Benchmark Problem Descriptions

Consistency among physical descriptions and implementations

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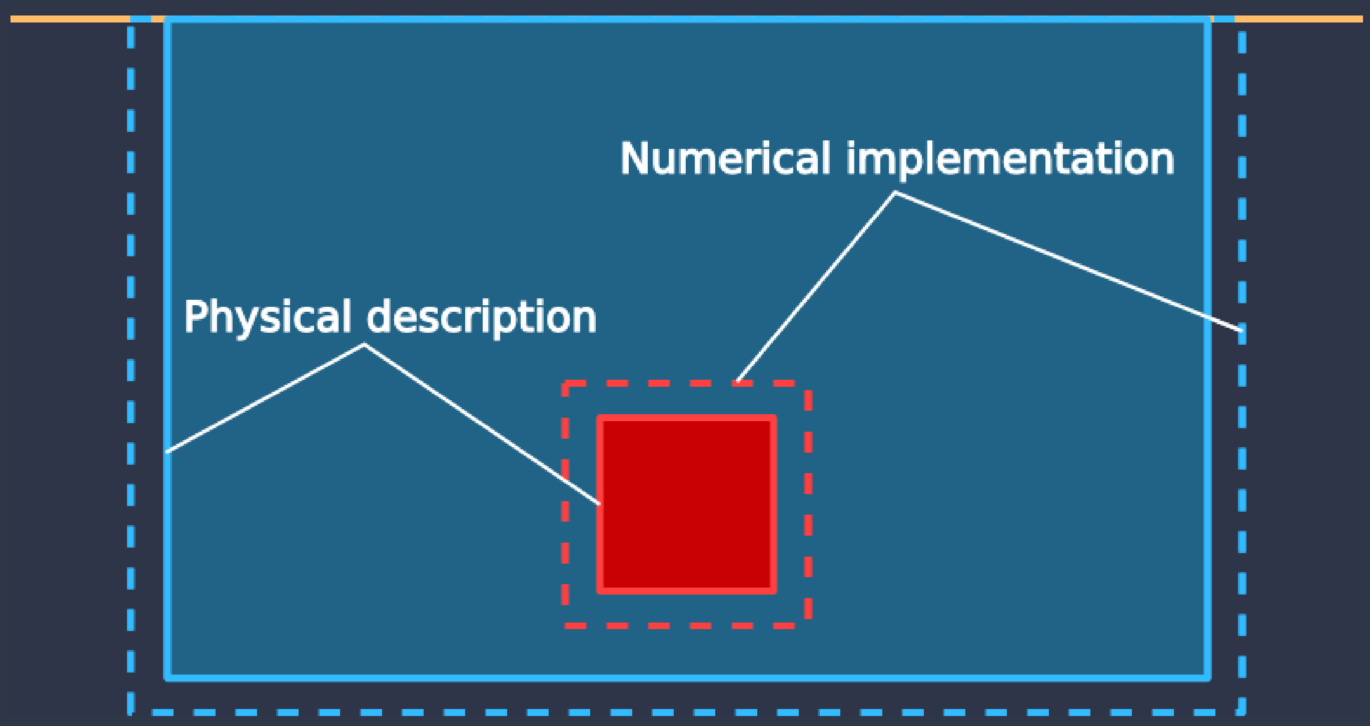
Inconsistencies in TPV8 Benchmark Description

What are the sizes of the rupture and nucleation patch?



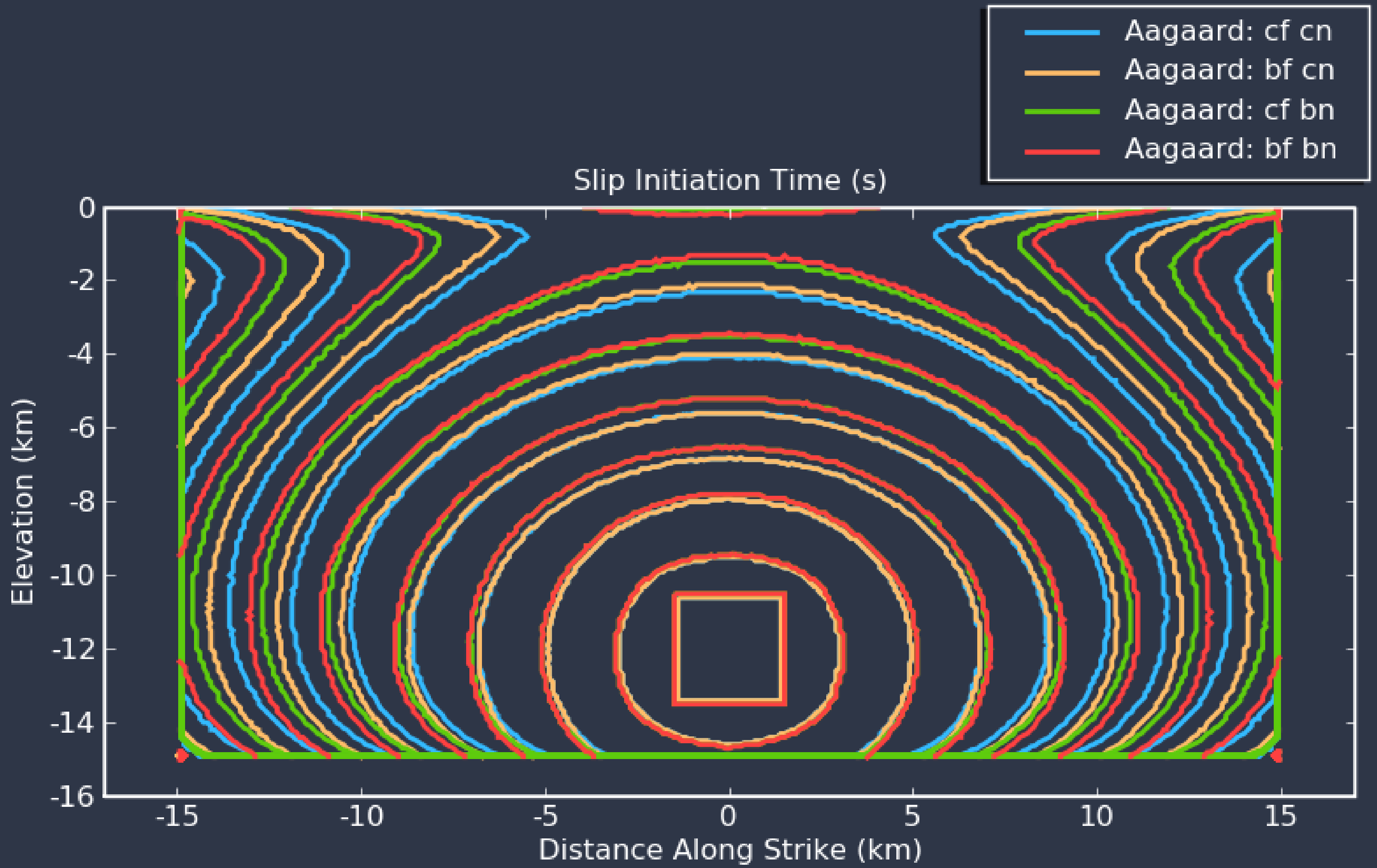
Why Does it Matter?

- Modelers interpret problem statement differently
$\Rightarrow$ Codes won't produce same results if they are solving different problems

- Problems can be discretization size dependent
$\Rightarrow$ Results for a code won't converge because problems change with discretization size

Examples of Different Interpretations of TPV8

Size of nucleation patch has strong affect on rupture speed



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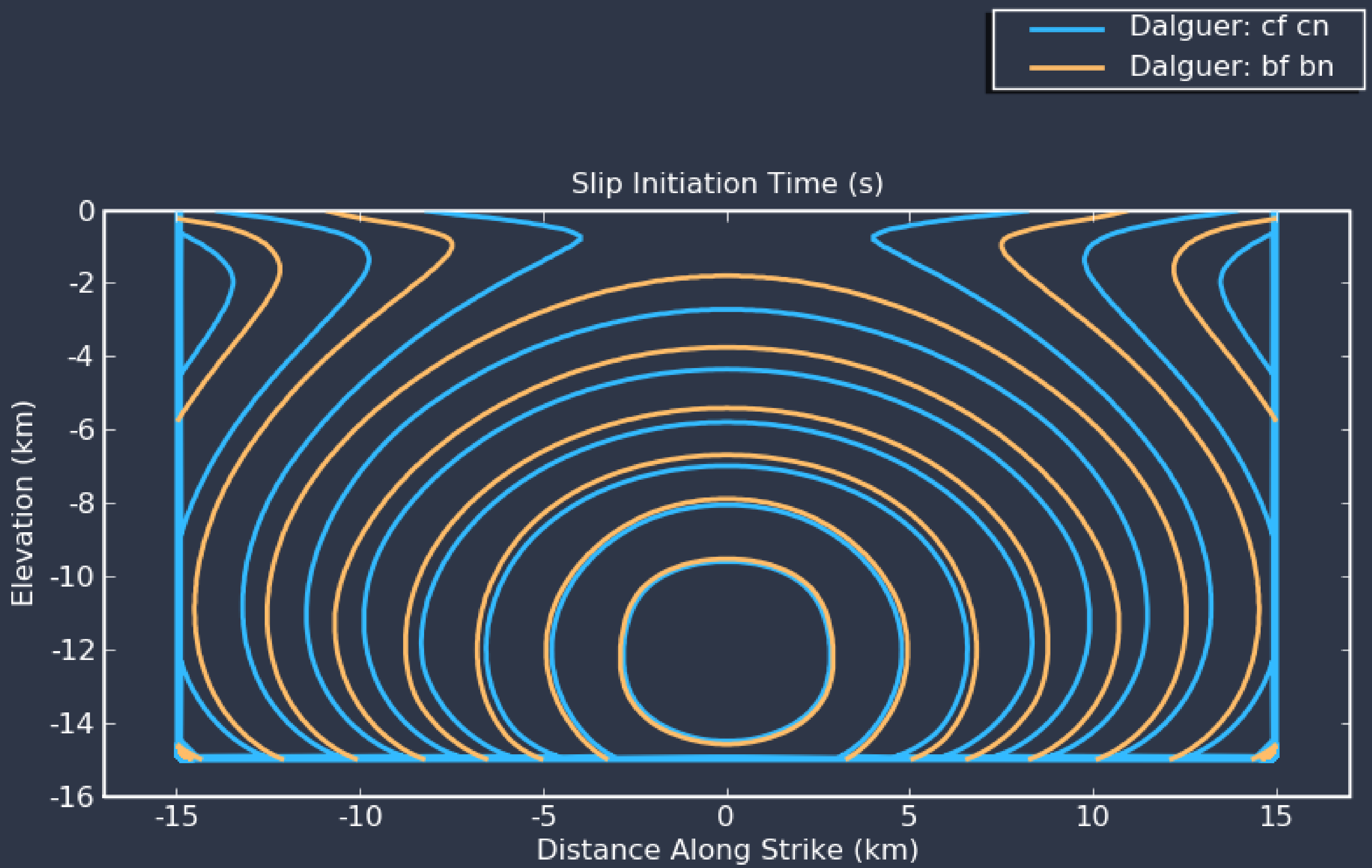

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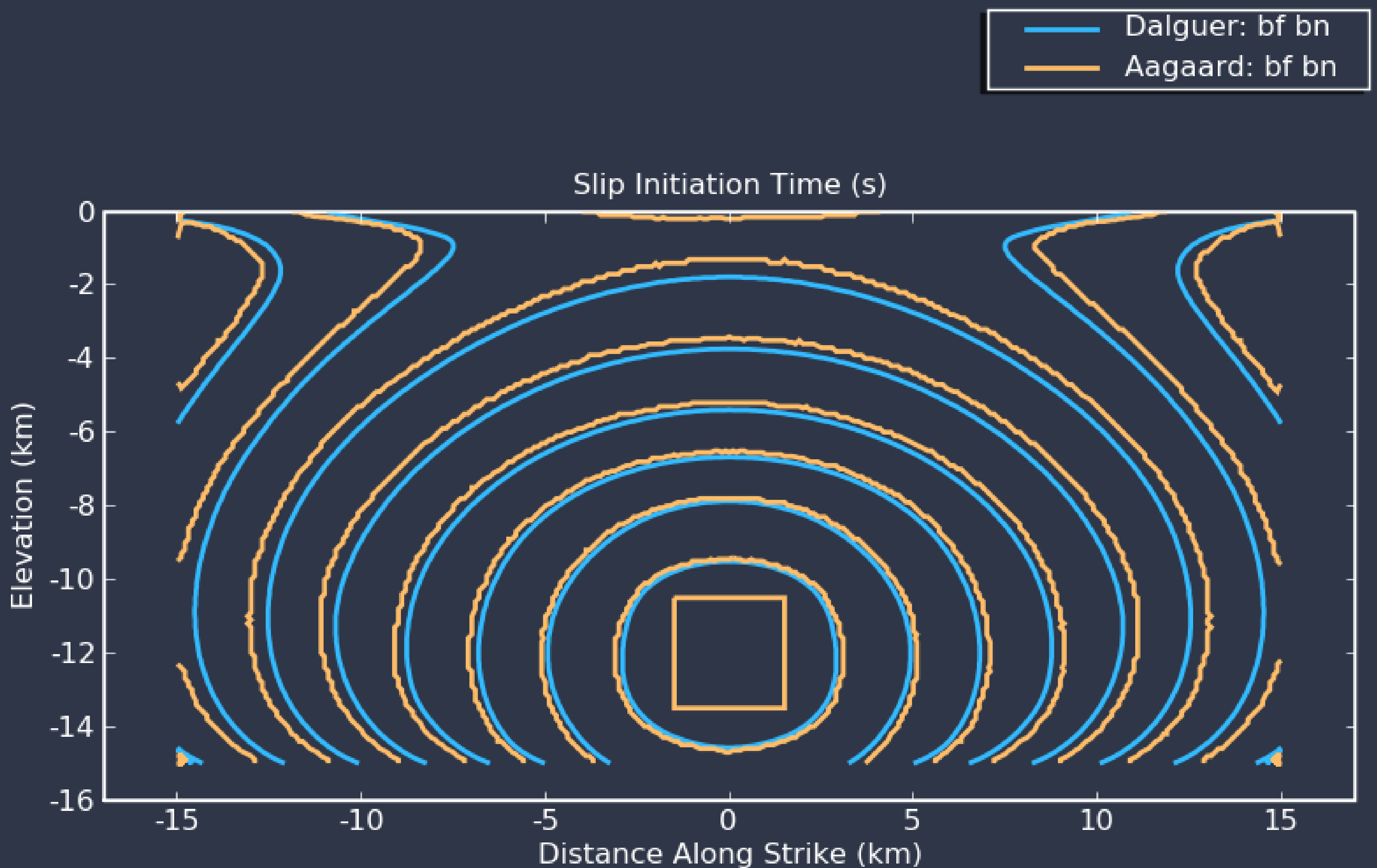
Examples of Different Interpretations of TPV8

Differences b/t codes smaller than differences among interpretations



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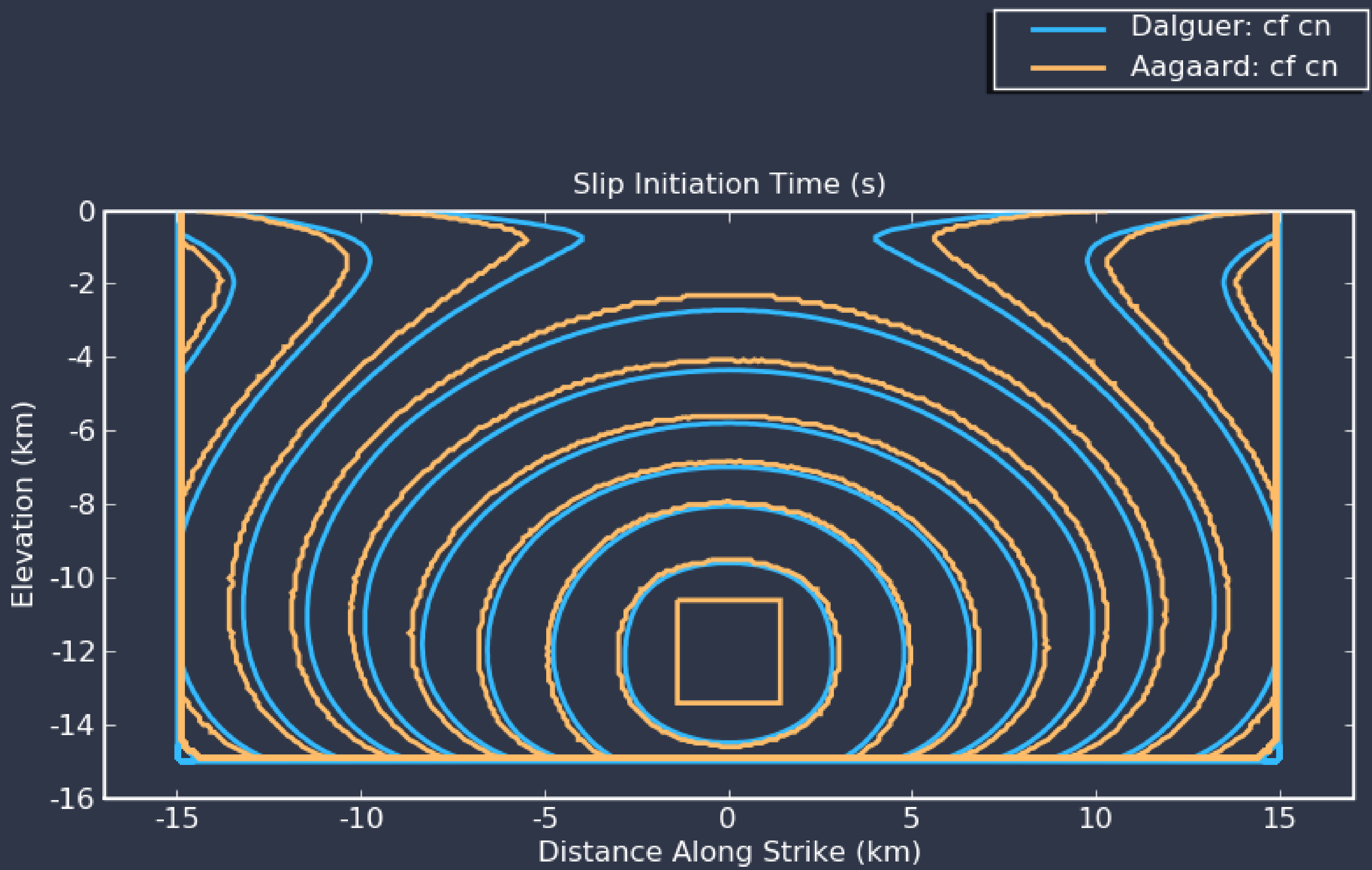
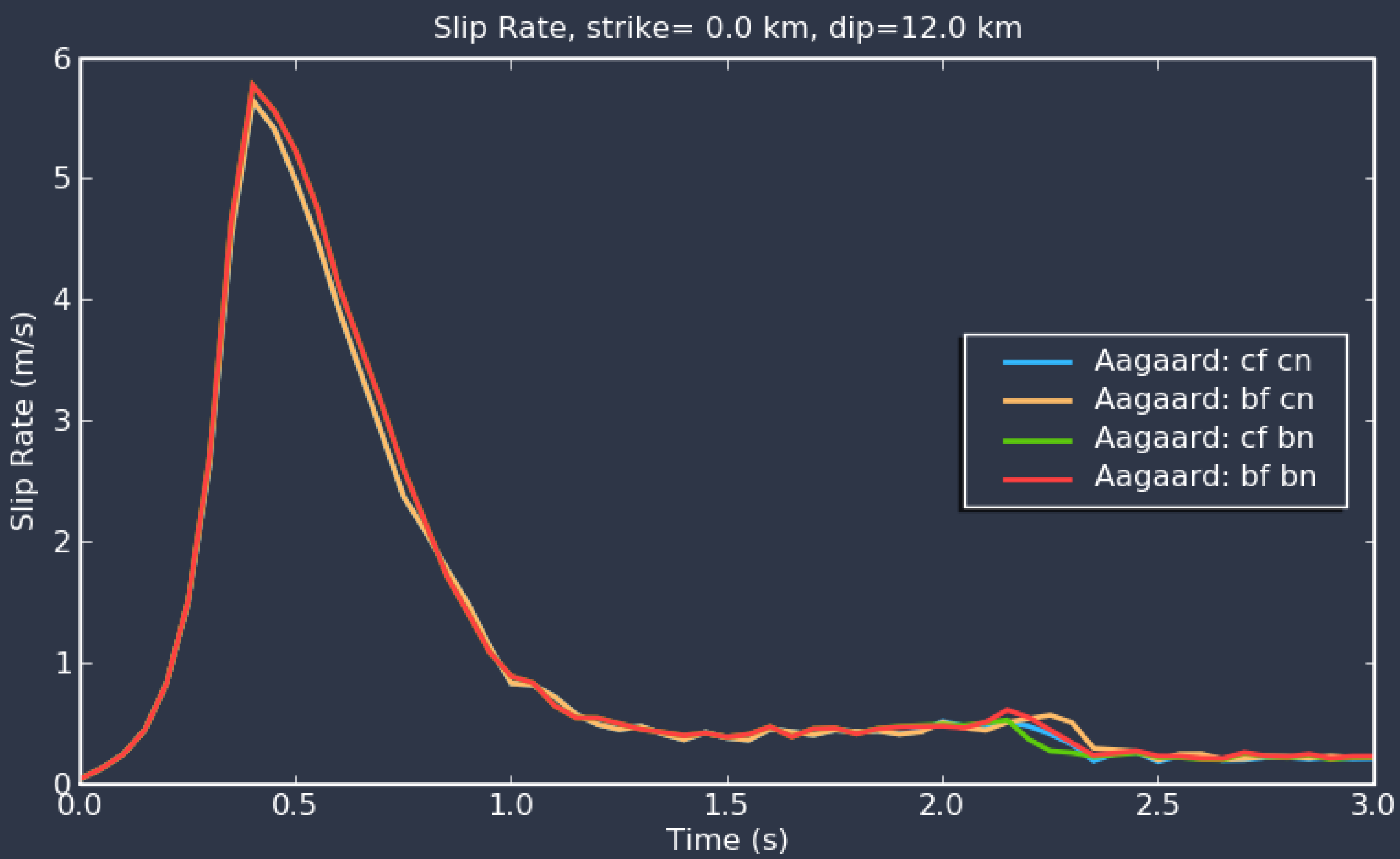
Examples of Different Interpretations of TPV8

Small, consistent differences when codes run same problems



Examples of Different Interpretations of TPV8

Slip rate sensitive to healing phases from bottom of fault



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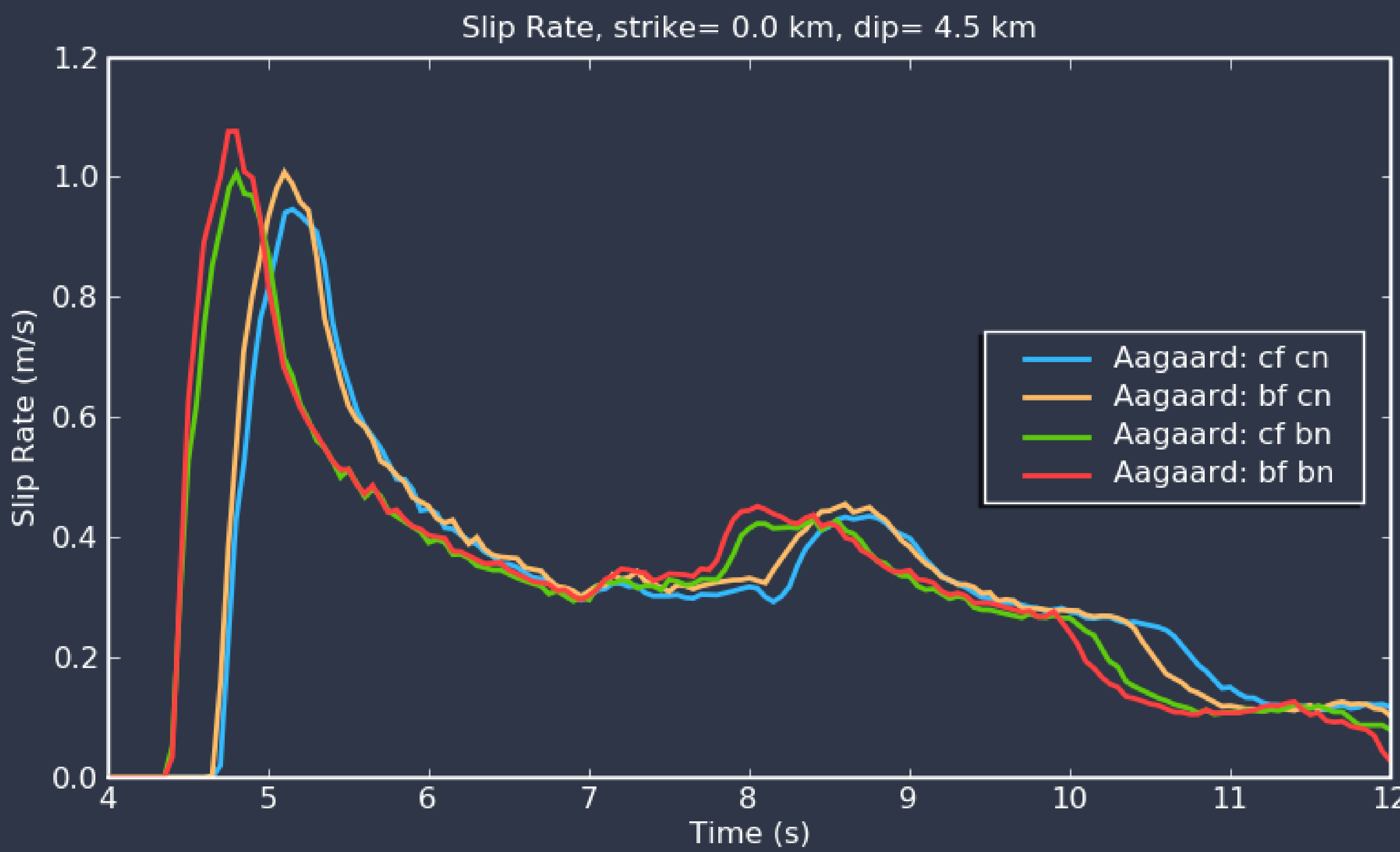
Examples of Different Interpretations of TPV8

Nucleation size has strong influence than size of fault



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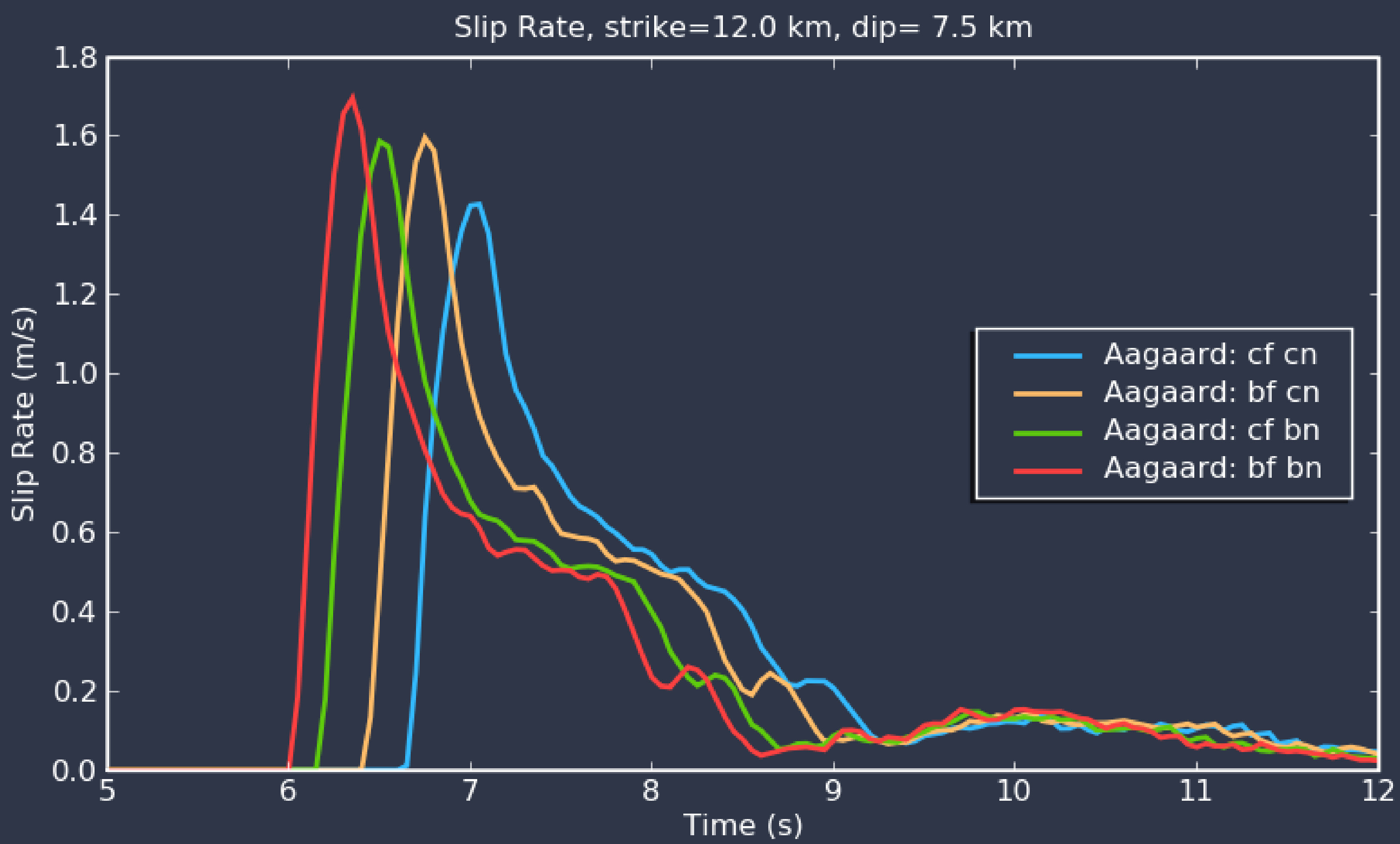
Examples of Different Interpretations of TPV8

Changes in rupture speed dominates slip rate time history



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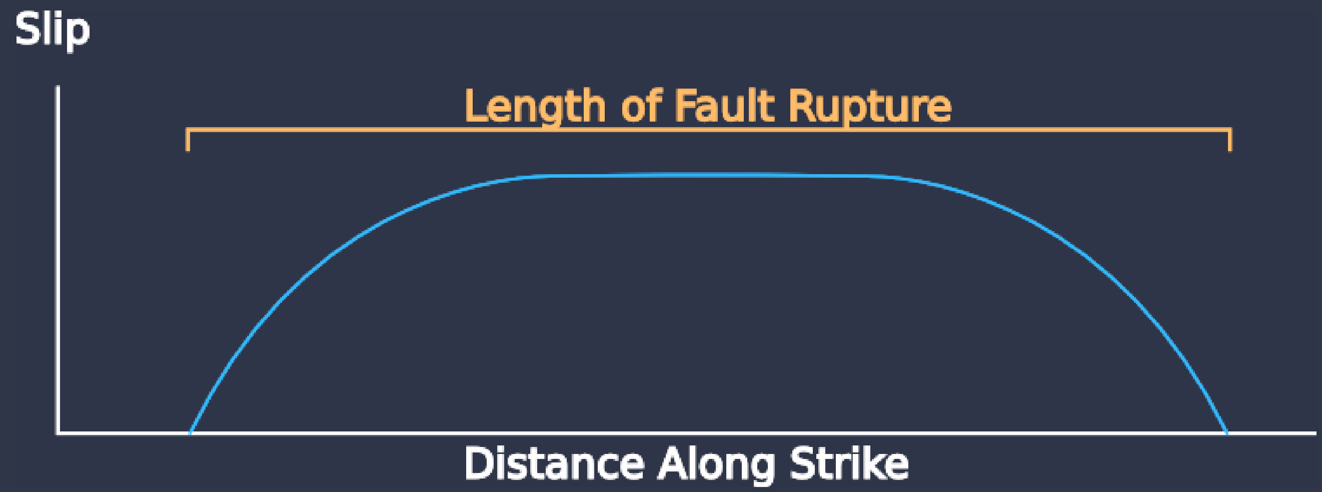
Physical Description of Fault Size

Fault is 30 km long and 15 km wide

Objective: Consistent descriptions of slip and displacement fields for continuum and (numerical) discretized models.

Slip is zero on border but can be nonzero anywhere inside border.



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Fault Size for Finite and Spectral Elements

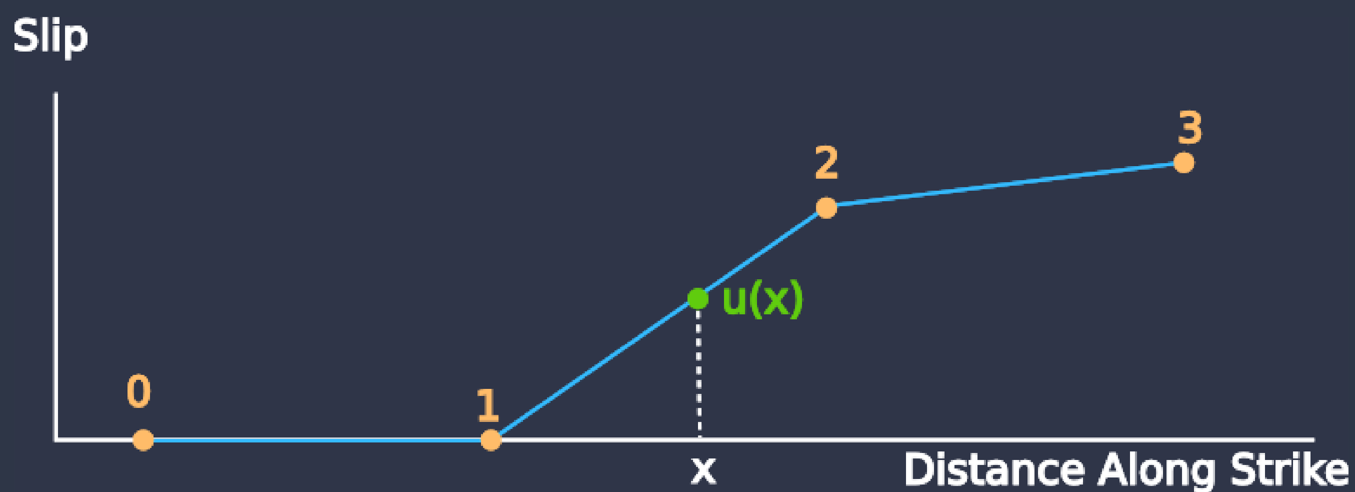
Field is interpolated over FE and SE cells using basis functions

$$u(x) = \sum_{i=1}^{n} \phi_i u_i$$

ϕ_i : interpolation (basis) functions

u_i : slip at vertices of finite-element cell

Edge of fault rupture is defined by vertex with zero slip ($u_1 = 0$).



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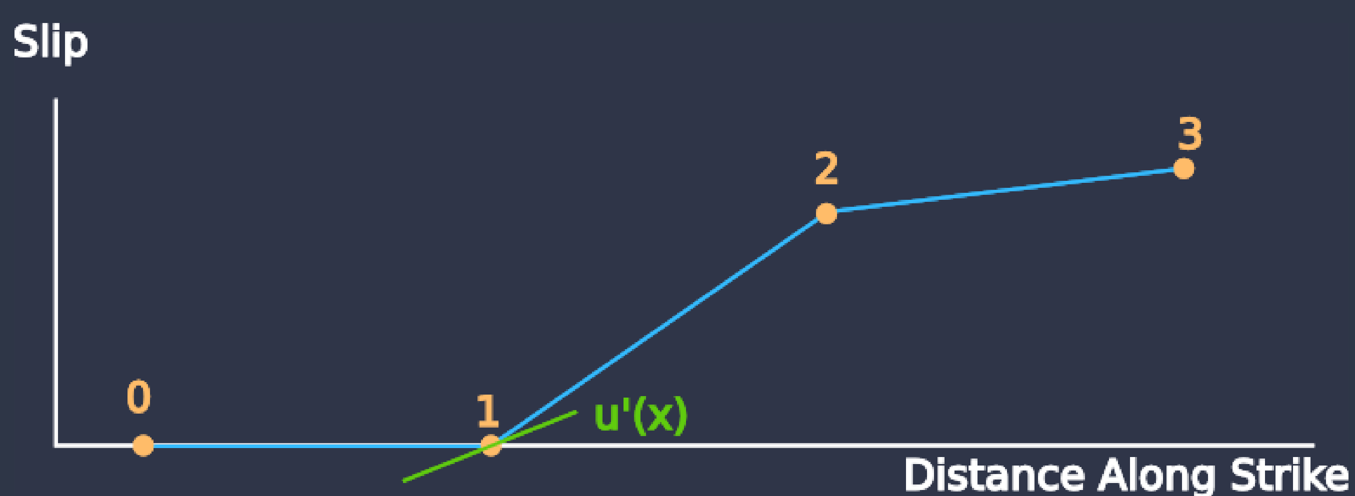
Fault Size for Finite Differences

Derivative at point is a function of field over molecule

$$u'(x) = \frac{u(x+\Delta x) + u(x-\Delta x)}{2\Delta x}$$

u_i : slip at vertices of finite-difference grid

Edge of fault rupture is defined by vertex with zero slip but nonzero gradient ($u'_1 > 0$).



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Proposed Fault Size Description

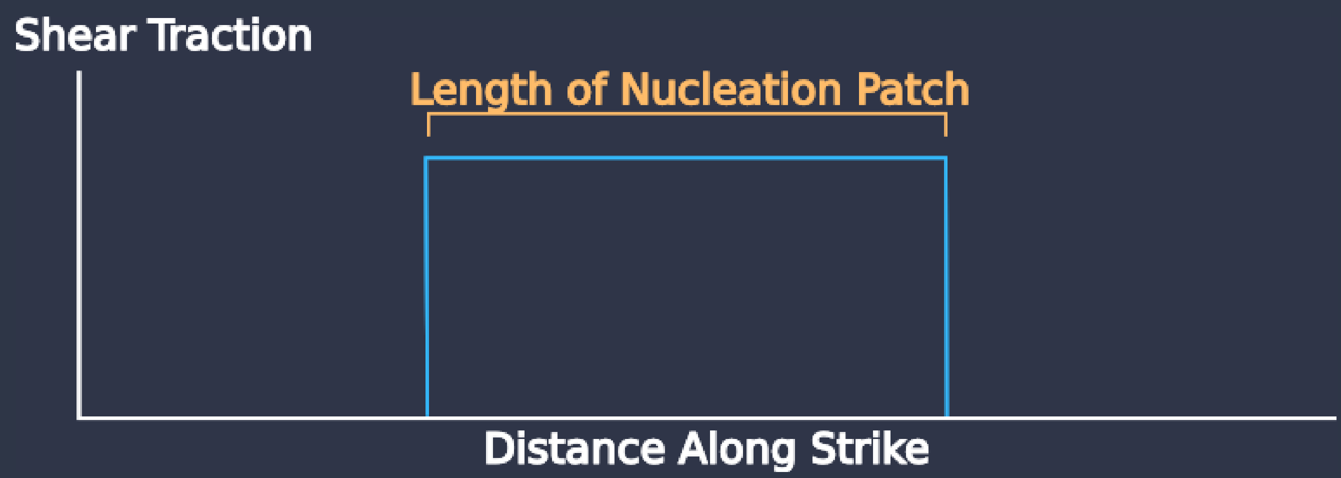
Vertices at edges of fault rupture should have zero slip

- Edges of fault rupture coincide with borders of physical description
- Vertices at a strike of ± 15 km and a down-dip distance of 15 km should have zero slip

Physical Definition of Nucleation Patch Size

Nucleation patch is 3 km by 3 km square

Shear traction has elevated value only inside nucleation patch.



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Nucleation Size for Finite and Spectral Elements

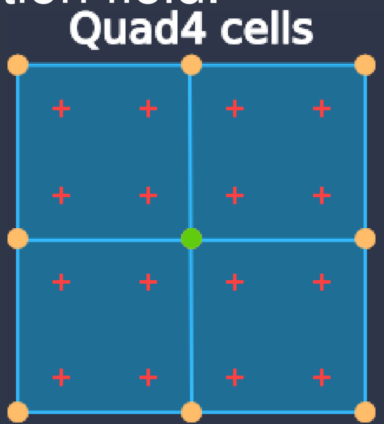
Integrate tractions over fault surface (cell faces)

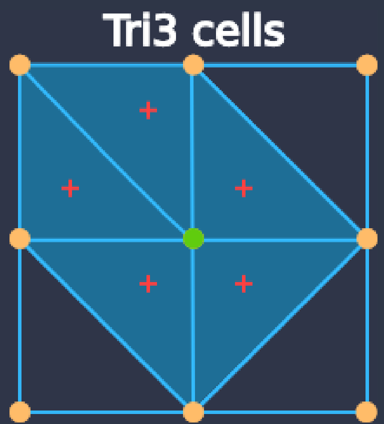
$$F^i = \int_{\Omega_e} T(\vec{x})\phi_i(\vec{x})d\Omega$$

$$F^i = \sum_q T(\vec{x}_q)\phi_i(\vec{x}_q)w_q|J(\vec{x}_q)|$$

$\vec{x}_q$: location of numerical integration (quadrature) points

Force at vertex depends on mesh topology and basis functions in addition to traction field.





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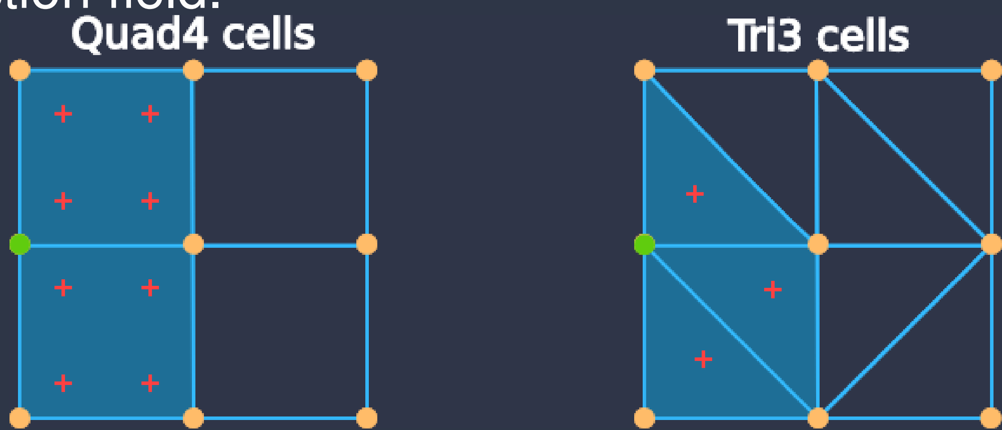

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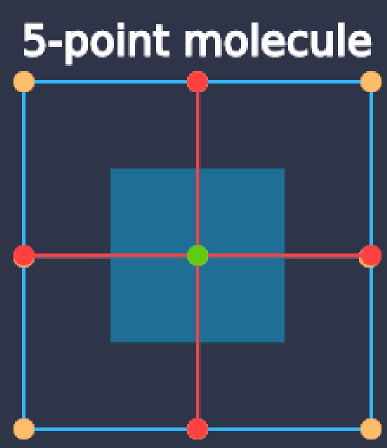
Nucleation Size for Finite Differences

Integration tractions over fault surface (tributary area of point)

$$F^i = \int_\Omega T(\vec{x}) d\Omega$$

Force at vertex depends on traction field. Does the integration depend on the FD molecule?



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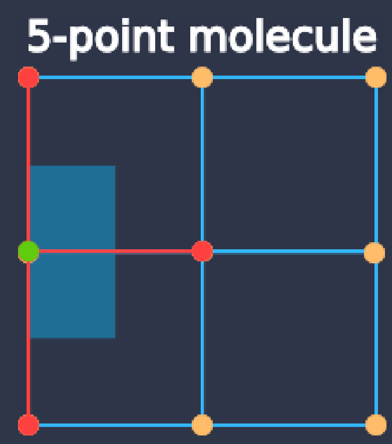

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Proposed Nucleation Size Description

Integrate tractions following numerical formulation of method

- “Smooth” nucleation patch is probably preferred
  - SPICE benchmark shows smooth nucleation patch reduces error
  - TPV9 may provide additional guidance
- Integration of tractions over nucleation patch in codes should match continuum description
  - Exact for uniform tractions in nucleation patch but can be tedious for non-rectangular shapes
  - Not exact (truncation error), but convergent for smooth nucleation patches