Branched Fault with Plasticity Benchmarks TPV18 through TPV21

Michael Barall

SCEC Dynamic Rupture Code Validation Workshop

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TPV18-21 Summary

Benchmarks			
Benchmark	*Dimension*	*Rupture Type*	*Material Properties*
TPV18	3D	Right-lateral, releasing branch.	Linear elastic.
TPV19	3D	Right-lateral, releasing branch.	Drucker-Prager plastic.
TPV20	3D	Left-lateral, restraining branch.	Linear elastic.
TPV21	3D	Left-lateral, restraining branch.	Drucker-Prager plastic.

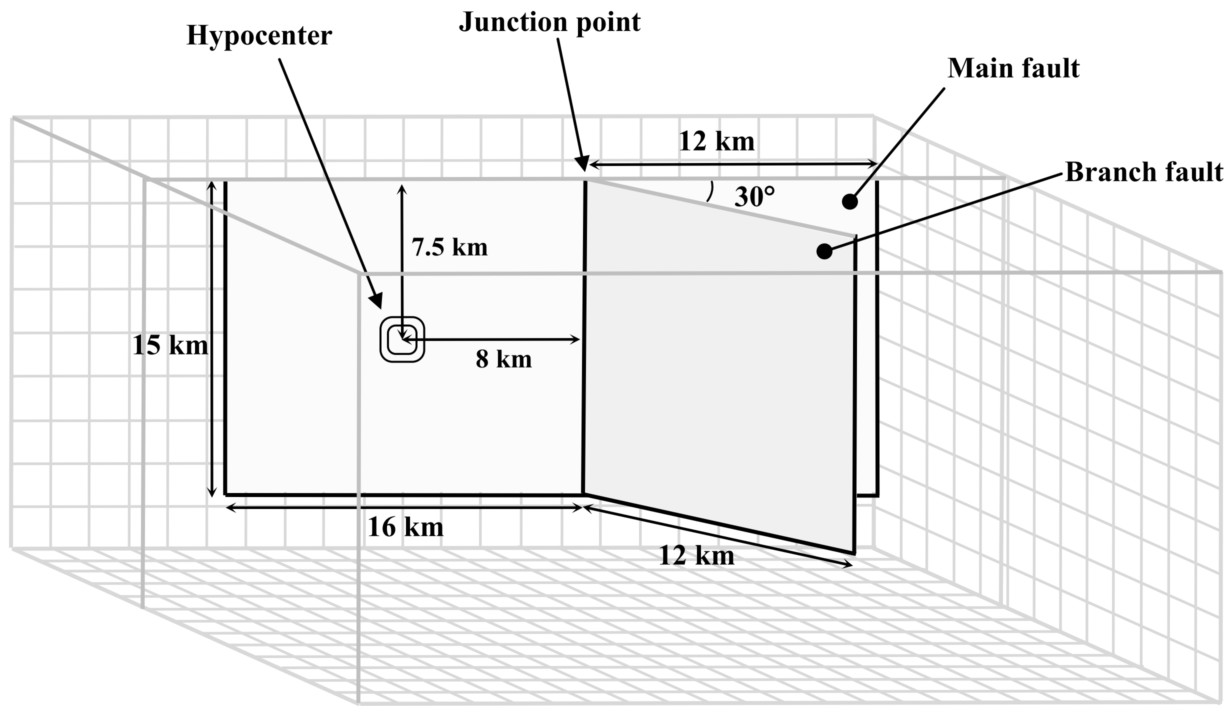
We reuse features from prior benchmarks wherever possible, so we build incrementally on prior work.

- Material properties are the same as TPV12 and TPV13.
- Fault geometry and station locations are the same as TPV14 and TPV15.
- Nucleation technique is the same as TPV16 and TPV17.

These are the most complicated benchmarks we have ever attempted!

Benchmark Description

TPV18-21 Fault Geometry



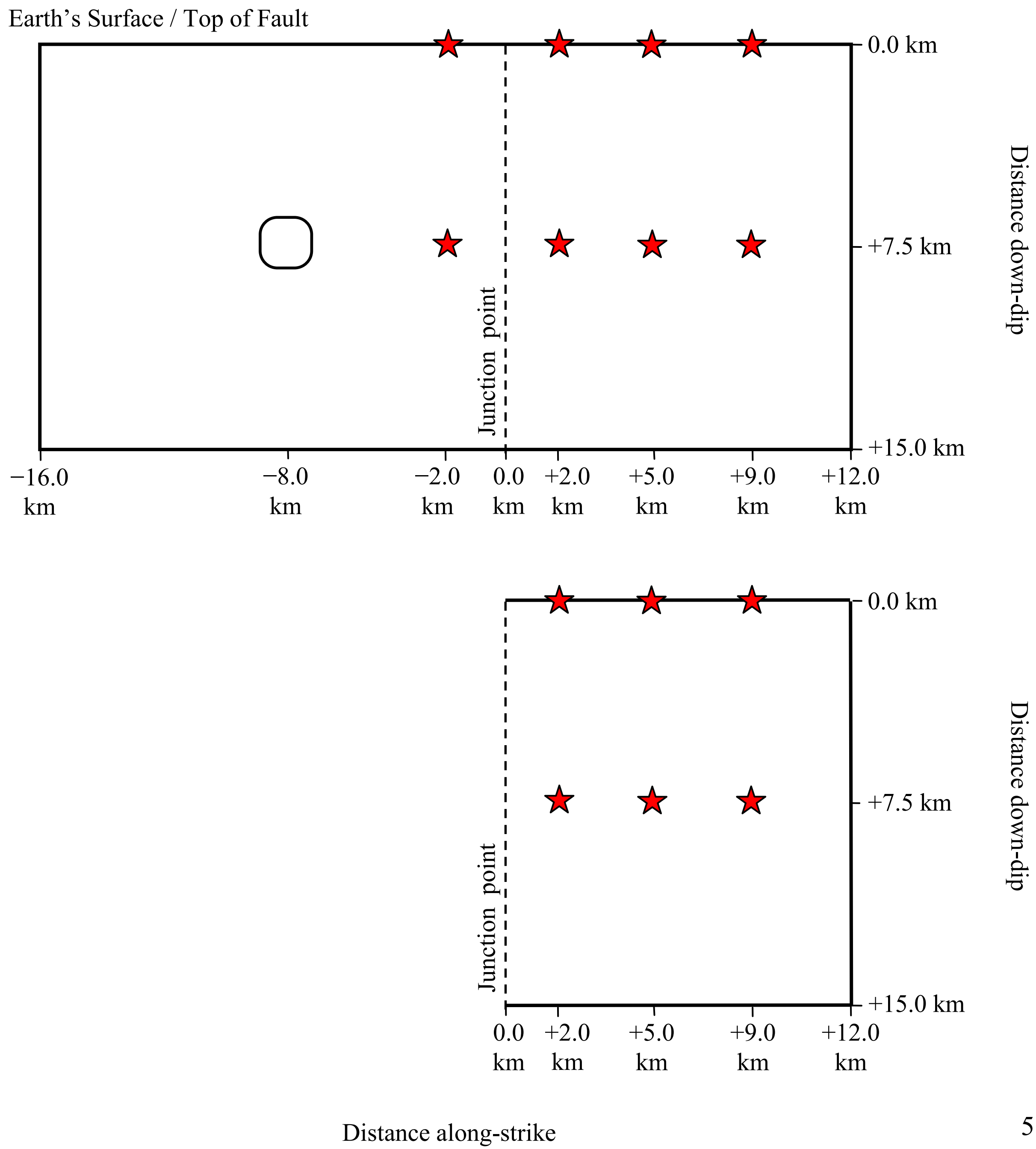
TPV18-21 have a branched, vertical, strike-slip fault.

The branch angle is 30 degrees.

On-Fault Stations

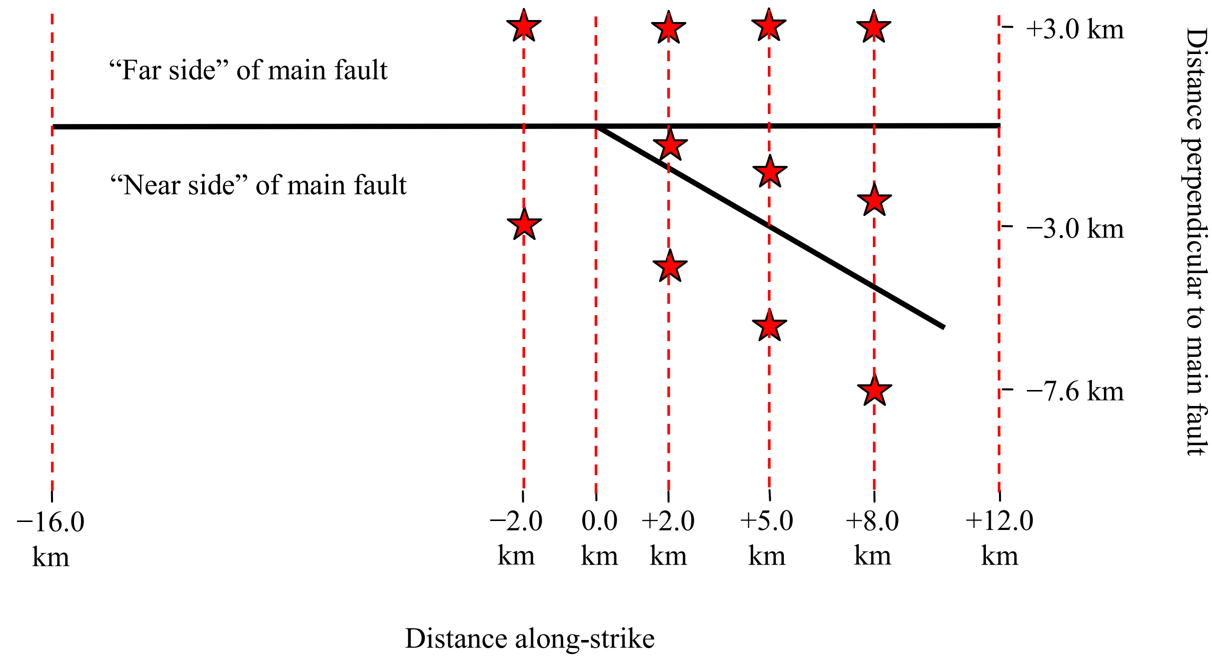
Modelers are asked to submit slip, slip rate, and stress as a function of time, for 8 stations on the main fault (top) and 6 stations on the branch fault (bottom).

In addition, modelers are asked to submit the time at which each point on the fault begins to slip, from which we construct rupture contour plots.



Off-Fault Stations

Modelers are asked to submit displacement and velocity as a function of time, for 11 stations on the earth's surface.



TPV18-21 Material Constitutive Parameters

Density $\rho = 2700.0 \text{ kg/m}^3$

$V_S = 3300.0 \text{ m/s}$

$V_P = 5716.0 \text{ m/s}$

Plastic cohesion $c = 5.0 \text{ MPa}$ (used for TPV19 and 21)

Bulk friction $\nu = 0.677$ (used for TPV19 and 21)

Fluid pressure $P_f = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$

All parameters are the same as TPV12-13.

Parameter selection follows our practice of reusing material from earlier benchmarks, so we build incrementally on prior work.

Drucker-Prager Plasticity with Yielding in Shear

Angle of friction $\phi = \tan^{-1} \nu$

Mean stress $\sigma_m = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$

Stress deviator $s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$

Second invariant of the stress deviator $J_2(\sigma) = \frac{1}{2}\sum_{i,j} s_{ij} s_{ji}$

Drucker-Prager yield stress $Y(\sigma) = \max(0,\ c \cos\phi - \left(\sigma_m + P_f\right) \sin\phi)$

Drucker-Prager yield function $F(\sigma) = \sqrt{J_2(\sigma)} - Y(\sigma)$

Drucker-Prager yield equation $F(\sigma) \leq 0$

Yielding in shear means that when you attempt to increase the deviatoric stress to exceed the yield stress, the stress tensor changes by an amount proportional to the stress deviator, reducing the deviatoric stress while leaving the mean stress unchanged.

The yield stress depends on the *effective mean stress* $\sigma_m + P_f$, which is the mean stress excluding fluid pressure.

This is the same form of plasticity used in TPV13.

Friction Parameters for TPV18-21

$$\mu_s = 0.60$$

$$\mu_d = 0.12$$

$$D_c = \begin{cases} 0.04\ \mathrm{m}\,, & \text{if } r \leq 360\ \mathrm{m} \\ r/9000\,, & \text{if } 360\ \mathrm{m} \leq r \leq 3600\ \mathrm{m} \\ 0.40\ \mathrm{m}\,, & \text{if } r \geq 3600\ \mathrm{m} \end{cases}$$

$$C_0 = \begin{cases} 0.20\ \mathrm{MPa} + (0.0006\ \mathrm{MPa/m})(3000\ \mathrm{m} - \text{depth})\,, & \text{if depth} \leq 3000\ \mathrm{m} \\ 0.20\ \mathrm{MPa}\,, & \text{if depth} \geq 3000\ \mathrm{m} \end{cases}$$

$$T = \begin{cases} r/(0.7\,V_S)\,, & \text{if } r \leq 720\ \mathrm{m} \\ (720\ \mathrm{m})/(0.7\,V_S) + (r - 720\ \mathrm{m})/(0.35\,V_S)\,, & \text{if } 720\ \mathrm{m} \leq r \leq 900\ \mathrm{m} \\ 1.0\mathrm{E}{+}9\,, & \text{if } r > 900\ \mathrm{m} \end{cases}$$

$r =$ distance from hypocenter

The static and dynamic coefficients of friction are much different than prior benchmarks.

Frictional cohesion C_0 tapers from 0.20 MPa at 3 km depth up to 2.00 MPa at the earth's surface.

The variation in critical slip distance D_c and time of force rupture T near the hypocenter nucleates the rupture.

Linear Slip-Weakening Friction

When the fault is sliding, the shear stress τ at a given point on the fault is given by:

$$\tau = C_0 + \mu \times \max(0,\ \sigma_n - P_f)$$

The *effective normal stress* is the total normal stress σ_n minus the fluid pressure P_f.

The time-varying coefficient of friction μ is given by:

$$\mu = \begin{cases} \mu_s + (\mu_d - \mu_s) \times D/D_c\ , & \text{if } \ D < D_c \ \text{ and } \ t < T \\ \mu_d\ , & \text{if } \ D \geq D_c \ \text{ or } \ t \geq T \end{cases}$$

where D is the total distance the node has slipped, and t is the time since the start of the earthquake. The effect is:

- The coefficient of friction declines linear from μ_s to μ_d as the fault slips by distance D_c.

- At time T, the coefficient of friction drops immediately to μ_d (if it is not already μ_d). This only happens within 360 m of the hypocenter.

Nucleation

Day Radius and the Problem of Nucleation

Day (1982) obtained the following formula, which gives the minimum radius R_D that a circular rupture must have, such that it is energetically favorable for the rupture to expand.

Shear modulus μ Static yield stress τ_s Slip-weakening critical distance D_c

$$R_D = \frac{7\pi\mu(\tau_s - \tau_d)D_c}{24(\tau - \tau_d)^2}$$

Day radius R_D Initial stress τ Dynamic sliding stress τ_d

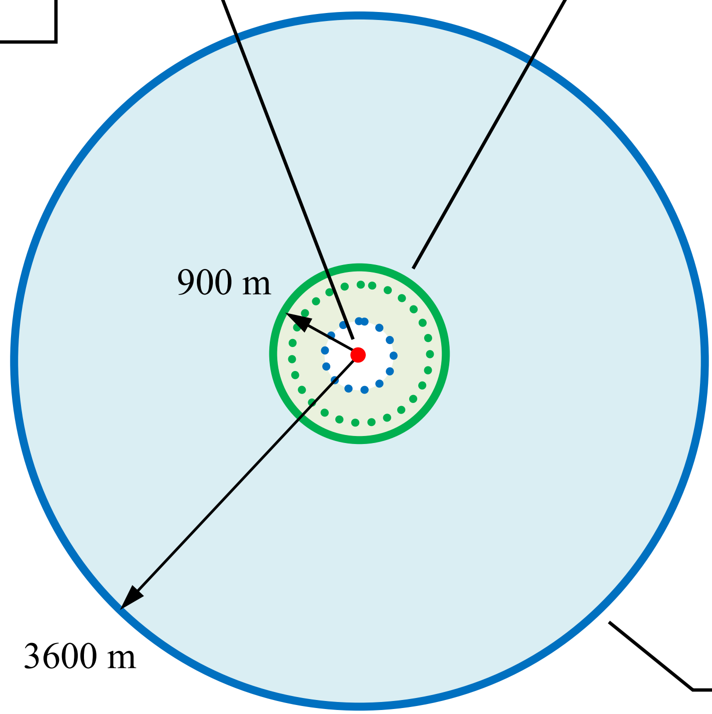
For typical parameter values used in spontaneous rupture simulations, the Day radius is about 3 to 4 km.

The nucleation problem is that, somehow, we must impose an artificial mechanism to get the size of the rupture up to the Day radius, at which point the rupture can be self-sustaining.

Two-Stage Nucleation Method

All of today's benchmarks use a new two-stage method of nucleation.

Hypocenter



Zone of forced rupture

Forced rupture propagates at $0.7V_S$ to dotted circle (720 m), then $0.35V_S$ to solid circle (900 m). Forced rupture immediately reduces friction coefficient to μ_d.

(Distances shown are for TPV18-21.)

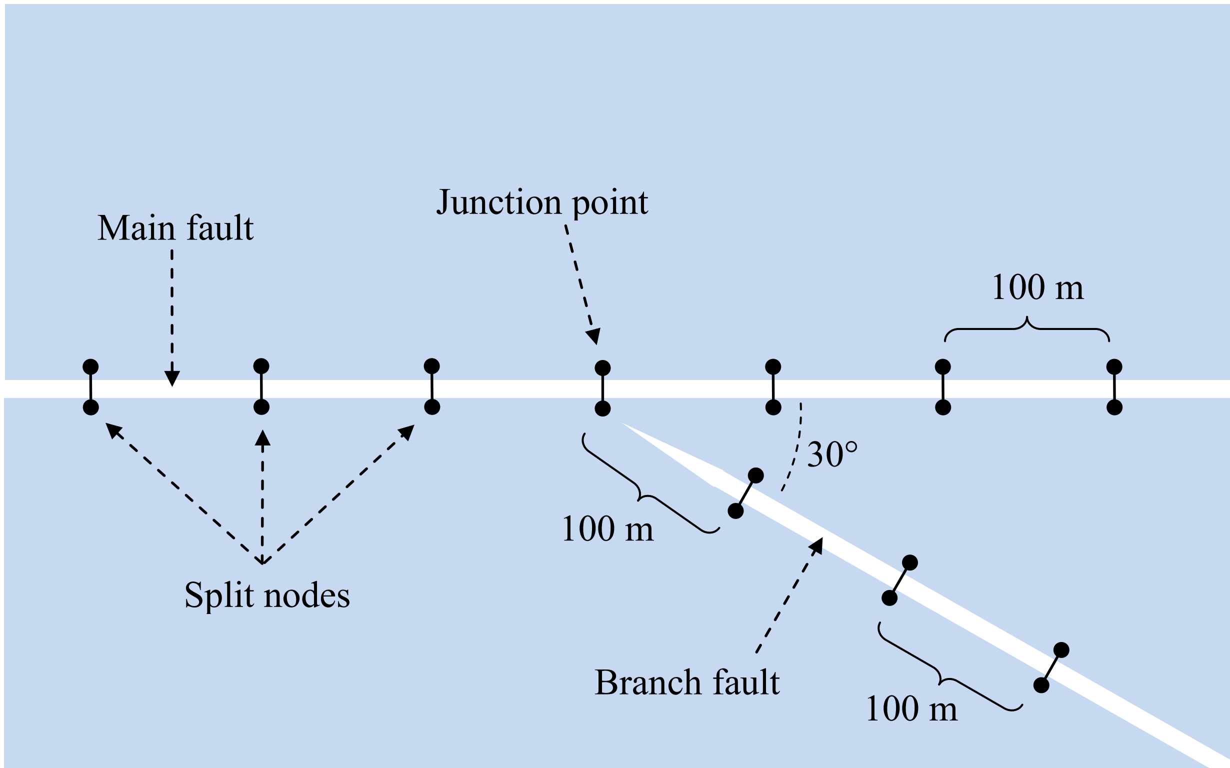
Zone of reduced fracture energy

D_C is linearly tapered from 0.04 m at dotted circle (360 m), to 0.40 m at solid circle (3600 m).

Because the Day radius is proportional to D_C, rupture propagation is energetically favorable once the rupture reaches the dotted blue circle.

Junction Point Behavior

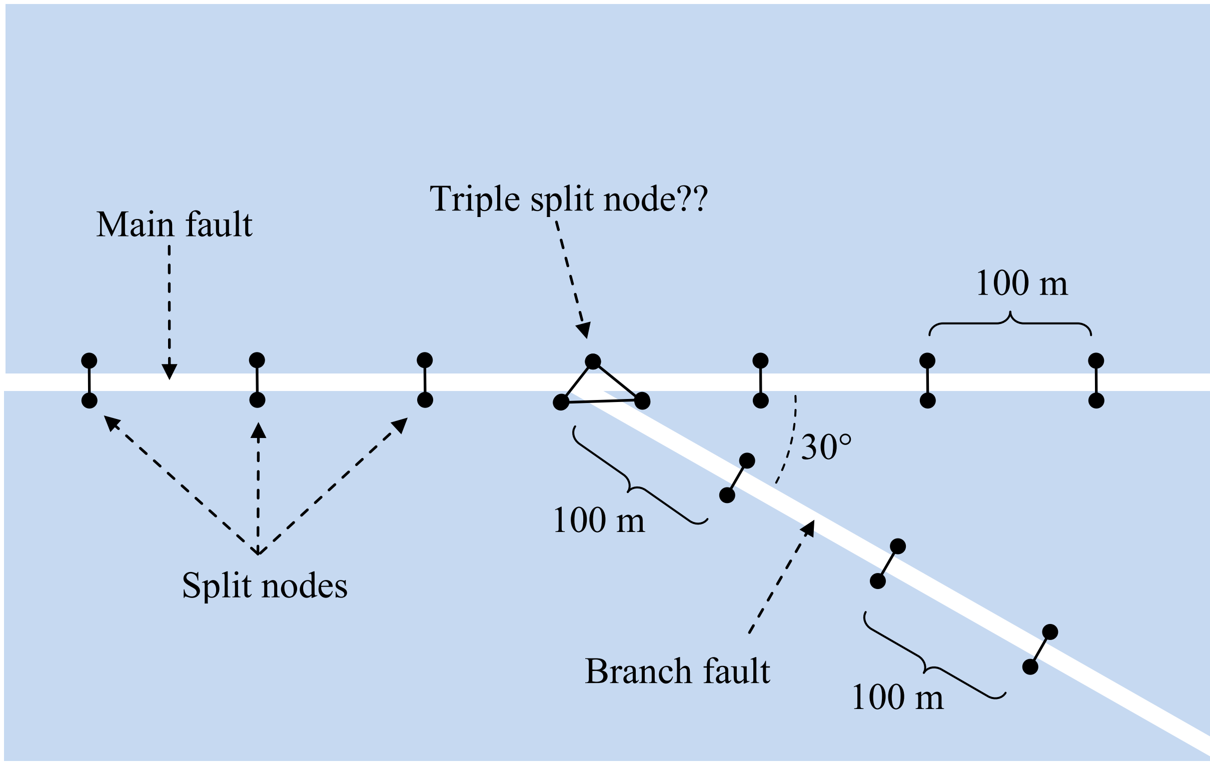
Junction Point Behavior



The boundary condition is that slip on the branch fault is zero at the junction point.

The picture shows a possible implementation. The main fault passes continuously through the junction point. The branch fault approaches the junction point, to within one element-size, but does not reach the junction point. The junction point behaves as an ordinary split-node on the main fault.

Could we let the branch fault reach the junction point?

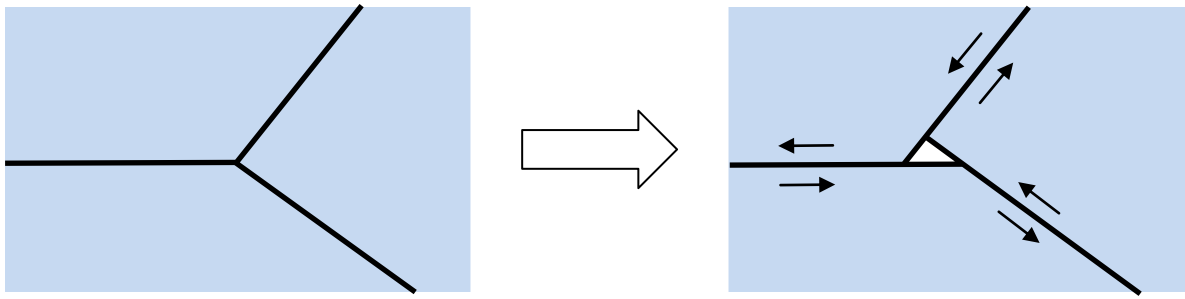


Challenges of defining a friction law for a triple split node:

- What strike-slip motions are permitted?
- Five possible states of dip-slip motion.
- Inability to calculate shear and normal stresses from nodal forces.

Because of these challenges, we do not use a triple split node in the benchmarks.

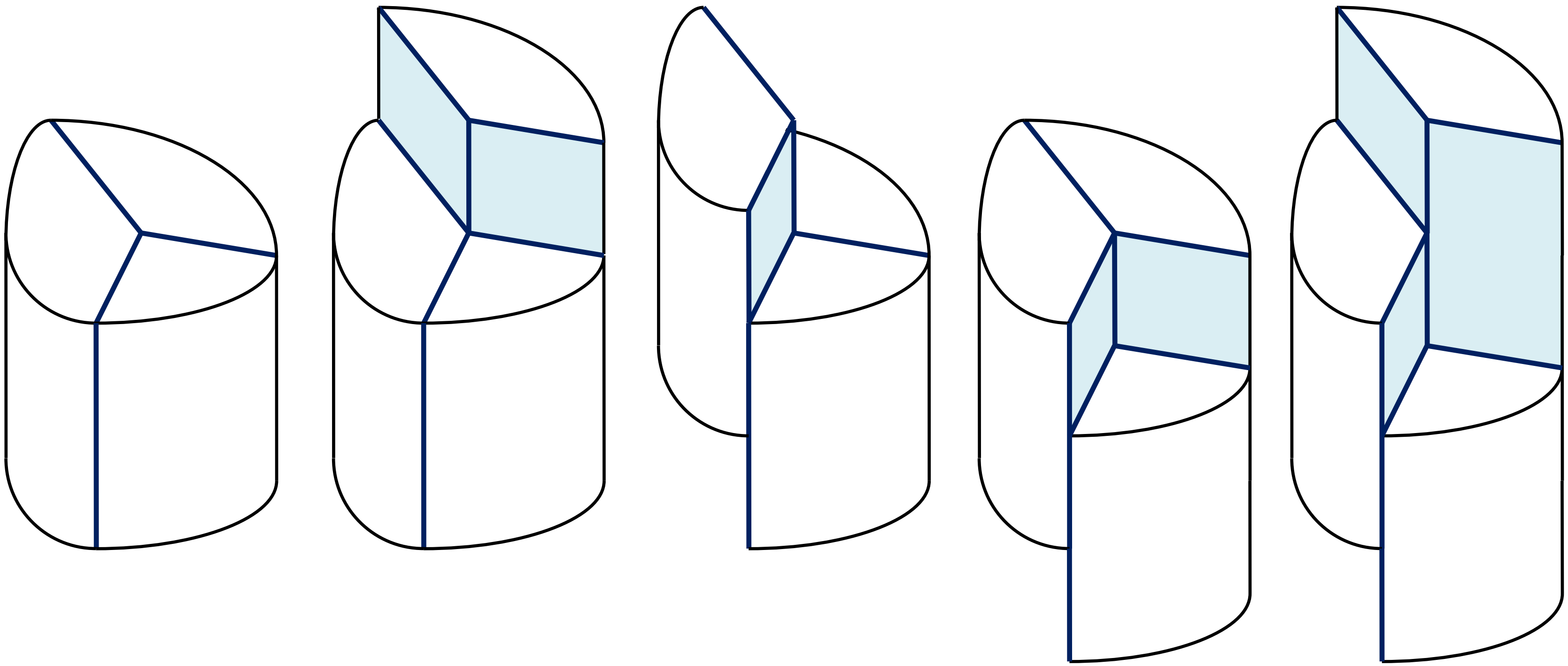
Strike-Slip Motion at a Branch



Joe Andrews proposed that at a branch, slip on all three legs is possible and would open a triangular void. But this requires the three legs to all point in different directions.

In our case, two legs are collinear, so the triangle disappears. We are left with no strike-slip motion on the branch fault at the junction point.

Five States of Dip-Slip Motion at a Branch

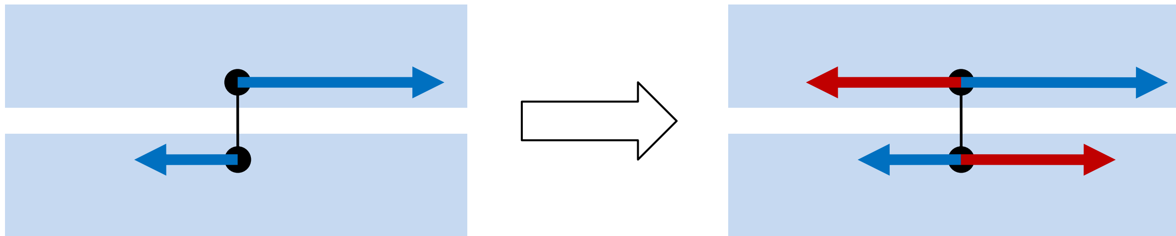


There is no geometric obstacle to dip-slip motion, but there are five possible states:

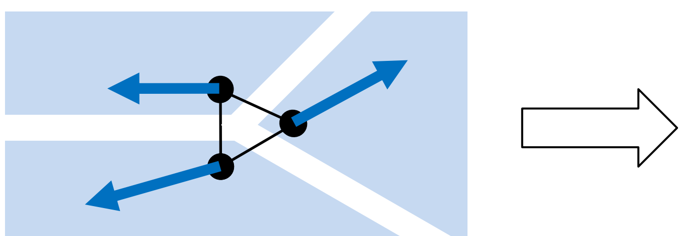
- No sliding, all three blocks are locked.
- One block sliding relative to the other two, which are locked.
- All three blocks sliding relative to each other.

Software would have to implement these states, and state-to-state transitions.

Traction-at-split-nodes does not work for a triple split node



Traction-at-split-nodes (Joe Andrews): Nodal forces (**blue**) are the forces exerted on each node by elastic deformation and acceleration. Given the nodal forces, the shear and normal stress on the fault can be computed by finding the relative force (**red**) that reduces the relative acceleration of the two nodes to zero. (Summing the red and blue vectors on each node produces the same total vector, hence the same acceleration assuming the two nodes have equal mass.)



Under-determined

For a triple split node, given the nodal forces it is not possible to compute shear and normal stress for each of the three legs of the fault. There is only enough information to compute two relative forces, not three.

Initial Stress State

Need to Specify Initial Stress Tensor

For a calculation with plasticity, we must specify the initial stress tensor throughout the model.

We choose to use a regional stress field, where the stress tensor varies only with depth, because it is simple to define. (Note that a regional stress field is not realistic; see Duan & Oglesby 2007.) Implications:

- Shear and normal stress on the faults is obtained from the stress tensor, and cannot be specified arbitrarily. (In TPV 14-15, we specified arbitrary shear and normal stresses, not consistent with a regional stress field.)

- We had to use friction coefficients with almost total stress drop, $\mu_s = 0.60$ and $\mu_d = 0.12$, to obtain stresses compatible with the friction coefficients. (TPV5 parameters $\mu_s = 0.677$ and $\mu_d = 0.525$ require the ratio of shear to normal stress be held within a narrow range around a high value, which can't be done for both faults.)

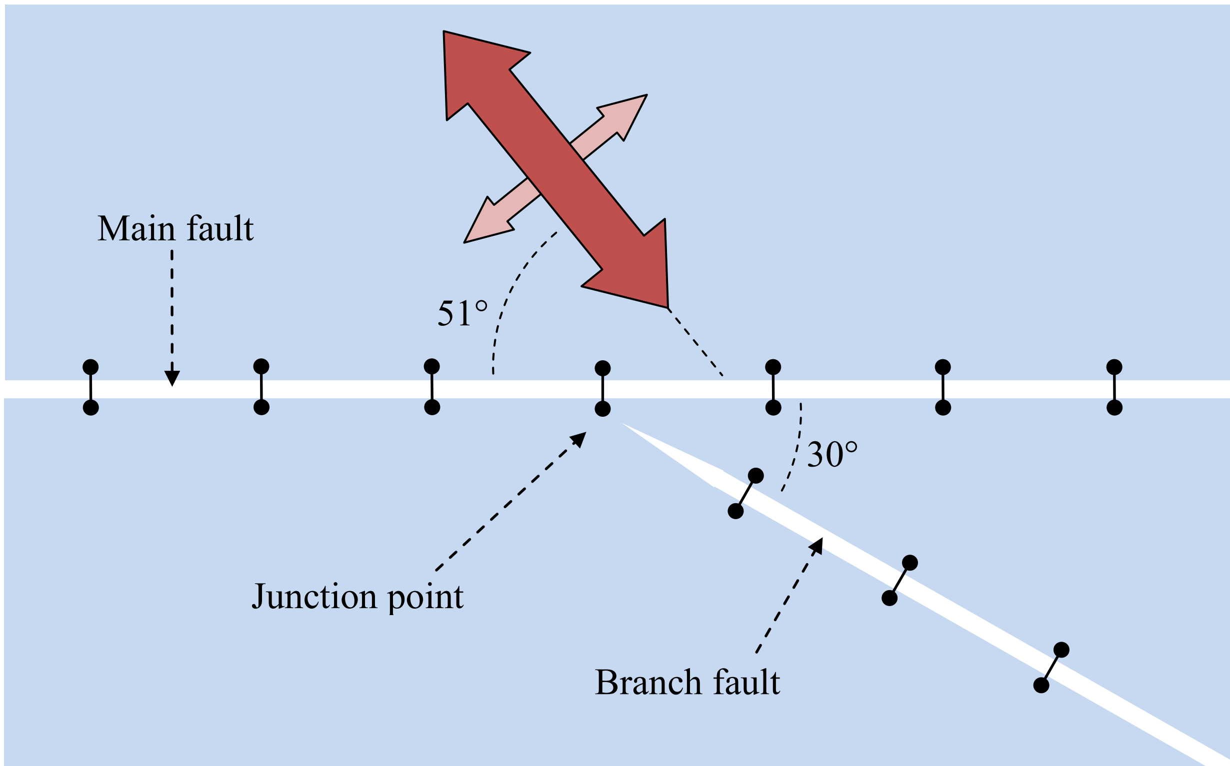
- Nucleation cannot be done by imposing a high-stress zone, because it would be inconsistent with the stress tensor.

- The model must include gravity to balance the vertical component of the stress tensor.

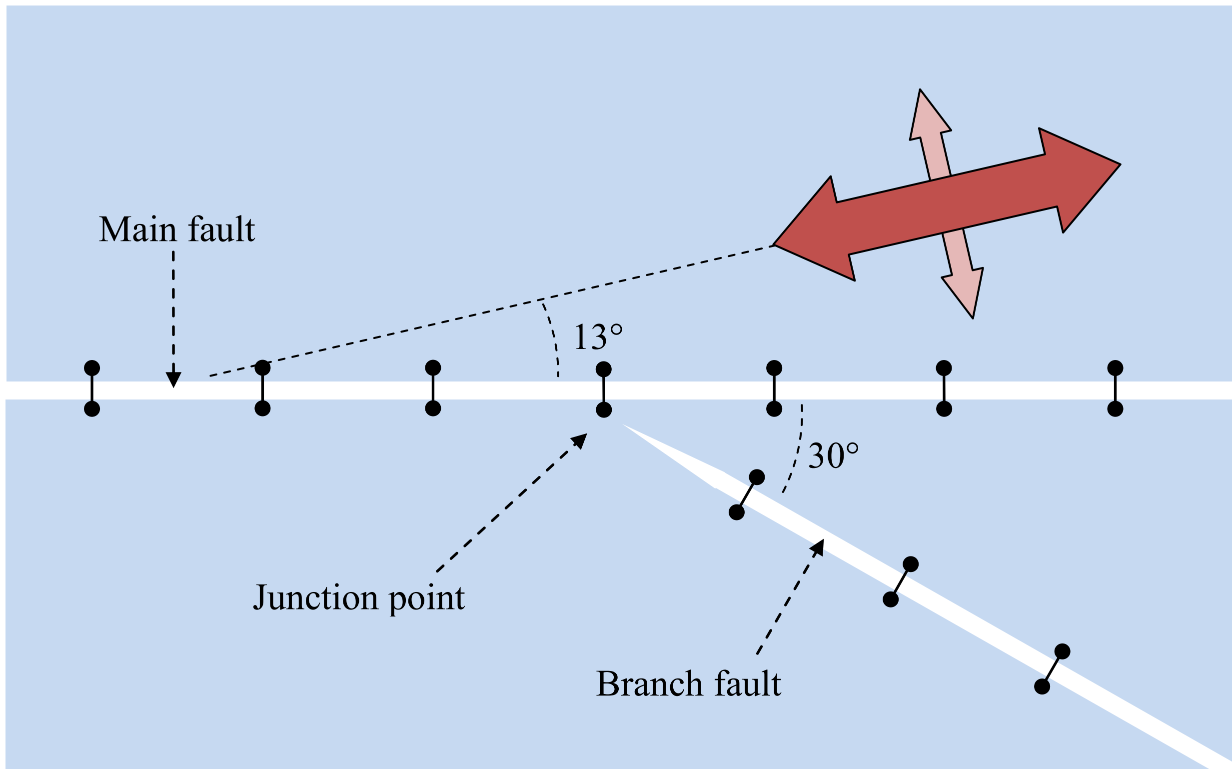
- The model must include fluid pressure.

- The initial stress tensor cannot exceed the plastic yield stress or be too close to it. But has to be close enough so that rupture causes some plastic yielding.

Principal Stress Axes for TPV18-19 (Right-Lateral)



Principal Stress Axes for TPV20-21 (Left-Lateral)



Stress Tensor Definition

We define a *total* stress tensor, which includes both elastic stress and fluid pressure.

Stress Tensor Components	
Component	*Definition*
σ_{11}	Compressive stress in the vertical direction. Negative values denote compression.
σ_{22}	Compressive stress in the direction parallel to the main fault. Negative values denote compression.
σ_{33}	Compressive stress in the direction perpendicular to the main fault. Negative values denote compression.
σ_{23}	Shear stress in the horizontal plane. Positive values denote right-lateral shear stress on the main fault.
σ_{13}	Shear stress in a vertical plane perpendicular to the main fault.
σ_{12}	Shear stress in a vertical plane parallel to the main fault.

The friction law and plastic yielding formula actually use an *effective* stress tensor which contains just the elastic stress. Its diagonal components are $\sigma_{11} + P_f$, $\sigma_{22} + P_f$, and $\sigma_{33} + P_f$.

Stress Tensor Definition (continued)

$$P_f = (1000\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(\text{depth in meters}) \qquad \text{---- Hydrostatic}$$

$$\sigma_{11} = -(2700\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(\text{depth in meters}) \qquad \text{---- Lithostatic}$$

$$\sigma_{22} = \begin{cases} b_{22}\left(\sigma_{11} + P_f\right) - P_f\,, & \text{if depth} \le 15000\ \mathrm{m} \\ \sigma_{11}\,, & \text{if depth} > 15000\ \mathrm{m} \end{cases}$$

$$\sigma_{33} = \begin{cases} b_{33}\left(\sigma_{11} + P_f\right) - P_f\,, & \text{if depth} \le 15000\ \mathrm{m} \\ \sigma_{11}\,, & \text{if depth} > 15000\ \mathrm{m} \end{cases}$$

$$\sigma_{23} = \begin{cases} b_{23}\left(\sigma_{11} + P_f\right), & \text{if depth} \le 15000\ \mathrm{m} \\ 0\,, & \text{if depth} > 15000\ \mathrm{m} \end{cases}$$

$$\sigma_{13} = 0$$

$$\sigma_{12} = 0$$

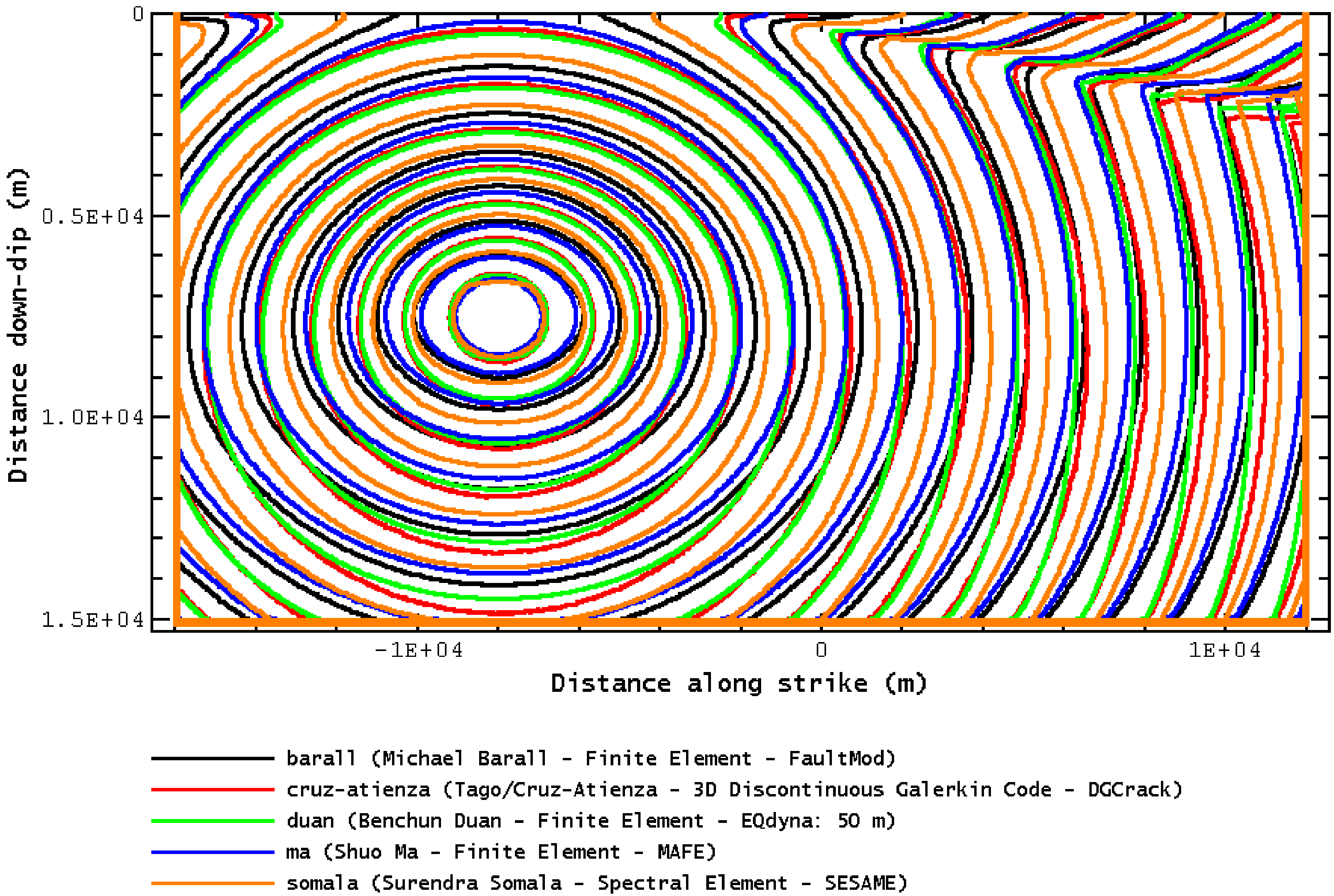
The coefficients b_{22}, b_{33} and b_{23} are ratios of the components of the effective stress tensor.

Stress Tensor Definition (continued)

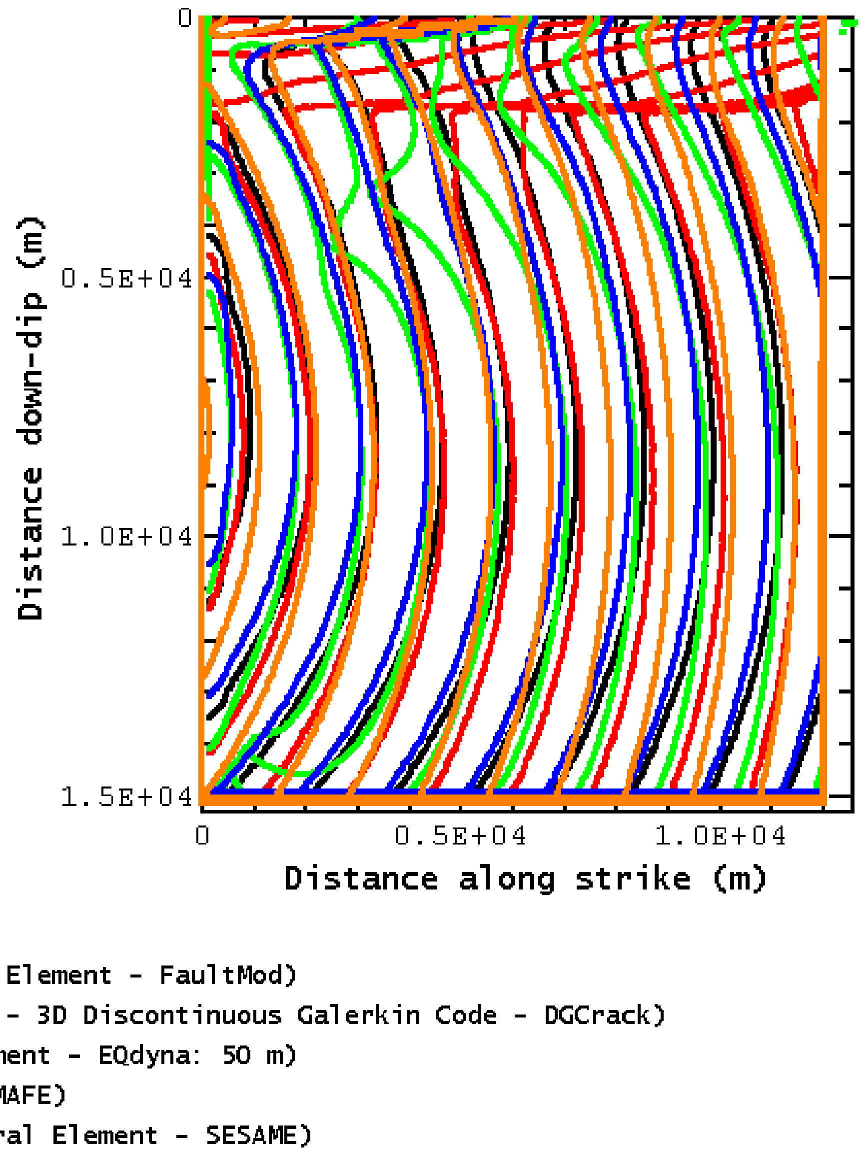
Initial Stress Tensor Coefficients, and Fault Stresses at 7500 m depth		
Coefficient	*Value for TPV18, TPV19*	*Value for TPV20, TPV21*
b_{22}	0.44327040	0.67738619
b_{33}	0.50911055	0.27499476
b_{23}	-0.15487679	0.09812971
τ_{main}	$19.4\ \text{MPa}$	$-12.3\ \text{MPa}$
$\sigma_{\text{main}}^{\text{eff}}$	$63.6\ \text{MPa}$	$34.4\ \text{MPa}$
$\tau_{\text{main}}/\sigma_{\text{main}}^{\text{eff}}$	0.304	0.357
τ_{branch}	$13.2\ \text{MPa}$	$-27.9\ \text{MPa}$
$\sigma_{\text{branch}}^{\text{eff}}$	$44.8\ \text{MPa}$	$57.5\ \text{MPa}$
$\tau_{\text{branch}}/\sigma_{\text{branch}}^{\text{eff}}$	0.296	0.485

Comparison of Contour Plots

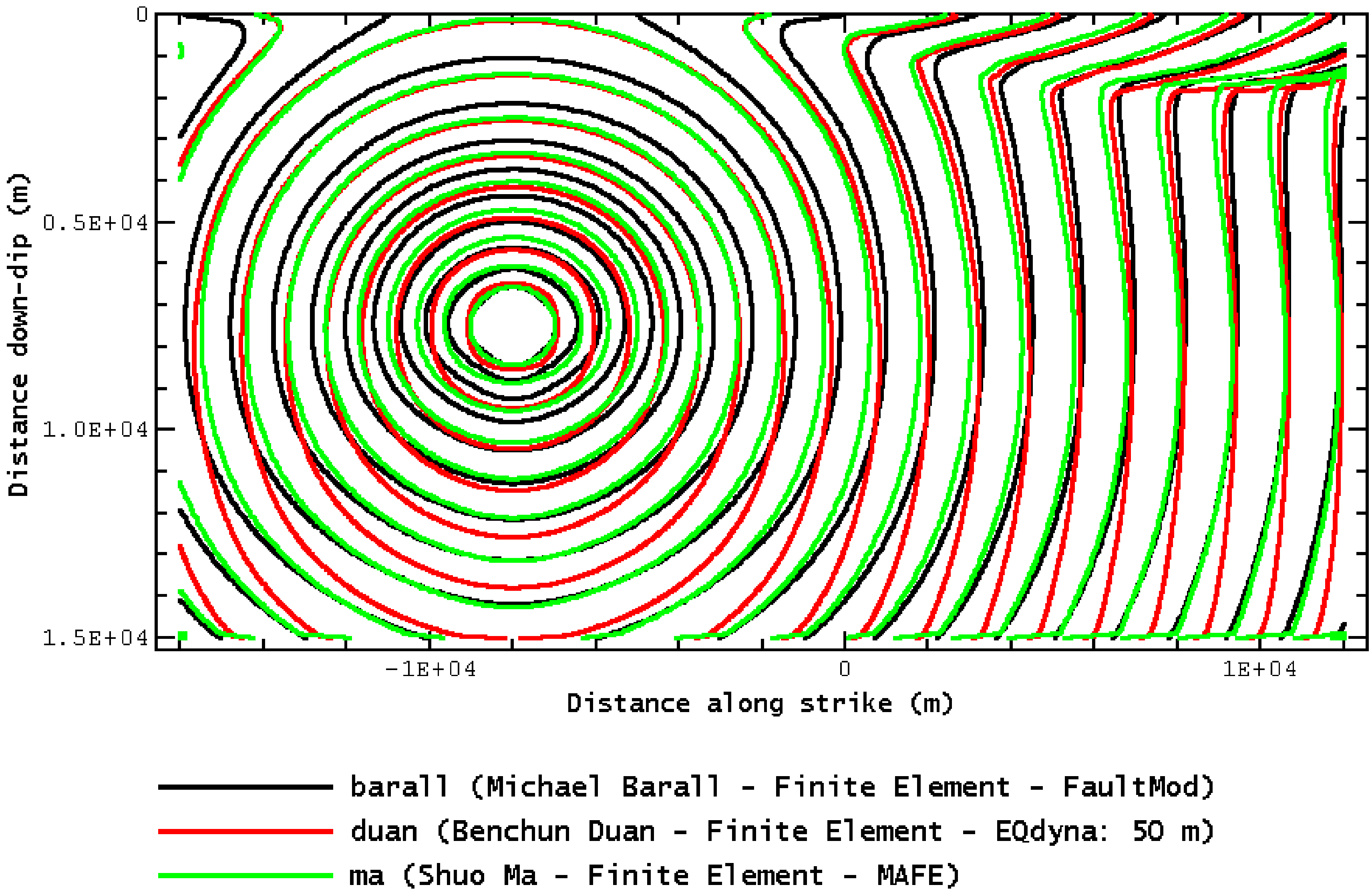
TPV18 (Right-Lateral, Elastic) — Main Fault Rupture Contours



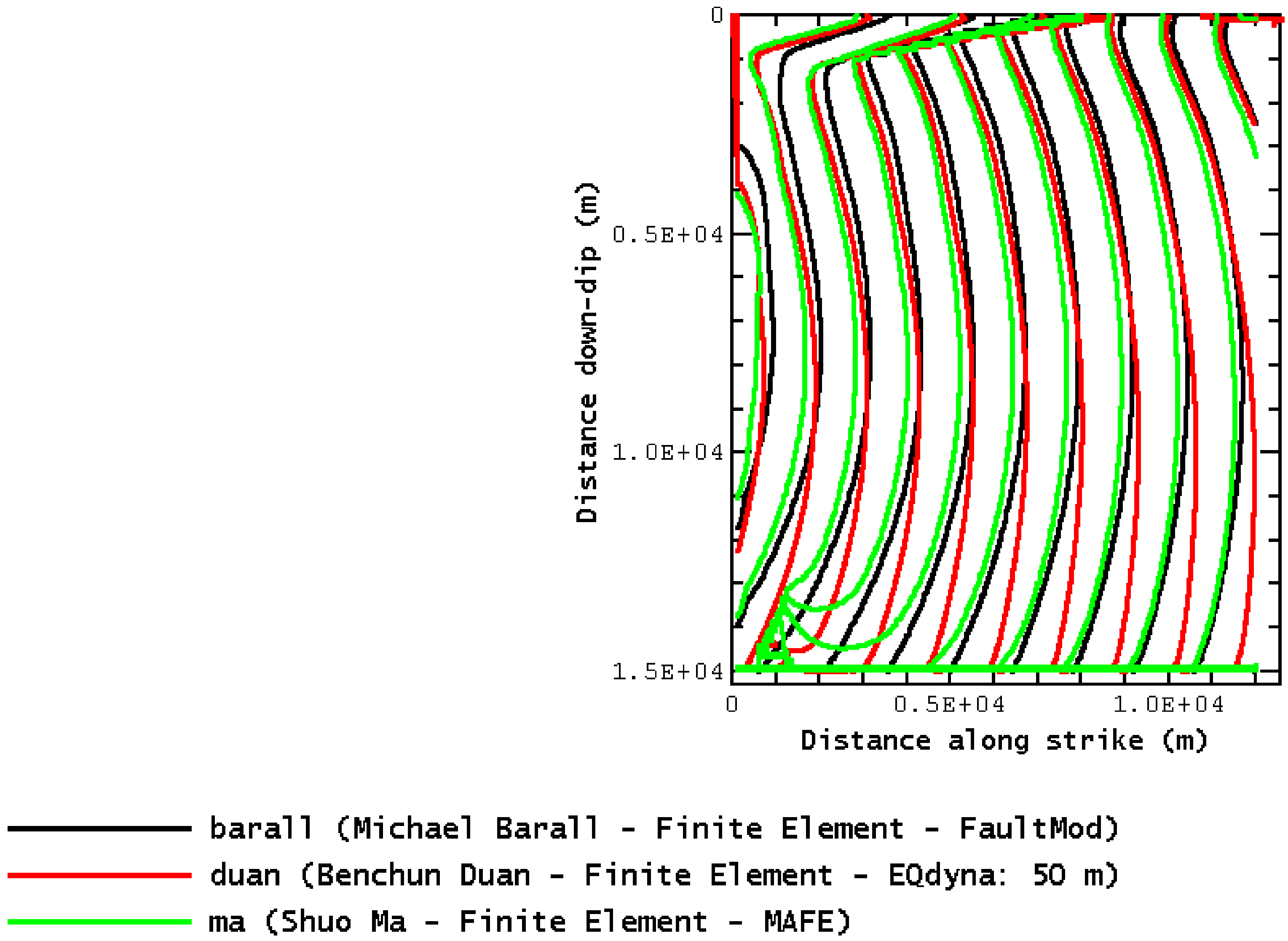
TPV18 (Right-Lateral, Elastic) — Branch Fault Rupture Contours



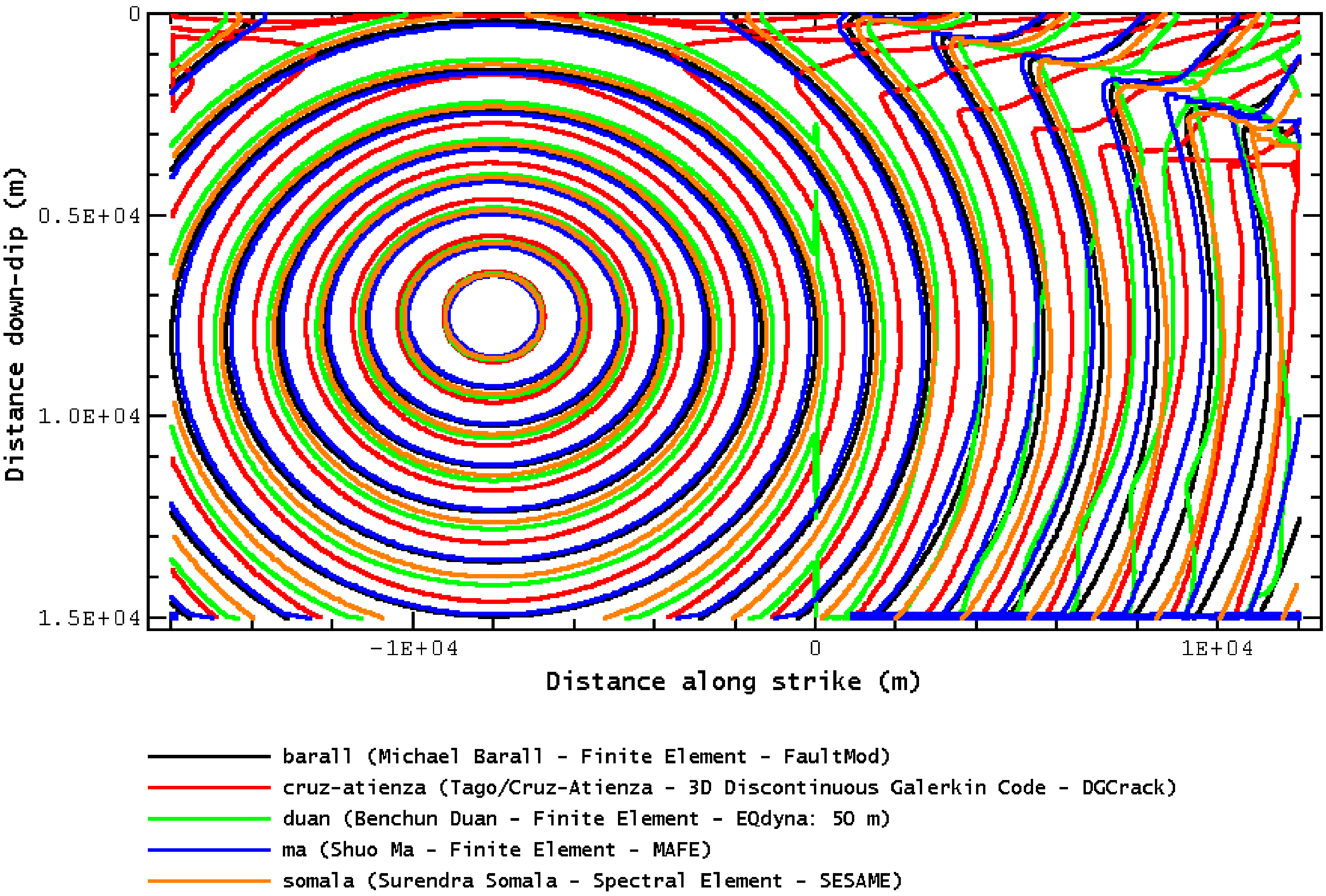
TPV19 (Right-Lateral, Plastic) — Main Fault Rupture Contours



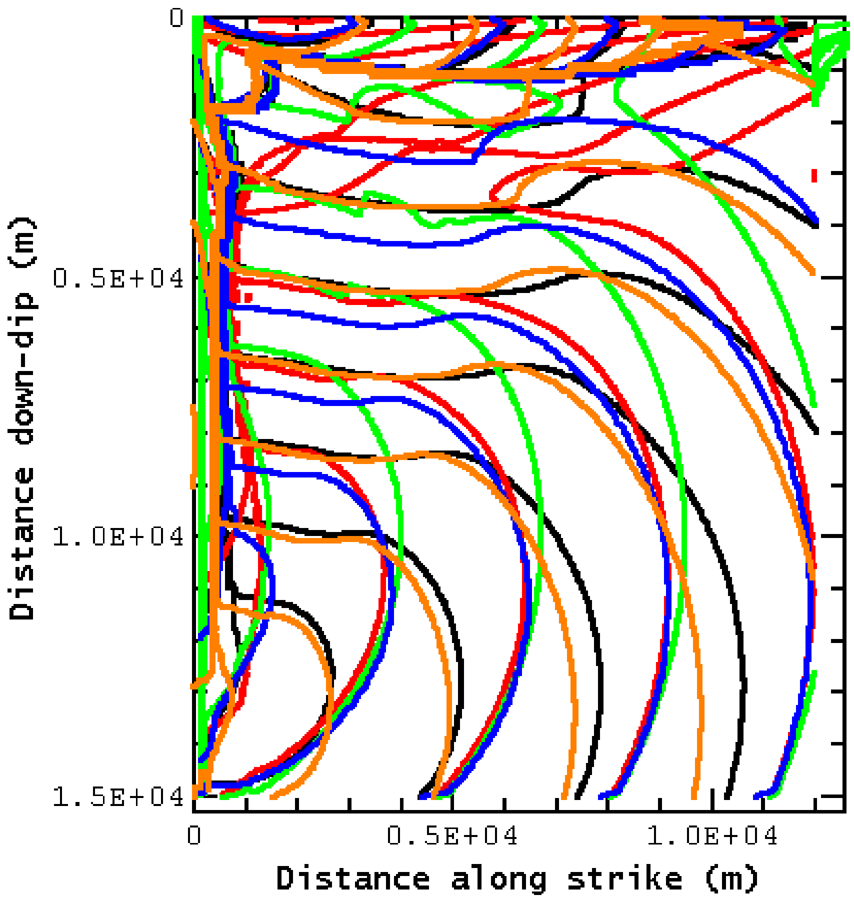
TPV19 (Right-Lateral, Plastic) — Branch Fault Rupture Contours



TPV20 (Left-Lateral, Elastic) — Main Fault Rupture Contours

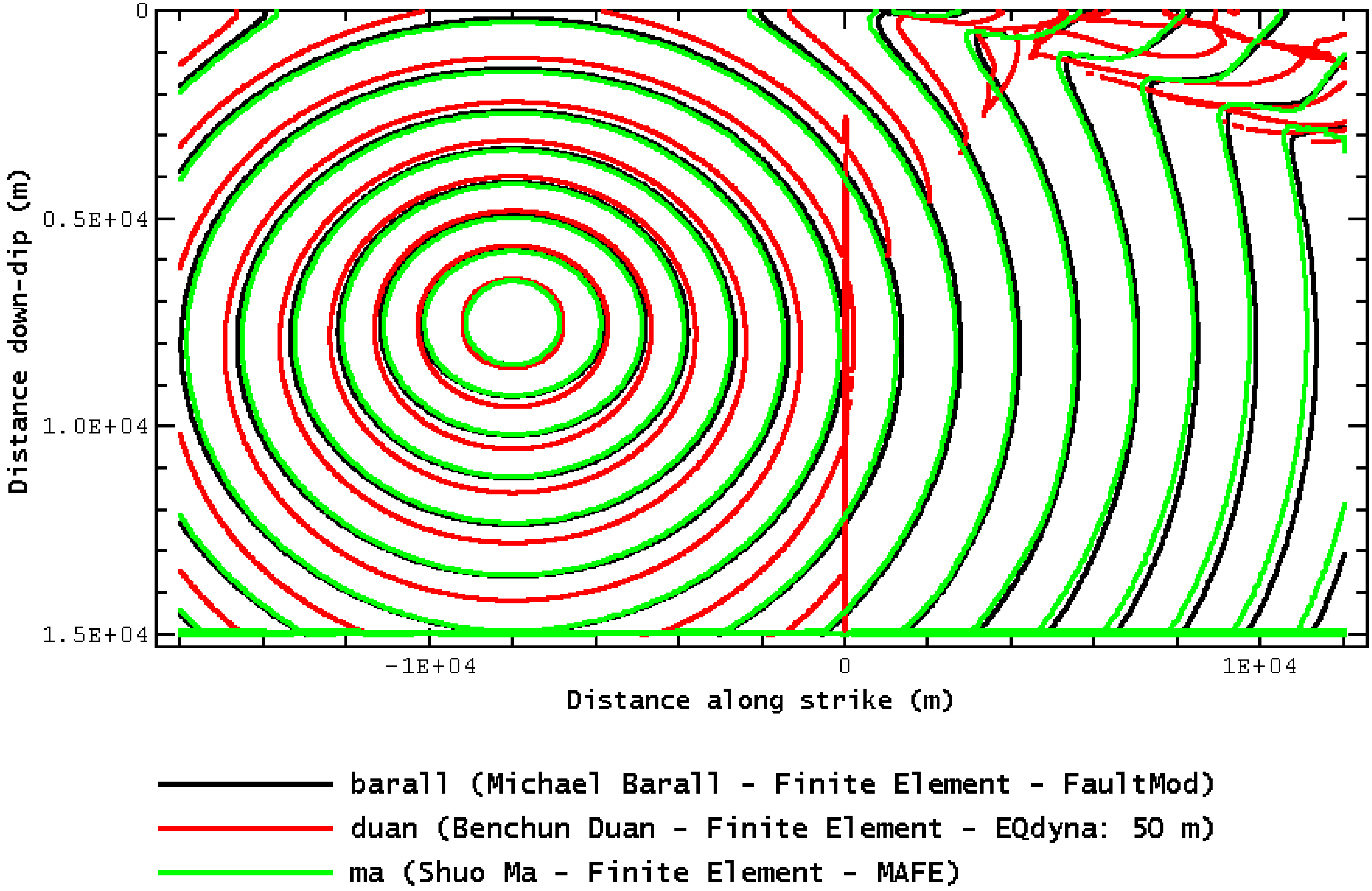


TPV20 (Left-Lateral, Elastic) — Branch Fault Rupture Contours

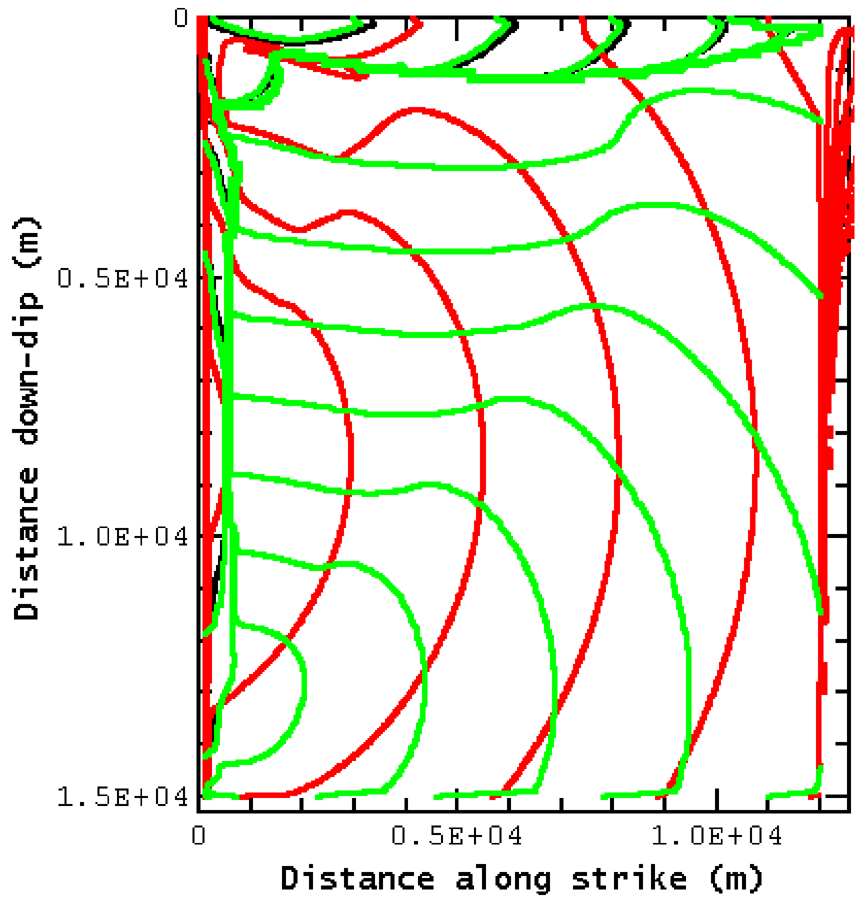


baral1 (Michael Barall - Finite Element - FaultMod)
cruz-atienza (Tago/Cruz-Atienza - 3D Discontinuous Galerkin Code - DGCrack)
duan (Benchun Duan - Finite Element - EQdyna: 50 m)
ma (Shuo Ma - Finite Element - MAFE)
somala (Surendra Somala - Spectral Element - SESAME)

TPV21 (Left-Lateral, Plastic) — Main Fault Rupture Contours

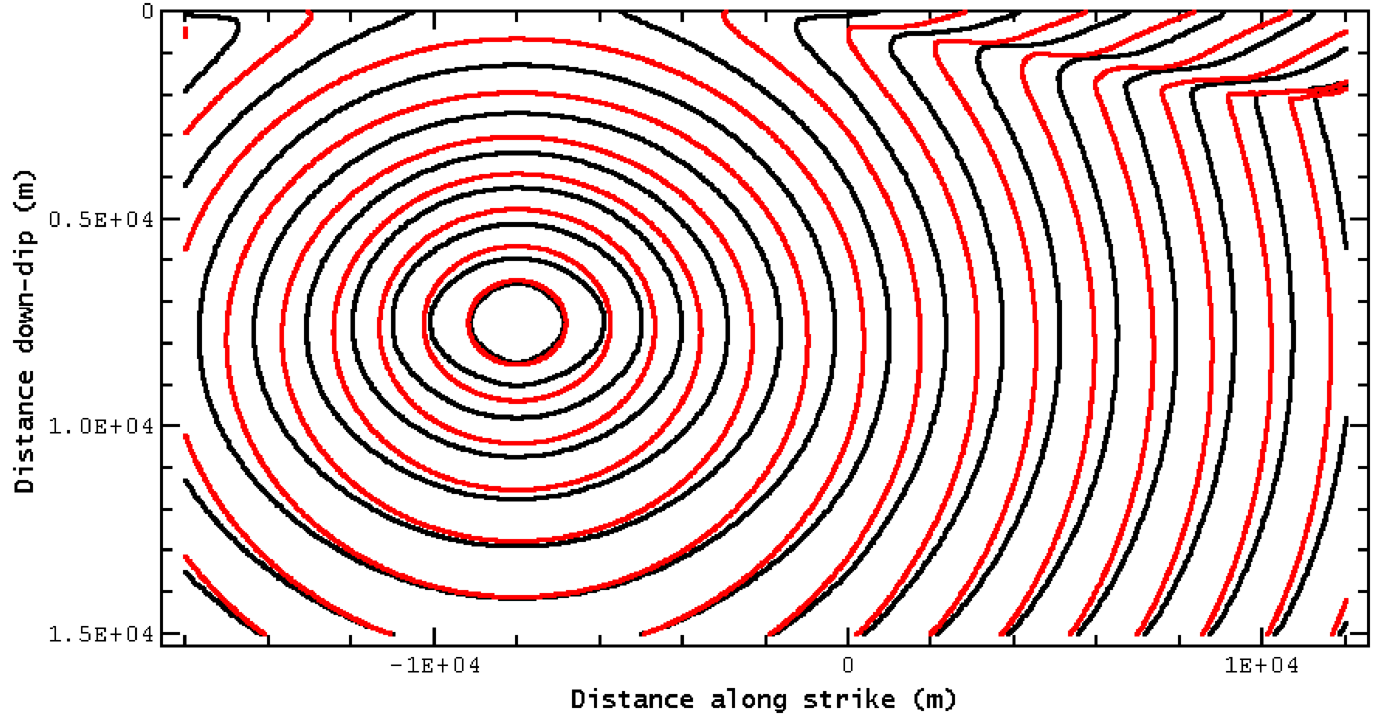


TPV21 (Left-Lateral, Plastic) — Branch Fault Rupture Contours

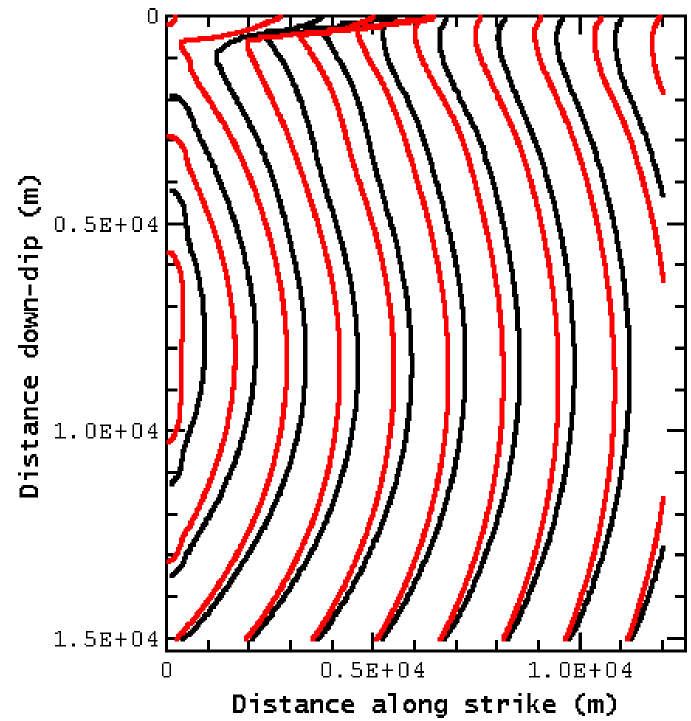


barall (Michael Barall - Finite Element - FaultMod)
duan (Benchun Duan - Finite Element - EQdyna: 50 m)
ma (Shuo Ma - Finite Element - MAFE)

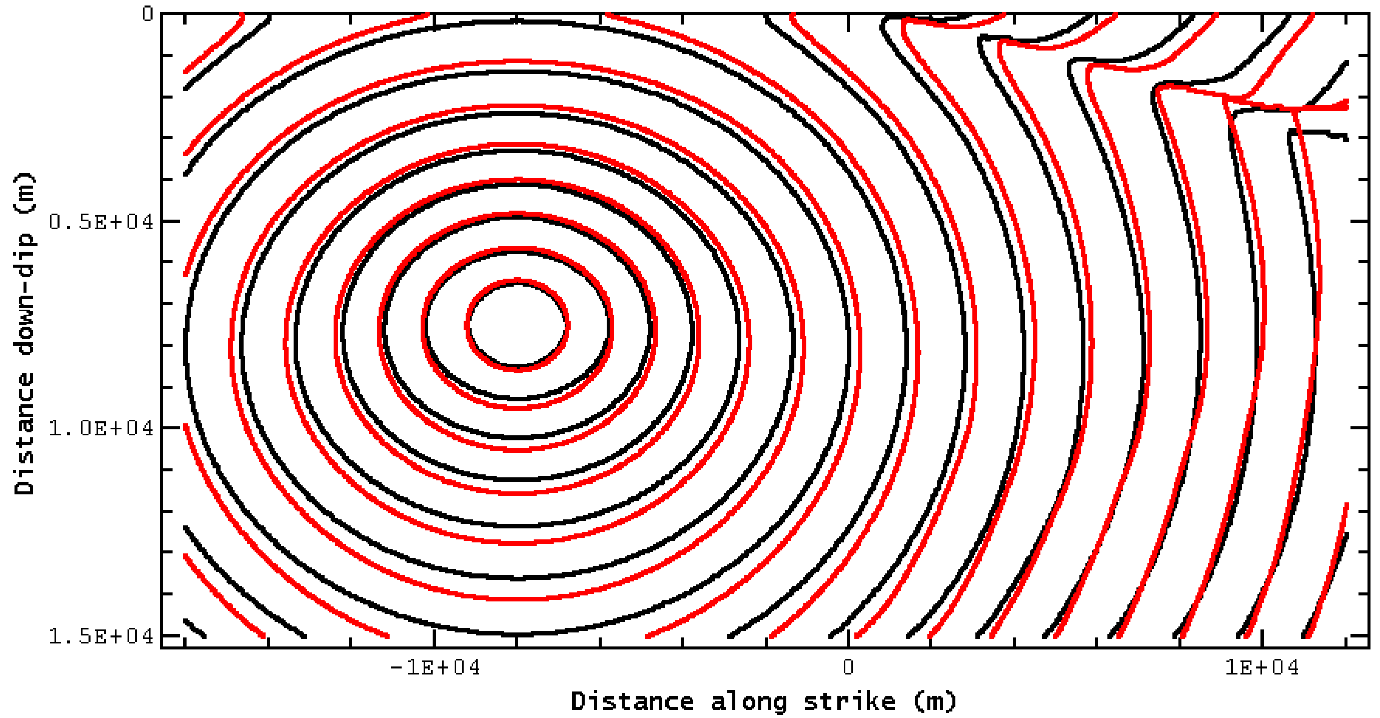
Resolution: 100m versus 50 m



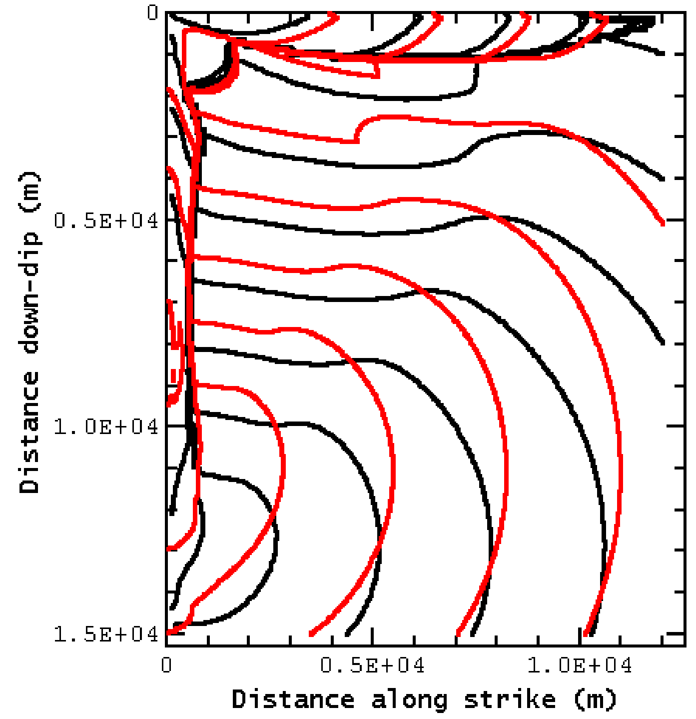
TPV18 (Right Lateral, Elastic):
100 m versus 50 m Resolution



barall (Michael Barall - Finite Element - FaultMod)
barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)

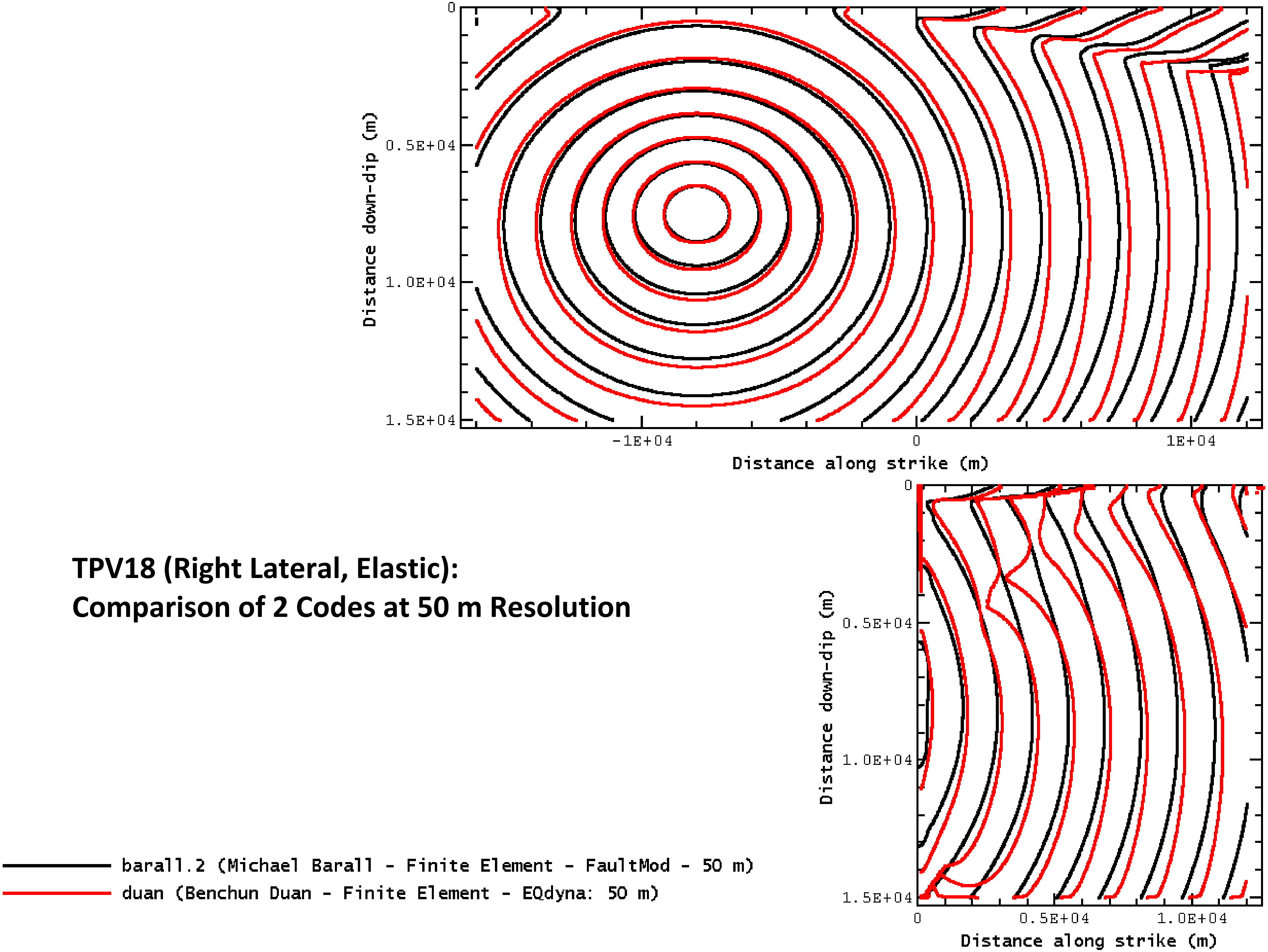


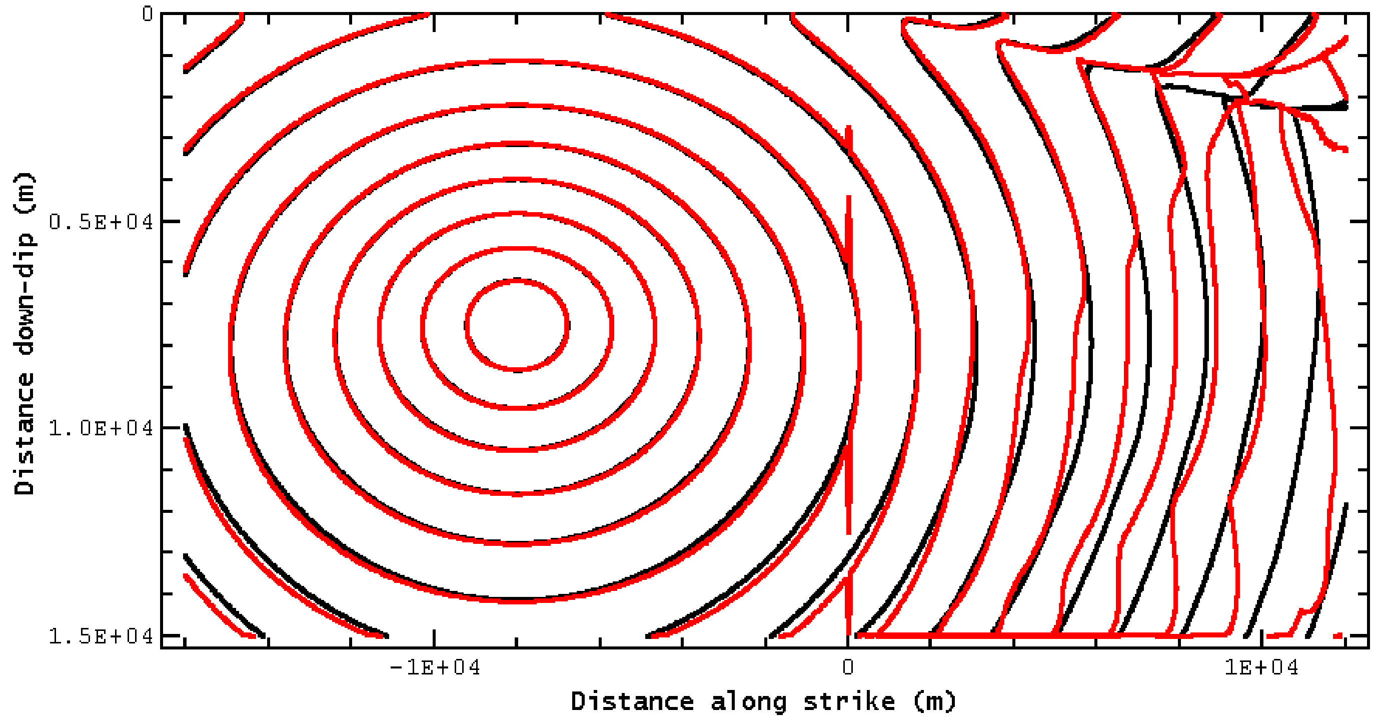
TPV20 (Left Lateral, Plastic):
100 m versus 50 m Resolution



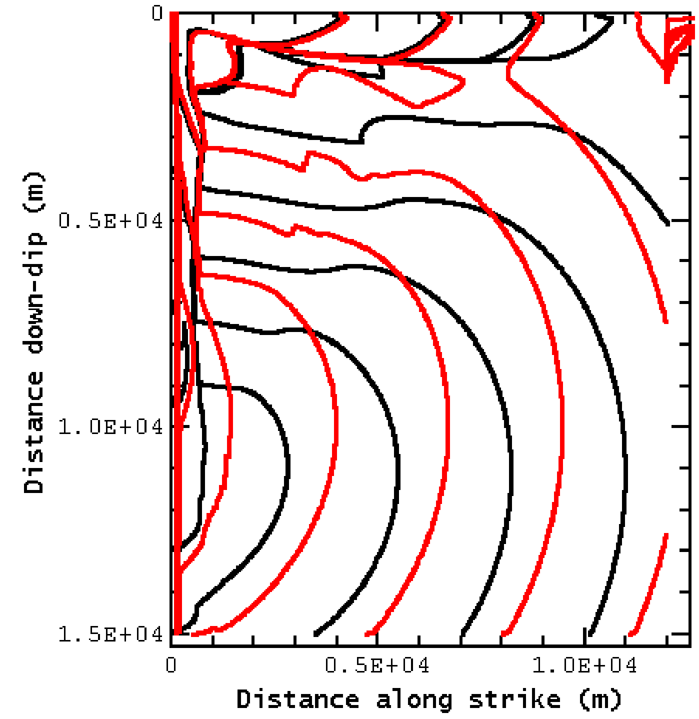
barall (Michael Barall - Finite Element - FaultMod)
barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)

TPV18 (Right Lateral, Elastic):
Comparison of 2 Codes at 50 m Resolution





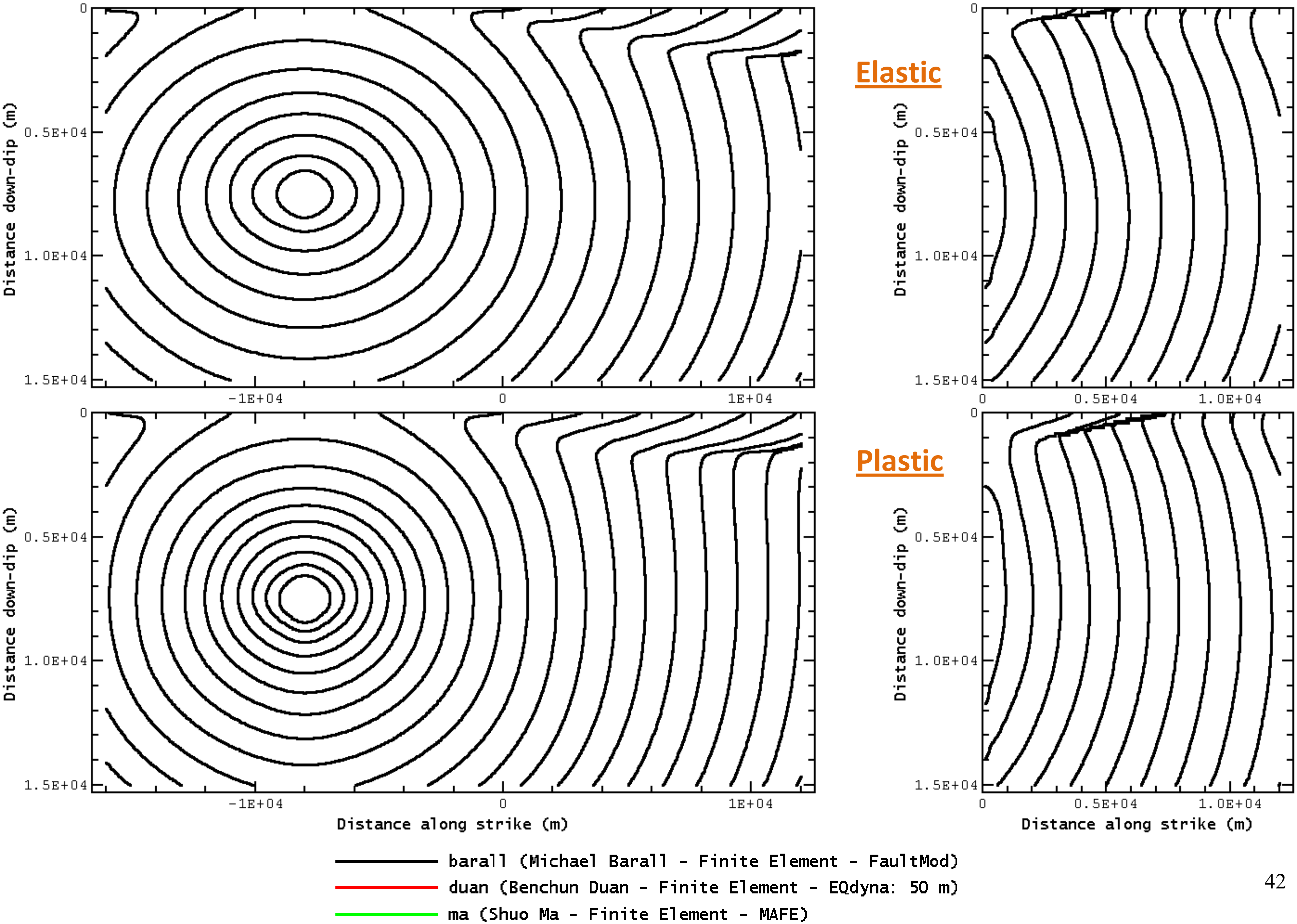
TPV20 (Left Lateral, Plastic):
Comparison of 2 Codes at 50 m Resolution



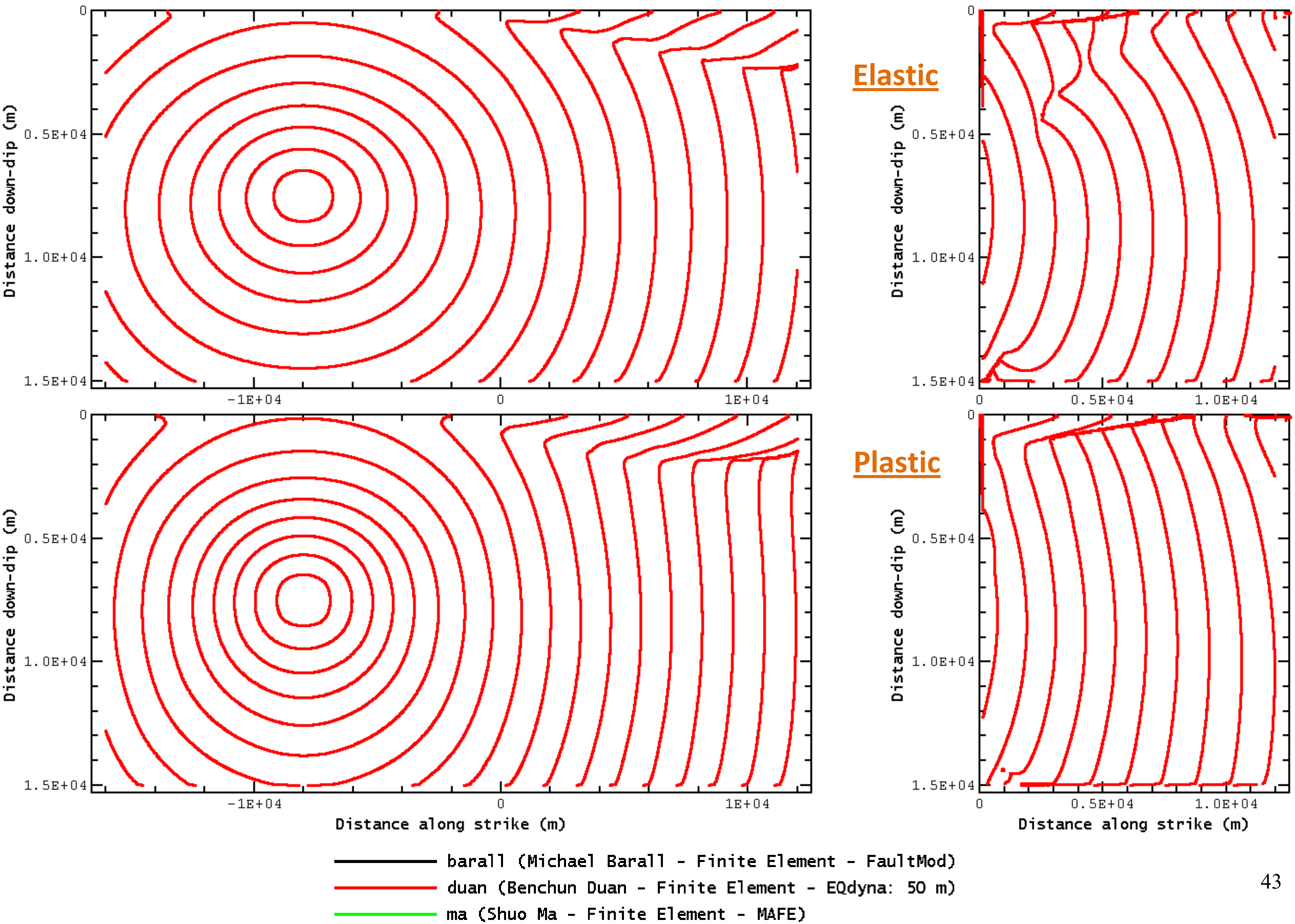
barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
duan (Benchun Duan - Finite Element - EQdyna: 50 m)

Comparison of Elastic versus Plastic Rupture Contours

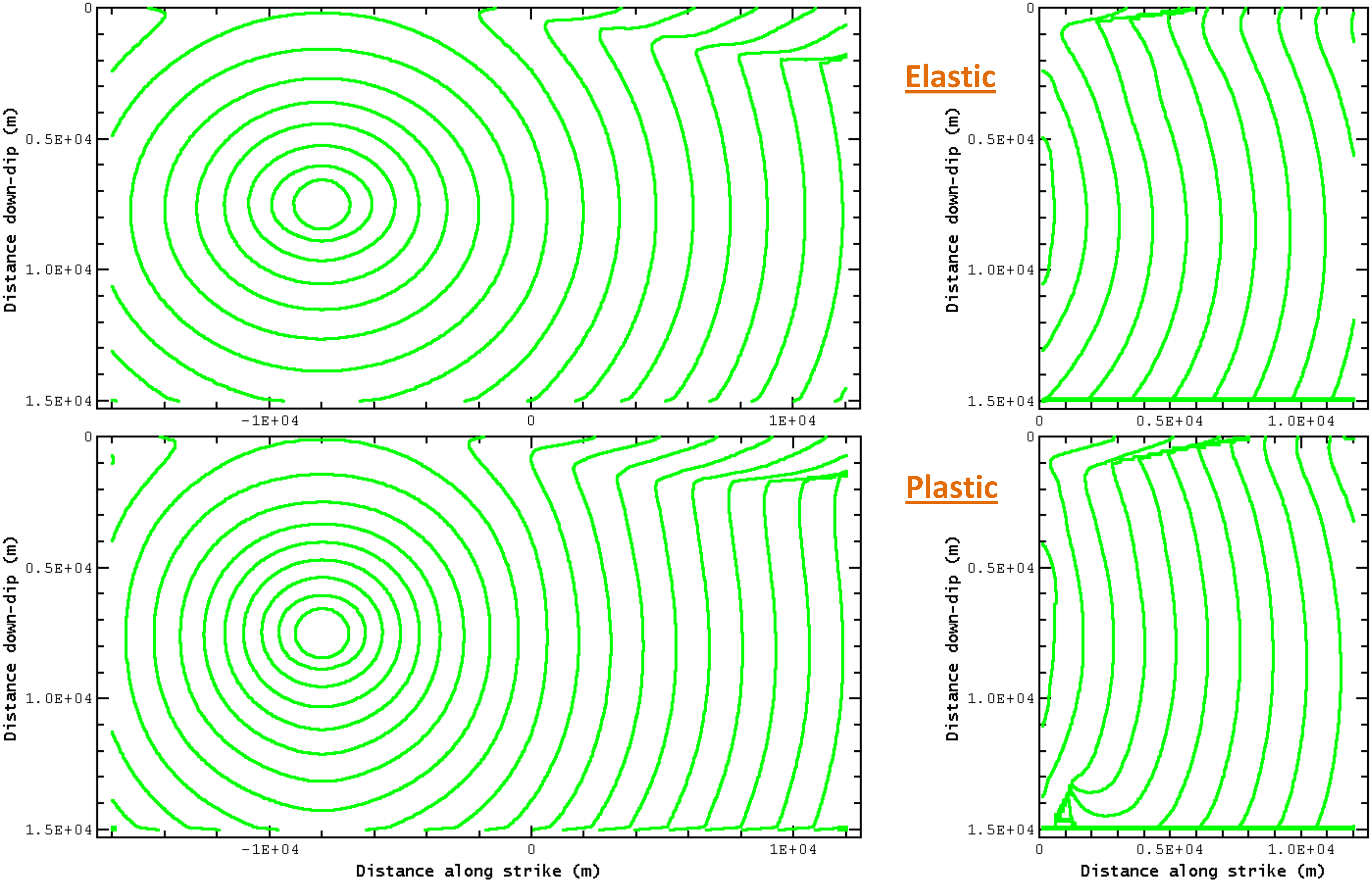
TPV18-19 (Right-Lateral) — Comparison of Elastic and Plastic Rupture Contours



TPV18-19 (Right-Lateral) — Comparison of Elastic and Plastic Rupture Contours

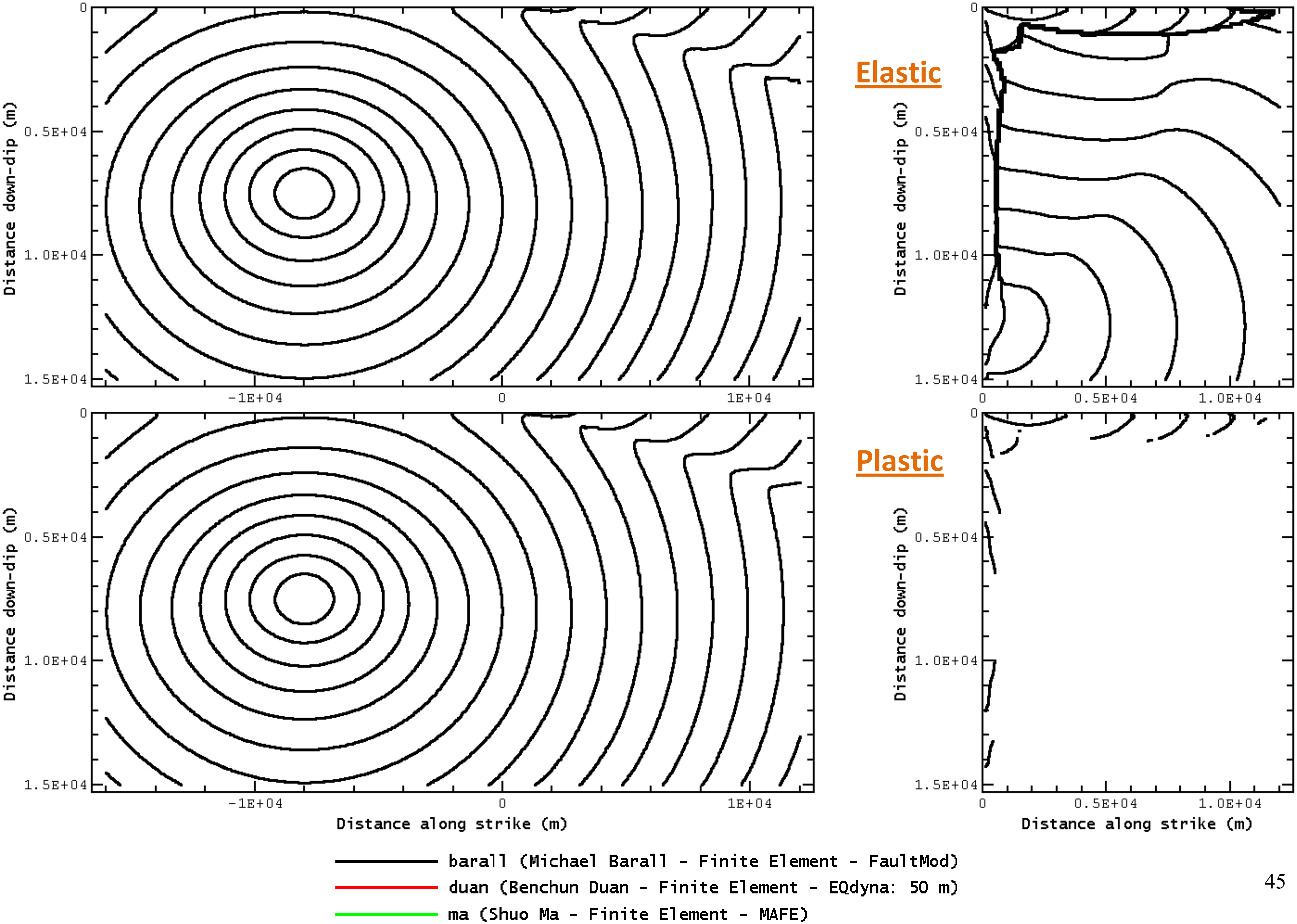


TPV18-19 (Right-Lateral) — Comparison of Elastic and Plastic Rupture Contours

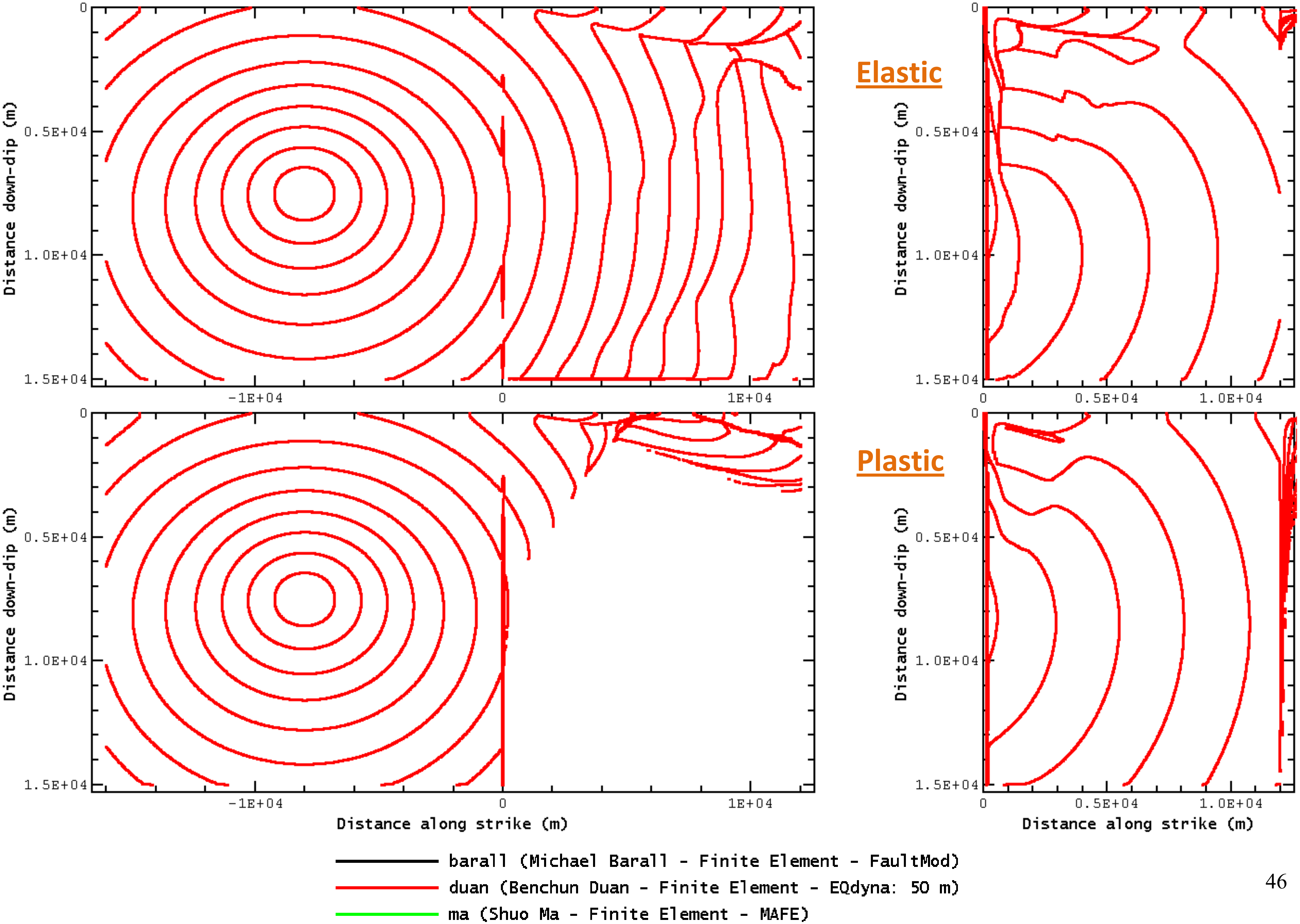


barall (Michael Barall - Finite Element - FaultMod)
duan (Benchun Duan - Finite Element - EQdyna: 50 m)
ma (Shuo Ma - Finite Element - MAFE)

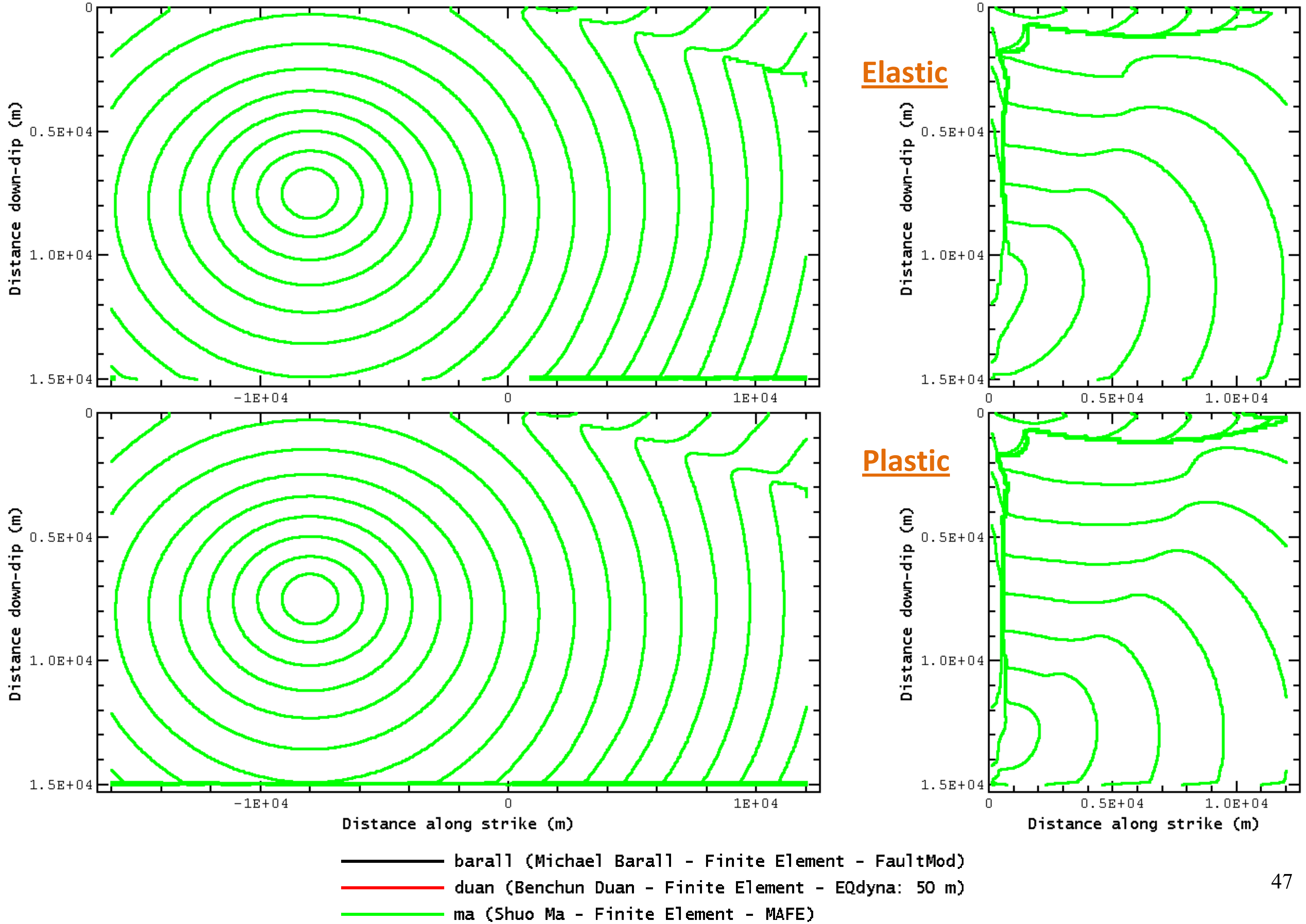
TPV20-21 (Left-Lateral) — Comparison of Elastic and Plastic Rupture Contours



TPV20-21 (Left-Lateral) — Comparison of Elastic and Plastic Rupture Contours

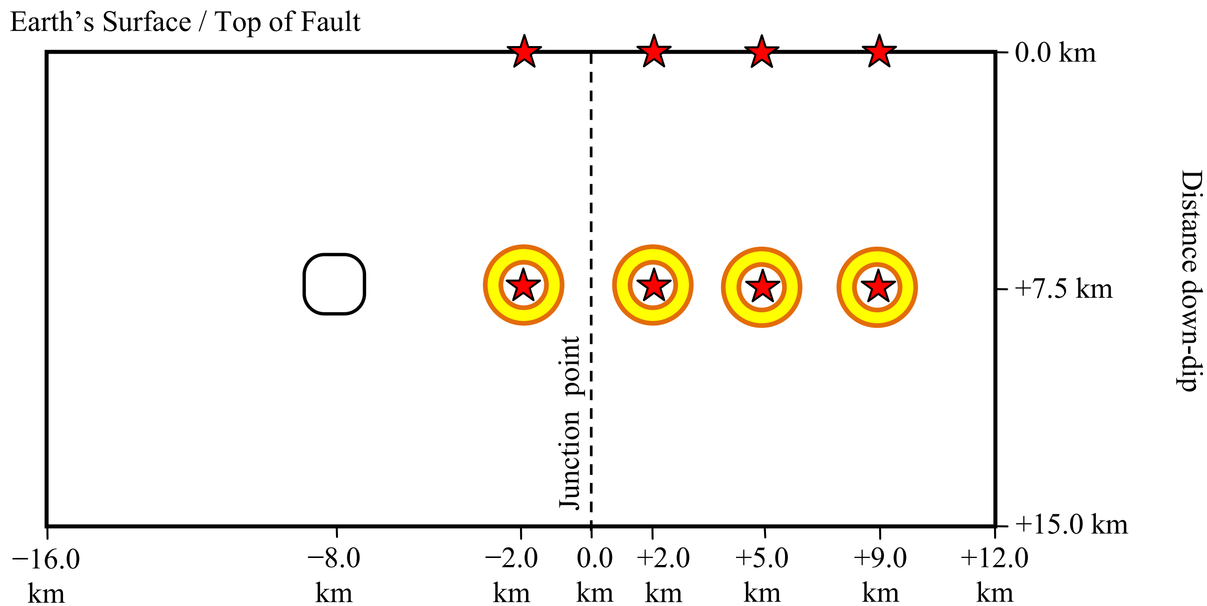


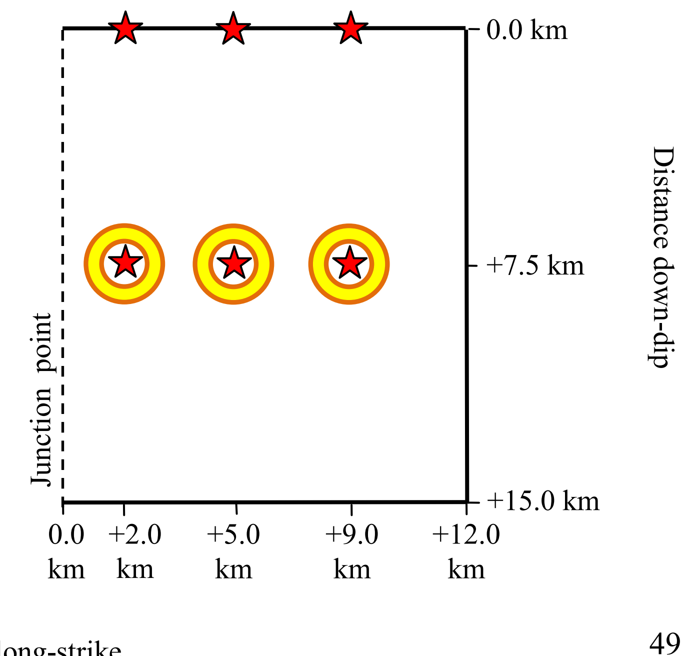
TPV20-21 (Left-Lateral) — Comparison of Elastic and Plastic Rupture Contours



Fault Slip Patterns

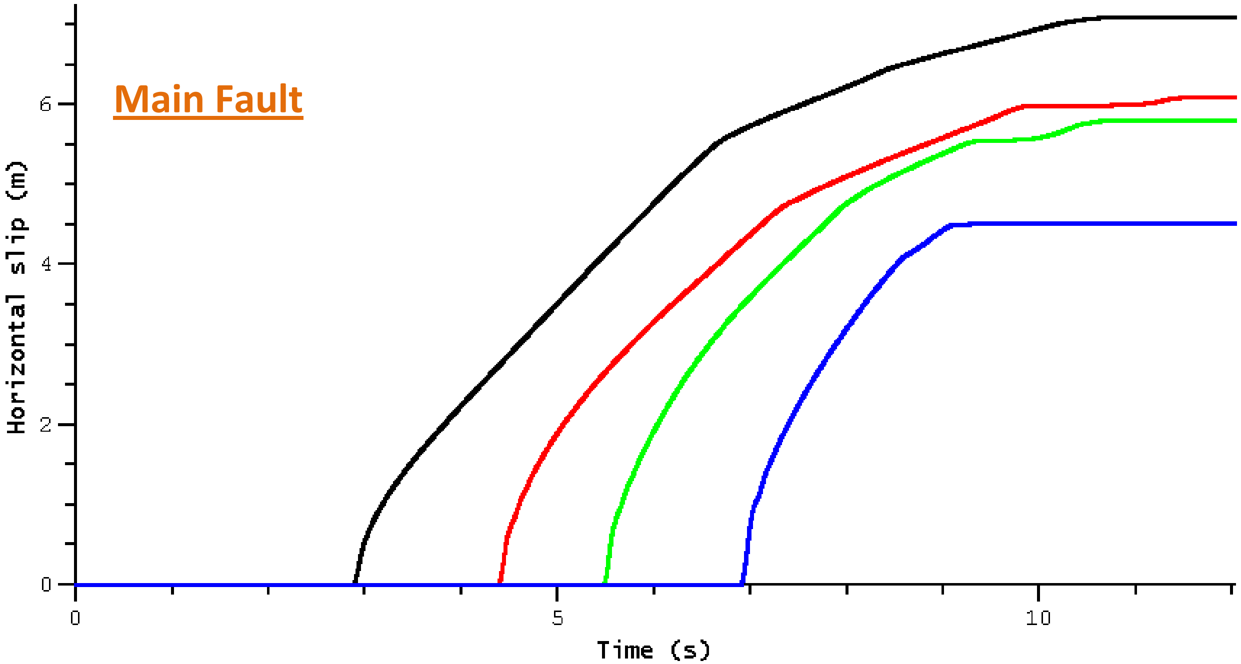
Stations for Slip Comparison

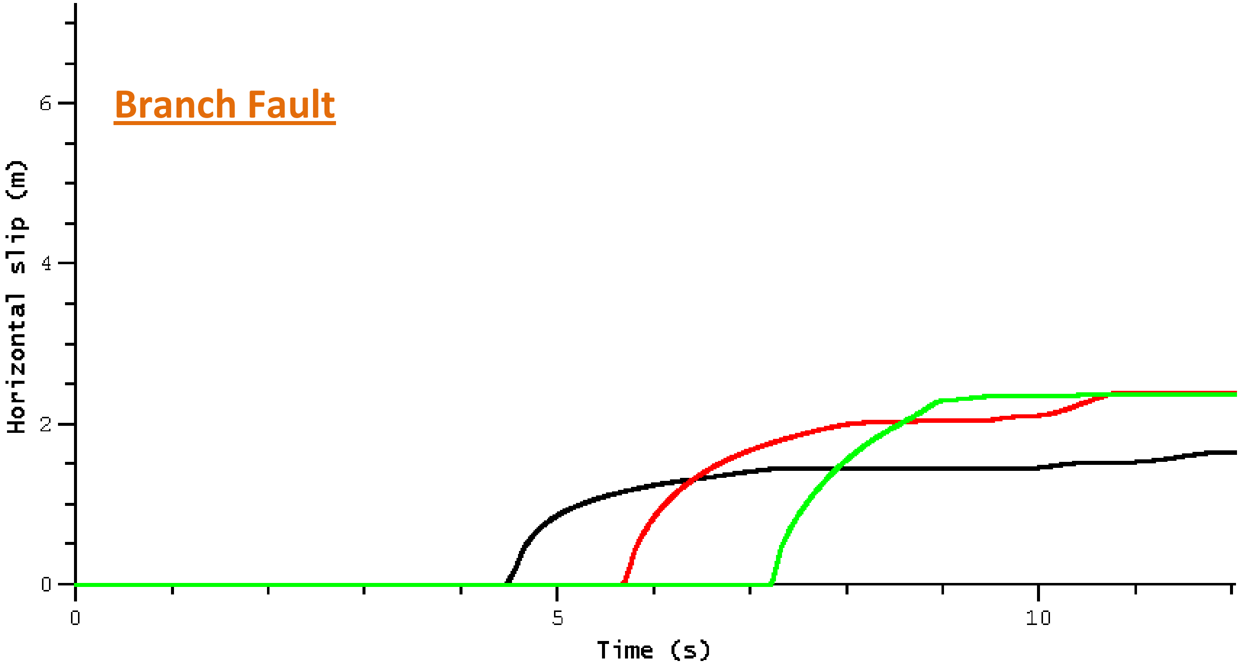




We will show slip and slip rate for the marked stations.

Slip rates are filtered with a 3 Hz low-pass filter.





TPV18
(Right-Lateral, Elastic)

Horizontal Slip

Code: Barall

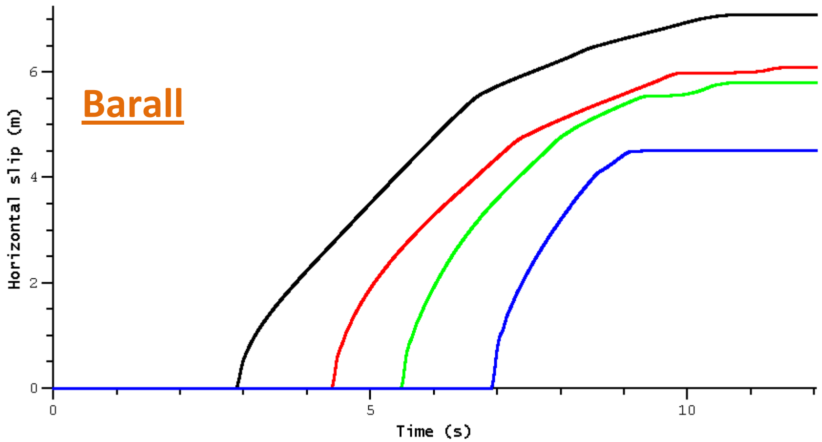
Each curve represents slip history at one station on the fault.

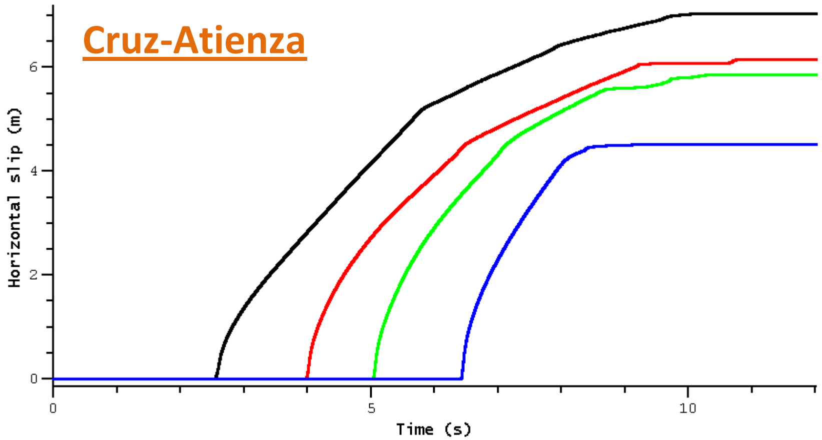
There is larger slip on the main fault than on the branch fault.

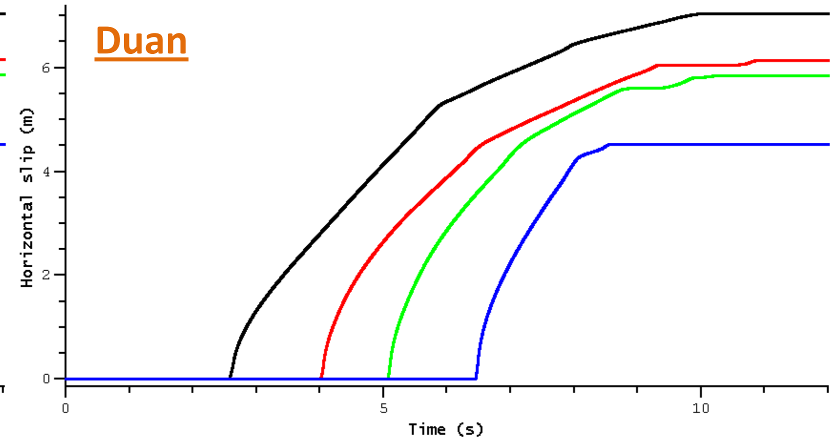
TPV18 (Right-Lateral, Elastic)

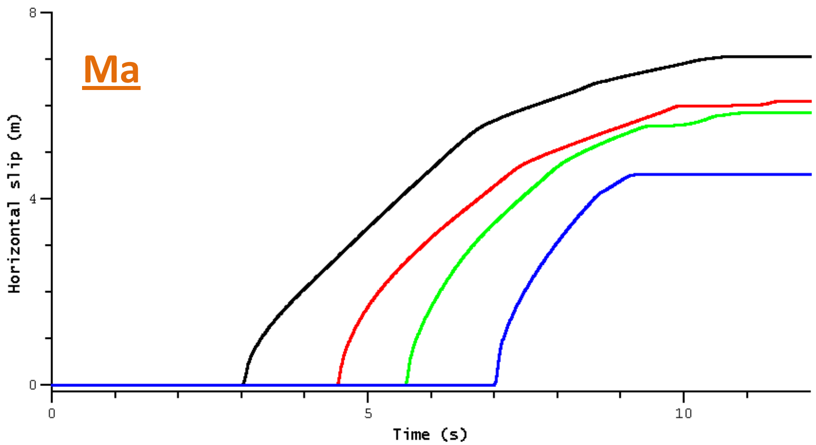
Horizontal Slip

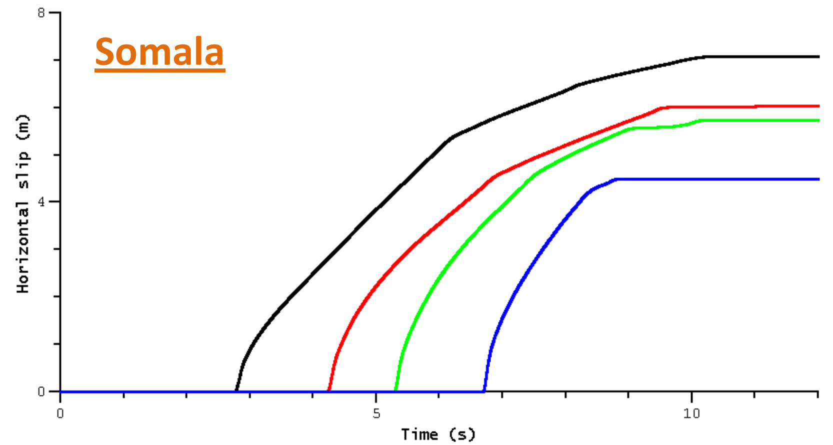
Main Fault







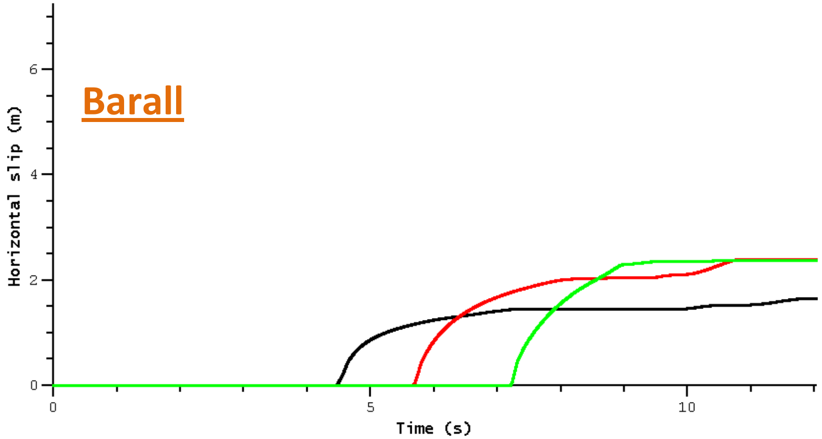


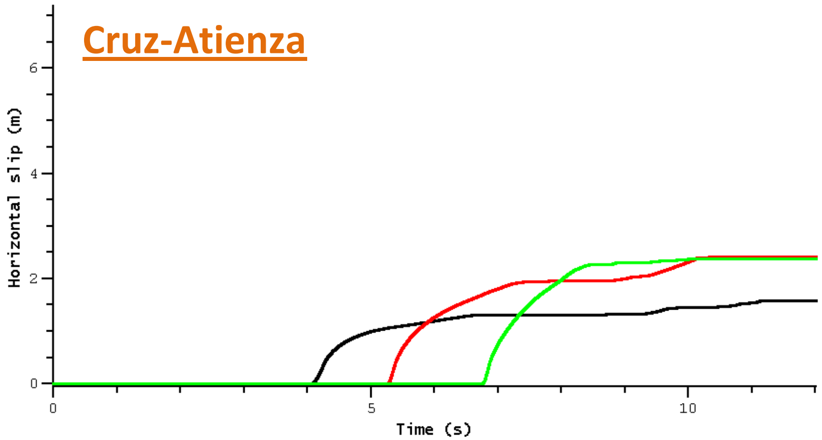


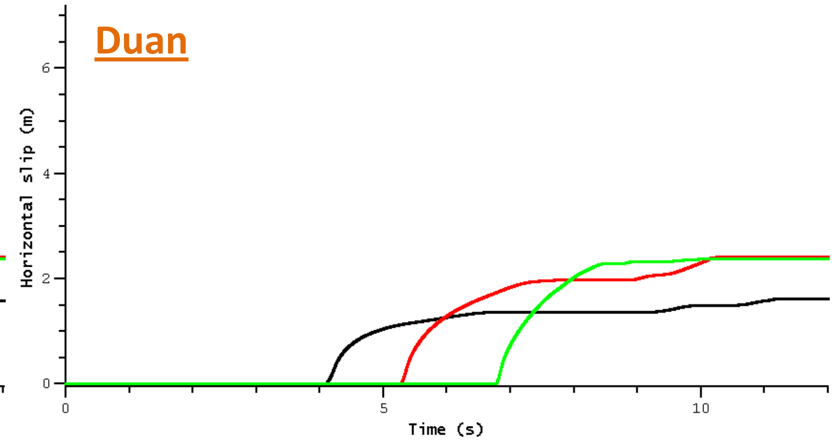
TPV18 (Right-Lateral, Elastic)

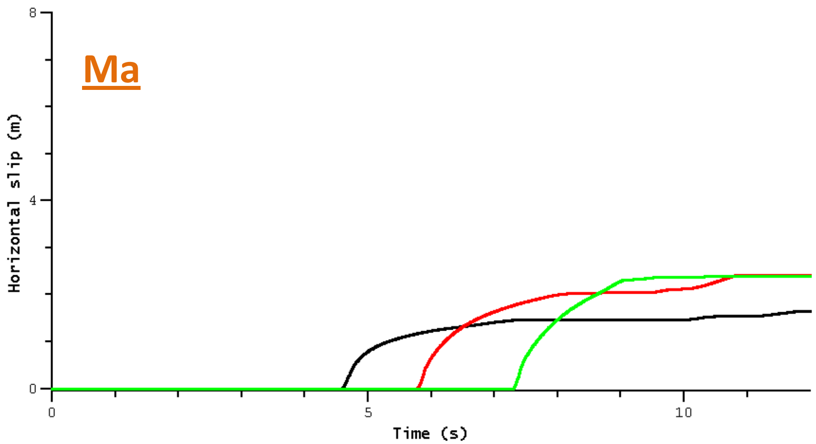
Horizontal Slip

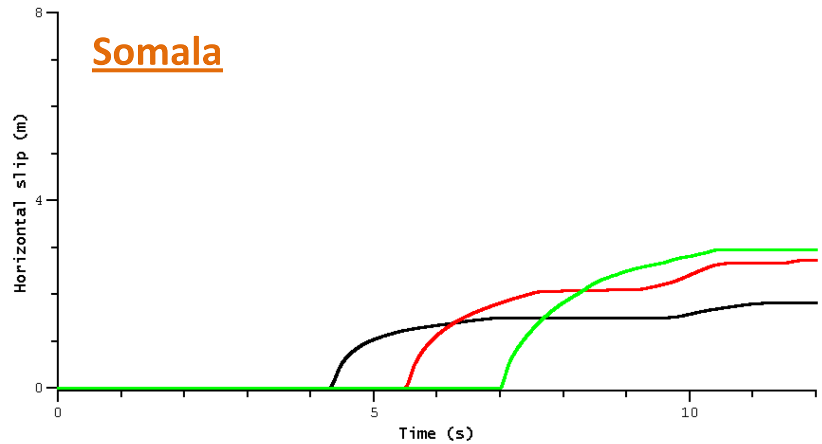
Branch Fault







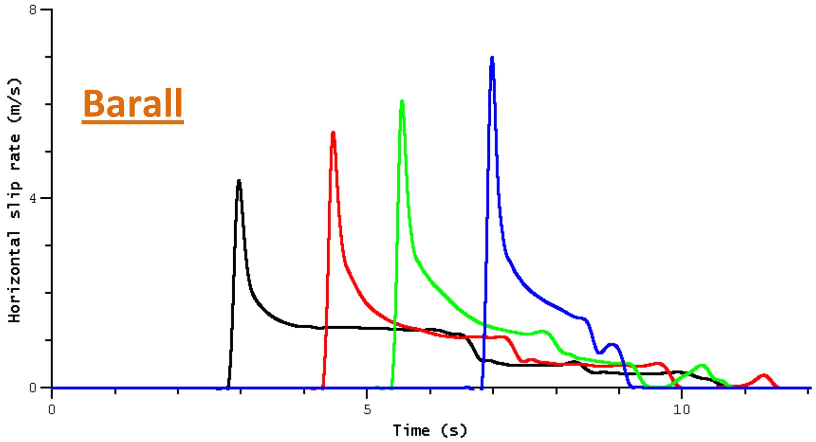


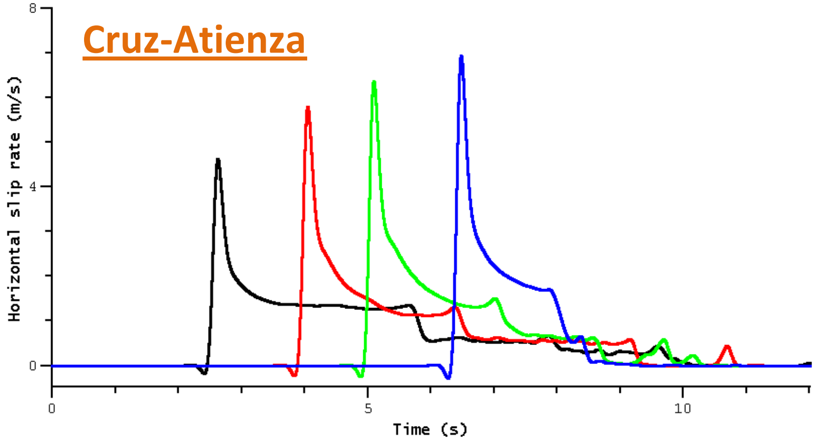


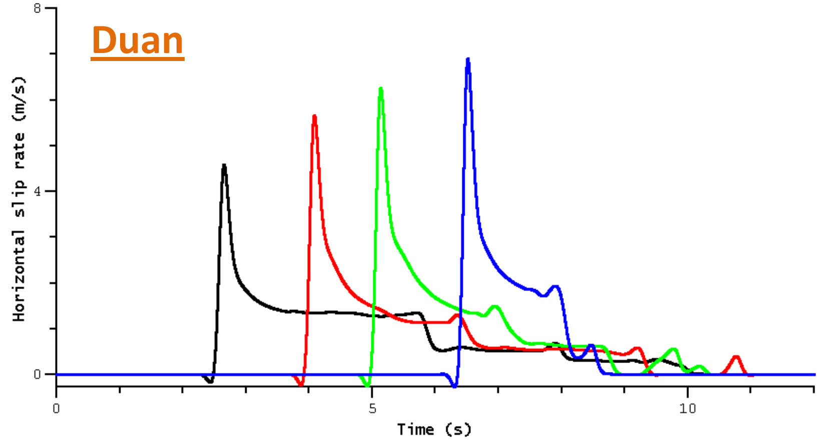
TPV18 (Right-Lateral, Elastic)

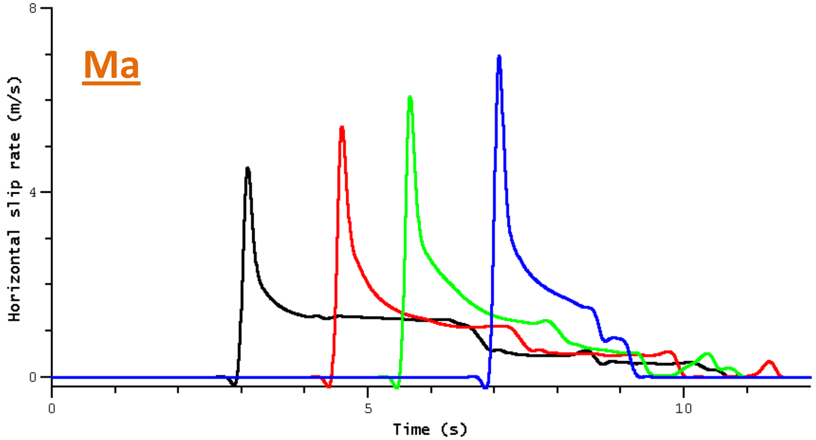
Horizontal Slip Rate

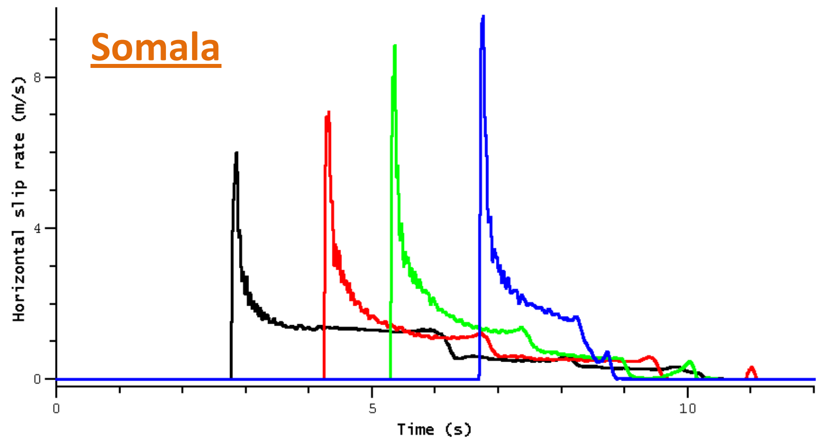
Main Fault







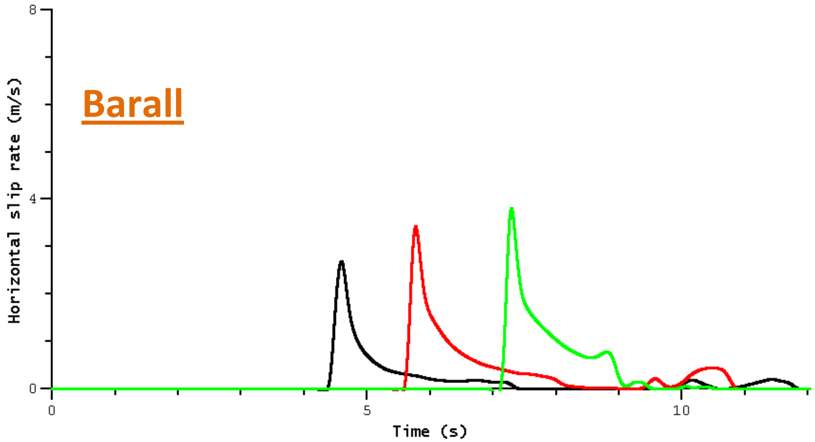


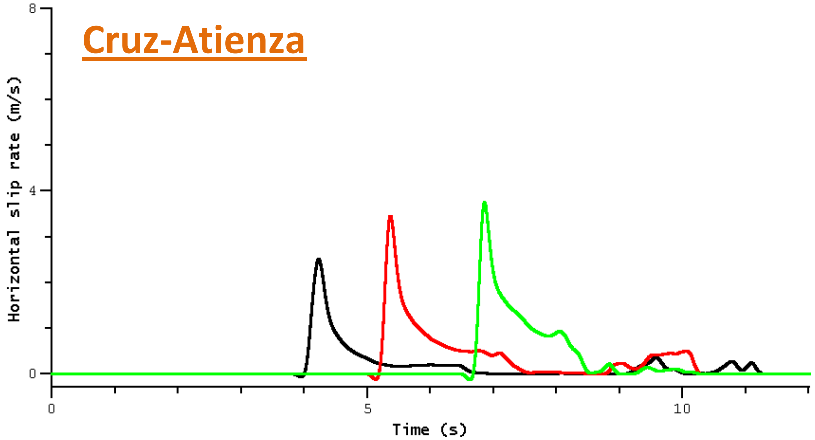


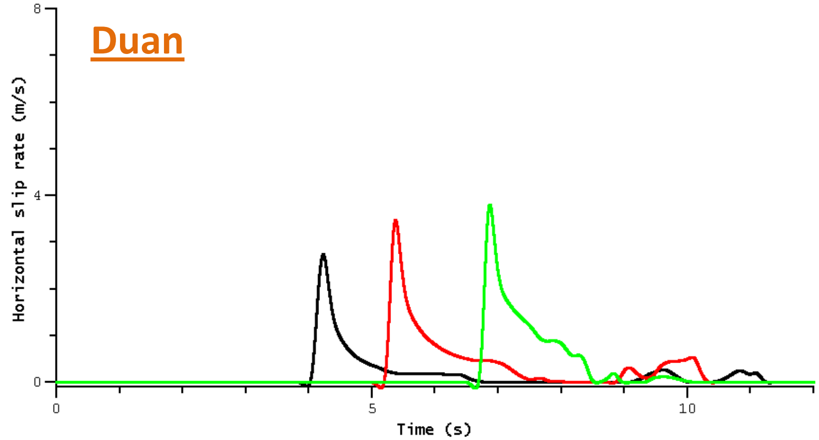
TPV18 (Right-Lateral, Elastic)

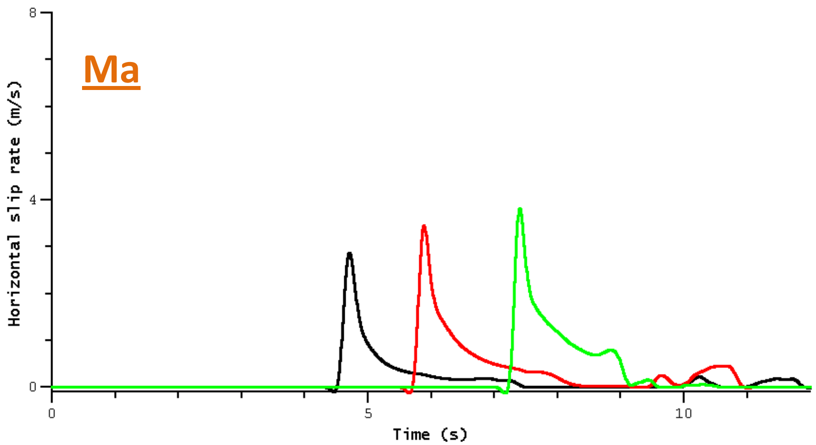
Horizontal Slip Rate

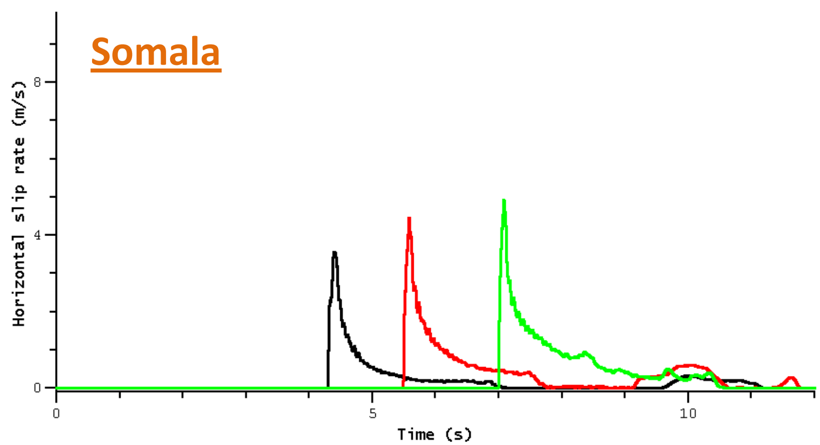
Branch Fault

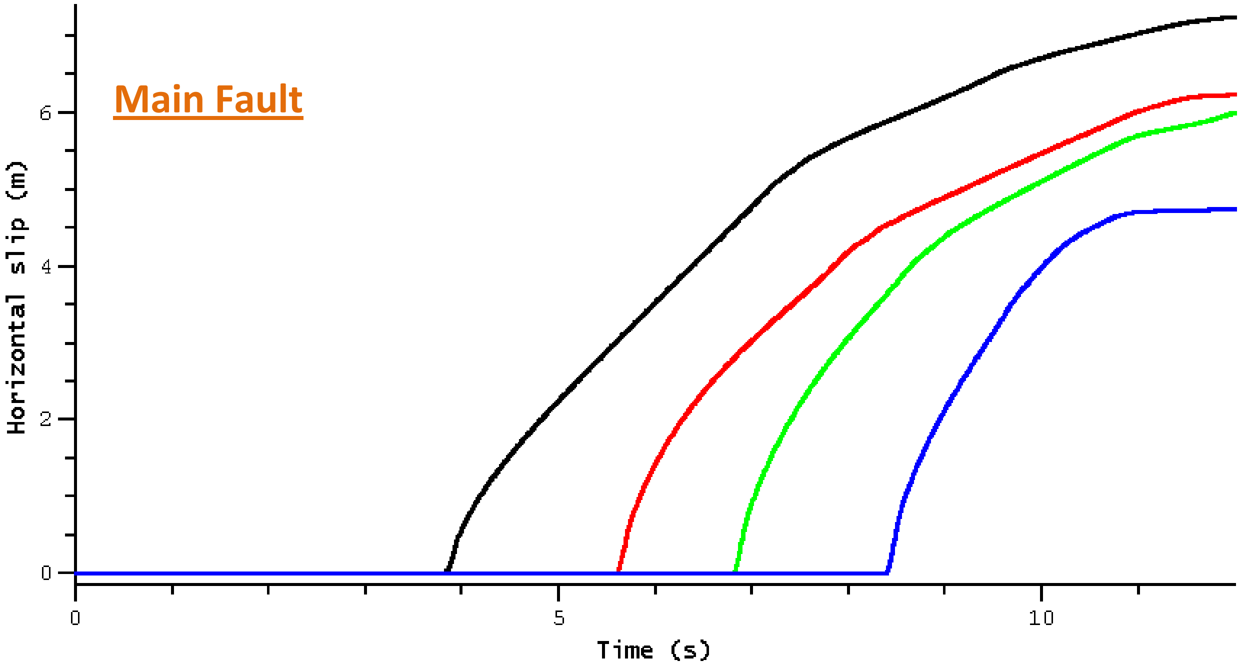








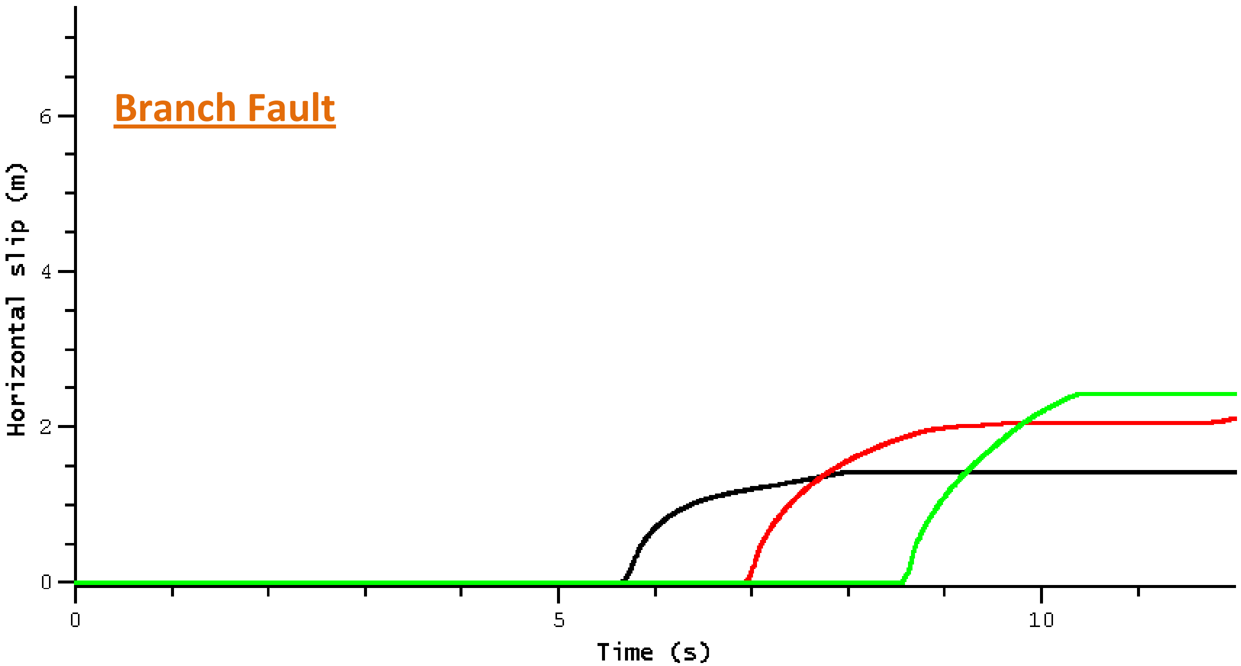




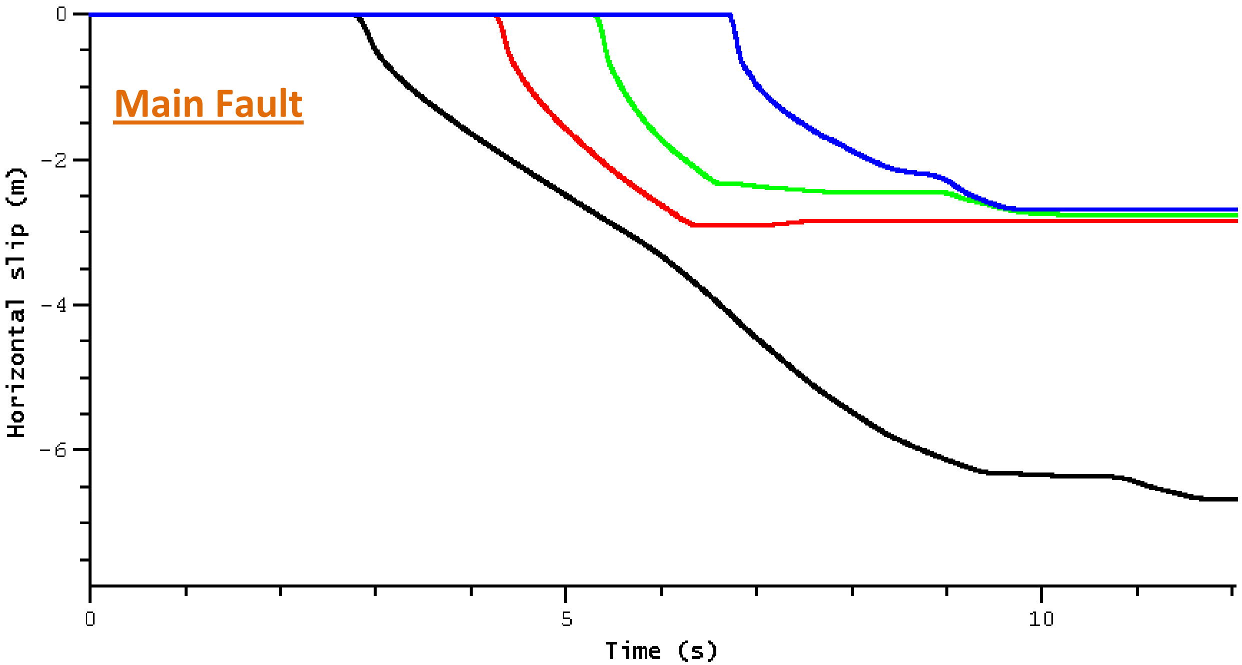
TPV19
(Right-Lateral, Plastic)

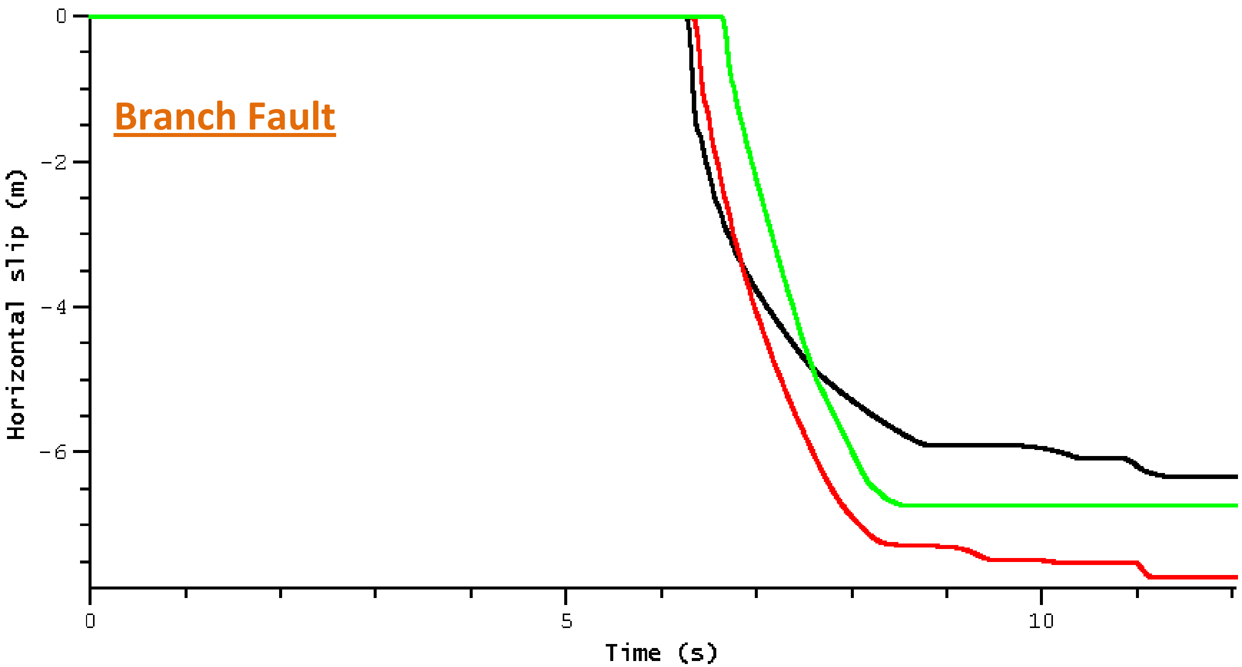
Horizontal Slip

Code: Ma



Curves for TPV19 are very similar to TPV18, except with slower rupture propagation and slightly lower peak slip rates.





TPV20
(Left-Lateral, Elastic)

Horizontal Slip

Code: Barall

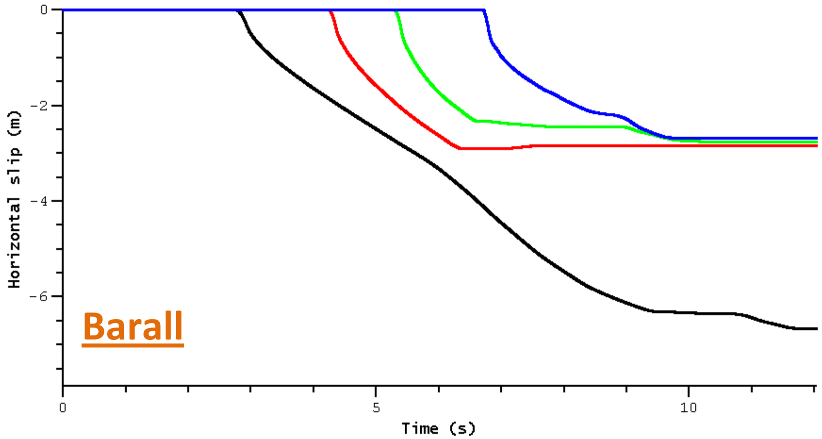
Each curve represents slip history at one station on the fault.

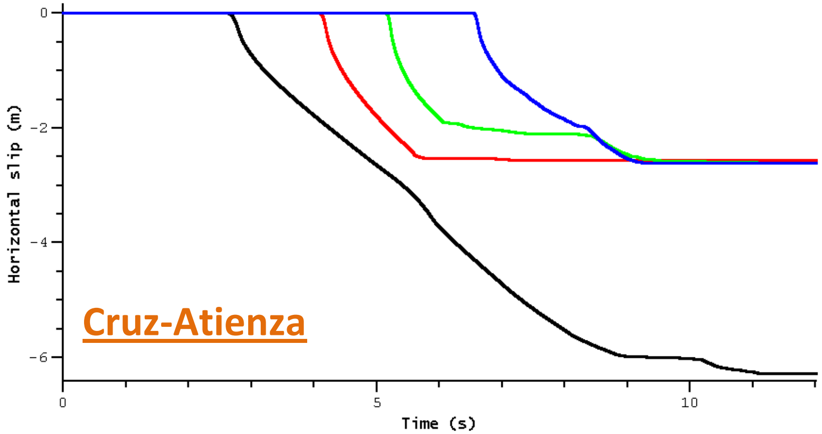
To the right of the junction point, there is larger slip on the branch fault than on the main fault.

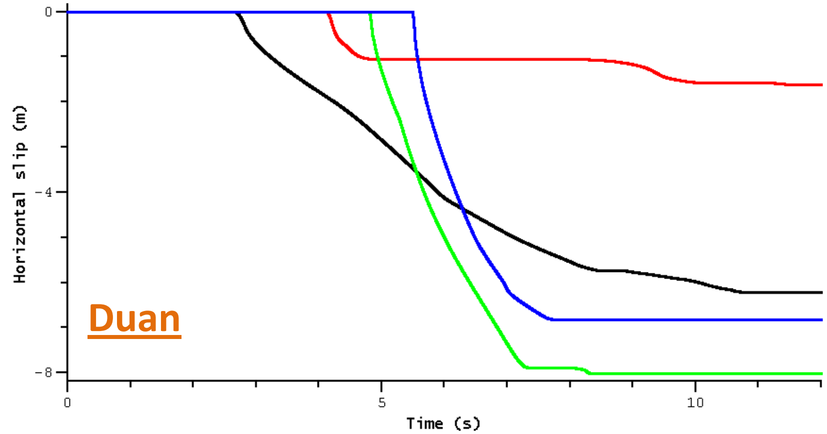
TPV20 (Left-Lateral, Elastic)

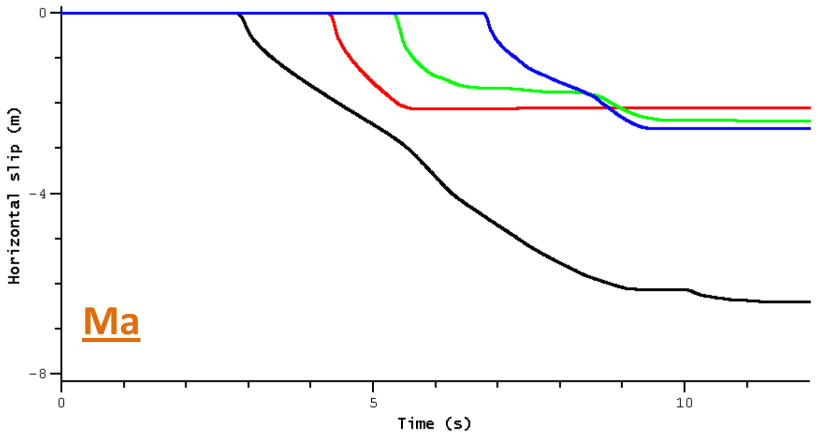
Horizontal Slip

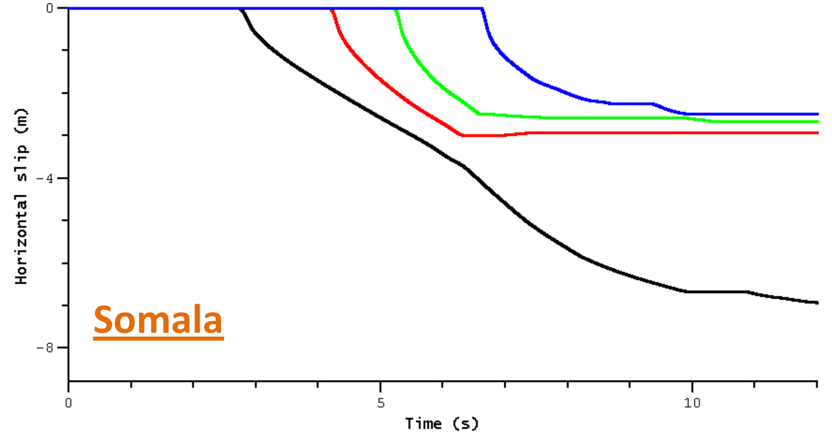
Main Fault







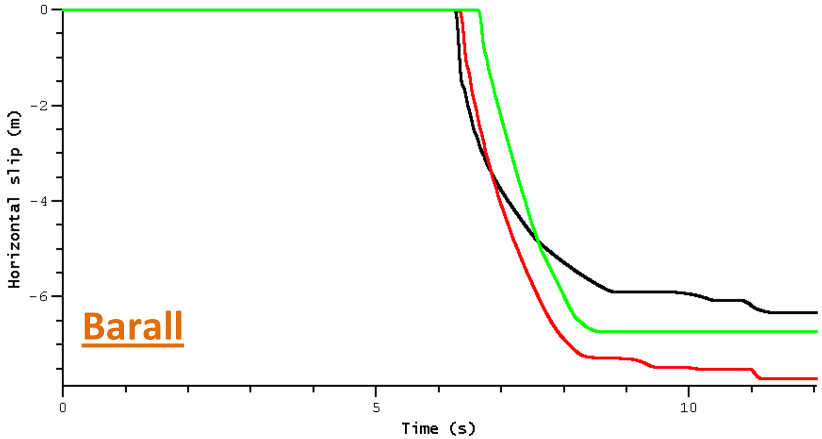


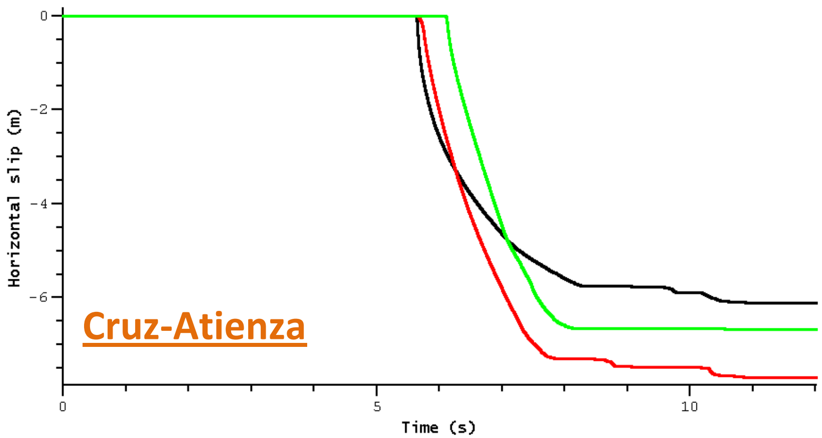


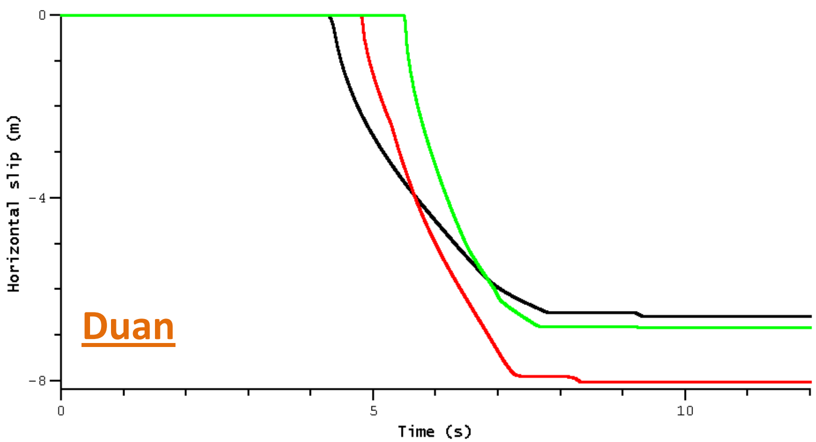
TPV20 (Left-Lateral, Elastic)

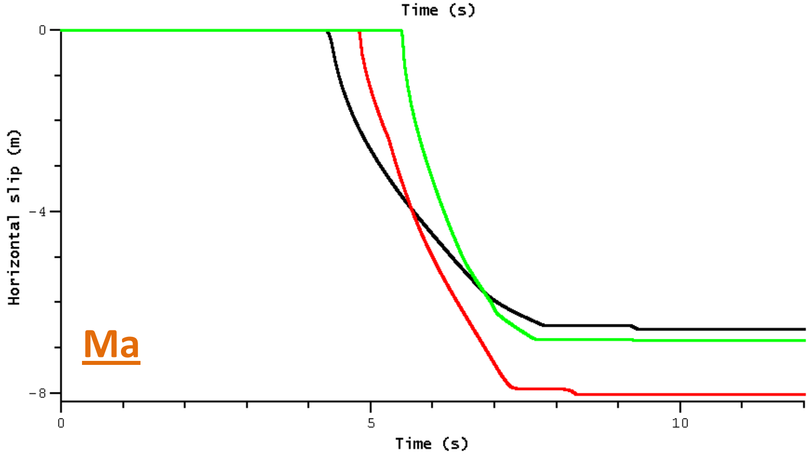
Horizontal Slip

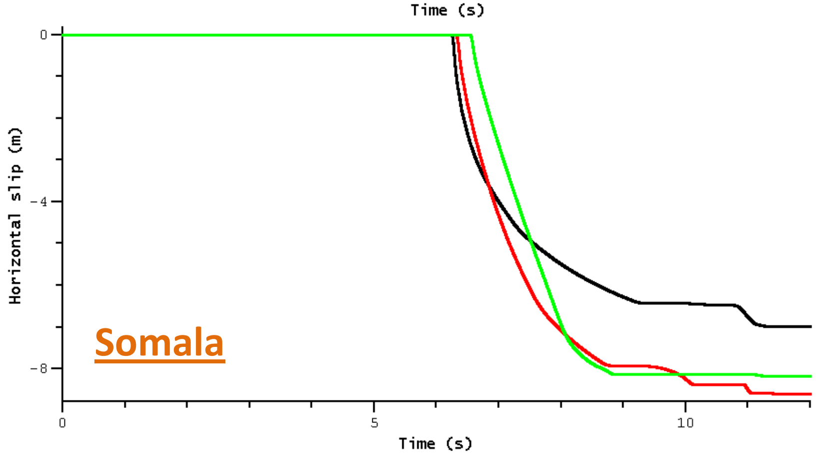
Branch Fault







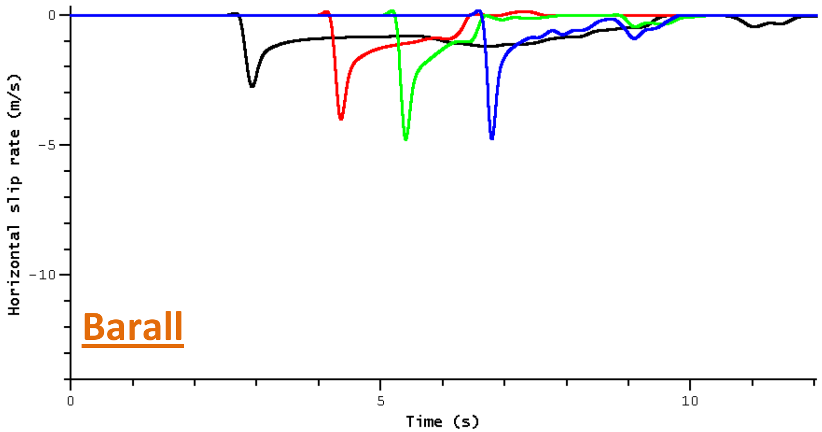


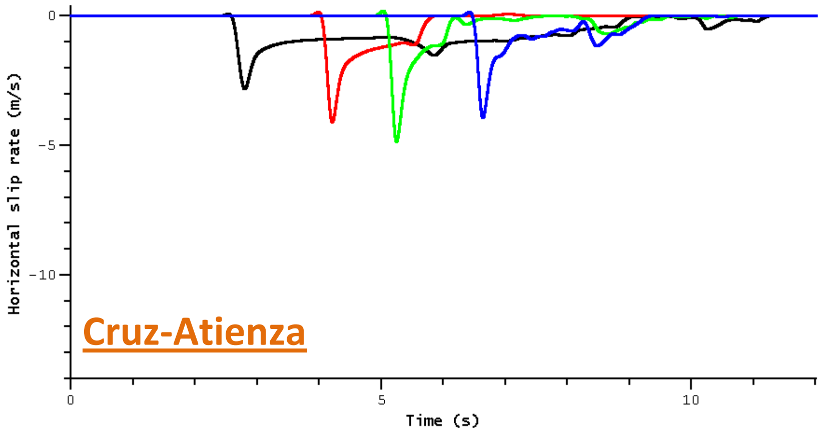


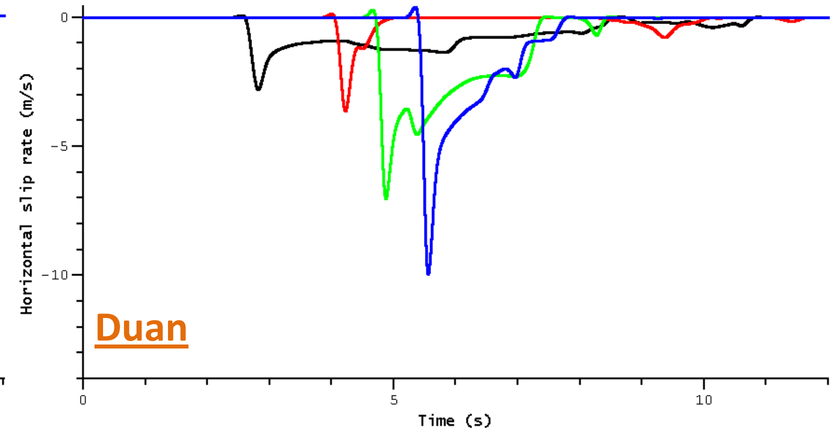
TPV20 (Left-Lateral, Elastic)

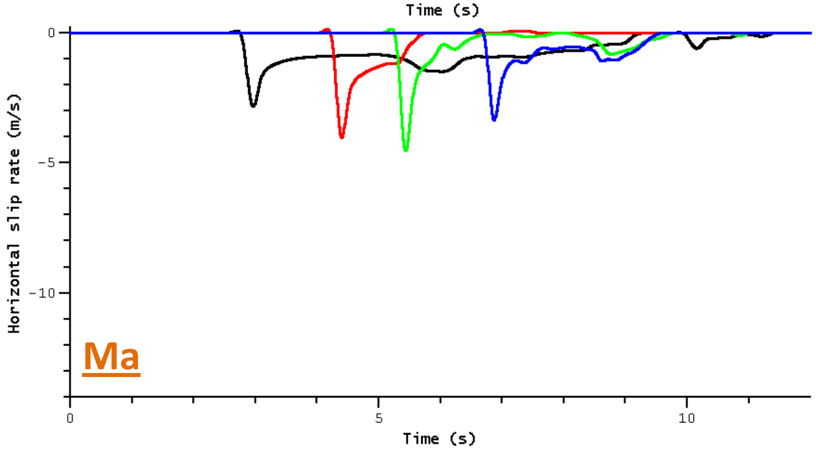
Horizontal Slip Rate

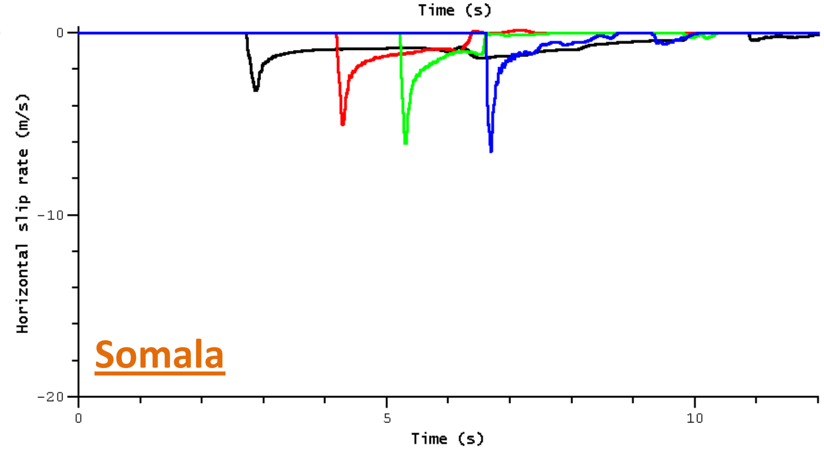
Main Fault







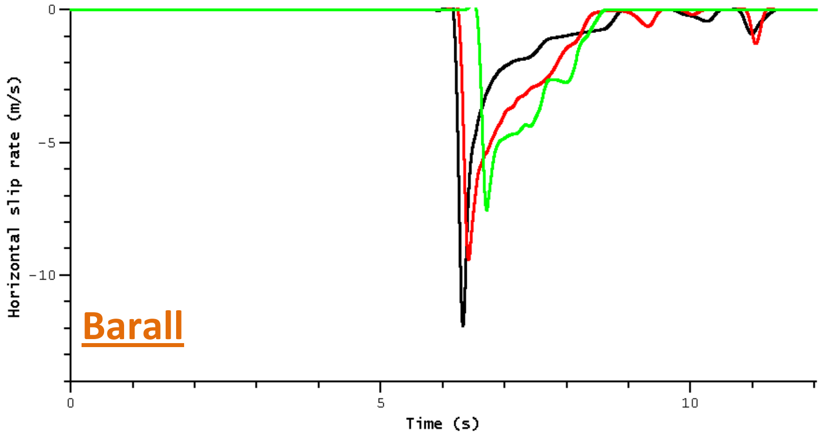


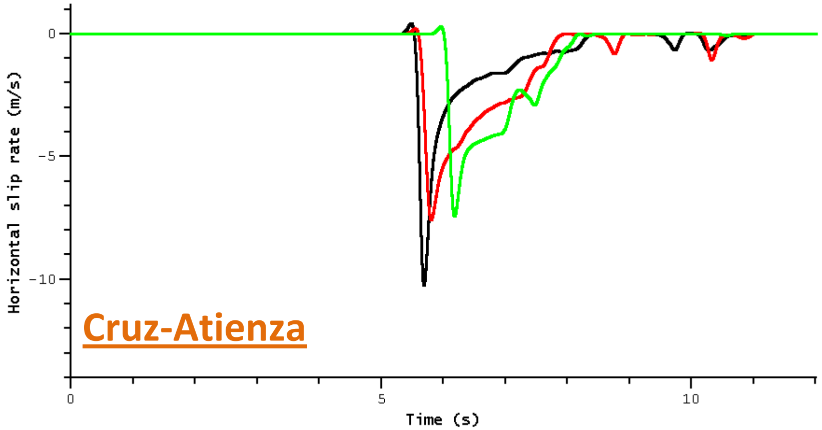


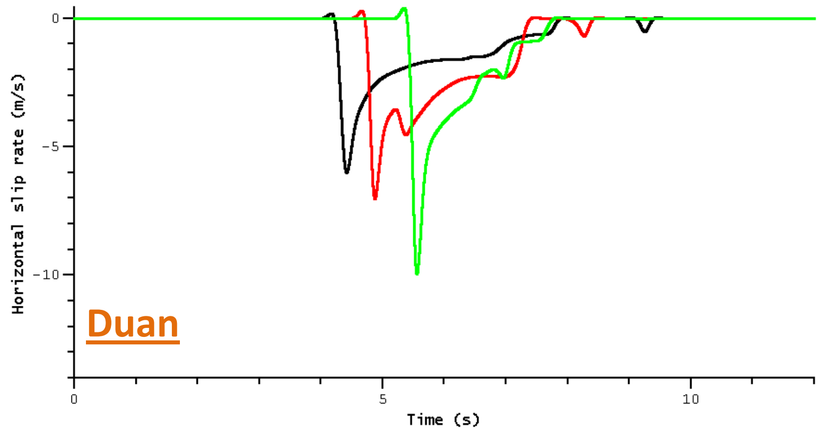
TPV20 (Left-Lateral, Elastic)

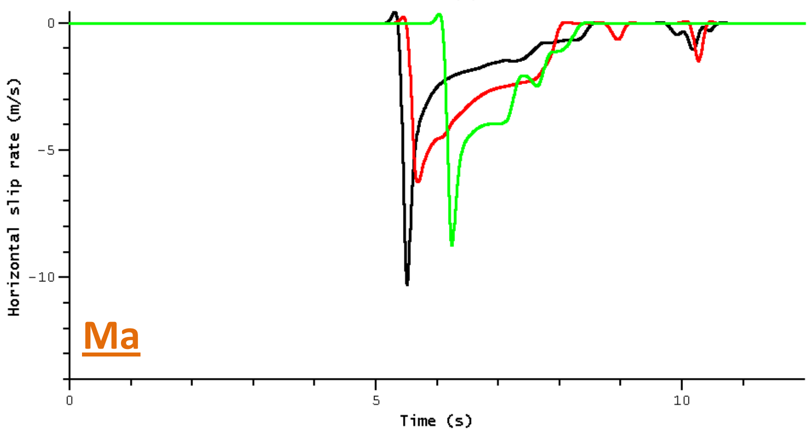
Horizontal Slip Rate

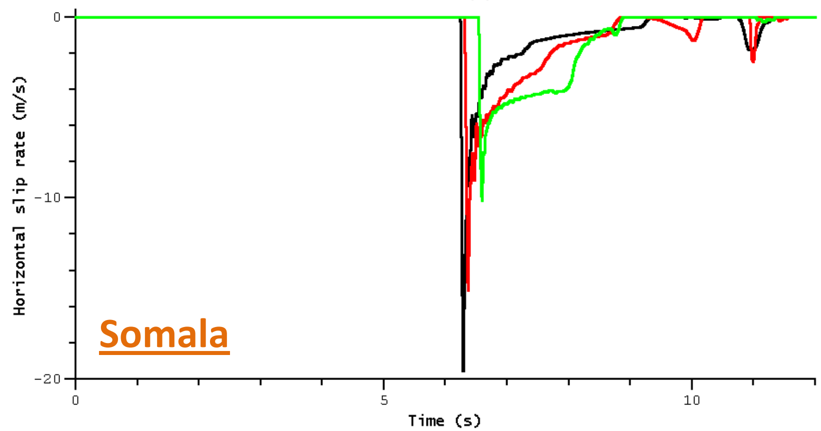
Branch Fault

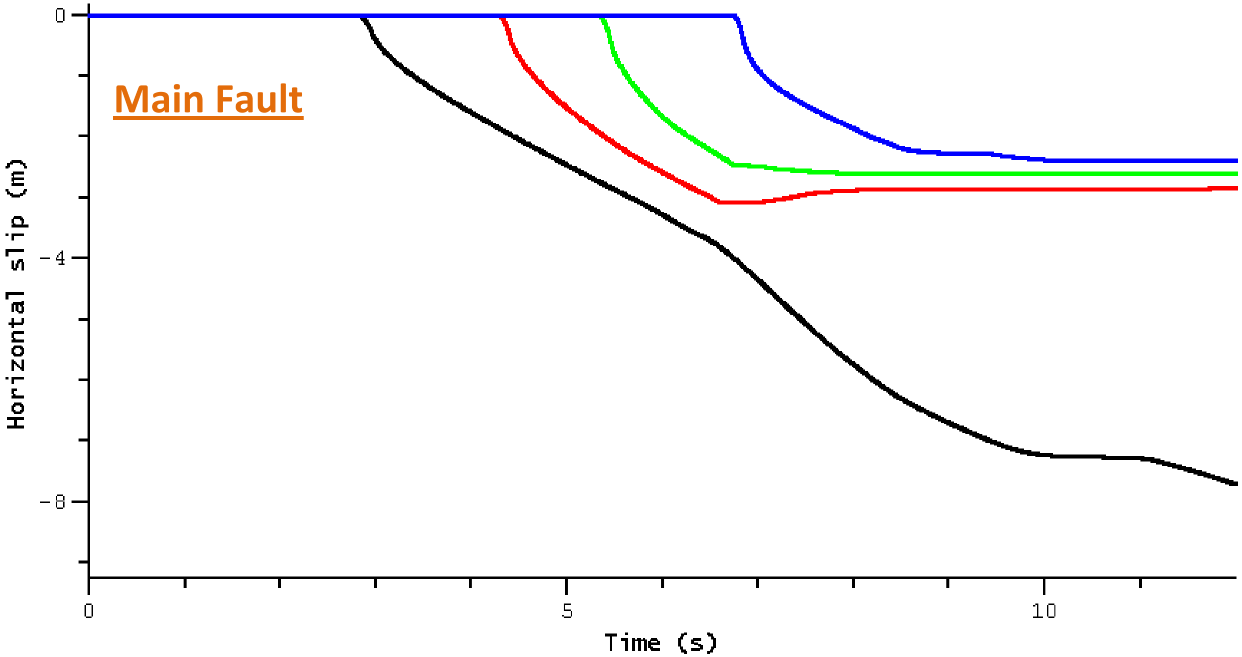


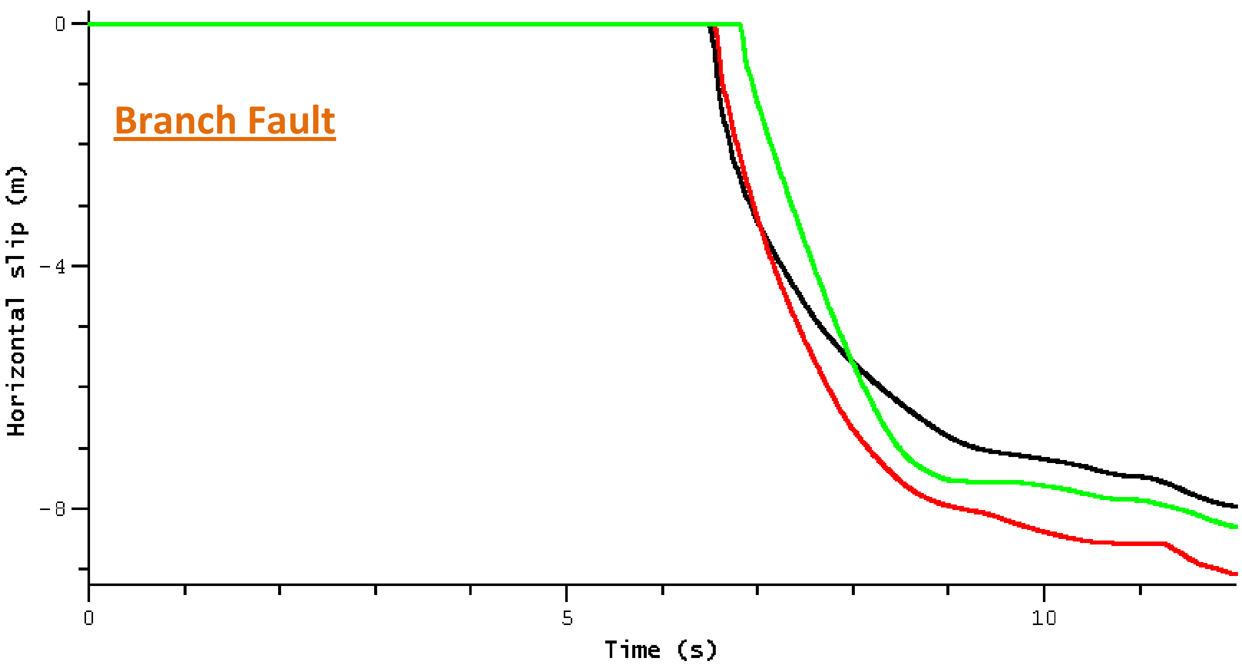










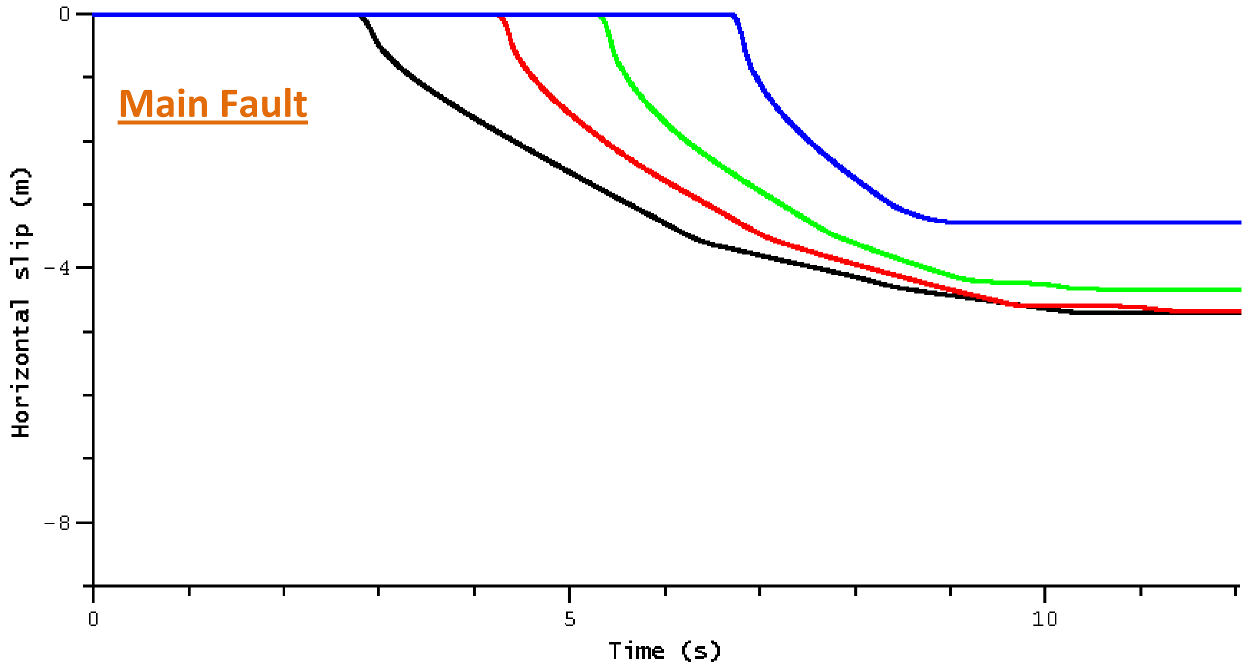


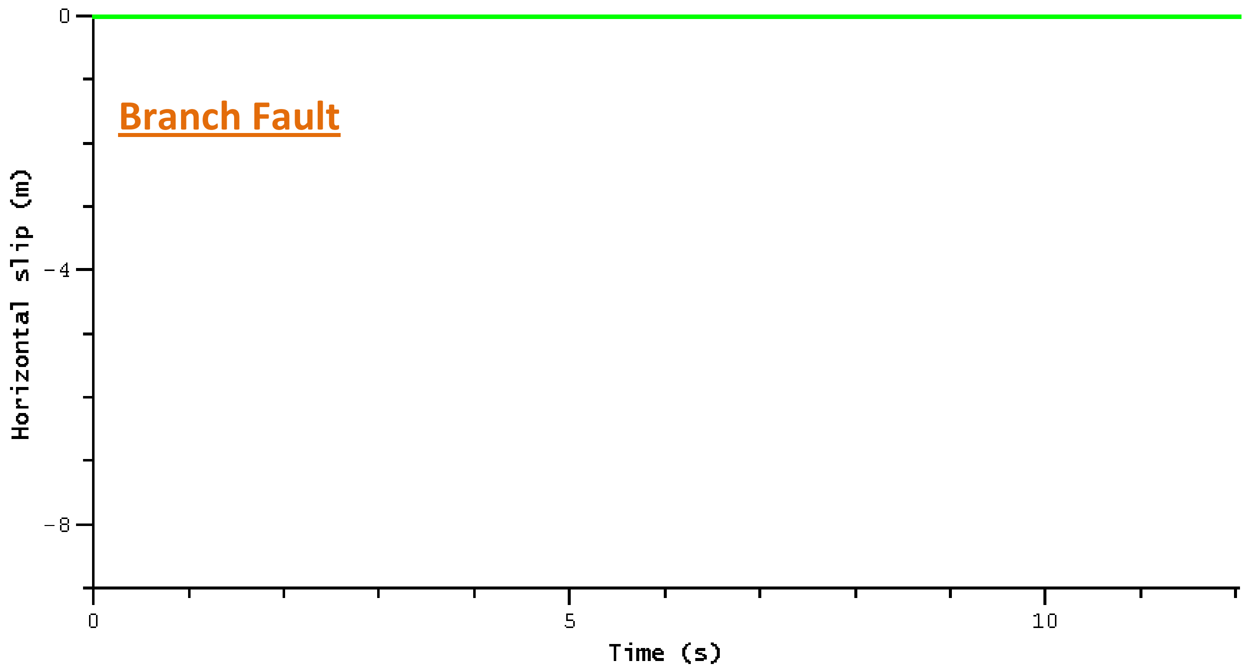
TPV21
(Left-Lateral, Plastic)

Horizontal Slip

Code: Ma

The Ma code ruptures both main and branch faults, producing results similar to the elastic case TPV20.



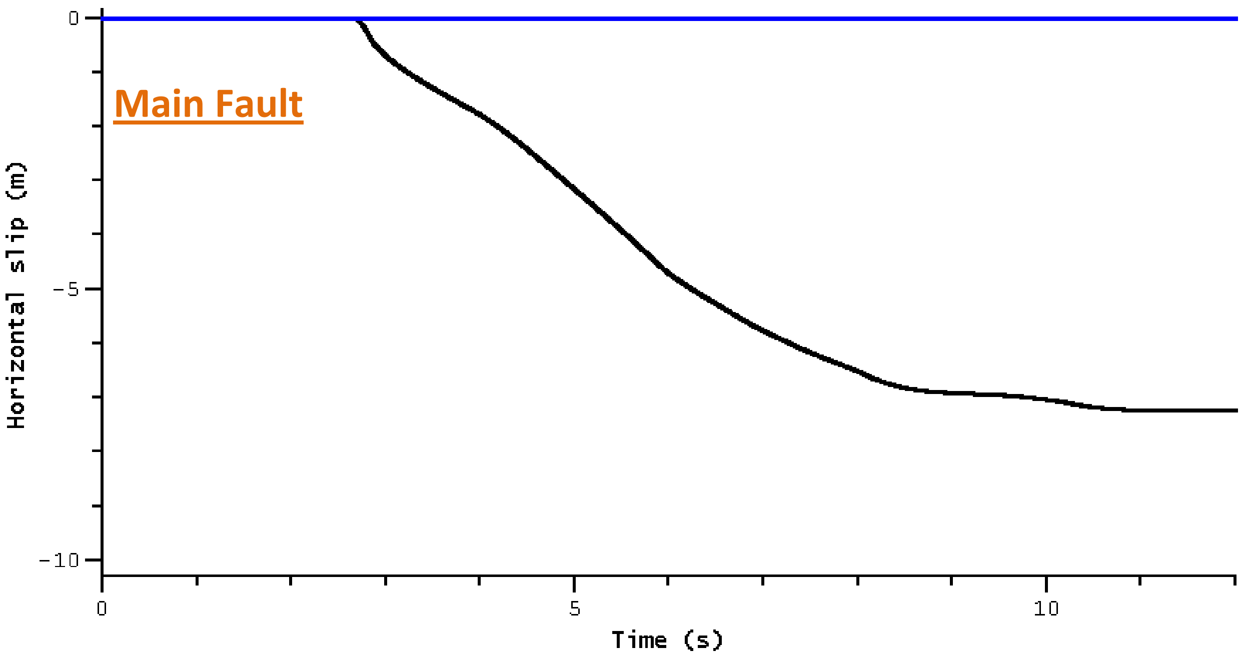


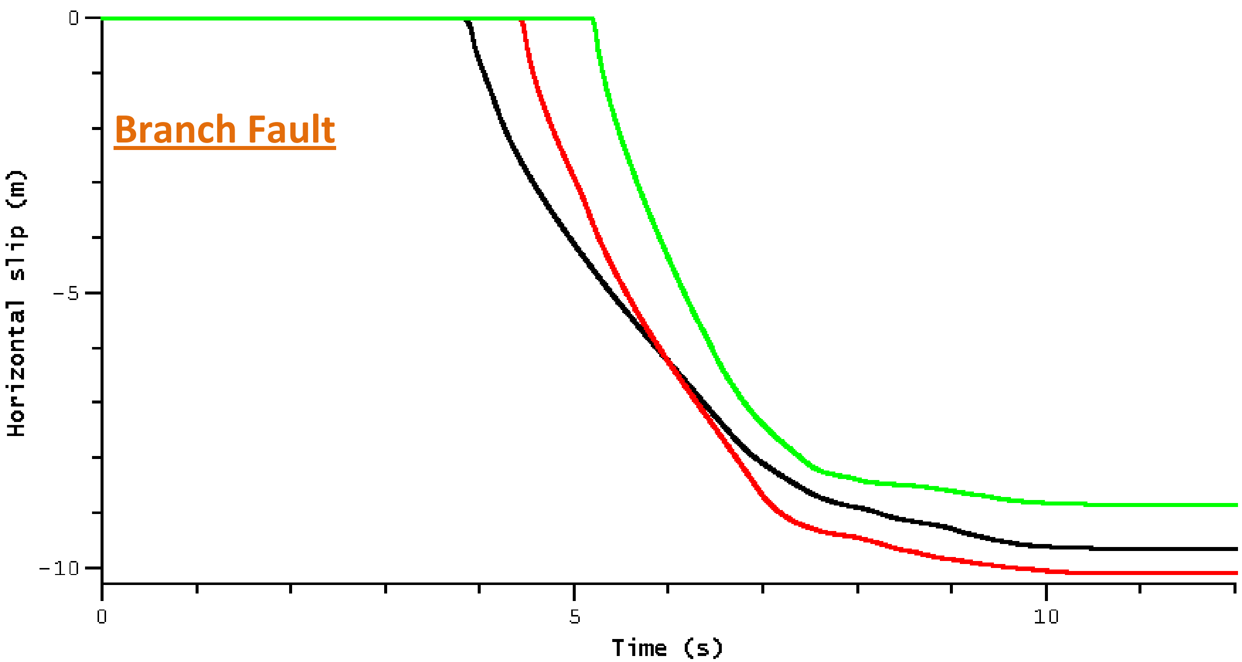
TPV21
(Left-Lateral, Plastic)

Horizontal Slip

Code: Barall

The Barall code ruptures the main fault but not the branch. The result is lower slip on the main fault than the other codes.





TPV21

(Left-Lateral, Plastic)

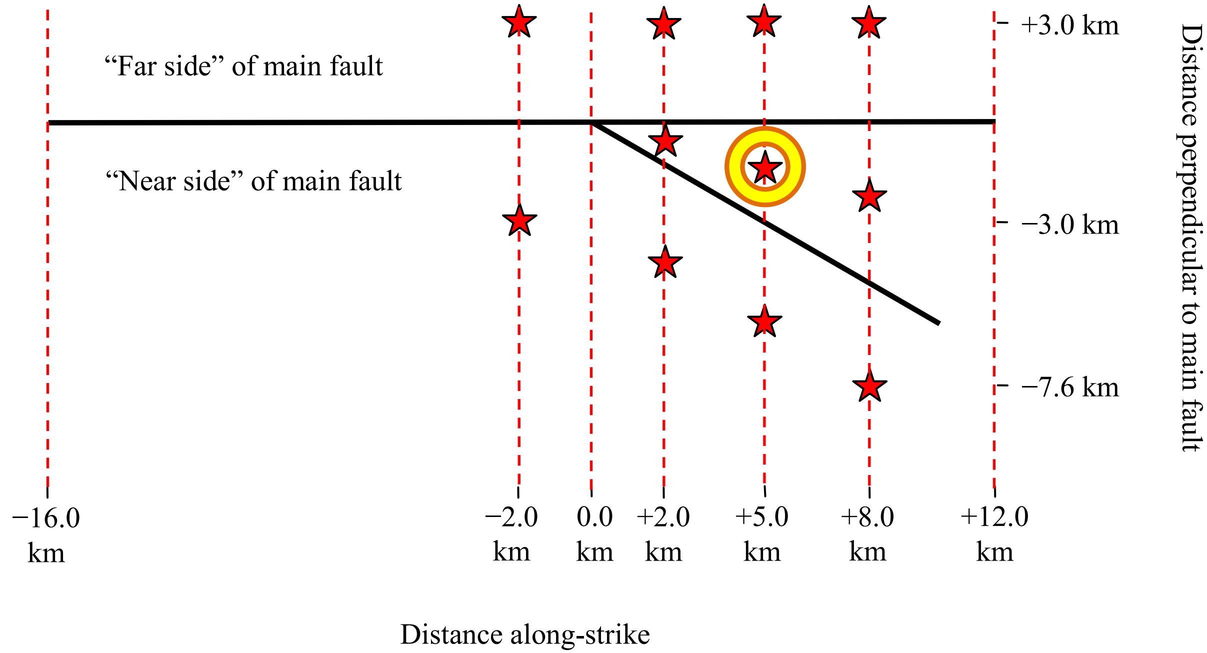
Horizontal Slip

Code: Duan

The Duan code does not rupture the main fault to the right of the junction point. The result is larger slip on the branch fault than the other codes.

Simulated Ground Motions

Station for Ground Motion Comparison

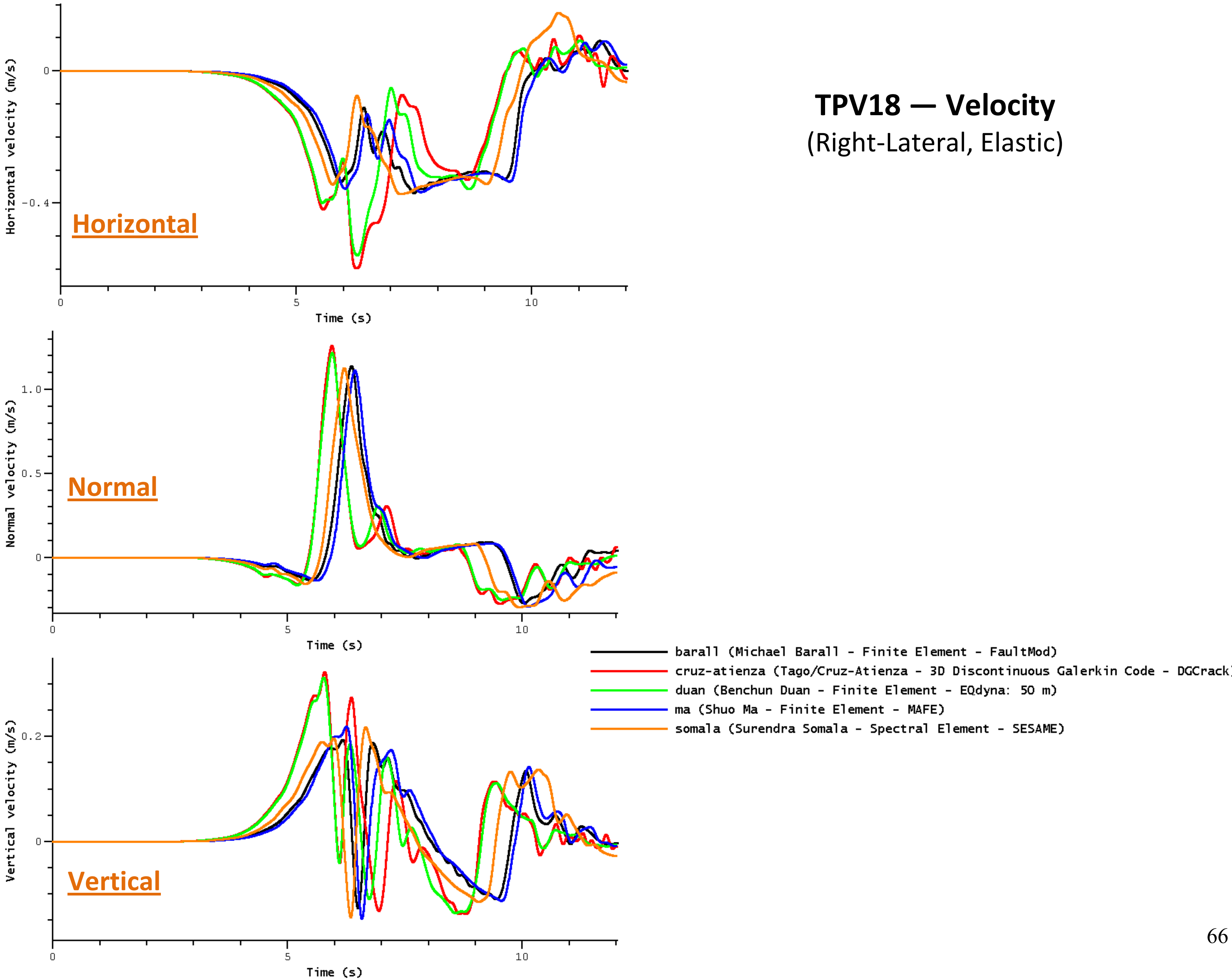


We will show velocity for the marked station.

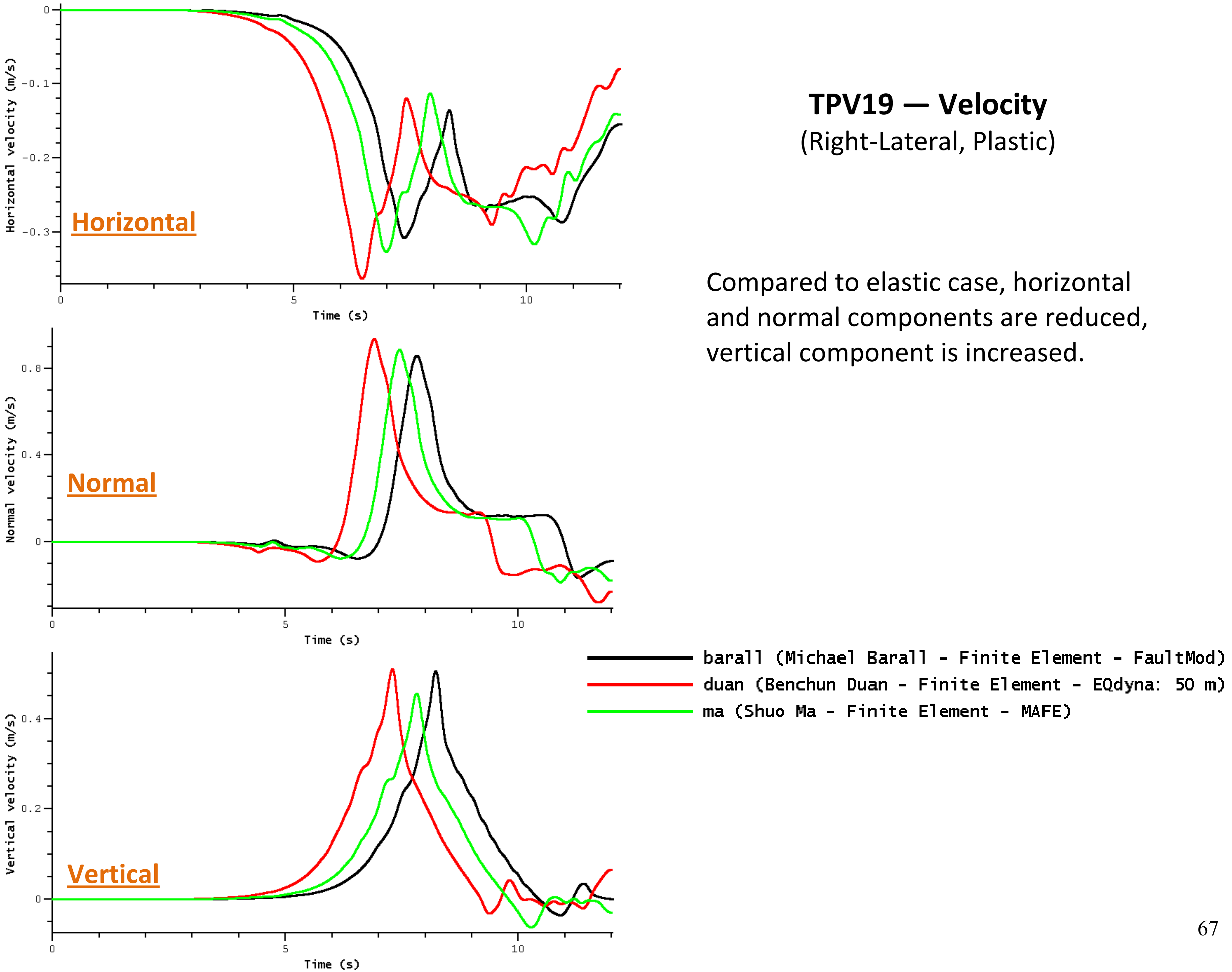
Velocity is filtered with a 3 Hz low-pass filter.

TPV18 — Velocity

(Right-Lateral, Elastic)



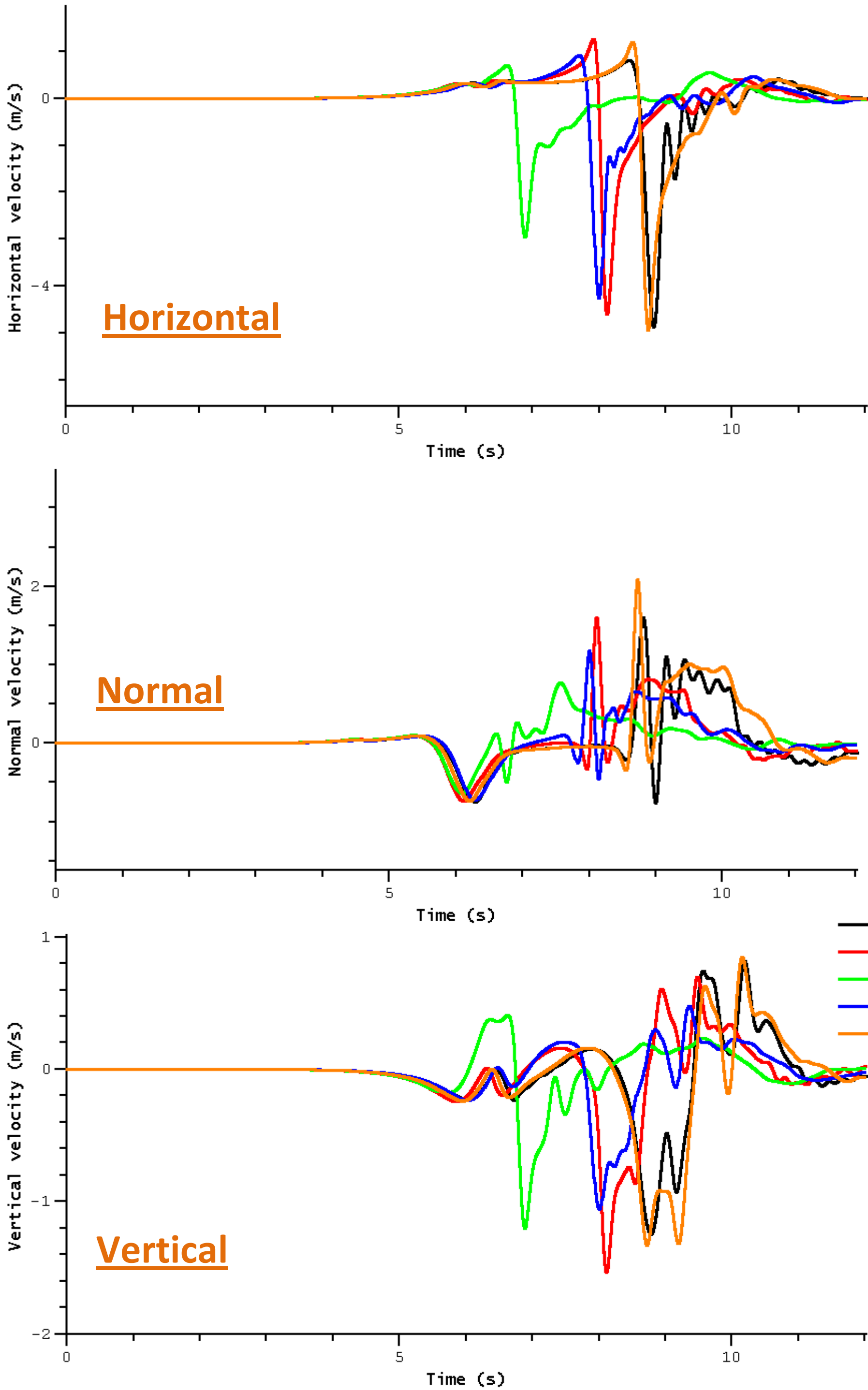
TPV19 — Velocity
(Right-Lateral, Plastic)



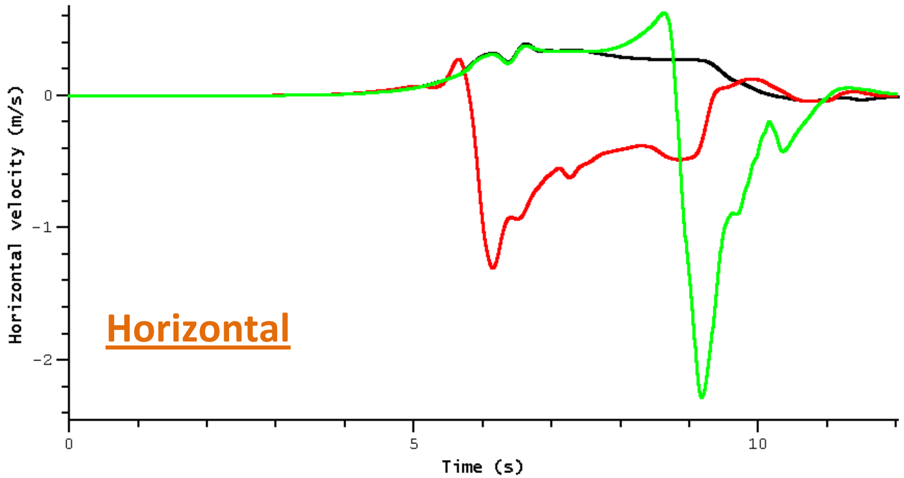
Compared to elastic case, horizontal and normal components are reduced, vertical component is increased.

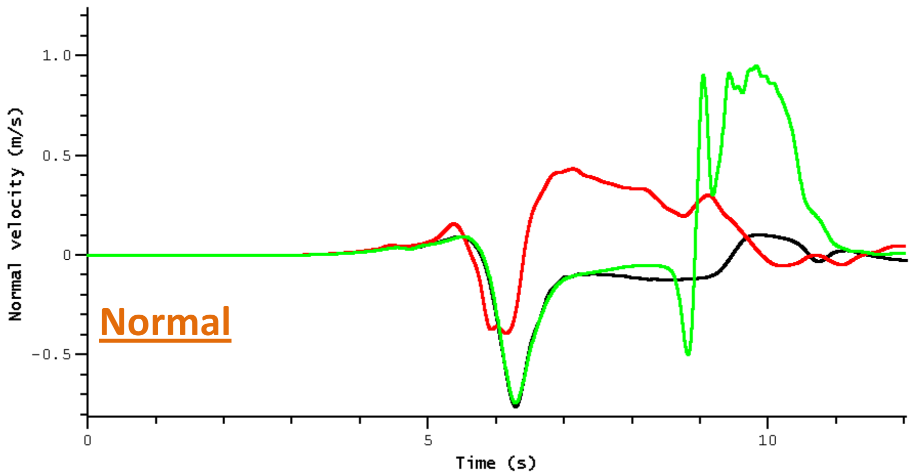
TPV20 — Velocity

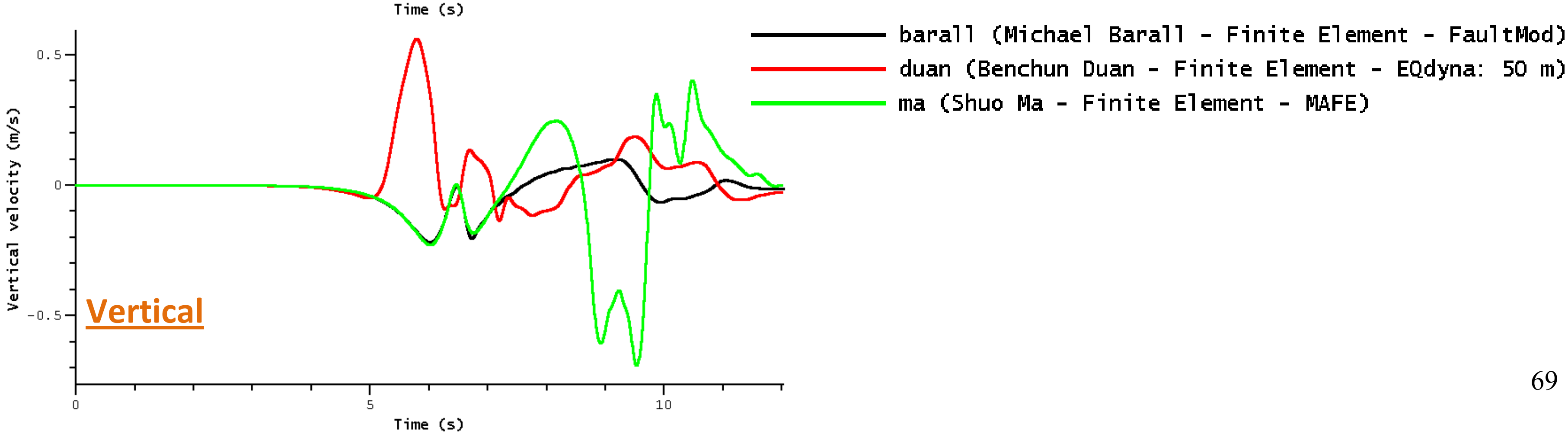
(Left-Lateral, Elastic)



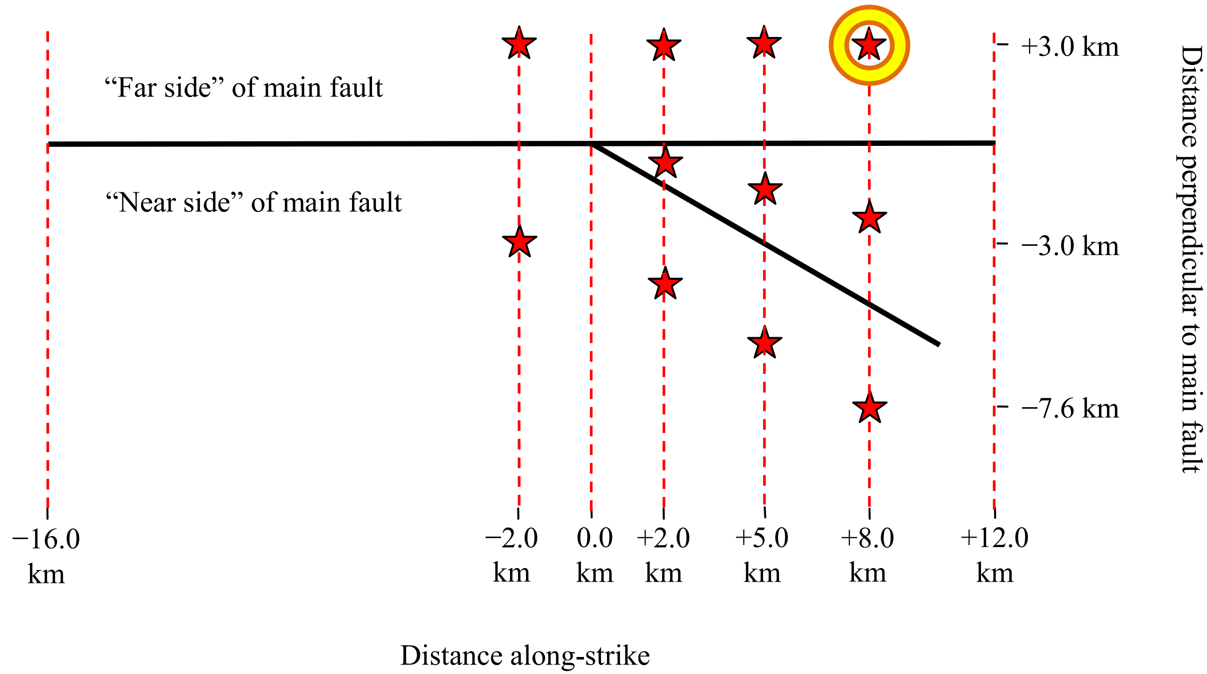
TPV21 — Velocity
(Left-Lateral, Plastic)







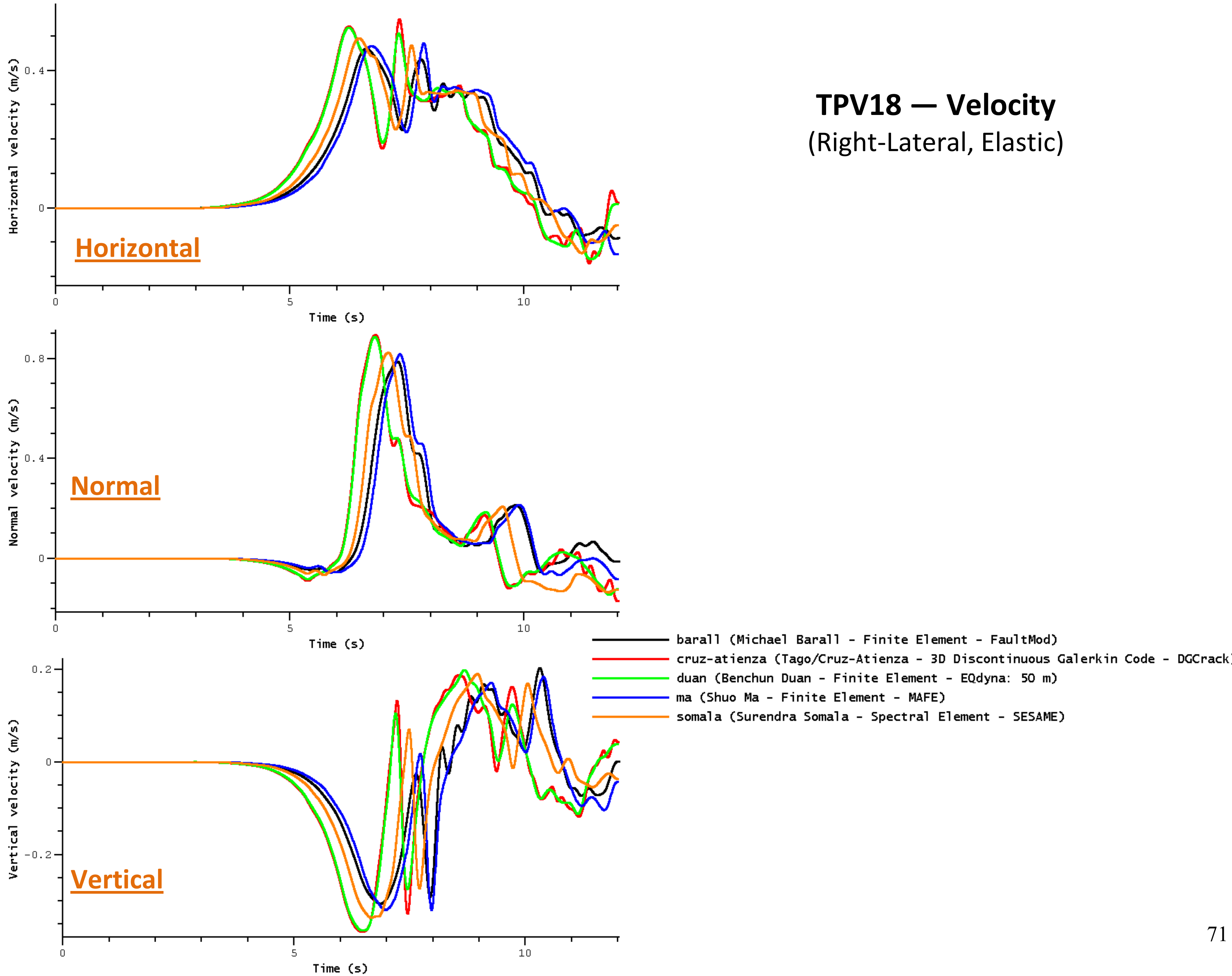
Station for Ground Motion Comparison

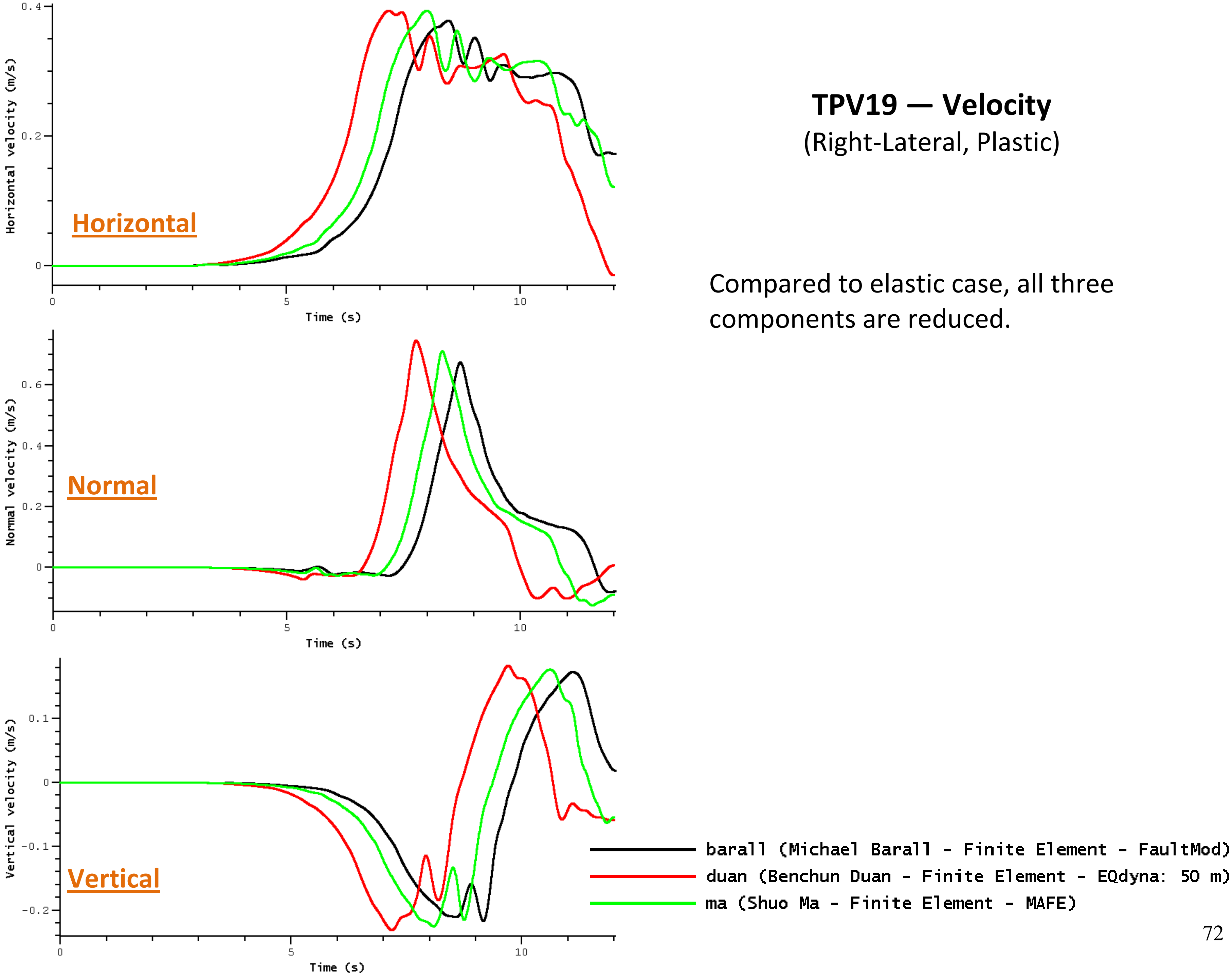


We will show velocity for the marked station.

Velocity is filtered with a 3 Hz low-pass filter.

TPV18 — Velocity
(Right-Lateral, Elastic)



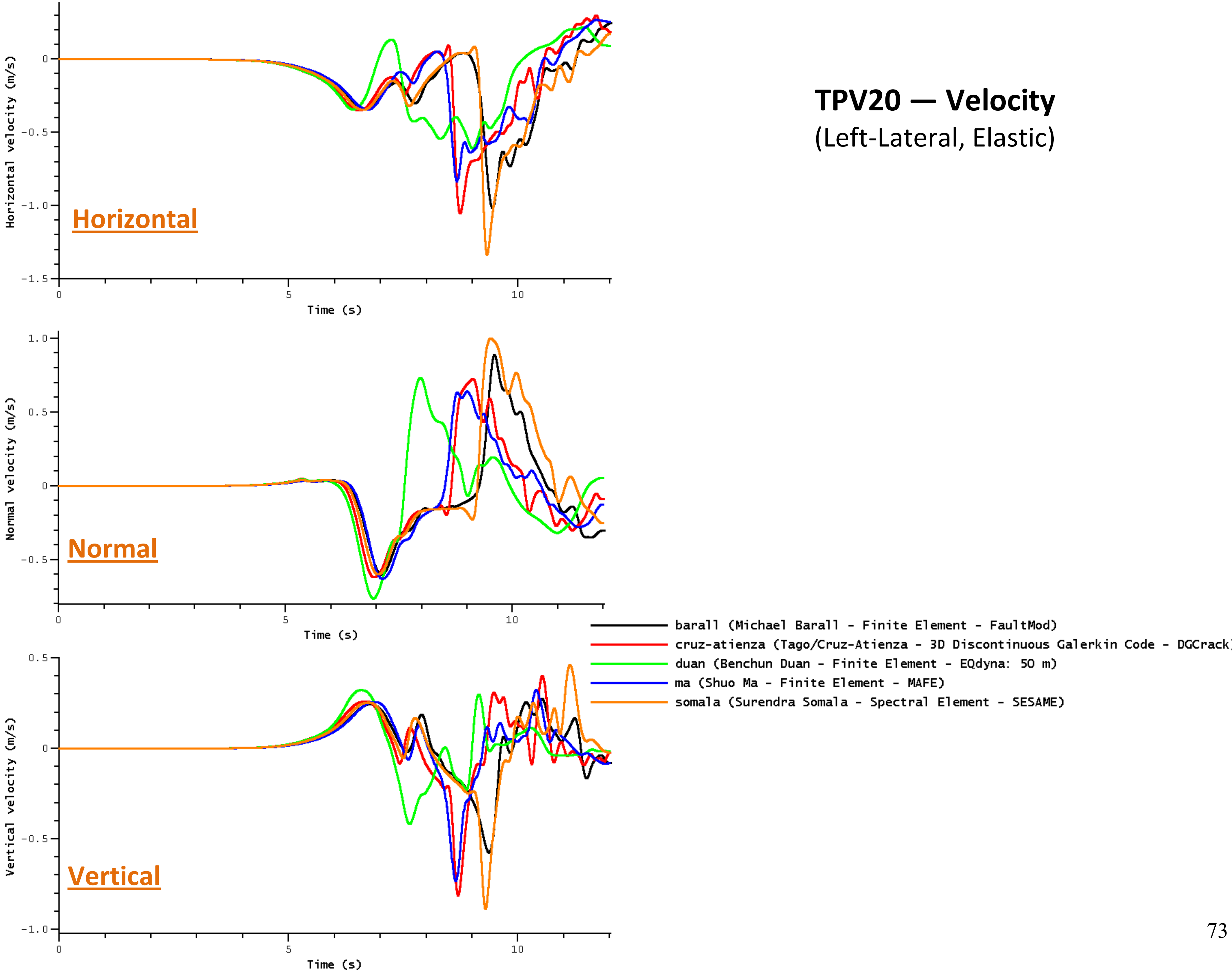


TPV19 — Velocity

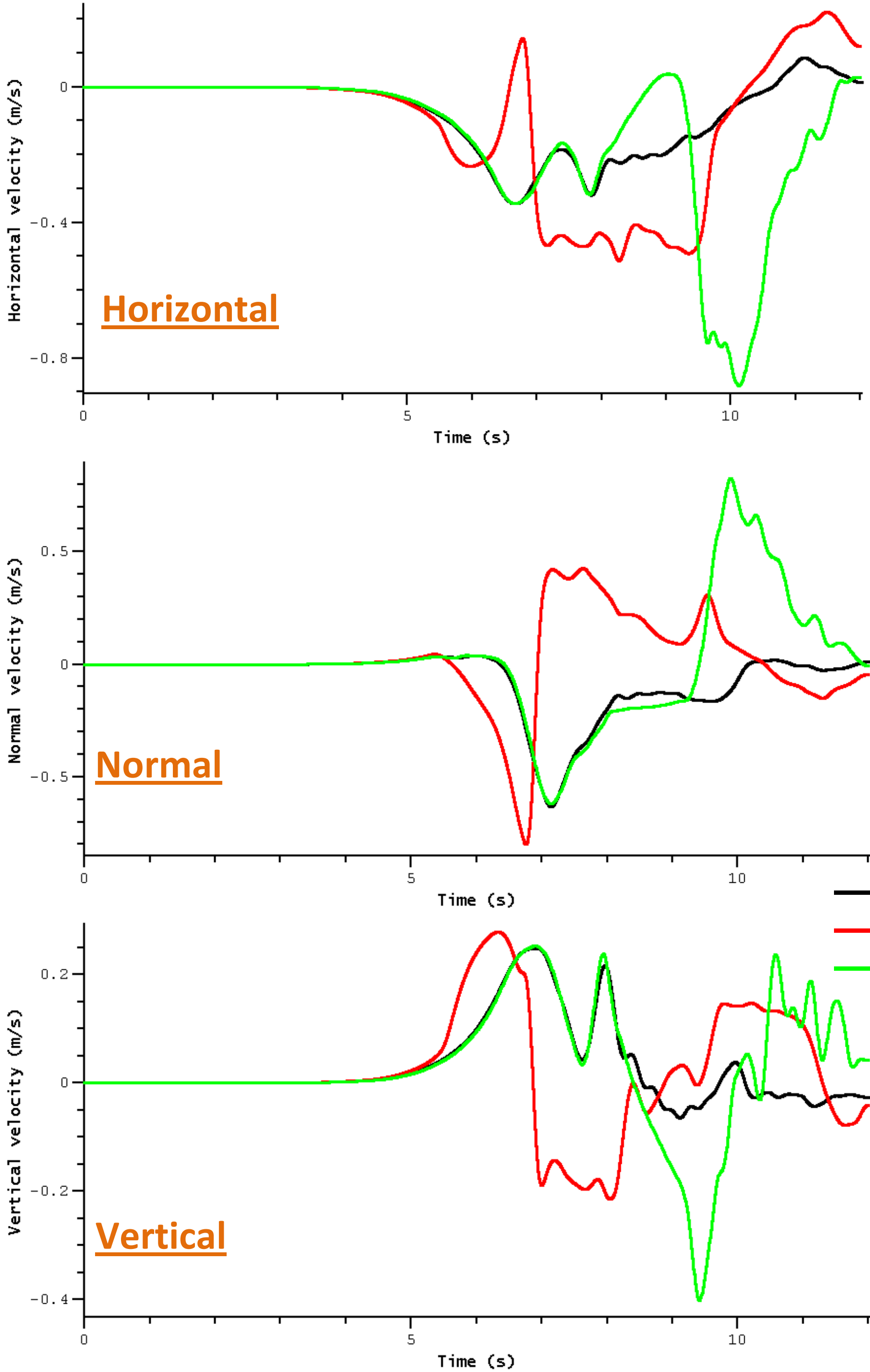
(Right-Lateral, Plastic)

Compared to elastic case, all three components are reduced.

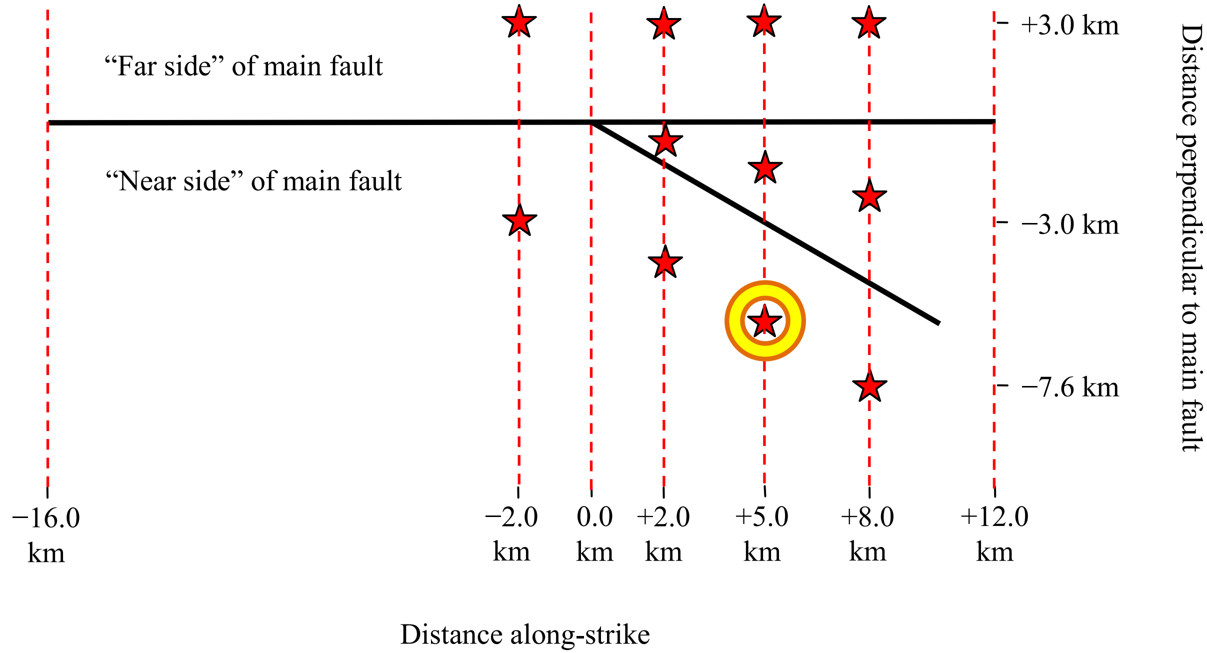
TPV20 — Velocity
(Left-Lateral, Elastic)



TPV21 — Velocity
(Left-Lateral, Plastic)



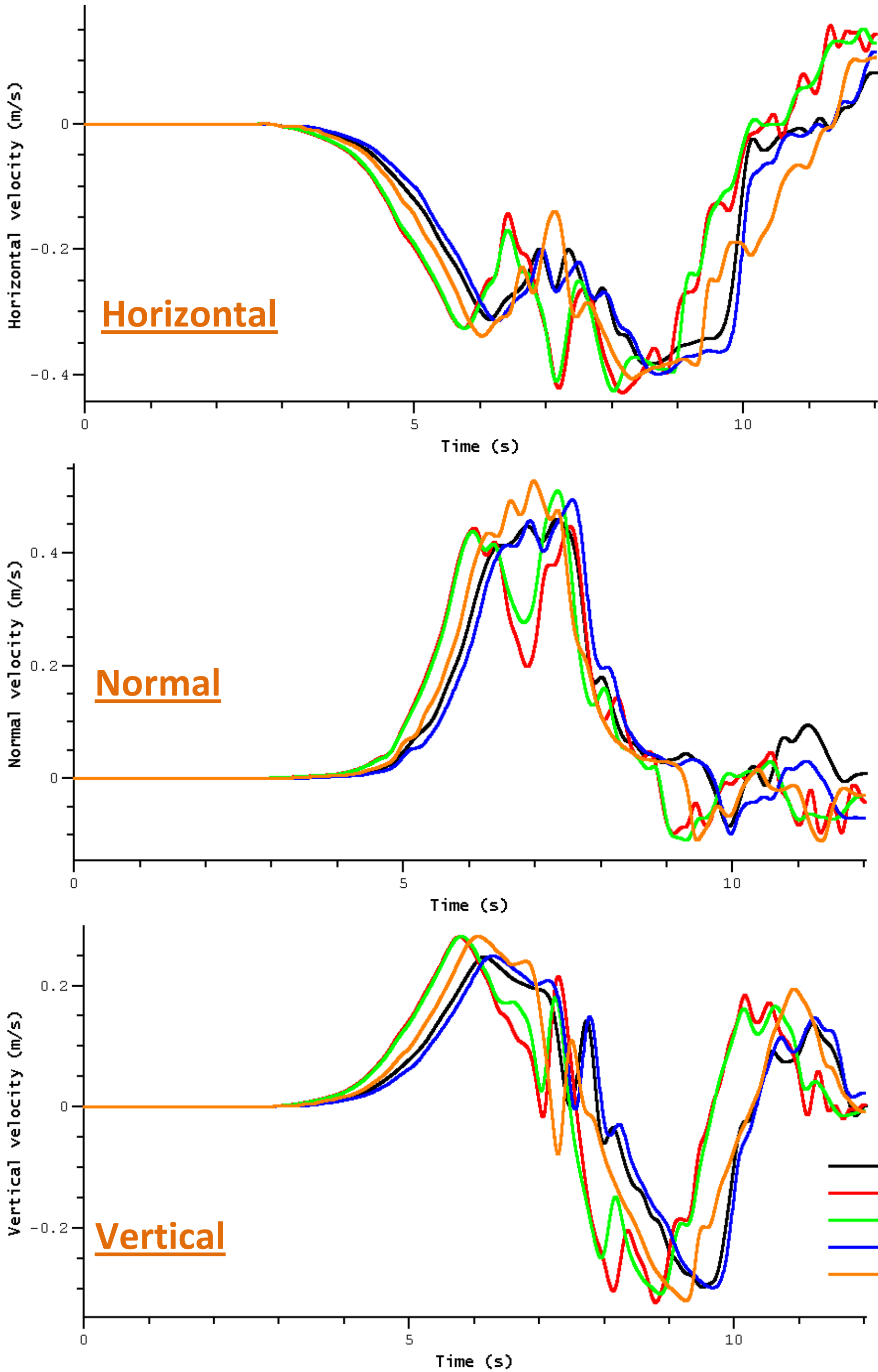
Station for Ground Motion Comparison

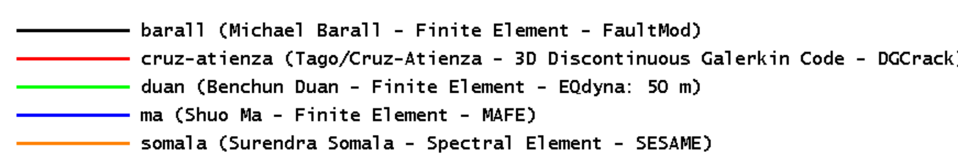


We will show velocity for the marked station.

Velocity is filtered with a 3 Hz low-pass filter.

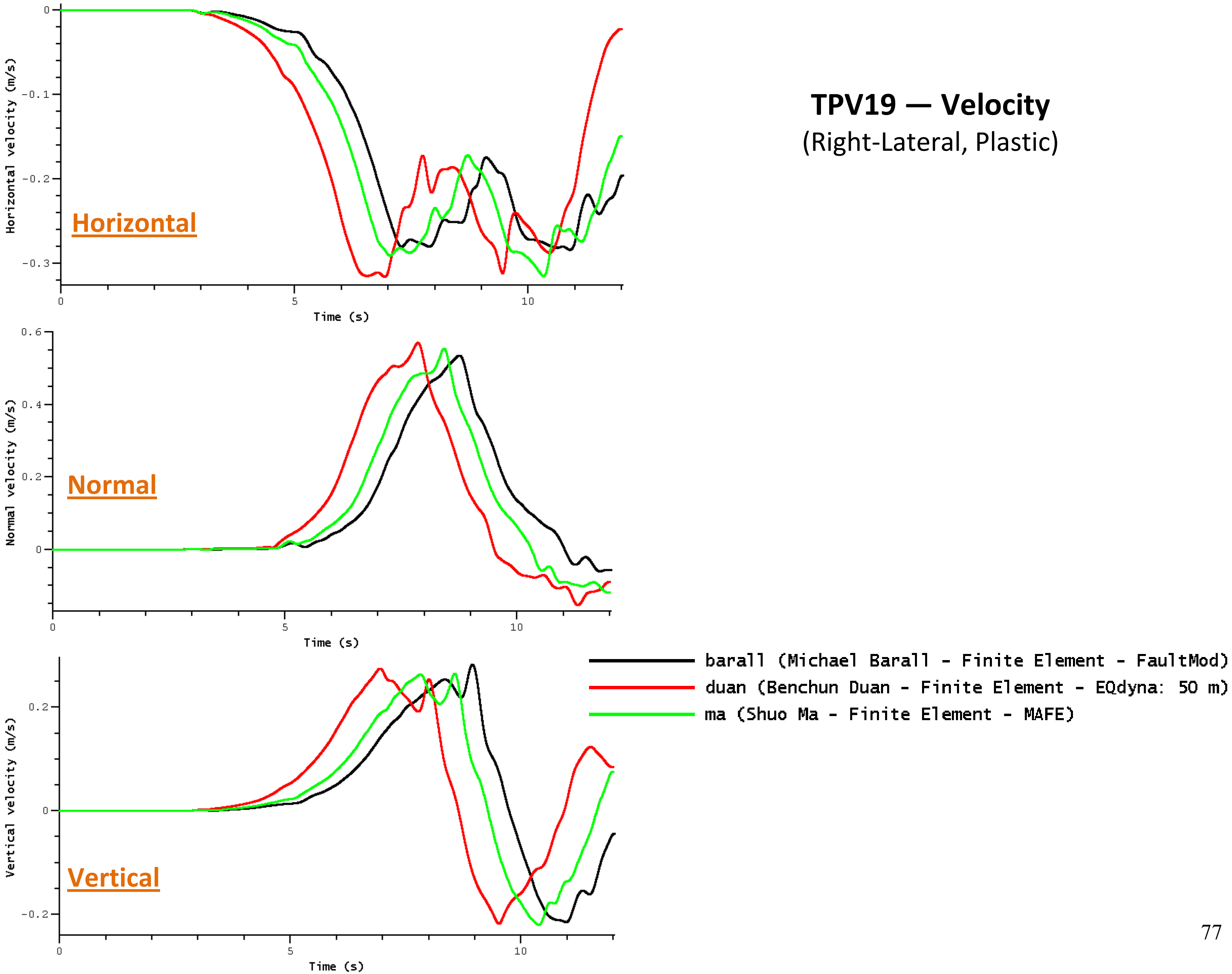
TPV18 — Velocity
(Right-Lateral, Elastic)



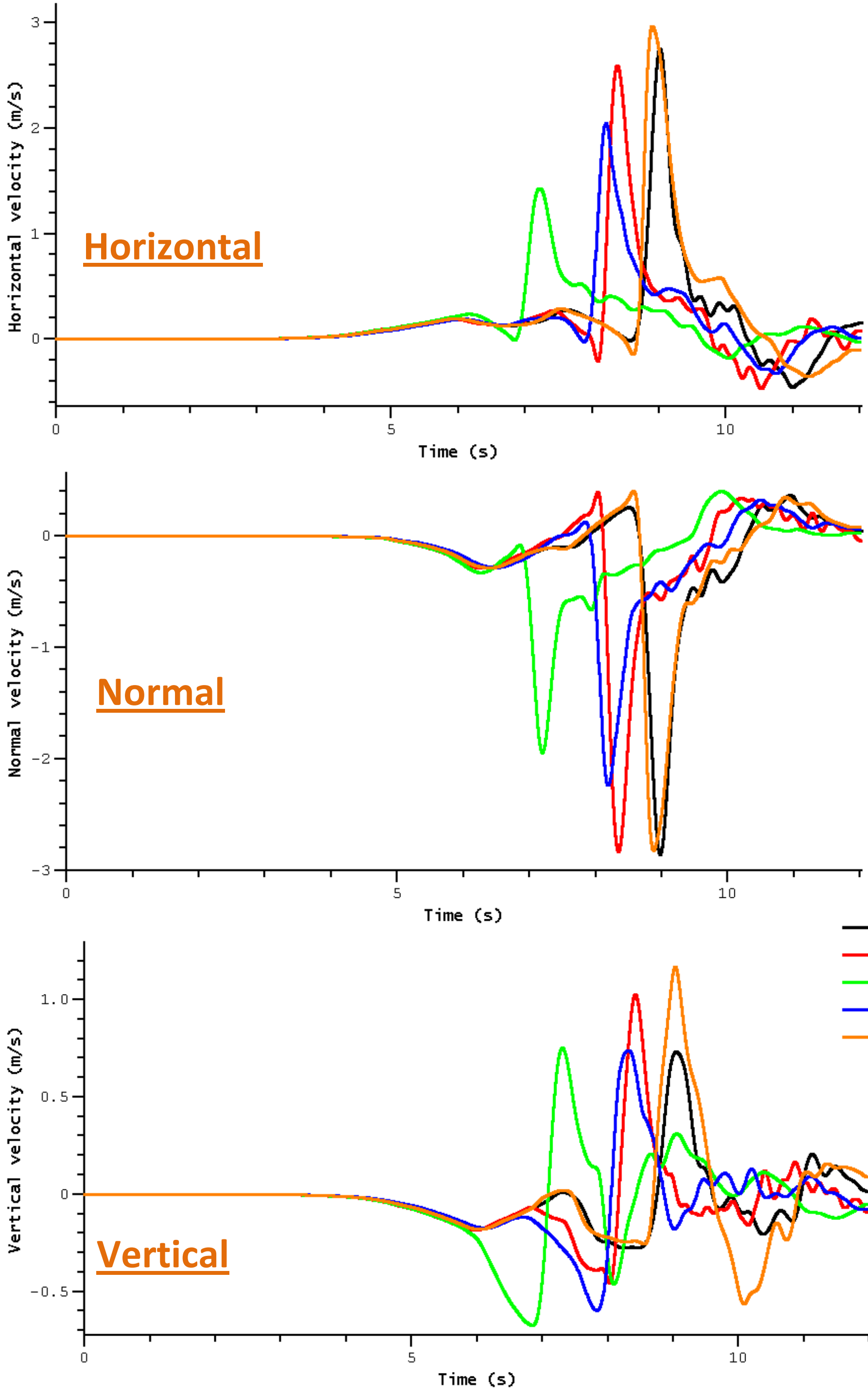


TPV19 — Velocity

(Right-Lateral, Plastic)

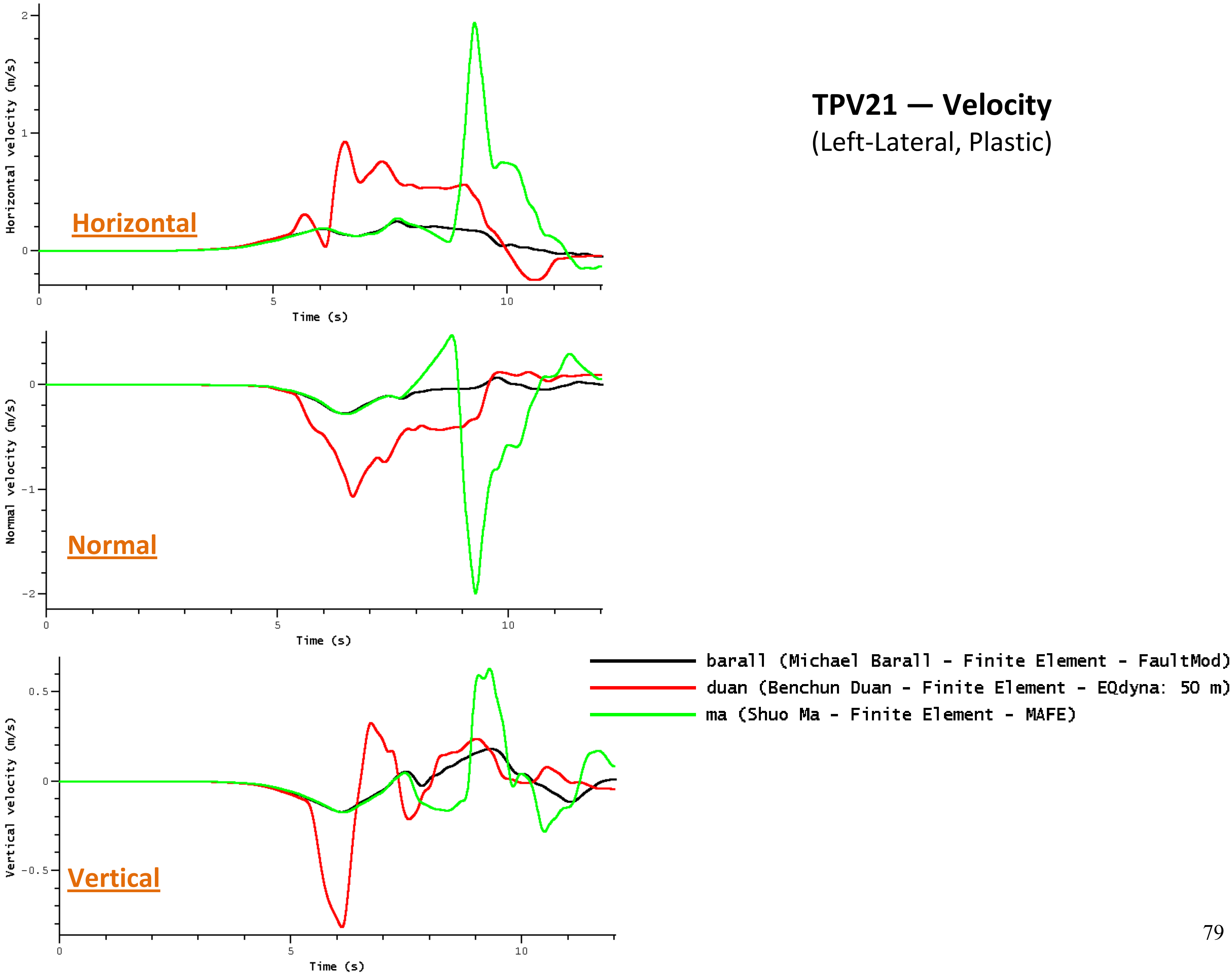


TPV20 — Velocity
(Left-Lateral, Elastic)



TPV21 — Velocity

(Left-Lateral, Plastic)



Conclusion

Conclusion

TPV18-21 are designed to explore the effects of plasticity on a branching fault.

Rupture contours do not agree very well, but there is some consistency in fault slip patterns.

A source of disagreement may be that 100 m is not sufficient resolution. Are there other sources of disagreement? Are we sure all modelers implemented the same problem?

TPV18-19 show that rupture propagation speed changes when plasticity is introduced.

TPV20-21 show that rupture propagation direction (main vs. branch fault) may change when plasticity is introduced, but codes disagree on how.

Question for the Group

Is it worthwhile to run TPV18-21 again after the workshop, to try to get better agreement?

We could reduce the recommended resolution to 50 m.

Are there suggestions for other changes?

Thank You