Benchmark Problem Descriptions

Consistency among physical descriptions and implementations

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March 10, 2008
Inconsistencies in TPV8 Benchmark Description

What are the sizes of the rupture and nucleation patch?
Why Does it Matter?

- Modelers interpret problem statement differently
  ⇒ Codes won’t produce same results if they are solving different problems

- Problems can be discretization size dependent
  ⇒ Results for a code won’t converge because problems change with discretization size
Examples of Different Interpretations of TPV8

Size of nucleation patch has strong affect on rupture speed
Examples of Different Interpretations of TPV8

Size of nucleation patch has strong affect on rupture speed

![Graph showing slip initiation time vs elevation and distance along strike](image)
Examples of Different Interpretations of TPV8

Differences b/t codes smaller than differences among interpretations
Examples of Different Interpretations of TPV8

Small, consistent differences when codes run same problems
Examples of Different Interpretations of TPV8

Slip rate sensitive to healing phases from bottom of fault

Slip Rate, strike= 0.0 km, dip=12.0 km

- Aagaard: cf cn
- Aagaard: bf cn
- Aagaard: cf bn
- Aagaard: bf bn
Examples of Different Interpretations of TPV8

Nucleation size has strong influence than size of fault

Slip Rate, strike= 0.0 km, dip= 4.5 km

- Blue: Aagaard: cf cn
- Orange: Aagaard: bf cn
- Green: Aagaard: cf bn
- Red: Aagaard: bf bn
Examples of Different Interpretations of TPV8

Changes in rupture speed dominates slip rate time history
Physical Description of Fault Size

Fault is 30 km long and 15 km wide

Objective: Consistent descriptions of slip and displacement fields for continuum and (numerical) discretized models.

Slip is zero on border but can be nonzero anywhere inside border.
Field is interpolated over FE and SE cells using basis functions

\[ u(x) = \sum_{i=1}^{n} \phi_i u_i \]

\( \phi_i \): interpolation (basis) functions

\( u_i \): slip at vertices of finite-element cell

Edge of fault rupture is defined by vertex with zero slip \((u_1 = 0)\).
Fault Size for Finite Differences

Derivative at point is a function of field over molecule

\[ u'(x) = \frac{u(x + \Delta x) + u(x - \Delta x)}{2\Delta x} \]

\[ u_i \]: slip at vertices of finite-difference grid

Edge of fault rupture is defined by vertex with zero slip but nonzero gradient \((u'_1 > 0)\).
Proposed Fault Size Description

Vertices at edges of fault rupture should have zero slip

- Edges of fault rupture coincide with borders of physical description
- Vertices at a strike of ±15 km and a down-dip distance of 15 km should have zero slip
Physical Definition of Nucleation Patch Size

Nucleation patch is 3 km by 3 km square

Shear traction has elevated value only inside nucleation patch.
Nucleation Size for Finite and Spectral Elements

Integrate tractions over fault surface (cell faces)

\[ F^i = \int_{\Omega_e} T(\vec{x}) \phi_i(\vec{x}) d\Omega \]

\[ F^i = \sum_q T(\vec{x}_q) \phi_i(\vec{x}_q) w_q |J(\vec{x}_q)| \]

\( \vec{x}_q \): location of numerical integration (quadrature) points

Force at vertex depends on mesh topology and basis functions in addition to traction field.
Nucleation Size for Finite and Spectral Elements

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Quad4 cells

Tri3 cells
Nucleation Size for Finite Differences

Integration tractions over fault surface (tributary area of point)

\[ F^i = \int_{\Omega} T(\vec{x}) \, d\Omega \]

Force at vertex depends on traction field. Does the integration depend on the FD molecule?
Nucleation Size for Finite Differences

Integration tractions over fault surface (tributary area of point)

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Proposed Nucleation Size Description

Integrate tractions following numerical formulation of method

- “Smooth” nucleation patch is probably preferred
  - SPICE benchmark shows smooth nucleation patch reduces error
  - TPV9 may provide additional guidance

- Integration of tractions over nucleation patch in codes should match continuum description
  - Exact for uniform tractions in nucleation patch but can be tedious for non-rectangular shapes
  - Not exact (truncation error), but convergent for smooth nucleation patches