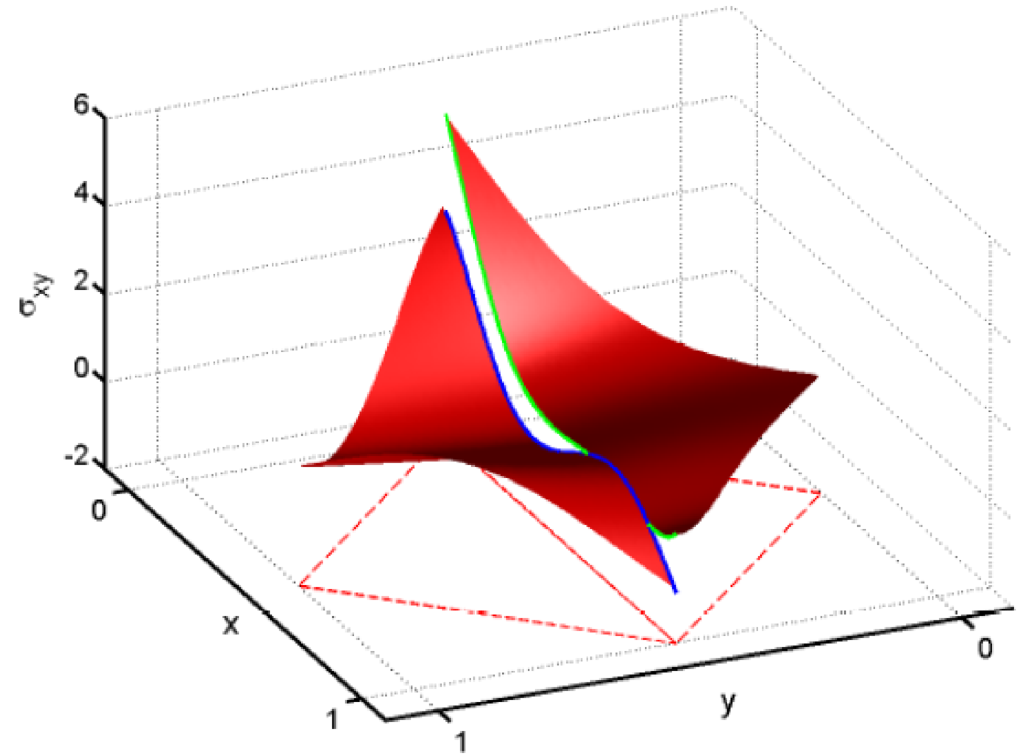
$$\frac{\partial Q_p}{\partial t} + A_{pq}\frac{\partial Q_q}{\partial x} + B_{pq}\frac{\partial Q_q}{\partial y} = 0$$ with $\mathbf{Q} = (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, u, v)^T$

Discontinuous Galerkin Approach

Numerical approximation of the solution:

$$\left(Q_h^{(m)}\right)_p(\xi,\eta,t) = \hat{Q}_{pl}^{(m)}(t)\Phi_l(\xi,\eta)$$

- Φ_l are othogonal basis functions
- diagonal mass matrix



Integrating the governing equations in space and time in the Discontinuous Galerkin (DG) framework gives

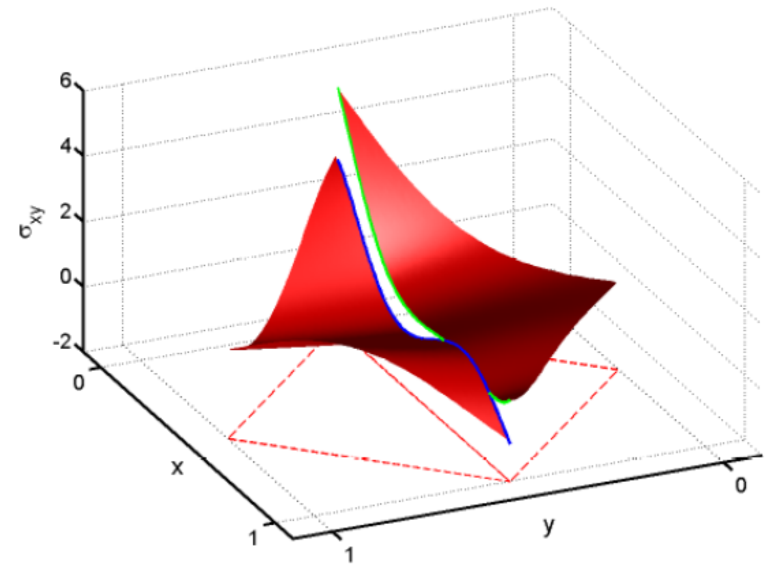
$$\int_t^{t+\Delta t}\int_{\mathcal{T}^{(m)}} \Phi_k \frac{\partial Q_p}{\partial t}\, dV\, dt + \sum_{j=1}^{3} \mathcal{F}_{pk}^{j} - \int_t^{t+\Delta t}\int_{\mathcal{T}^{(m)}} \left(\frac{\partial \Phi_k}{\partial x} A_{pq} + \frac{\partial \Phi_k}{\partial y} B_{pq}\right) Q_q\, dV\, dt = 0$$

where the numerical flux is given by

$$\mathcal{F}_{pk} = A_{pr} \int_t^{t+\Delta t}\int_S \Phi_k \tilde{Q}_r\, dS\, dt$$

Riemann problem

Standard wave propagation!



The state of the variables at the interface are given as

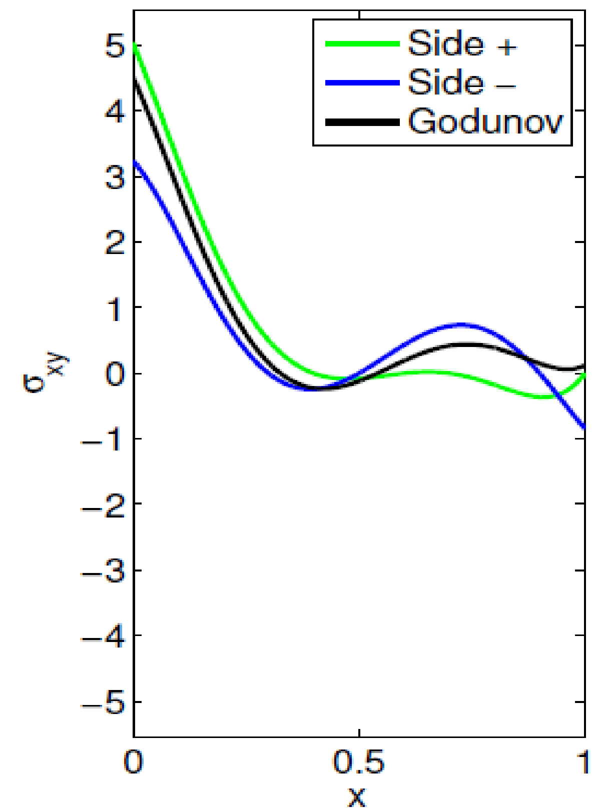
$$2\sigma_{xx}^G = (\sigma_{xx}^- + \sigma_{xx}^+) + \frac{\lambda + 2\mu}{c_p}(u^- - u^+) \ ,$$

$$2\sigma_{yy}^G = \frac{\lambda}{c_p}(u^- - u^+) + \frac{\lambda}{\lambda + 2\mu}(\sigma_{xx}^- + \sigma_{xx}^+) + 2\sigma_{yy}^+ \ ,$$

$$2\sigma_{xy}^G = (\sigma_{xy}^- + \sigma_{xy}^+) + \frac{\mu}{c_s}(v^- - v^+) \ ,$$

$$2u^G = (u^- + u^+) + \frac{c_p}{\lambda + 2\mu}(\sigma_{xx}^- - \sigma_{xx}^+) \ ,$$

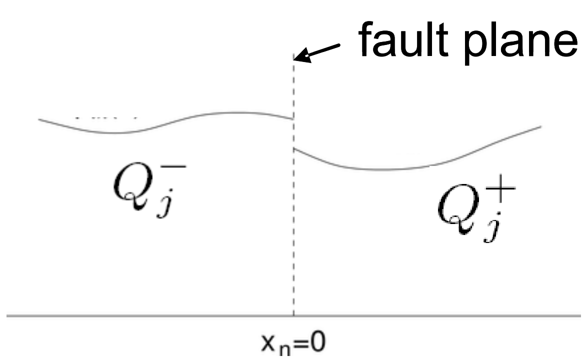
$$2v^G = (v^- + v^+) + \frac{c_s}{\mu}(\sigma_{xy}^- - \sigma_{xy}^+) \ ,$$



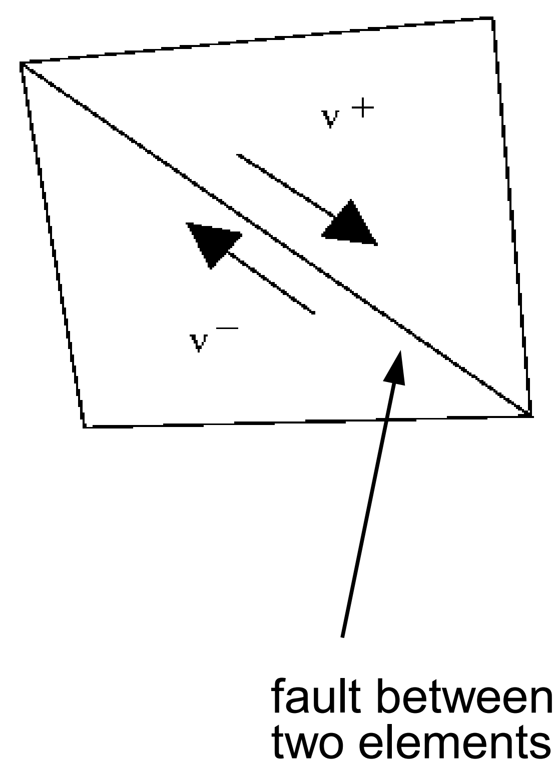
Dynamic Rupture within the Discontinuous Galerkin Approach

Treat dynamic rupture as a <u>boundary condition</u> using the flux term - solve inverse Riemann problem:

- flux provides information at element interface
- solve friction law
- in case of slip, impose traction and fault parallel velocities

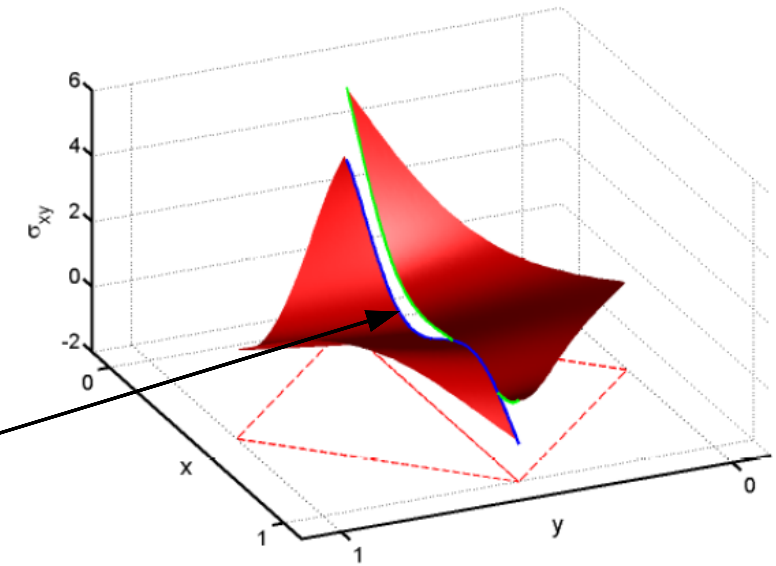


(de la Puente *et al.*, 2009)



Riemann problem

At fault interface!



The state of the variables at the interface are given as

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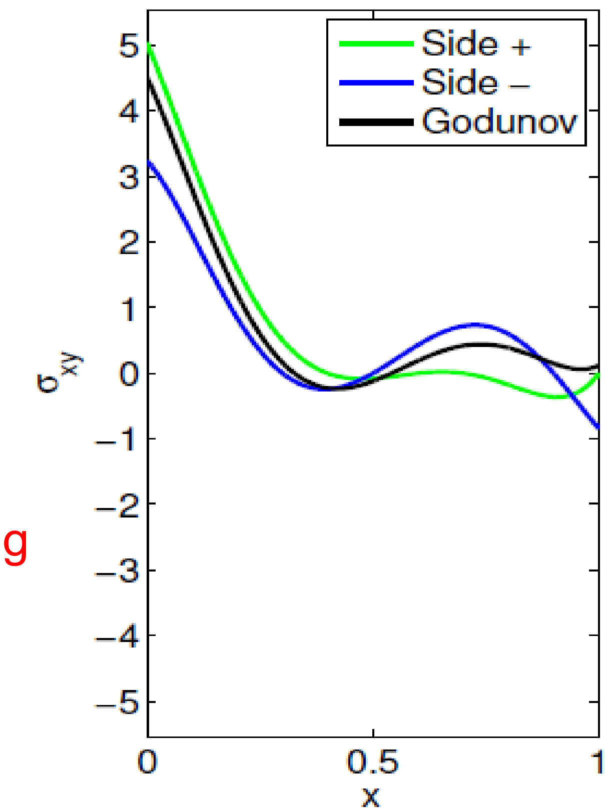
$$2\sigma_{xy}^G = (\sigma_{xy}^- + \sigma_{xy}^+) + \frac{\mu}{c_s}(v^- - v^+) \ ,$$

$$2u^G = (u^- + u^+) + \frac{c_p}{\lambda + 2\mu}(\sigma_{xx}^- - \sigma_{xx}^+) \ ,$$

$$2v^G = (v^- + v^+) + \frac{c_s}{\mu}(\sigma_{xy}^- - \sigma_{xy}^+) \ ,$$

Take imposed values regarding failure criterion!



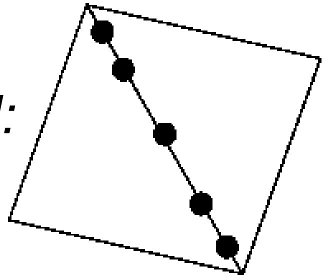
To get the imposed state vector Q_{il} we follow three steps:

1. Evaluate failure criterion & impose traction $\tilde{\sigma}_{xy}$

Substitute the Godunov state σ_{xy}^{G} from linear elasticity with an <u>imposed traction</u> $\tilde{\sigma}_{xy}$ at the fault considering the Coulomb failure criterion!

$$\tilde{\sigma}_{xy,il} = \min\left\{\sigma_{xy,il}^{G}, \mu_{f,il}(\sigma_{xx,il}^{G} + \sigma_{xx}^{0}) - \sigma_{xy}^{0}\right\}$$

<u>Side note:</u>
This is done for each Gaussian integration point *il:*



To get the imposed state vector Q_{il} we follow three steps:

1. Evaluate failure criterion & impose traction $\tilde{\sigma}_{xy}$

2. Compute fault parallel velocities and slip rate

The imposed traction provides boundary conditions for the slip rates (velocities) on both sides!

$$\tilde{v}^{+} = v^{+} + \frac{c_s}{\mu}\left(\tilde{\sigma}_{xy} - \sigma_{xy}^{+}\right)$$

$$\tilde{v}^{-} = v^{-} - \frac{c_s}{\mu}\left(\tilde{\sigma}_{xy} - \sigma_{xy}^{-}\right)$$

The <u>imposed slip rate</u> is given by

$$\Delta\tilde{v} = \tilde{v}^{+} - \tilde{v}^{-} = \left(v^{+} - v^{-}\right) + \frac{c_s}{\mu}\left[2\tilde{\sigma}_{xy} - \left(\sigma_{xy}^{-} + \sigma_{xy}^{+}\right)\right]$$

$$\Delta\tilde{v} = \frac{2c_s}{\mu}\left(\tilde{\sigma}_{xy} - \sigma_{xy}^{G}\right)$$

➔ $\Delta\tilde{v} \neq 0$ only if $\tilde{\sigma}_{xy} \neq \sigma_{xy}^{G}$

To get the imposed state vector Q_{il} we follow three steps:

1. Evaluate failure criterion & impose traction $\tilde{\sigma}_{xy}$

2. Compute fault parallel velocities and slip rate

3. Compute slip Δd and update friction coefficent

$$\mu_f = \begin{cases} \mu_s - \dfrac{\mu_s - \mu_d}{D_c} \Delta d & \text{if } \Delta d < D_c \, , \\ \mu_d & \text{if } \Delta d \geq D_c \, . \end{cases}$$

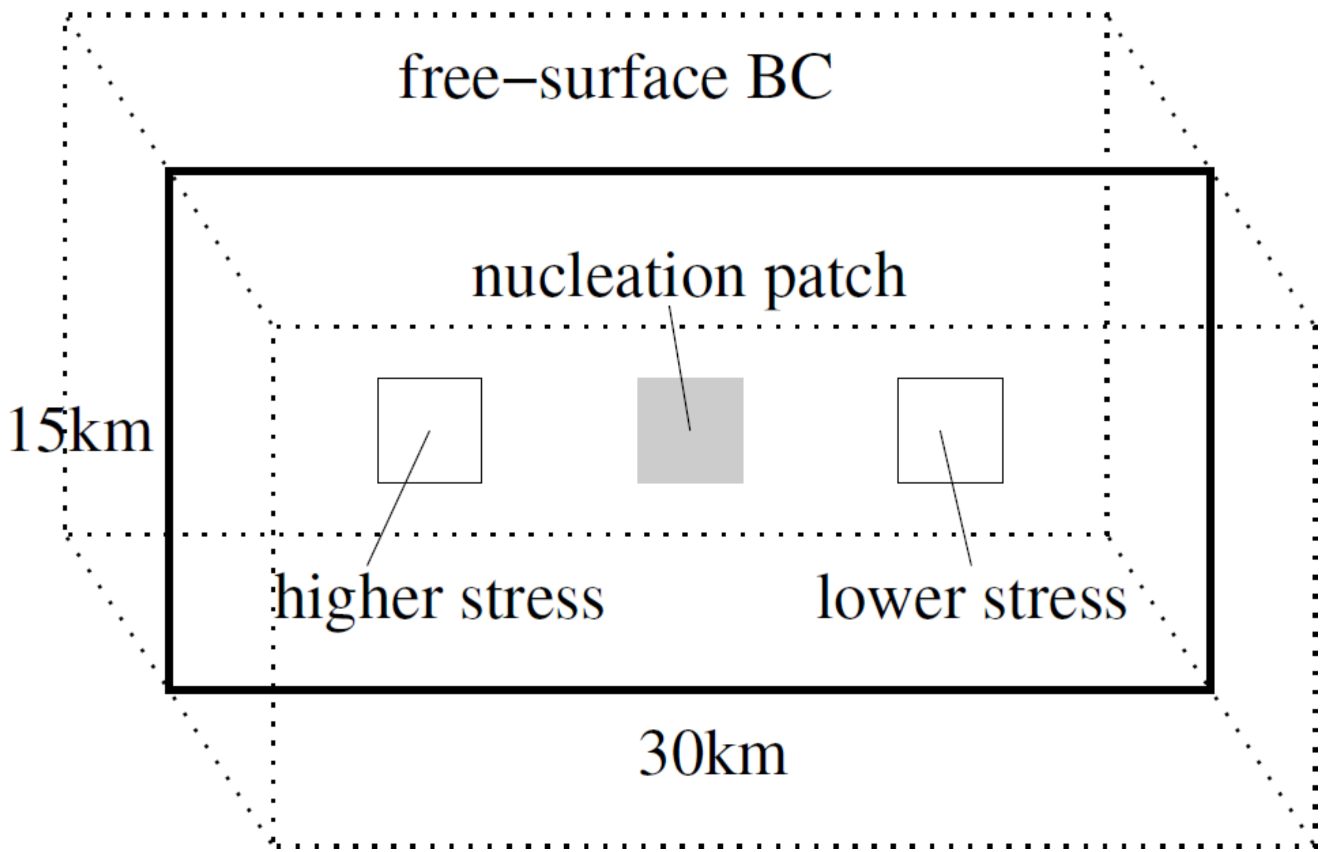
Gauss-Integration of flux:

$$\mathcal{F}_{pk} \;=\; A_{pr} \sum_{i=1}^{3N} \sum_{l=1}^{N+1} \omega_i^S \omega_l^T \Phi_k(\boldsymbol{\xi}_i) \tilde{Q}_{r,il}$$

Verification – TPV5 SCEC Test Case

(Harris *et al.*, 2009)

- spontaneous rupture propagation on a straight fault
- homogeneous fullspace
- linear slip weakening friction
- two patches of lower and higher initial stress

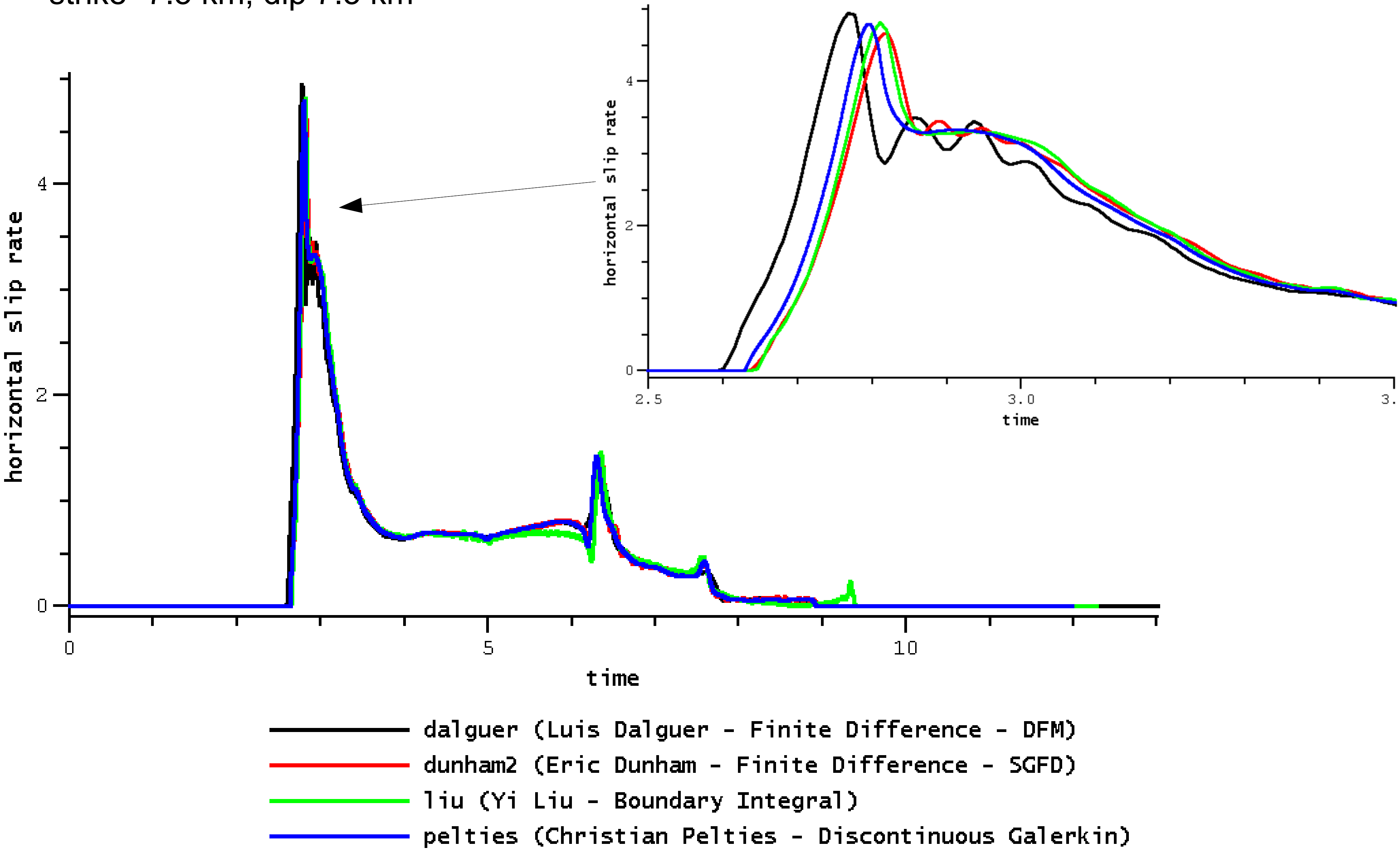


Mesh is aligned to the patches!

ADER-DG method order 4 and 200m triangles at the fault (tetrahedrons of 4000m in bulk)

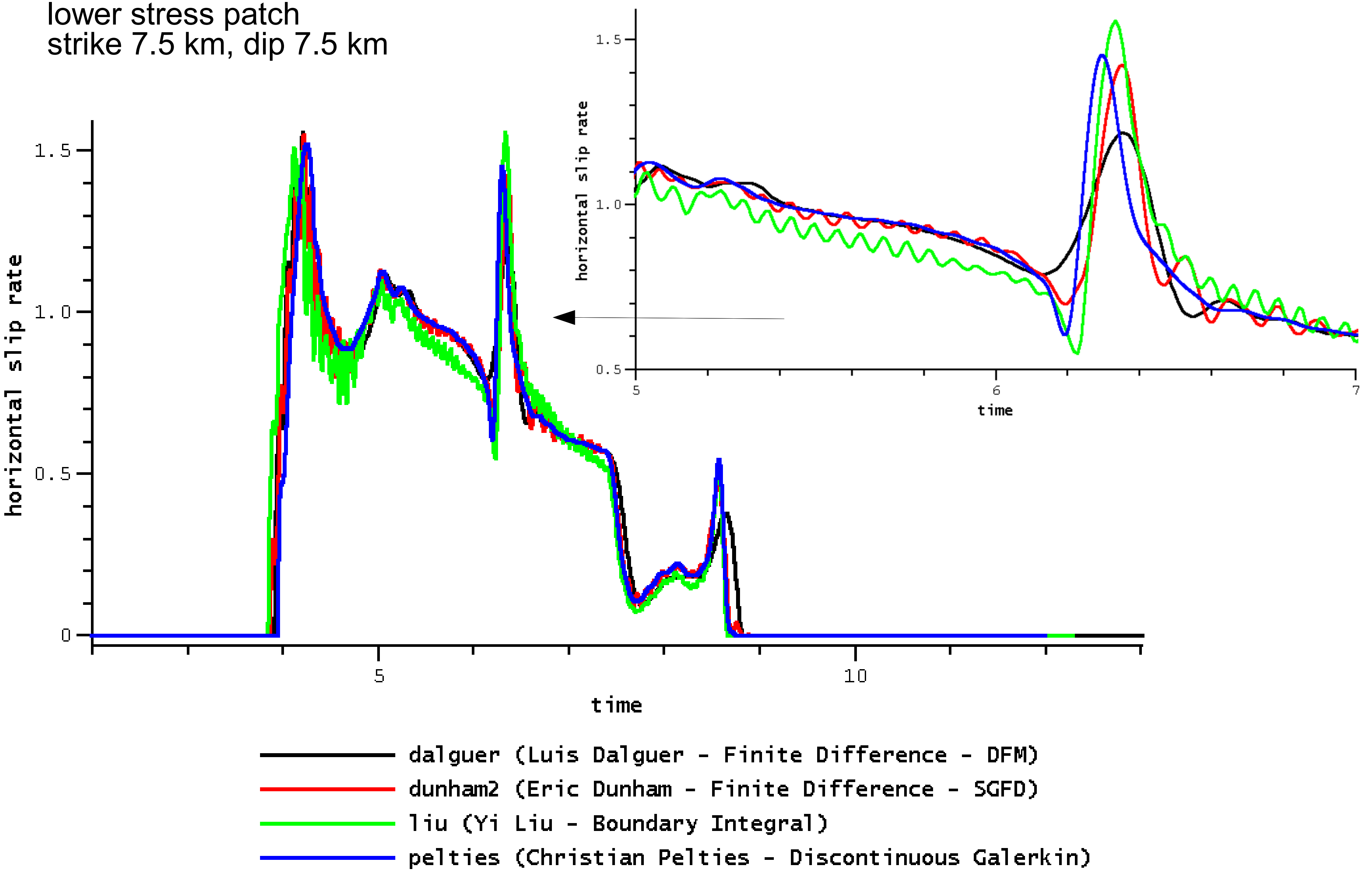
Verification – TPV5 SCEC Test Case

higher stress patch
strike -7.5 km, dip 7.5 km



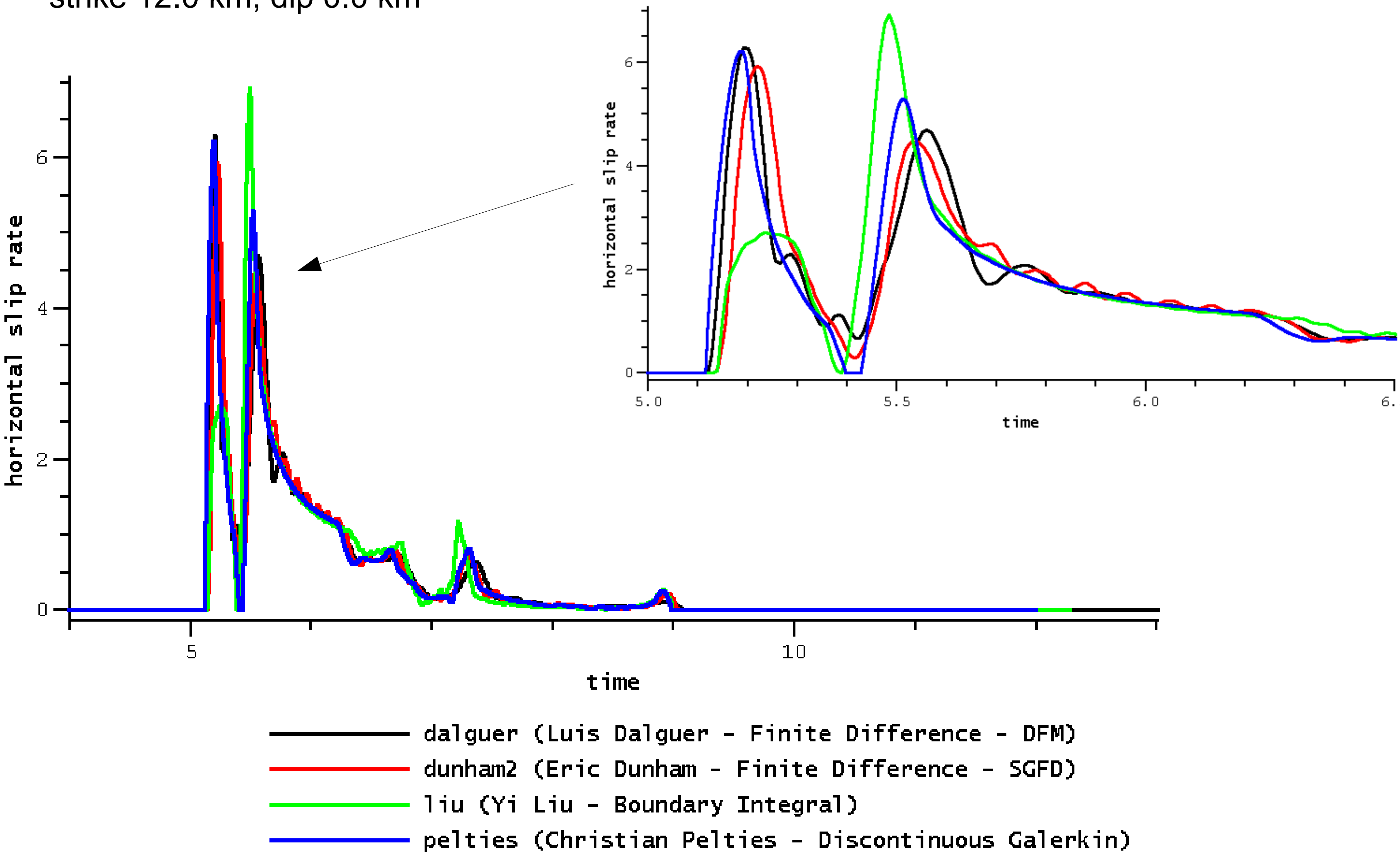
Verification – TPV5 SCEC Test Case

lower stress patch
strike 7.5 km, dip 7.5 km



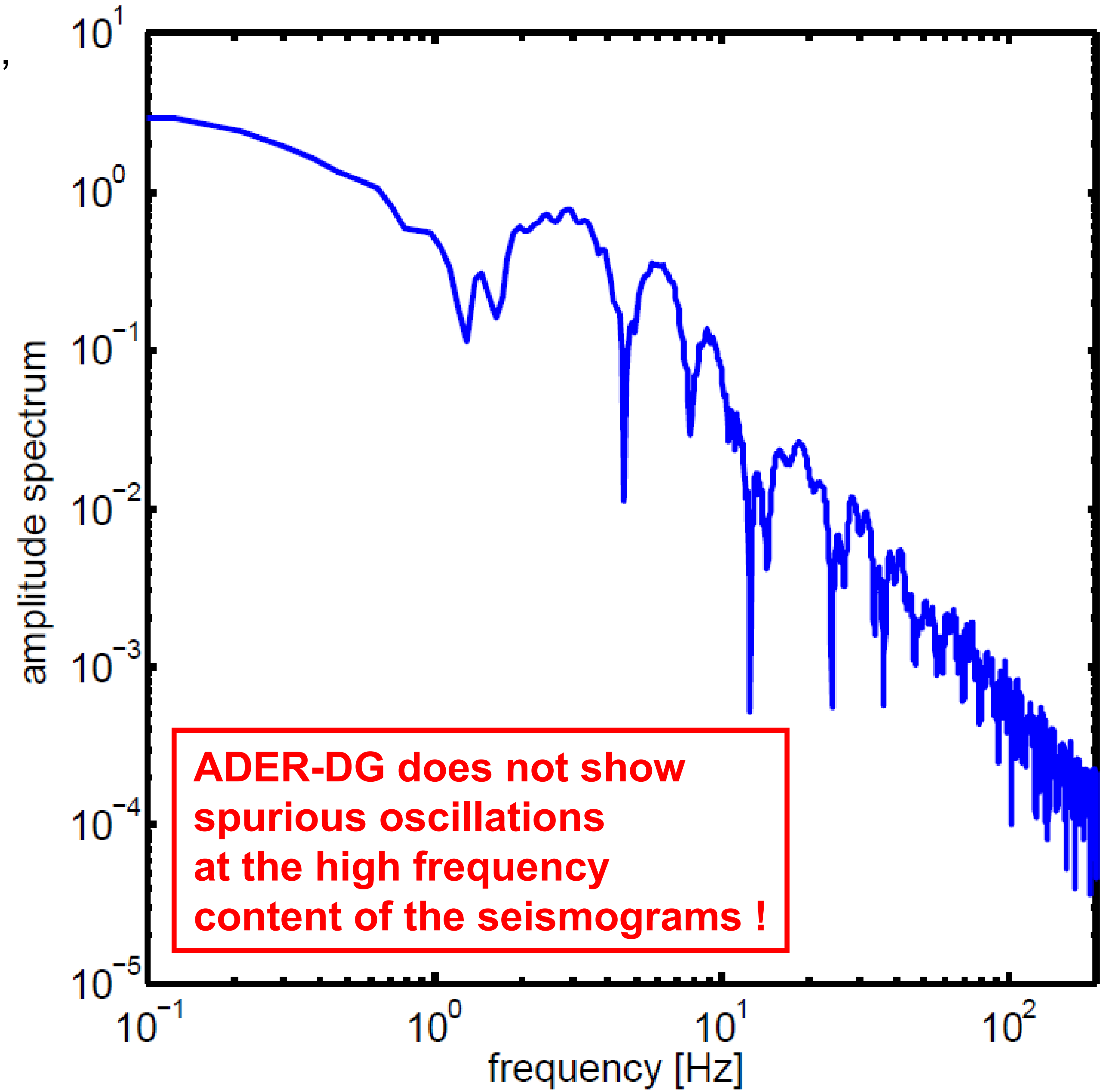
Verification – TPV5 SCEC Test Case

free-surface
strike 12.0 km, dip 0.0 km



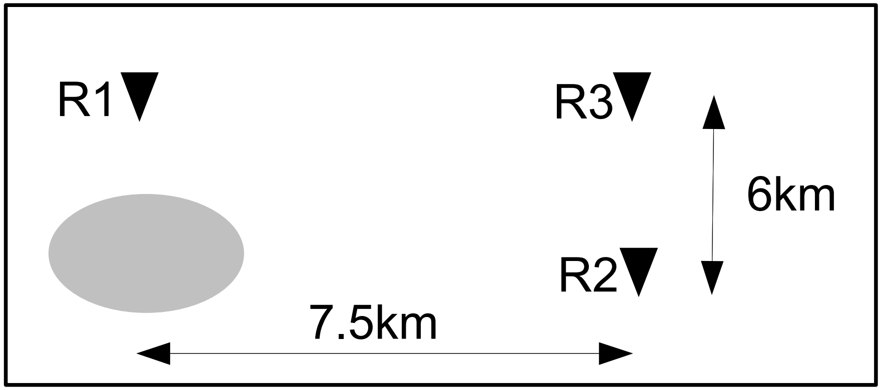
Verification – TPV5 SCEC Test Case

free-surface
strike 12.0 km,
dip 0.0 km



Smooth Elliptical Nucleation Zone Benchmark

Upper right quarter of fault with elliptical nucleation zone:

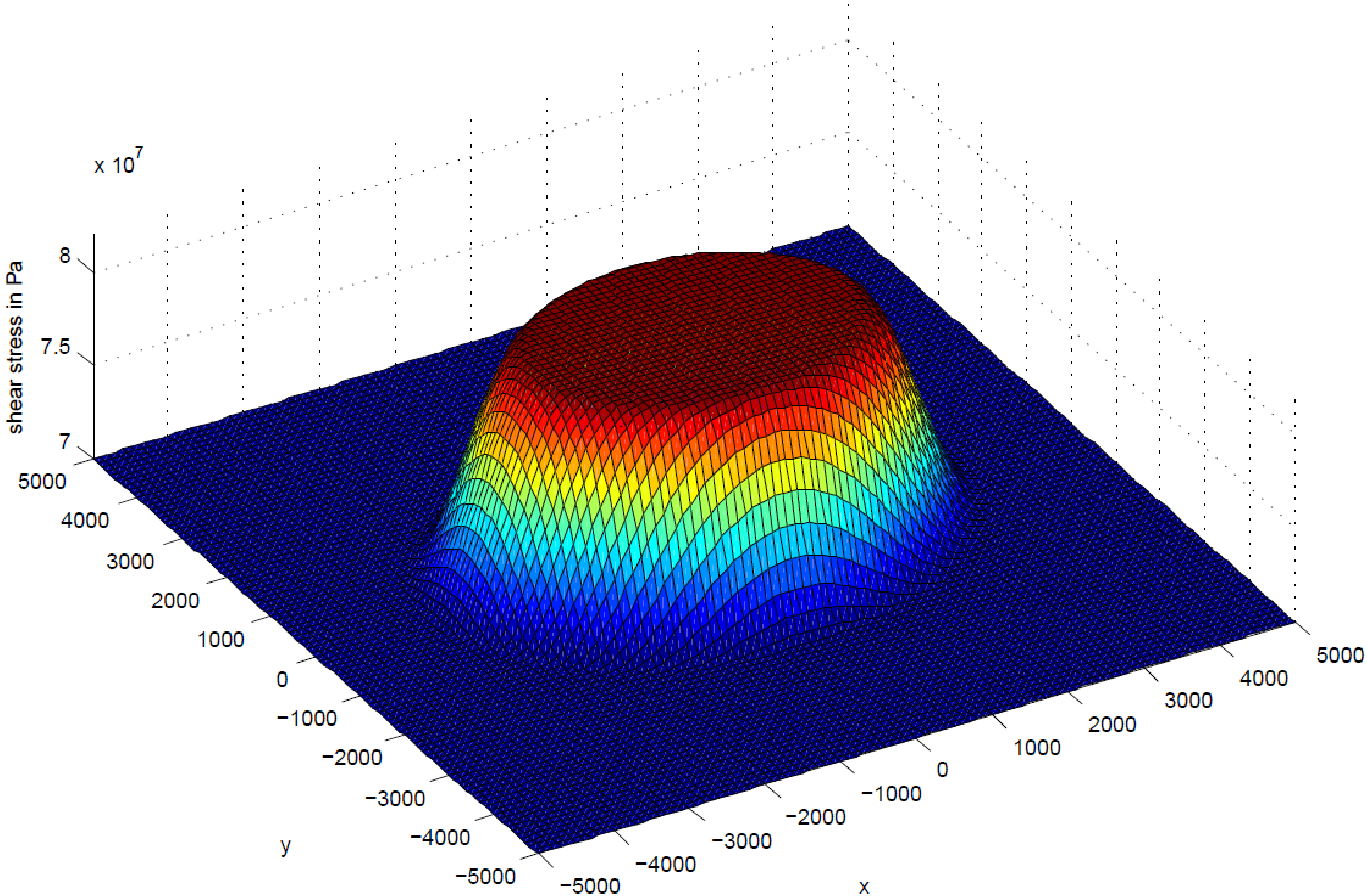


FEM-ASA: adaptive smoothing algorithm

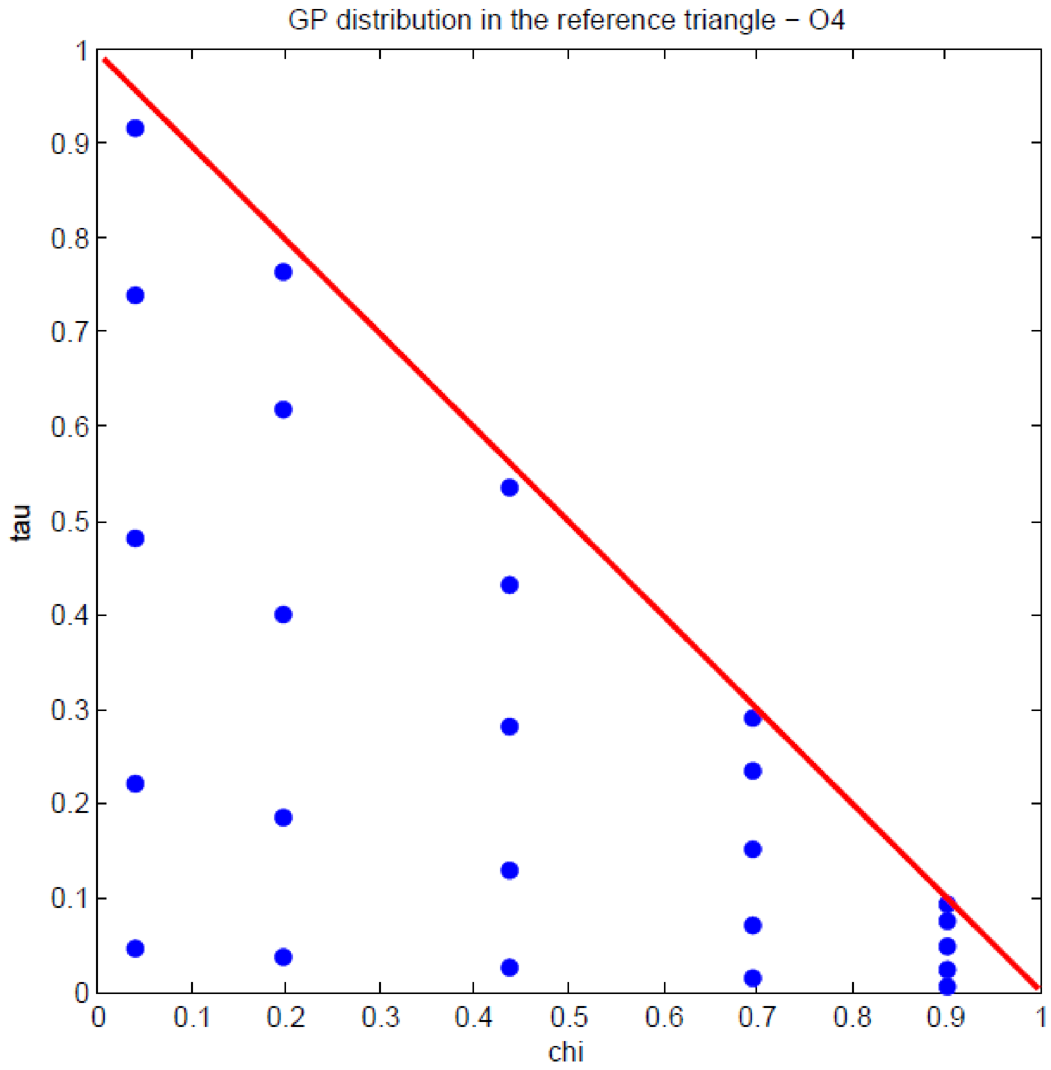
- Motivation: Reduce spurious high-frequency oscillations for FEM using TSN
- No Kelvin-Voigt damping necessary
- Applicable to standard FEM and FDM
- Ref:

Galis, M., P. Moczo, J. Kristek, and M. Kristekova (2010), An adaptive smoothing algorithm in the TSN modeling of rupture propagation with the linear slip-weakening friction law, *Geophys. J. Int.*, *180*, 418–432.

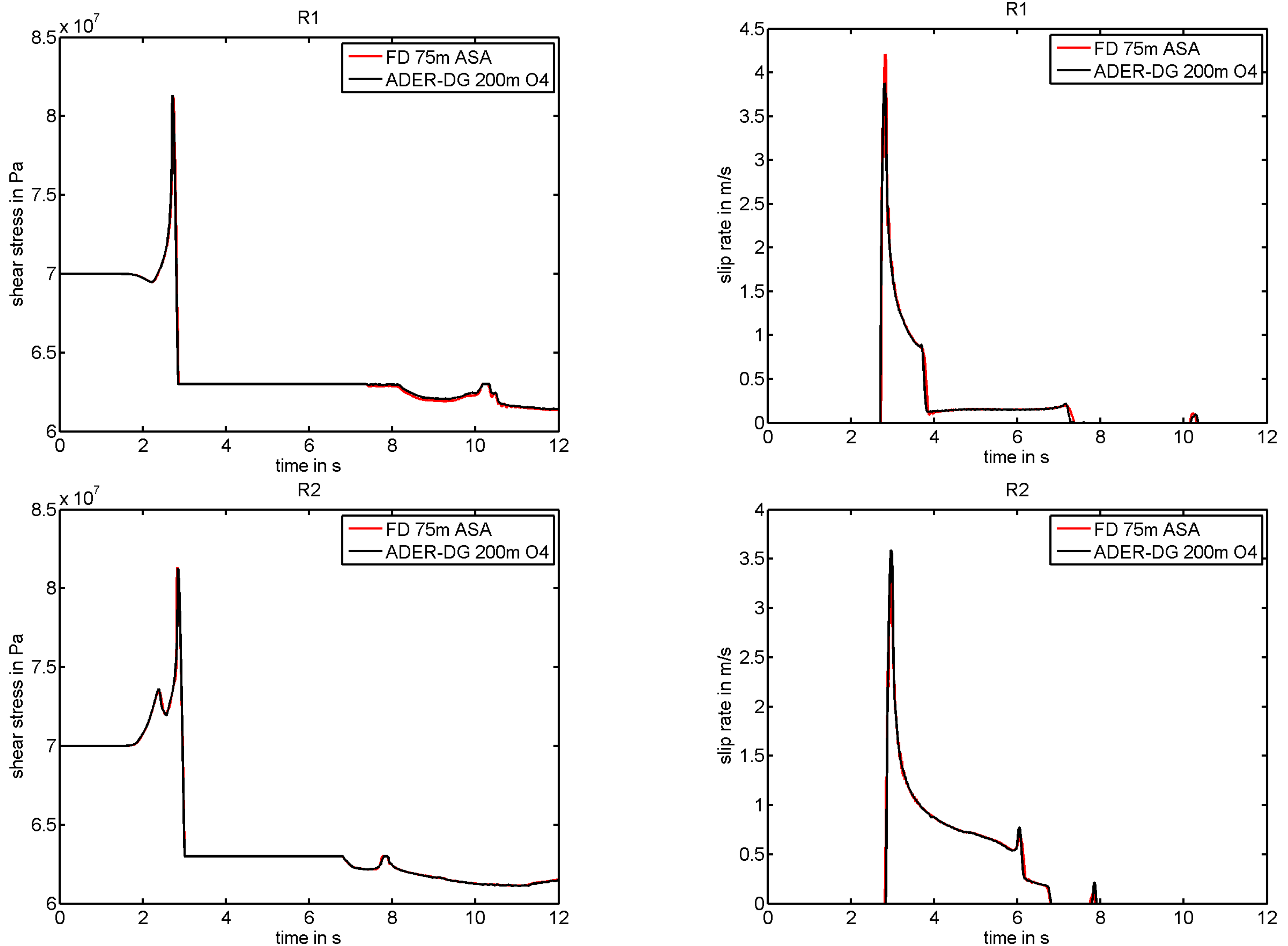
Smooth Elliptical Nucleation Zone



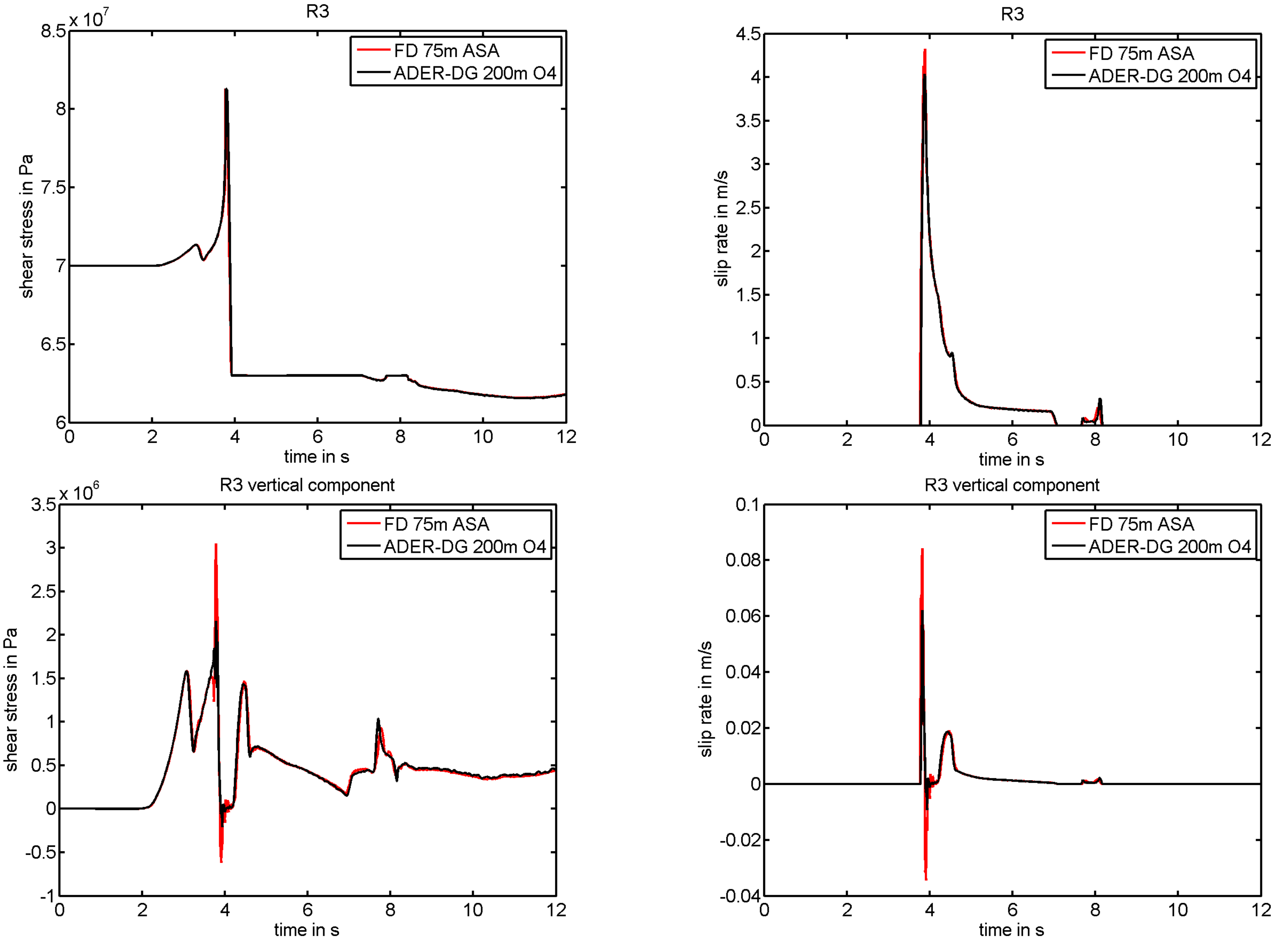
Gaussian Integration Point Location in the Reference Element



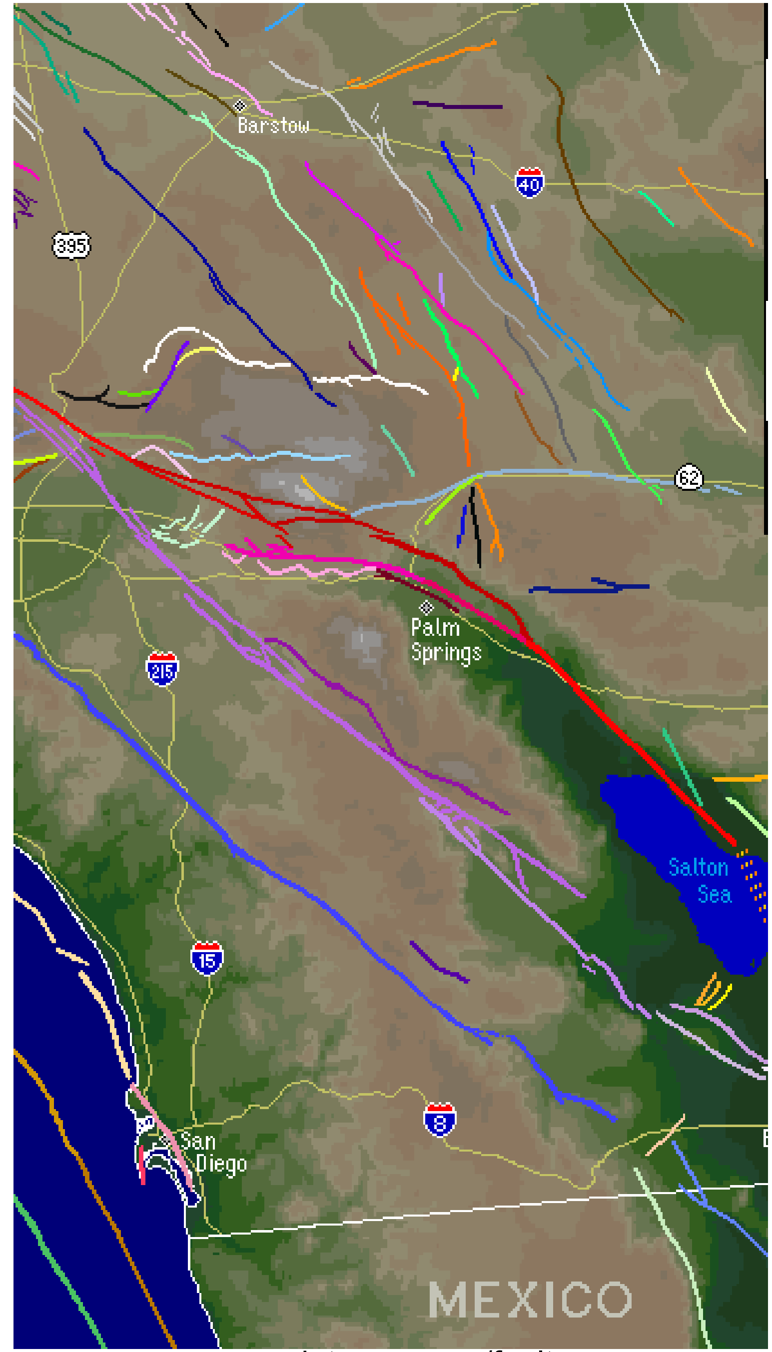
Smooth Elliptical Nucleation Zone Benchmark



Smooth Elliptical Nucleation Zone Benchmark

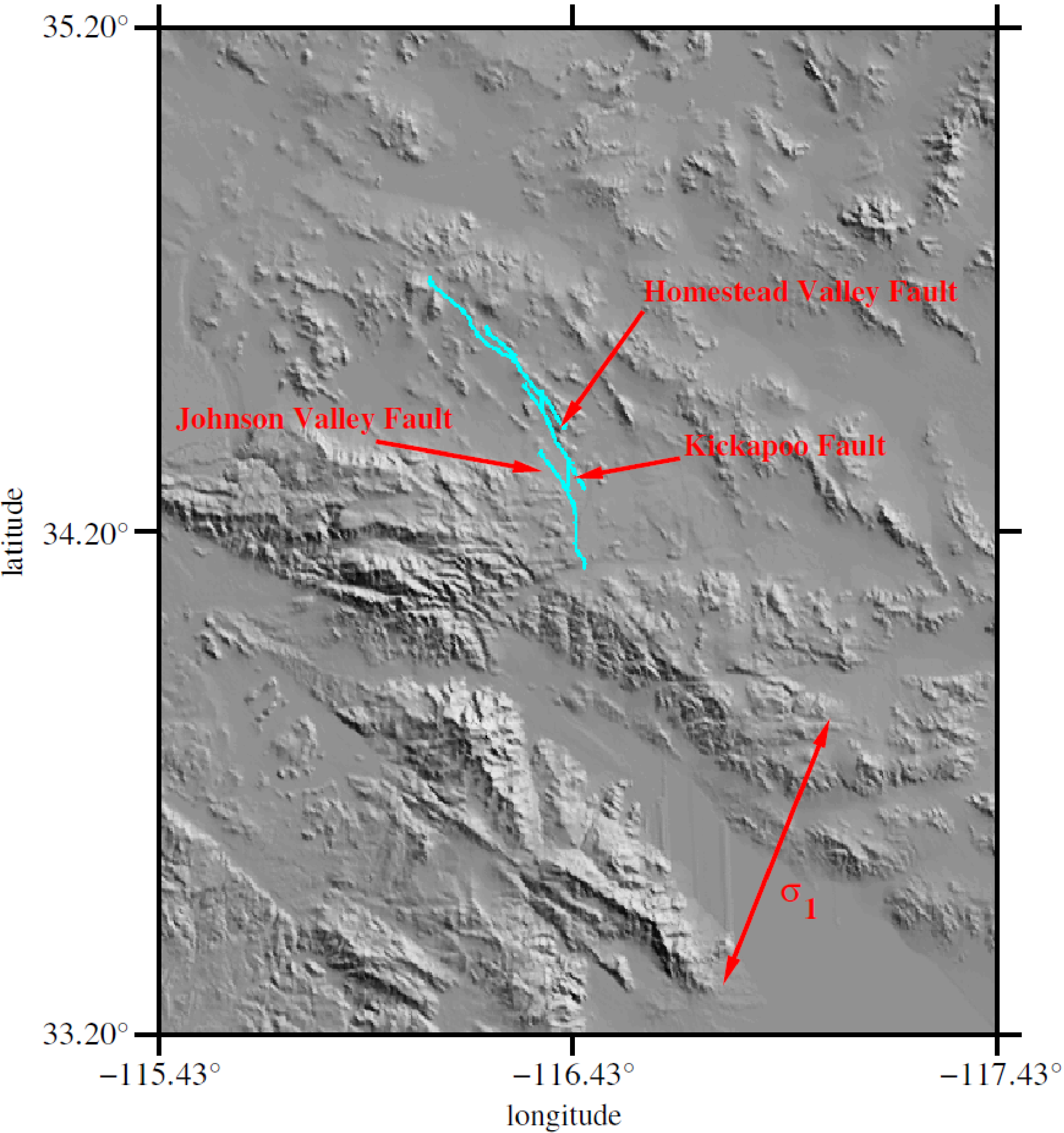


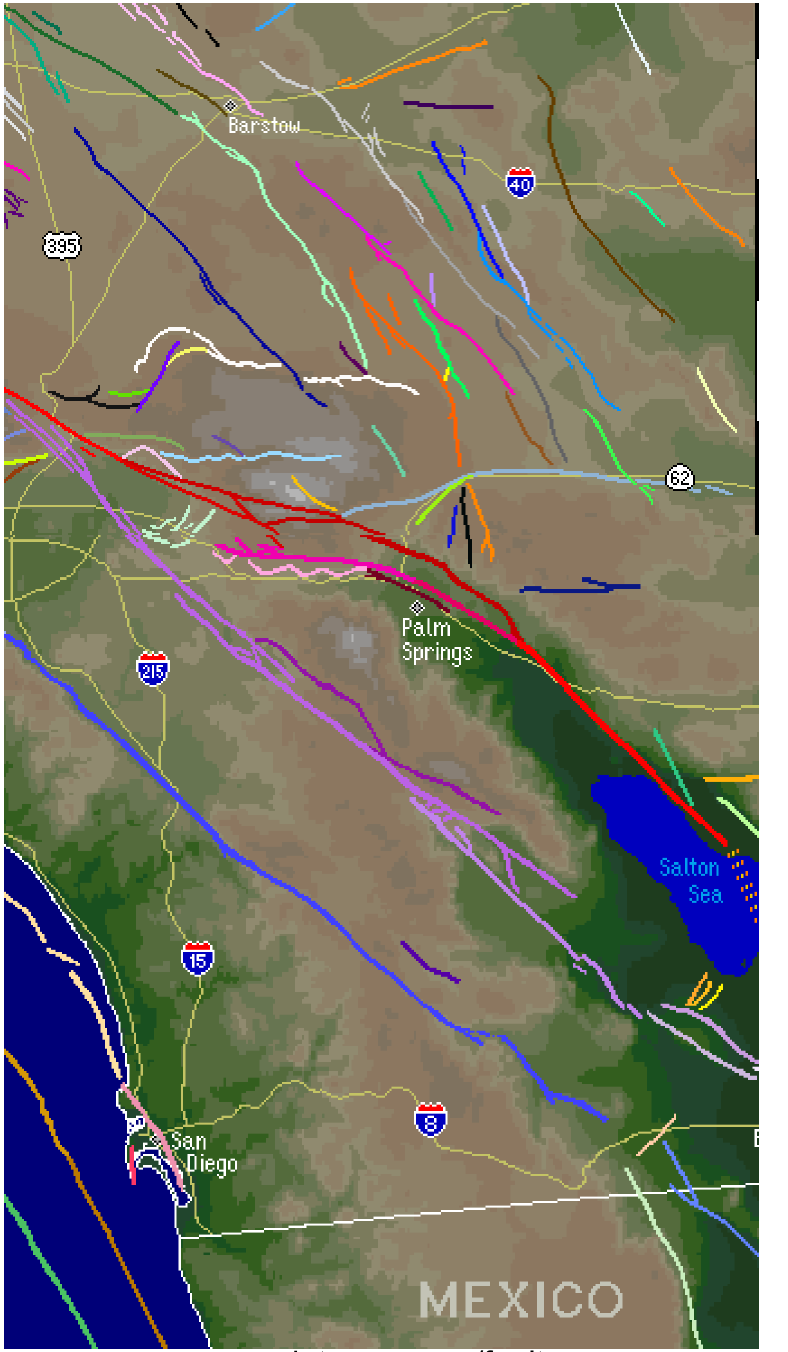
Application to the Landers Earthquake Fault System



www.data.scec.org/faults

Application to the Landers Earthquake Fault System





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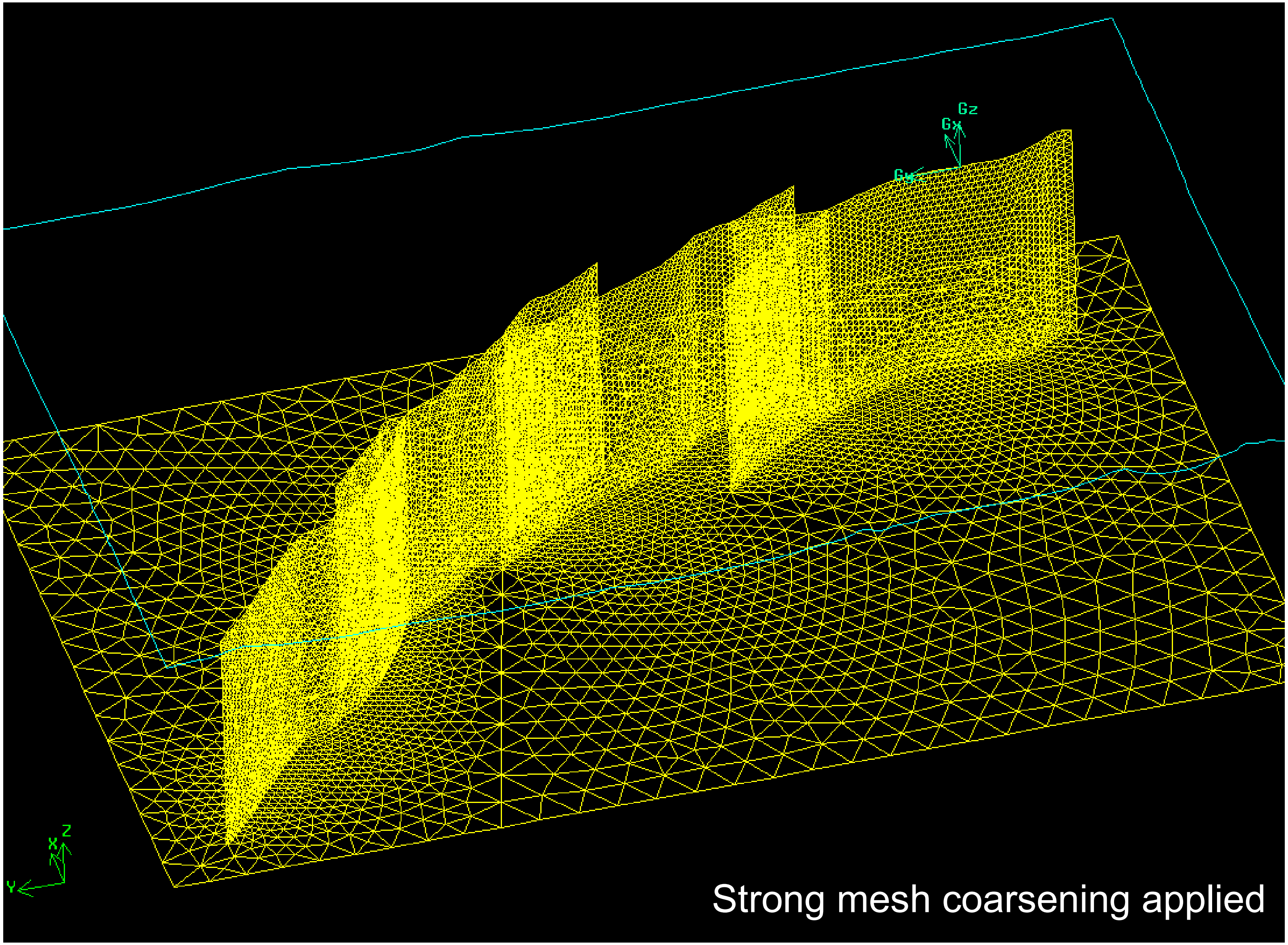

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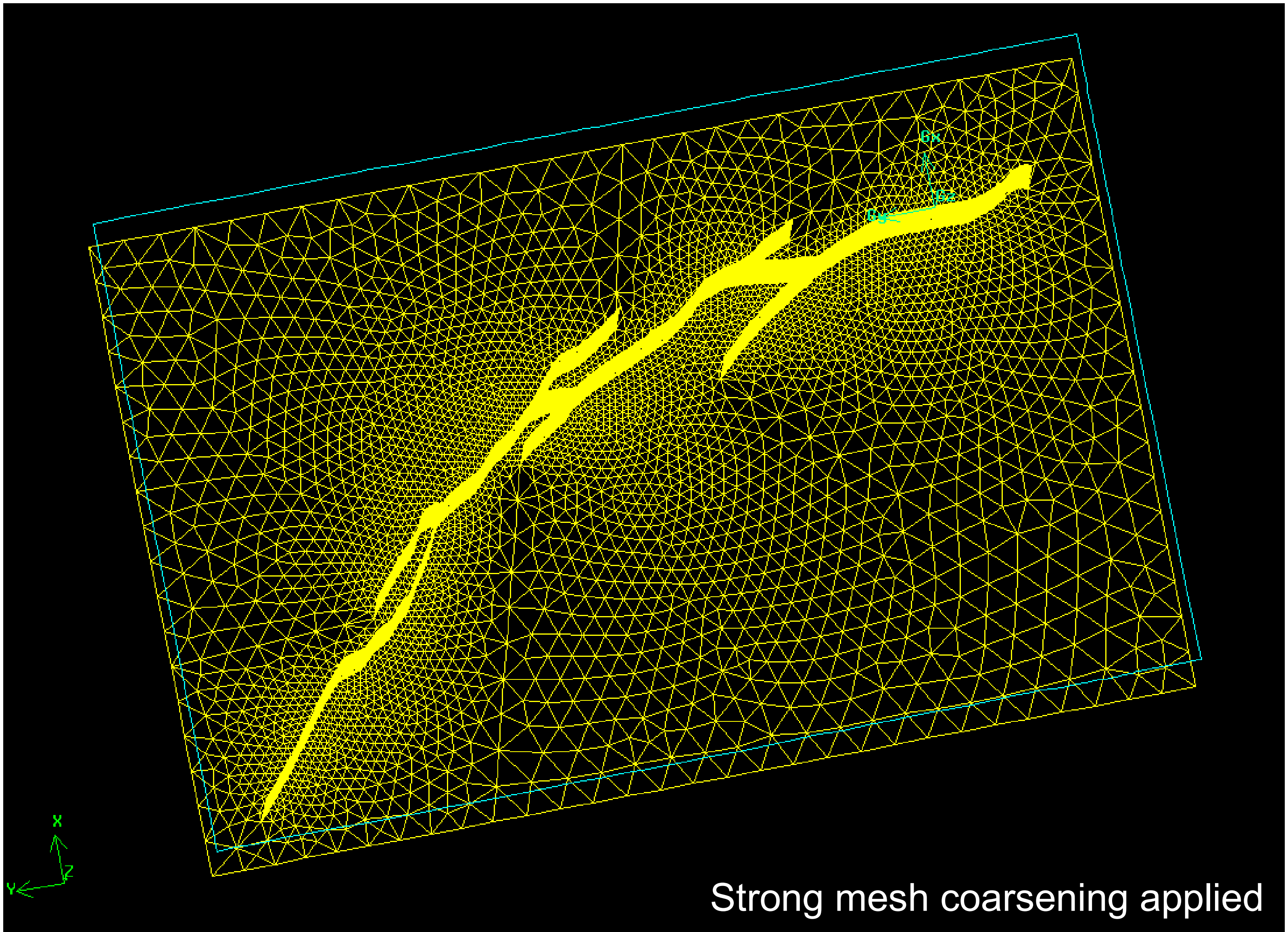
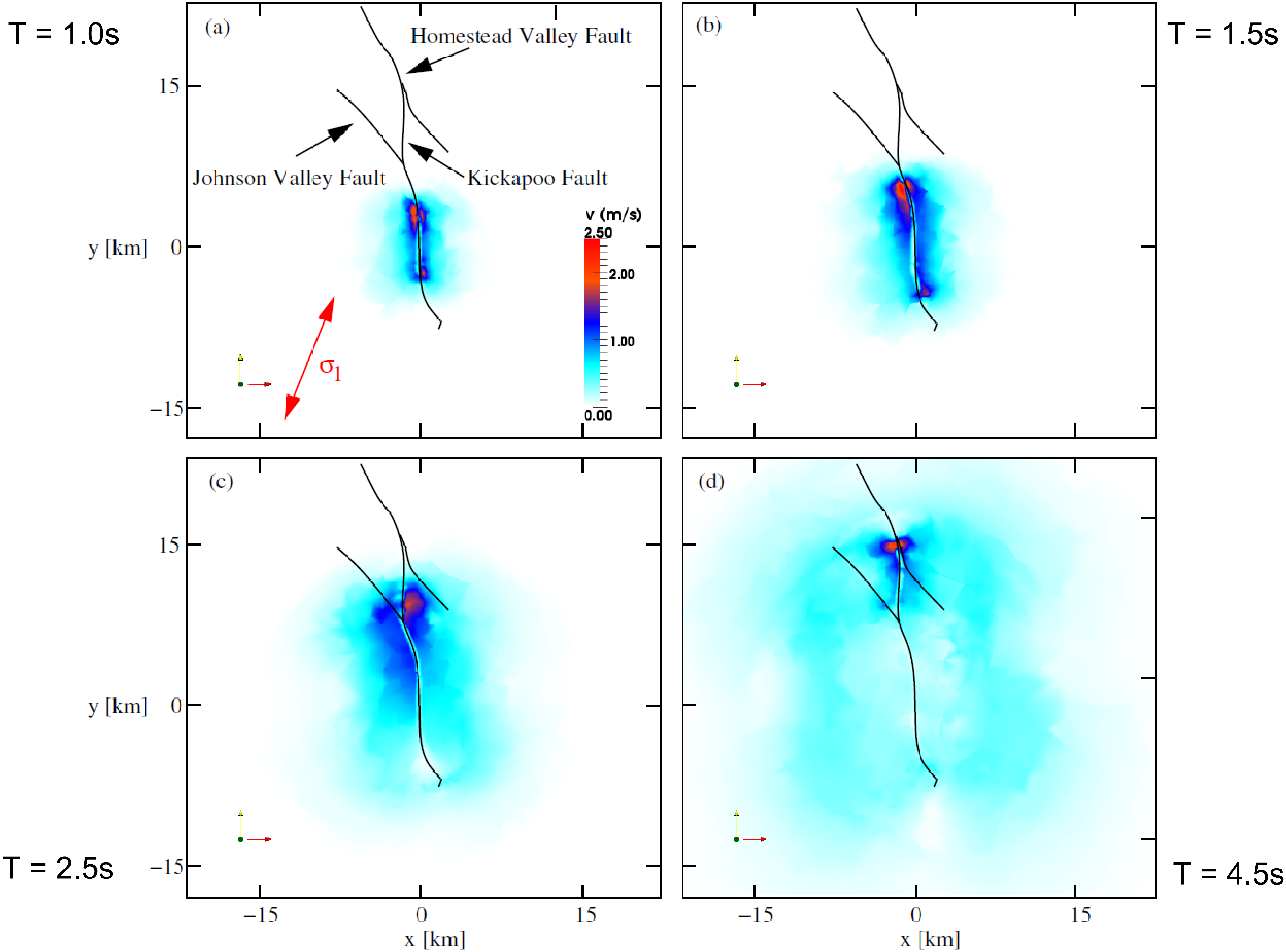
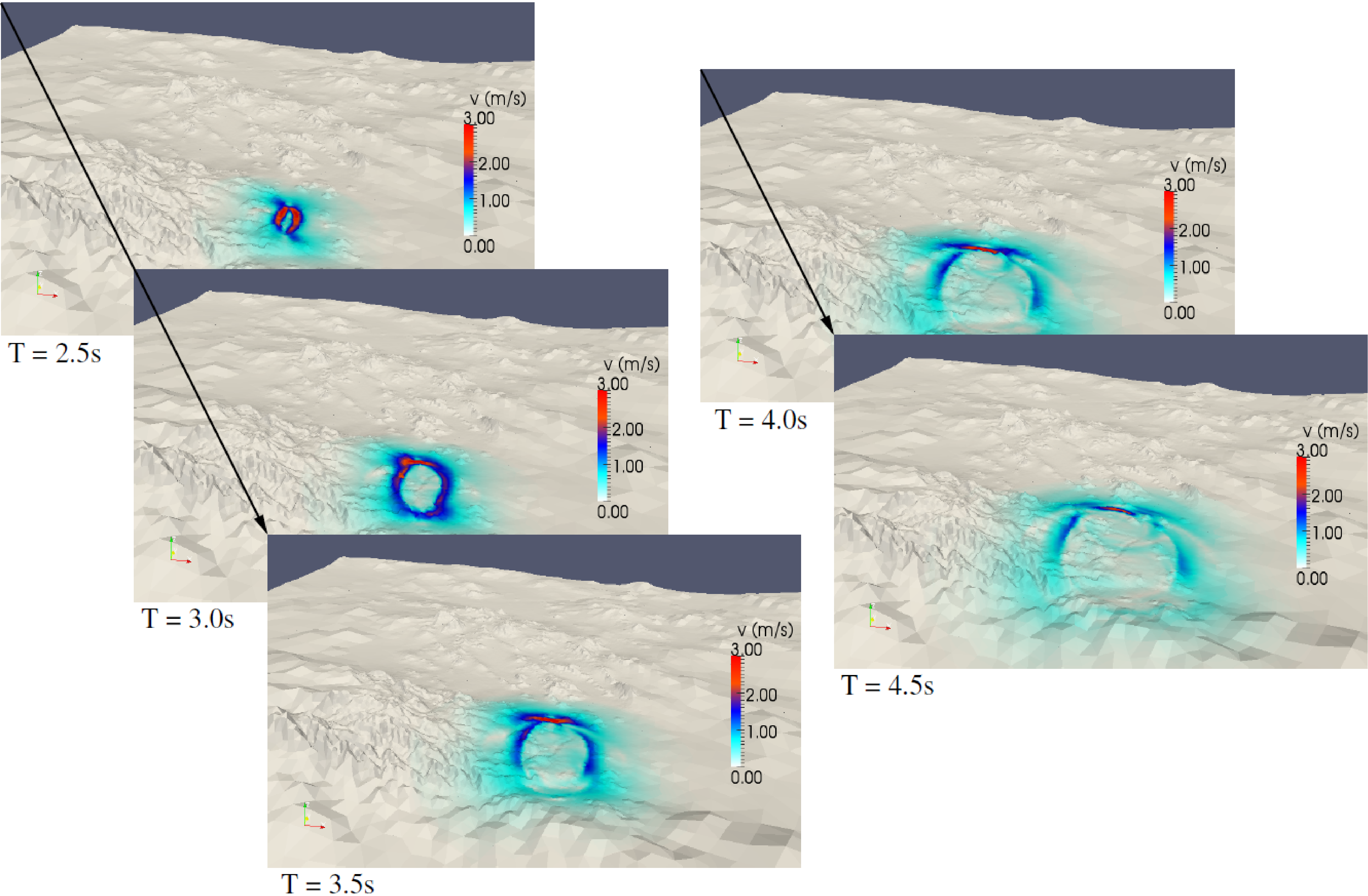




T = 1.0s
T = 1.5s
T = 2.5s
T = 4.5s
(a)
(b)
(c)
(d)
Homestead Valley Fault
Johnson Valley Fault
Kickapoo Fault
v (m/s)
2.50
2.00
1.00
0.00
σ1
y [km]
x [km]
15
0
−15

Application to the Landers Earthquake Fault System



Summary

- New approach of implementing dynamic rupture via fluxes (J. de la Puente '09)
- Verification with the SCEC test (TPV3, TPV5) + smooth nucleation zone
- Bimaterial applications under Prakash-Clifton regularization (not presented)
- Application to complex fault structures with branches (1992 Landers)
- Method should allow surface rupture, fault branching, curved and kinked faults
- No spurious high-frequency contributions in the slip rate spectra
- *hp*-adaptivity help to adjust the resolution independently to fault and bulk

Outlook:

- Completion of TPV 16/17 and TPV 18-21 is essential
- More advanced friction laws: rate-and-state
- Applications

Pelties et al., (2011), Three-Dimensional Dynamic Rupture Simulation with a High-order Discontinuous Galerkin Method on Unstructured Tetrahedral Meshes, JGR – Solid Earth, doi:10.1029/2011JB008857, in print.