(This paper will be submitted to JGR in february 2012)

DGCrack: a 3D hp-Adaptive Discontinuous Galerkin Method for Modeling Earthquake Dynamics

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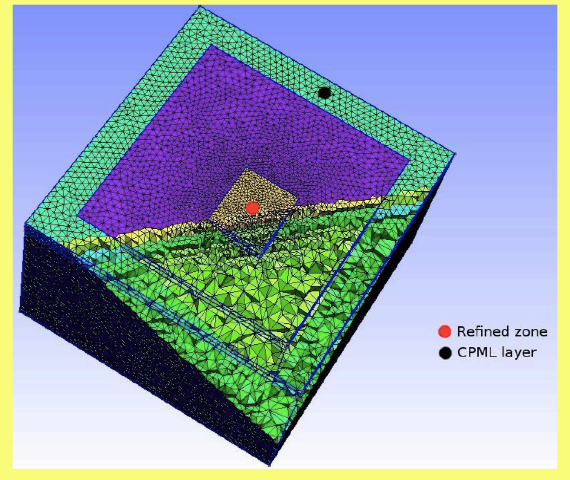
February 6, 2012

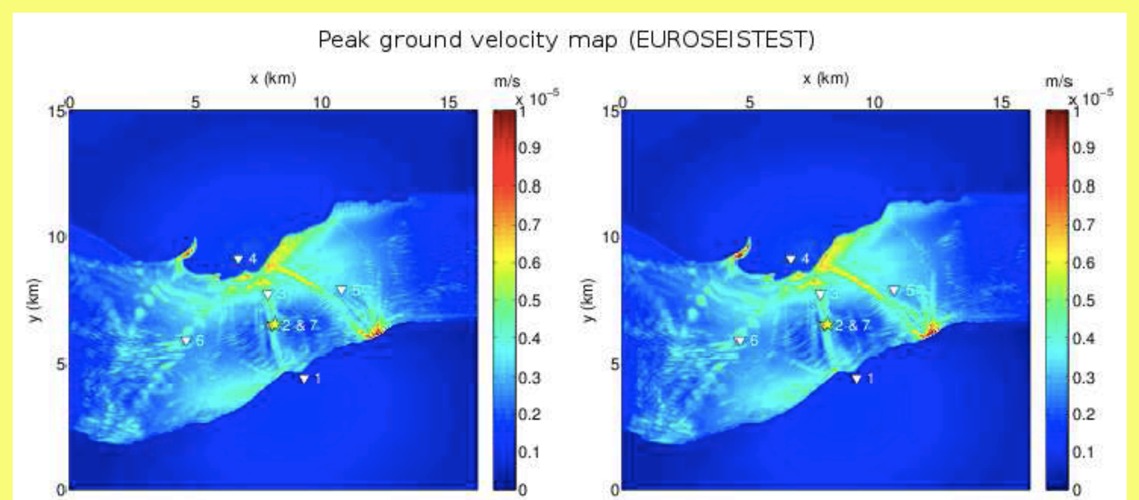
DGCrack Model Overall Features

1. Convolutional Perfectly Matched Layers (CPML) absorbing boundary conditions.

2. H-adaptivity: unstructured mesh refinement depending on model properties and geometry.

3. P-adaptivity: different approximation orders in every tetrahedron (i.e. P0, P1 and P2).





Etienne et al., 2010

4. Viso-elastic solver for intrinsic attenuation Q (Tago et al., 2010)

5. Parallel implementation using MPI for supercom-puting with 85% scalability (Etienne et al., 2010)

Velocity-Stress Discontinuous Galerkin Scheme for Elastic Wave Propagation

Equation of Motion

$$
\begin{aligned}
\rho^i(\underline{\underline{I}}_3 \otimes \underline{\underline{\mathcal{K}}}^i)\frac{\underline{\hat{v}}^i_{m+\frac{1}{2}} - \underline{\hat{v}}^i_{m-\frac{1}{2}}}{\triangle t} \quad = \quad & - \sum_{\theta \in \{x,y,z\}} (\underline{\underline{M}}_\theta \otimes \underline{\underline{\mathcal{E}}}^i_\theta)\underline{\hat{\sigma}}^i_m \\
& + \frac{1}{2} \sum_{k \in N^i} [(\underline{\underline{\mathcal{P}}}^{ik} \otimes \underline{\underline{\mathcal{F}}}^{ik})\underline{\hat{\sigma}}^i_m + (\underline{\underline{\mathcal{P}}}^{ik} \otimes \underline{\underline{\mathcal{G}}}^{ik})\underline{\hat{\sigma}}^k_m]
\end{aligned}
$$

Constitutive Equation

$$
\begin{aligned}
(\underline{\underline{\Lambda}}^i \otimes \underline{\underline{\mathcal{K}}}^i)\frac{\underline{\hat{\sigma}}^i_{m+1} - \underline{\hat{\sigma}}^i_m}{\triangle t} \quad = \quad & - \sum_{\theta \in \{x,y,z\}} (\underline{\underline{N}}_\theta \otimes \underline{\underline{\mathcal{E}}}^i_\theta)\underline{\hat{v}}^i_{m+\frac{1}{2}} \\
& + \frac{1}{2} \sum_{k \in N^i} [(\underline{\underline{\mathcal{Q}}}^{ik} \otimes \underline{\underline{\mathcal{F}}}^{ik})\underline{\hat{v}}^i_{m+\frac{1}{2}} + (\underline{\underline{\mathcal{Q}}}^{ik} \otimes \underline{\underline{\mathcal{G}}}^{ik})\underline{\hat{v}}^k_{m+\frac{1}{2}}]
\end{aligned}
$$

Discontinuous Galerkin Scheme with Explicit Fluxes Across the Fault

Equation of Motion (velocity scheme)

$$
\begin{aligned}
\underline{\hat{v}}^{i}_{m+\frac{1}{2}} \;=\; & \underline{\hat{v}}^{i}_{m-\frac{1}{2}} + \frac{\Delta t}{\rho^{i}} \left(\underline{I}_{3} \otimes \underline{\underline{\mathcal{K}}}^{i}\right)^{-1} \Bigg[- \sum_{\theta \in \{x,y,z\}} \left(\underline{\underline{M}}_{\theta} \otimes \underline{\underline{\mathcal{E}}}^{i}_{\theta}\right) \underline{\hat{\sigma}}^{i}_{m} \\
& + \frac{1}{2} \sum_{\substack{k \in N^{i} \\ \mathcal{T}^{i} \cap \mathcal{T}^{k} \not\subset \Gamma}} \left[\left(\underline{\underline{\mathcal{P}}}^{ik} \otimes \underline{\underline{\mathcal{F}}}^{ik}\right) \underline{\hat{\sigma}}^{i}_{m} + \left(\underline{\underline{\mathcal{P}}}^{ik} \otimes \underline{\underline{\mathcal{G}}}^{ik}\right) \underline{\hat{\sigma}}^{k}_{m} \right] + \delta^{\gamma} \left(\underline{I}_{3} \otimes \underline{\underline{\mathcal{F}}}^{ik}\right) \underline{\mathbb{F}}^{\gamma i}_{m} \Bigg]
\end{aligned}
$$

Flux across the fault in terms of fault tractions

$$
\underline{\mathbb{F}}^{\gamma i}_{m} = \left(\underline{\hat{T}}^{ik}_{N}\right)_{m} + \left(\underline{\hat{T}}^{ik}_{T}\right)_{m}
$$

Discontinuous Galerkin Scheme with Explicit Fluxes Across the Fault

Constitutive Equation (stress scheme)

$$\begin{aligned}\underline{\hat{\sigma}}^i_{m+1} &= \underline{\hat{\sigma}}^i_m + \frac{\Delta t}{\rho^i}(\underline{\underline{\Lambda_S}}^i \otimes \underline{\underline{\mathcal{K}}}^i)^{-1}\Bigg[-\sum_{\theta\in\{x,y,z\}}(\underline{\underline{N}}_{S\theta} \otimes \underline{\underline{\mathcal{E}}}^i_\theta)\underline{\hat{v}}^i_{m+\frac{1}{2}} \\ &\quad + \frac{1}{2}\sum_{\substack{k\in N^i \\ \mathcal{T}^i\cap\mathcal{T}^k\not\subset\Gamma}}[((\underline{\underline{\mathcal{P}}}^{ik})^T \otimes \underline{\underline{\mathcal{F}}}^{ik})\underline{\hat{v}}^i_{m+\frac{1}{2}} + ((\underline{\underline{\mathcal{P}}}^{ik})^T \otimes \underline{\underline{\mathcal{G}}}^{ik})\underline{\hat{v}}^k_{m+\frac{1}{2}}] \\ &\quad + \delta^\gamma(\underline{\underline{I}}_6 \otimes \underline{\underline{\mathcal{F}}}^{ik})\underline{\mathbb{G}}^{\gamma i}_{m+\frac{1}{2}}\Bigg]\end{aligned}$$

Flux across the fault in terms of fault velocity

$$\underline{\mathbb{G}}^{\gamma i}_{m+\frac{1}{2}} = (\underline{\underline{P}}^{ik})^T\,\underline{\hat{V}}^i_{m+\frac{1}{2}}$$

Fluxes Across the Fault

Velocity scheme: flux in terms of fault traction

$$\underline{\mathbb{F}}_m^{\gamma i} = (\underline{\hat{T}}_N^{ik})_m + (\underline{\hat{T}}_T^{ik})_m$$

Stress scheme: flux in terms of velocity

$$\underline{\mathbb{G}}_{m+\frac{1}{2}}^{\gamma i} = (\underline{\underline{P}}^{ik})^T \underline{\hat{v}}_{m+\frac{1}{2}}^{i}$$

Computation of Tangential Tractions

$$\begin{aligned} \tau_c - \|\underline{T}_T\| &\geq 0 \\ \tau_c \underline{V} - \underline{T}_T \|\underline{V}\| &= 0 \end{aligned}$$

Day et al., 2005

Jump conditions imply that shear tractions must force continuity of tangential velocities when no rupture

$$(\underline{V}^{ik})_{m+\frac{1}{2}} = 0$$

Computation of Tangential Tractions

Shear tractions forcing continuity of tangential velocity

$$(\underline{\tilde{T}}_T^{ik})_m = (\underline{I}_3 \otimes \underline{\underline{\mathcal{F}}}^e)^{-1}(\underline{I}_3 \otimes \underline{\underline{\check{\mathcal{K}}}}^e)\frac{(-\underline{V}_{m-\frac{1}{2}}^{ik} - (\underline{R}_T^i)_m + (\underline{R}_T^k)_m)\rho^i\rho^k V^i V^k}{\Delta t S^{ik}(\rho^i V^i + \rho^k V^k)}$$

Terms excluding the fault tractions

$$\begin{aligned} \underline{R}_m^i &= \frac{\Delta t}{\rho^i}(\underline{I}_3 \otimes \underline{\underline{\mathcal{K}}}^i)^{-1}\Bigg[-\sum_{\theta\in\{x,y,z\}}(\underline{\underline{M}}_\theta \otimes \underline{\underline{\mathcal{E}}}_\theta^i)\underline{\hat{\sigma}}_m^i \\ &\quad + \frac{1}{2}\sum_{\substack{k\in N^i \\ \mathcal{T}^i\cap\mathcal{T}^k\not\subseteq\Gamma}}[(\underline{\underline{\mathcal{P}}}^{ik}\otimes\underline{\underline{\mathcal{F}}}^{ik})\underline{\hat{\sigma}}_m^i + (\underline{\underline{\mathcal{P}}}^{ik}\otimes\underline{\underline{\mathcal{G}}}^{ik})\underline{\hat{\sigma}}_m^k]\Bigg] \end{aligned}$$

Dynamic Rupture Boundary Conditions

Shear tractions boundary conditions

$$(\underline{\hat{T}}^{ik}_T)_m = \begin{cases} (\underline{\tilde{T}}^{ik}_T)_m & \text{if } \|(\underline{\tilde{T}}^{ik}_T)_m\| < \tau_c \\ \dfrac{(\underline{\tilde{T}}^{ik}_T)_m}{\|(\underline{\tilde{T}}^{ik}_T)_m\|}\tau_c & \text{if } \|(\underline{\tilde{T}}^{ik}_T)_m\| \geq \tau_c \end{cases}$$

Frictional fault strength

$$\tau_c = -\sigma_N \mu$$

Linear slip-weakening friction law

$$\mu(U) = \mu_d + (\mu_s - \mu_d)\left(1 - \frac{U}{\delta_0}\right) H\left(1 - \frac{U}{\delta_0}\right)$$

Rupture Propagation Across Fault Element Nodes

(Predictor – Corrector Scheme)

To allow rupture propagation throughout the fault elements and accurately satisfying the BC we used a Predictor-Corrector iterative scheme

Prediction of shear tractions only considering the broken nodes in the element

$$(\underline{\tilde{T}}_T^{ik})_m = (\underline{\underline{I}}_3 \otimes \underline{\underline{\mathcal{F}}}^e)^{-1}(\underline{\underline{I}}_3 \otimes \underline{\underline{\check{\mathcal{K}}}}_*^e)\frac{(-\underline{V}_{m-\frac{1}{2}}^{ik} - (\underline{R}_T^i)_m + (\underline{R}_T^k)_m)\rho^i\rho^k V^i V^k}{\Delta t S^{ik}(\rho^i V^i + \rho^k V^k)}$$

The mass matrix is built only using the unbroken element nodes

Computation of Normal Tractions

$$(\underline{V}_N^{ik})_{m+\frac{1}{2}} = 0$$

Neither crack opening nor mass interpenetration

Normal tractions forcing continuity of normal velocity

$$(\hat{\underline{T}}_N^{ik})_m = (\underline{\underline{I}}_3 \otimes \underline{\underline{\mathcal{F}}}^e)^{-1}(\underline{\underline{I}}_3 \otimes \underline{\underline{\check{\mathcal{K}}}}^e)\frac{(-(\underline{V}_N^{ik})_{m-\frac{1}{2}} - (\underline{R}_N^i)_m + (\underline{R}_N^k)_m)\rho^i\rho^k V^i V^k}{\Delta t S^{ik}(\rho^i V^i + \rho^k V^k)}$$

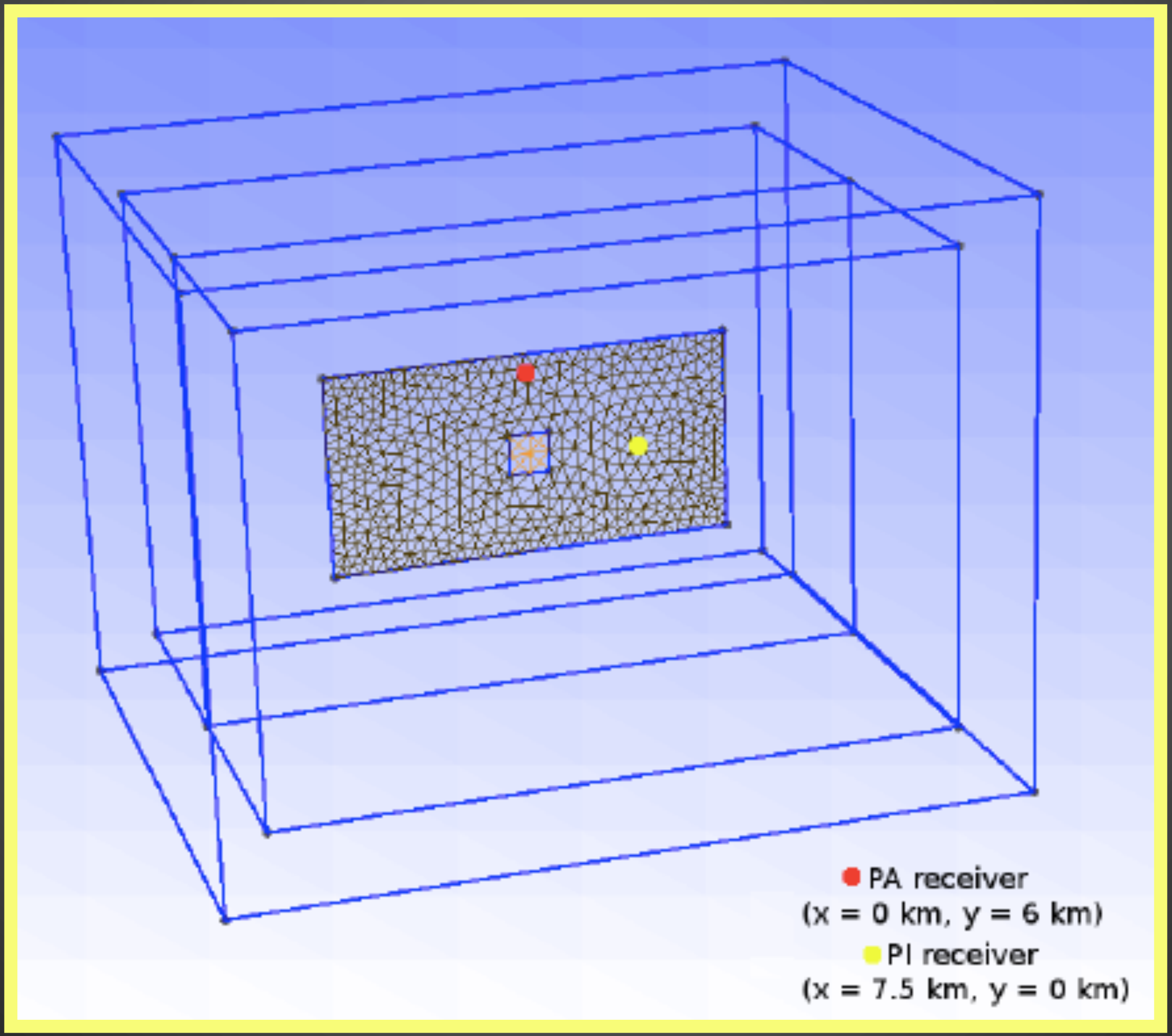
Terms excluding the fault tractions

$$\begin{aligned}\underline{R}_m^i &= \frac{\Delta t}{\rho^i}(\underline{\underline{I}}_3 \otimes \underline{\underline{\mathcal{K}}}^i)^{-1}\Bigg[-\sum_{\theta\in\{x,y,z\}}(\underline{\underline{M}}_\theta \otimes \underline{\underline{\mathcal{E}}}_\theta^i)\underline{\hat{\sigma}}_m^i \\ &\quad + \frac{1}{2}\sum_{\substack{k\in N^i \\ \mathcal{T}^i\cap\mathcal{T}^k\not\subset\Gamma}}[(\underline{\underline{\mathcal{P}}}^{ik} \otimes \underline{\underline{\mathcal{F}}}^{ik})\underline{\hat{\sigma}}_m^i + (\underline{\underline{\mathcal{P}}}^{ik} \otimes \underline{\underline{\mathcal{G}}}^{ik})\underline{\hat{\sigma}}_m^k]\Bigg]\end{aligned}$$

DGCrack Model Verification

(The Problem Version 3)

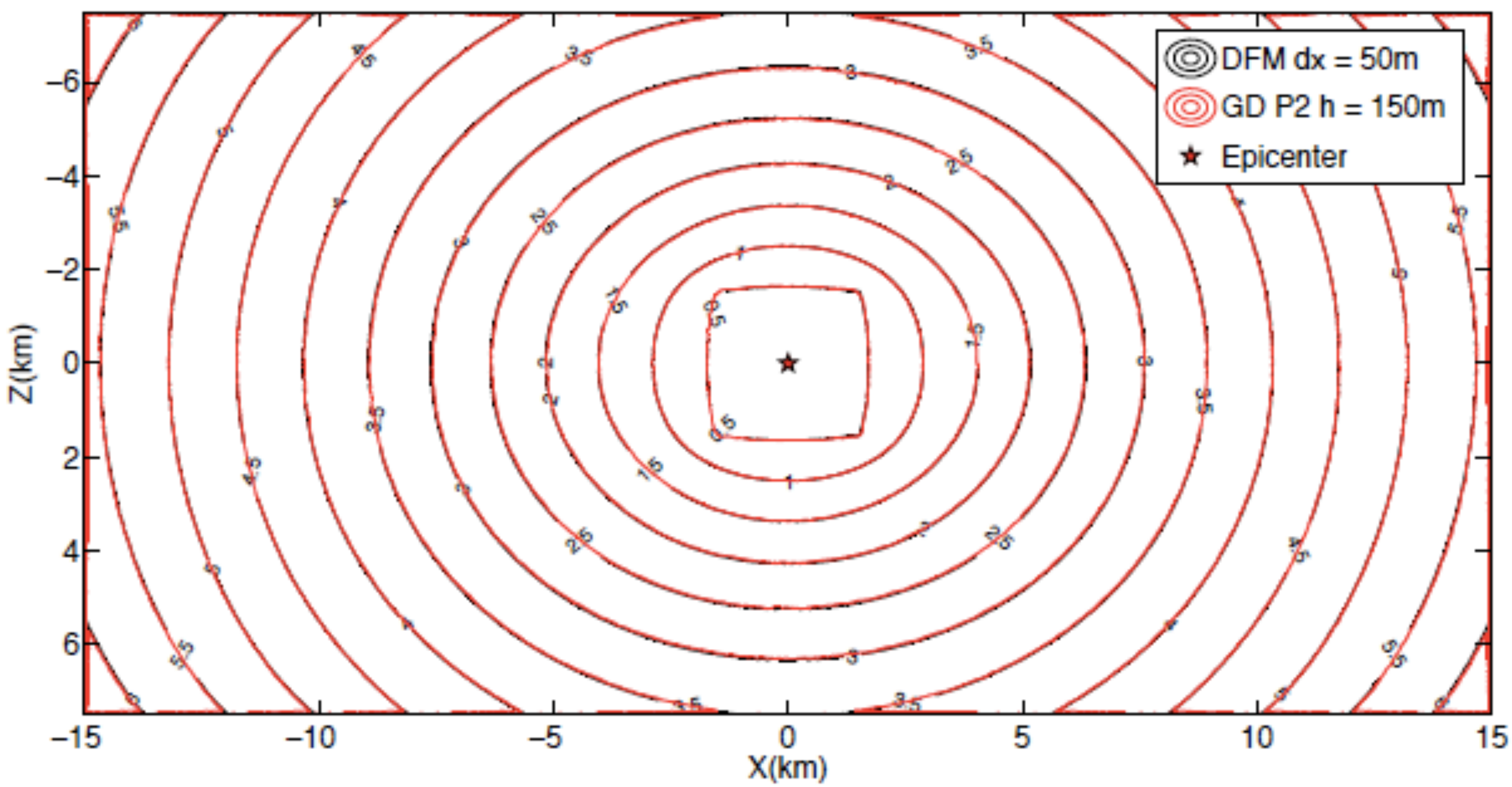
Strike-slip rupture in a fullspace



DGCrack Model Verification

(The Problem Version 3)

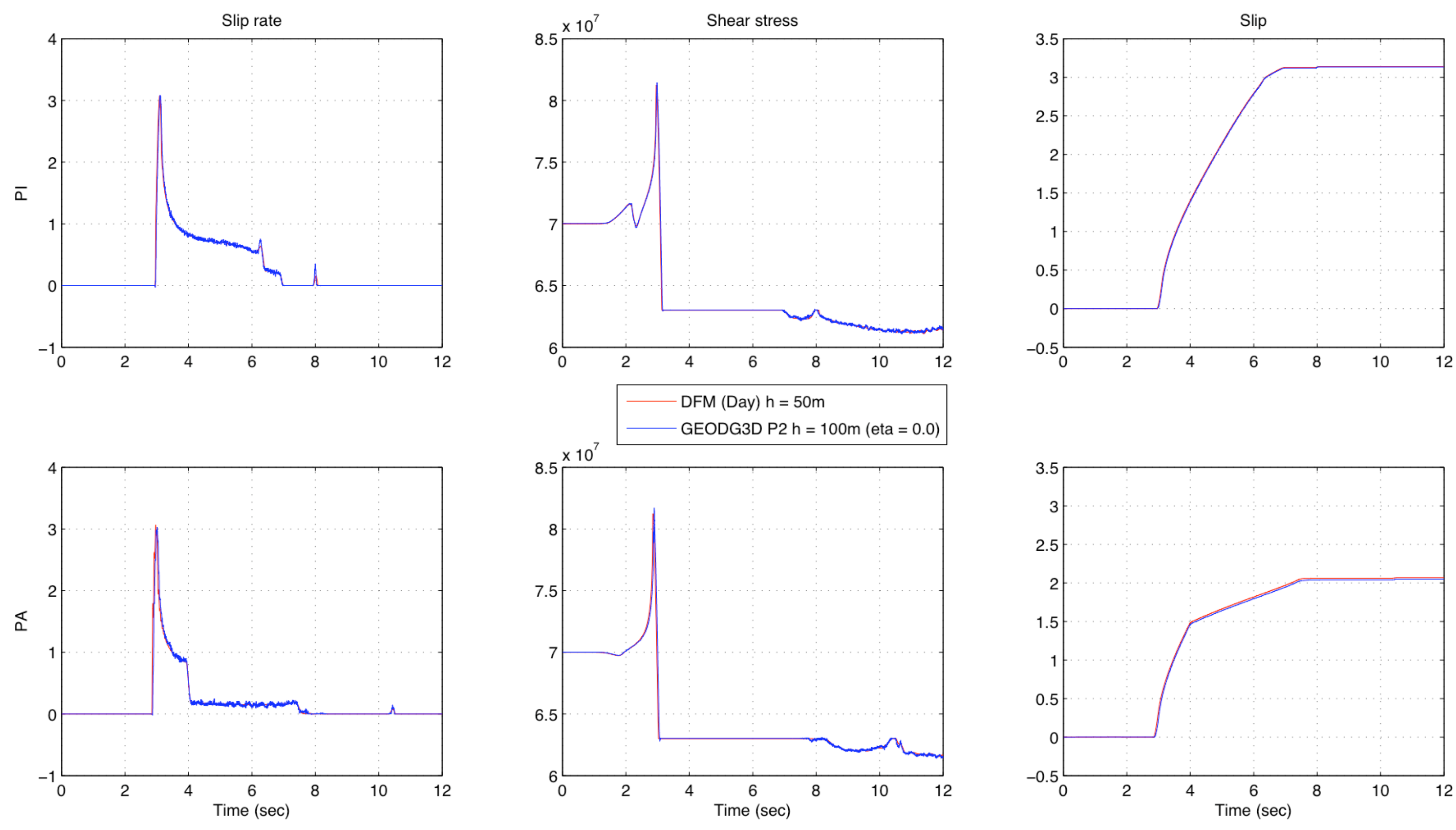
Rupture times comparison for h = 150 m



DGCrack Model Verification

(The Problem Version 3)

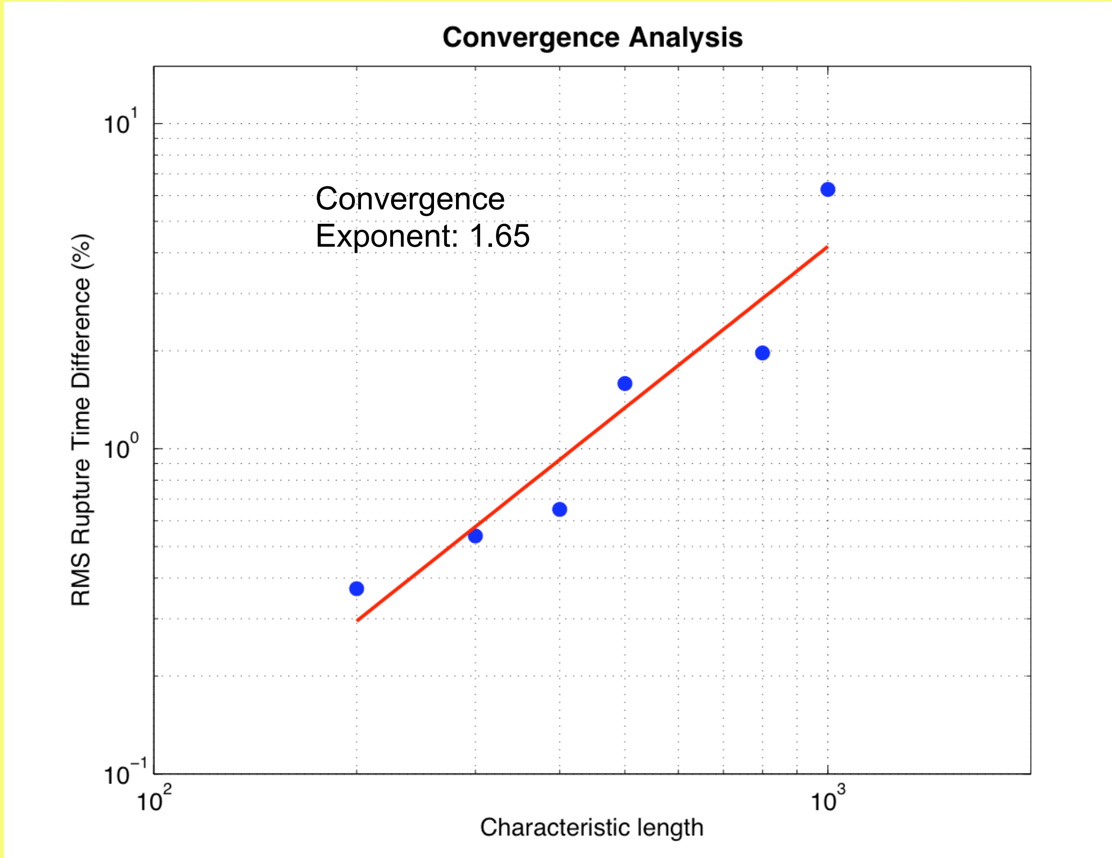
Fault waveforms comparison for h = 100 m (No Damping)



Rupture time metric (RMS%) below 0.7% for h < 500 m

DGCrack Model Convergence

(The Problem Version 3)

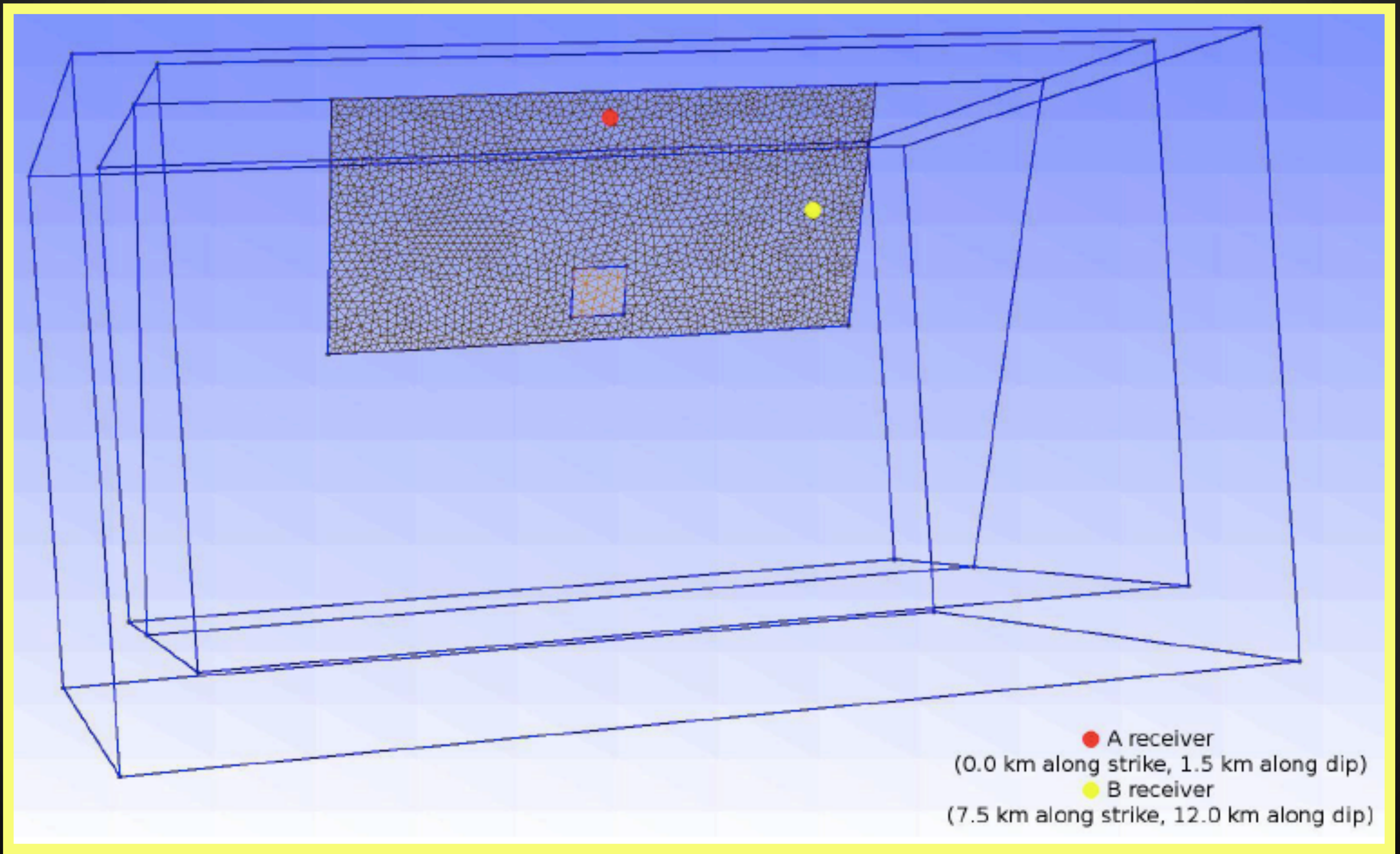


Rupture time error below 1.0% for h < 500 m

DGCrack Model Verification

(The Problem Version 10)

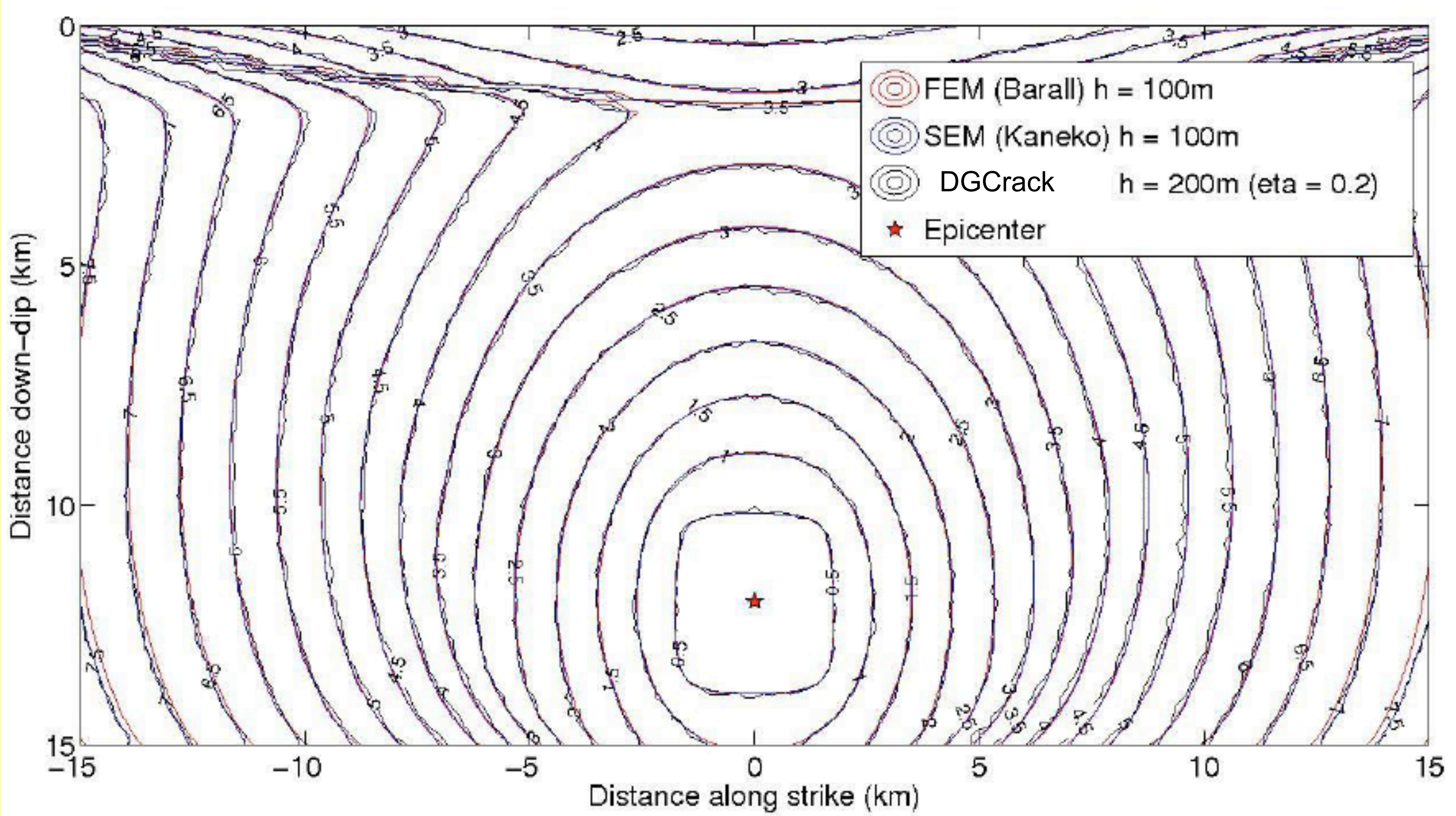
Normal faulting over a 60° dipping fault reaching the free surface



DGCrack Model Verification

(The Problem Version 10)

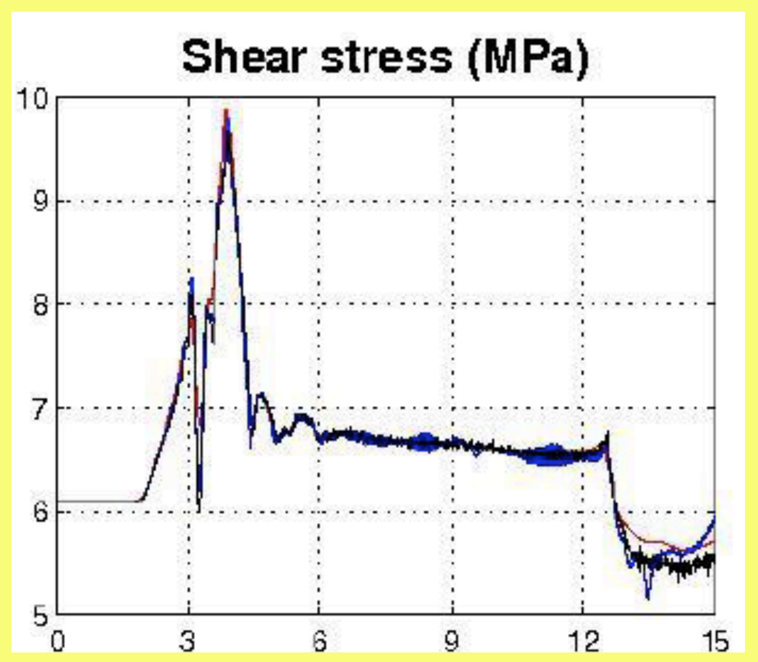
Rupture times comparison for h = 200 m

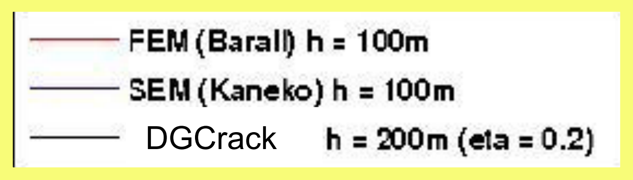


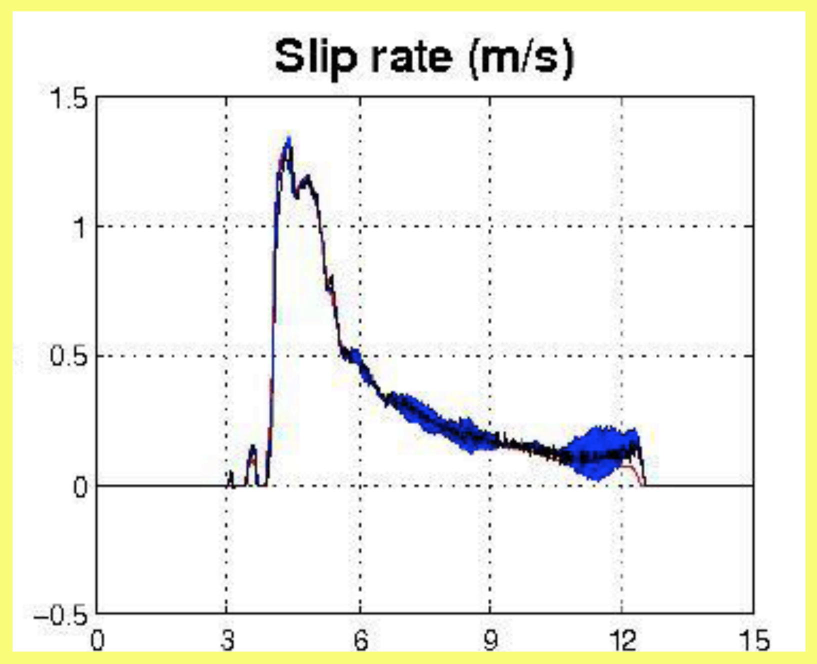
DGCrack Model Verification

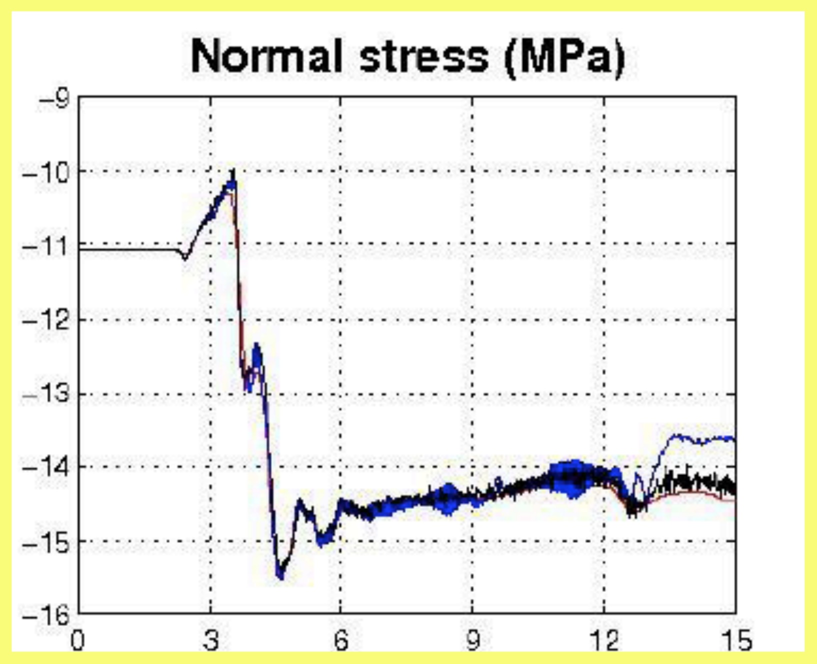
(The Problem Version 10)

Fault waveforms comparison for h = 200 m



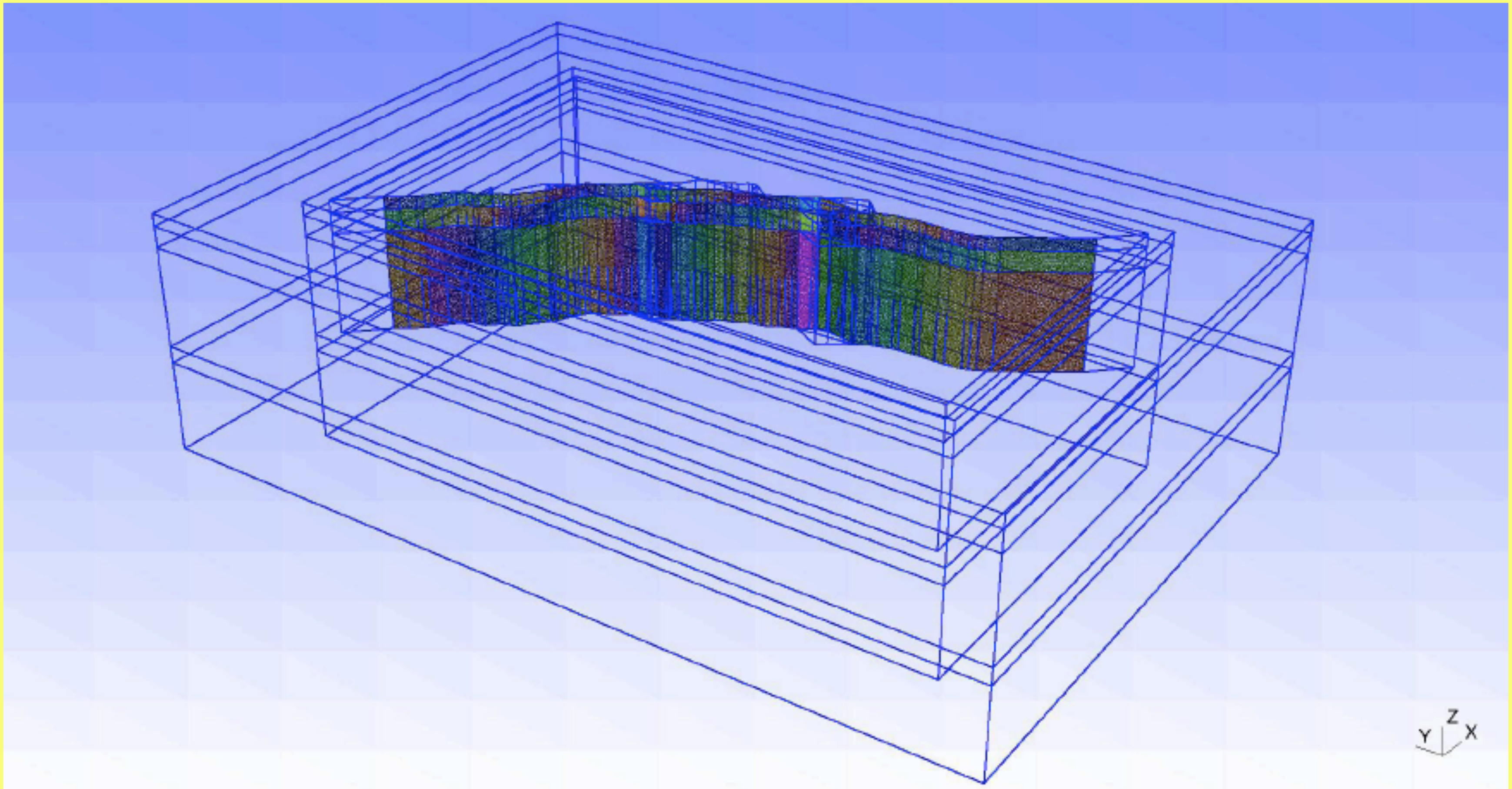






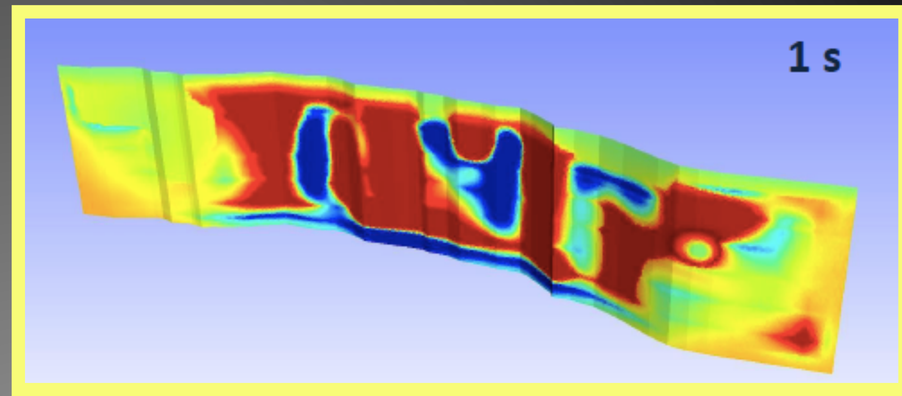
1992 Landers Earthquake Modeling

Non-structured discretization of the FCM with mesh refinement in a layered medium

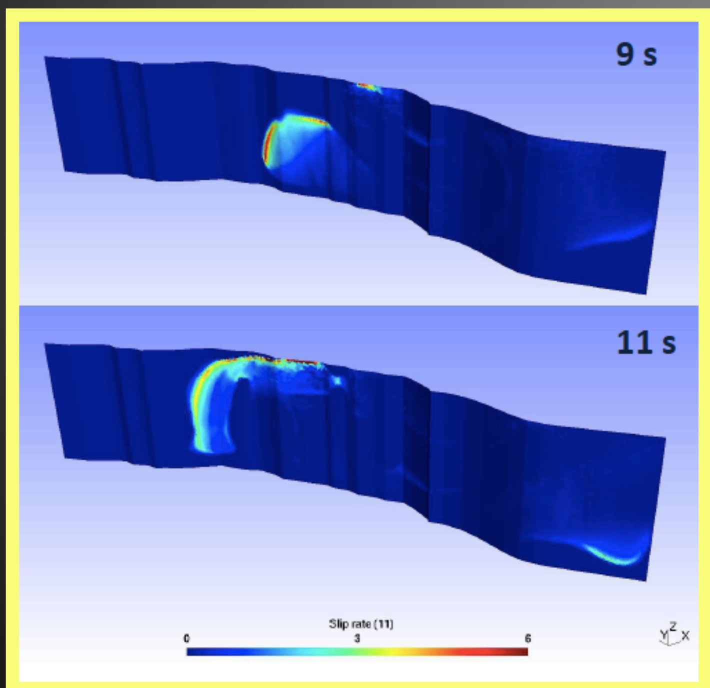


1992 Landers Earthquake Modeling

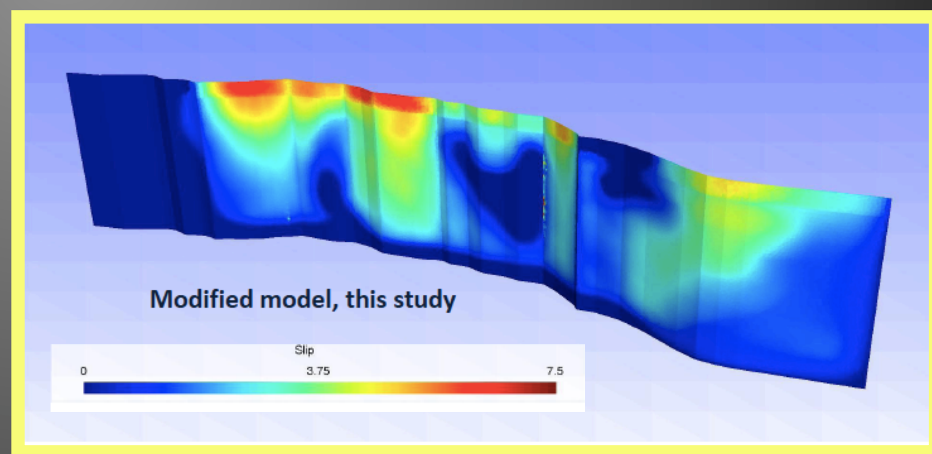
Initial Shear Stress



Slip rate snapshots



Final Slip



Conclusions

1. We have introduced a novel 3D scheme for dynamic rupture modeling based on a hp-adaptive Discontinuous Galerkin method (DGCrack)

2. The method converges with a 1.65 exponential rate and finds RMS% errors for rupture times (TPV3) below 1.0% for h < 500 m

3. The algorithm is accurate and very efficient thanks to both the unstructured mesh refinement and the approximation order adaptivity

4. DGCrack handles irregular fault geometries with accurate and stable approximations of both shear and normal fault tractions

5. Rate- and state dependent friction with thermal pressurization is now being integrated to DGCrack