TPV101/102: Earthquake Ruptures with Rate-and-State Friction

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Rate-and-State Friction

shear strength

friction coefficient

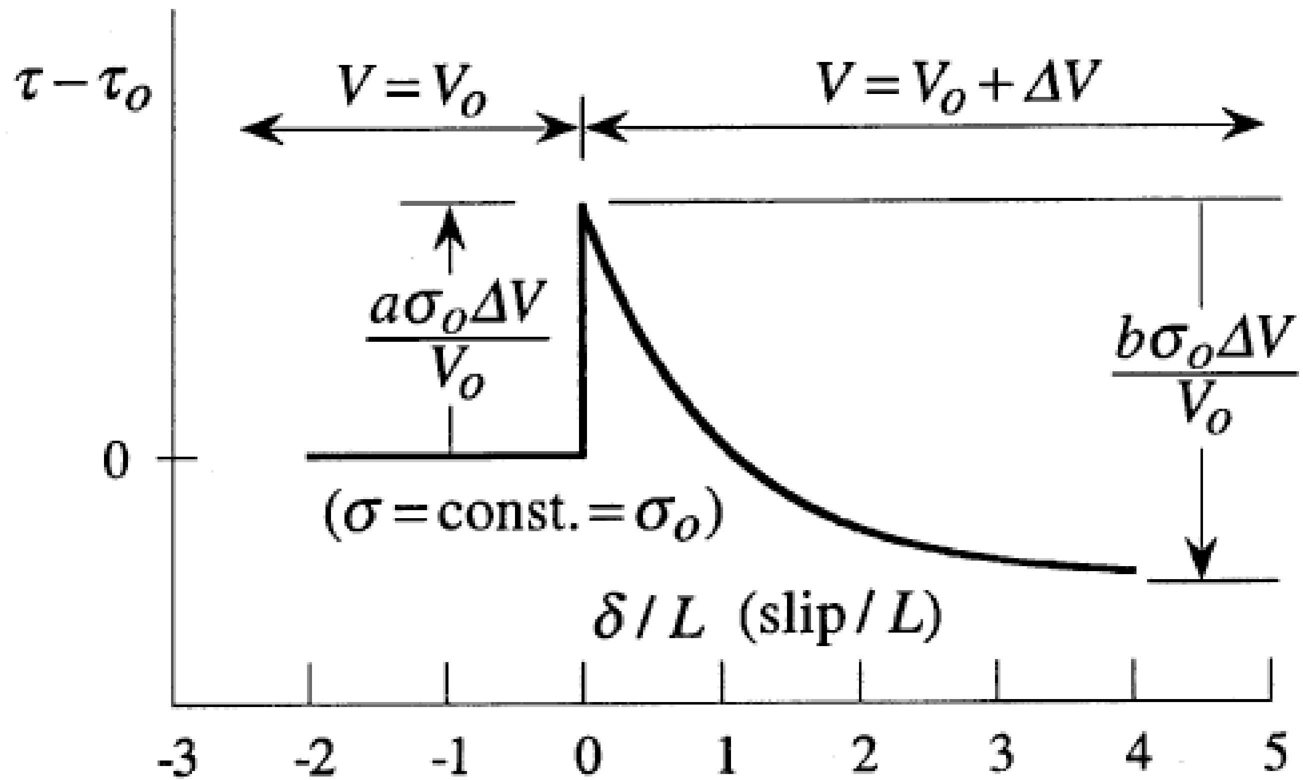
normal stress
(positive in compression)

$$\tau = f\left(V,\psi\right)\sigma$$

slip velocity

state variable

$$\frac{d\psi}{dt} = G\left(V,\psi\right)$$

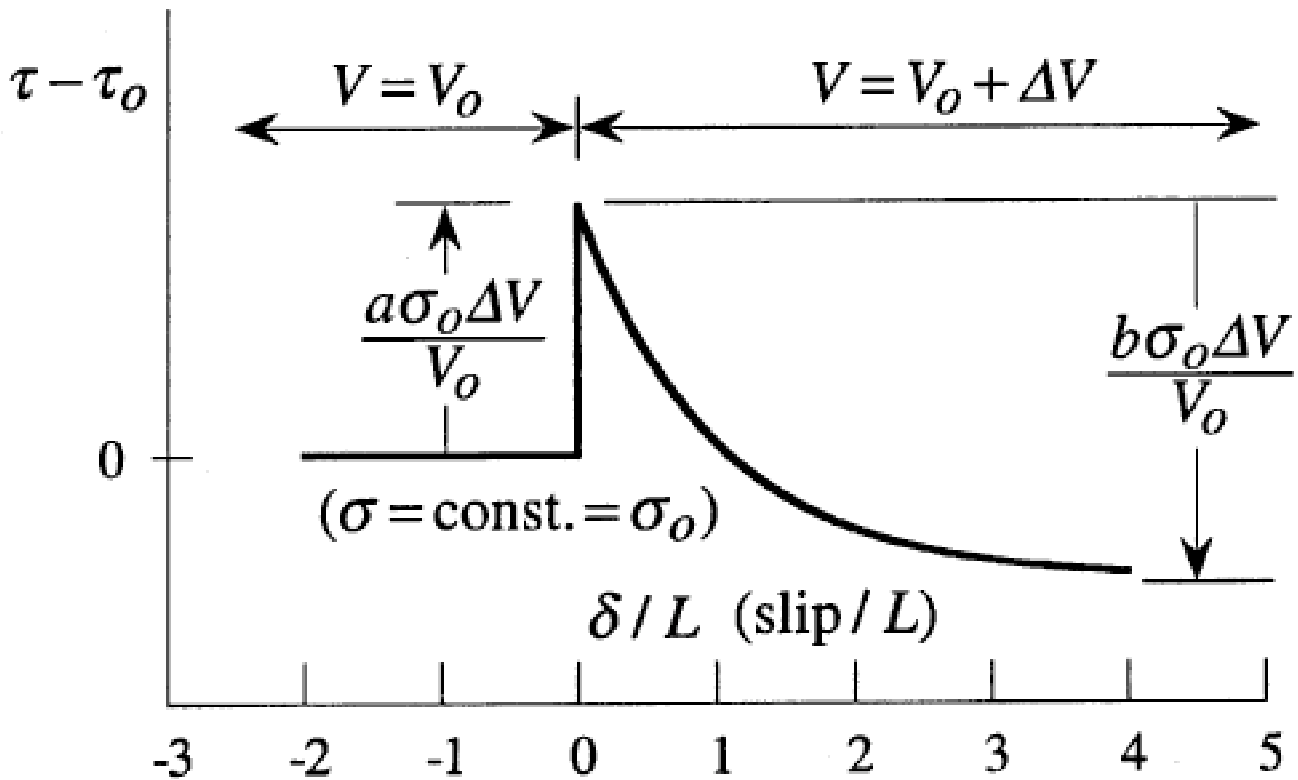


[Rice, Lapusta, Ranjith, J. Mech. Phys. Solids, 2001]

Rate-and-State Friction

$$\frac{d\tau}{dt} = \boxed{\frac{a\sigma}{V}\frac{dV}{dt}} - \boxed{\frac{V}{L}\left[\tau - f_{ss}(V)\sigma\right]}$$

direct effect　　　evolution (with slip) toward steady state

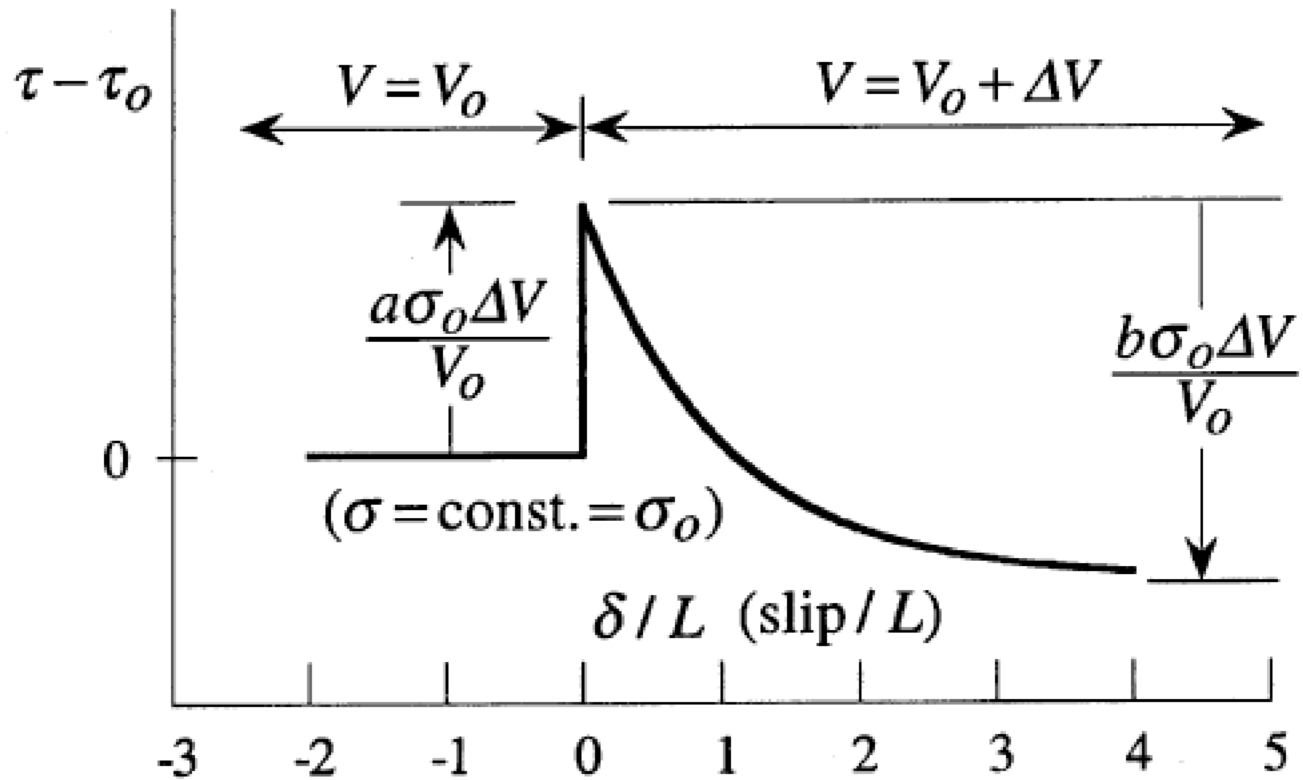


[Rice, Lapusta, Ranjith, J. Mech. Phys. Solids, 2001]

Steady State Friction Depends on Velocity

$$f_{ss}(V) = f_0 - (b-a)\ln\frac{V}{V_0}$$

- no physical meaning for f_0 and V_0
- b-a>0 velocity weakening
- b-a<0 velocity strengthening
- stronger V-dependence at large V
- not valid at V=0



[Rice, Lapusta, Ranjith, J. Mech. Phys. Solids, 2001]

Physical Basis for *a* (not *b*)

$$V \propto \exp\left(-\frac{E-\tau_c\Omega}{kT}\right)$$

$$\ln V \propto -\frac{E}{kT}+\frac{\tau_c\Omega}{kT}=-\frac{E}{kT}+\frac{(\tau\sigma_c/\sigma)\Omega}{kT}$$

$$=-\frac{E}{kT}+\left(\frac{\sigma_c\Omega}{kT}\right)\frac{\tau}{\sigma}=-\frac{E}{kT}+\frac{\tau}{a\sigma}$$

$$\frac{\tau}{\sigma}\propto a\ln V+\frac{E}{\sigma_c\Omega}, \qquad a=\frac{kT}{\sigma_c\Omega}$$

not valid at V=0

[Rice, Lapusta, Ranjith, J. Mech. Phys. Solids, 2001]

Introduction of State Variable

$$\frac{\tau}{\sigma} \propto a \ln V + \frac{E}{\sigma_c \Omega}, \qquad a = \frac{kT}{\sigma_c \Omega}$$

$$f = a \ln \frac{V}{V_0} + f_0 + \psi$$

$$= a \ln \frac{V}{V_0} + f_0 + b \ln \frac{V_0 \theta}{L}$$

[Rice, Lapusta, Ranjith, J. Mech. Phys. Solids, 2001]

Different Forms of State Evolution

$$f = a \ln \frac{V}{V_0} + f_0 + b \ln \frac{V_0 \theta}{L}$$

strength increases with ln(t)
in stationary contact

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L} \qquad \text{(ageing law)}$$

$$f = a \ln \frac{V}{V_0} + f_0 + \psi$$

$$\frac{d\psi}{dt} = -\frac{V}{L}\left(\psi + b \ln \frac{V}{V_0}\right) \qquad \text{(slip law)}$$

no evolution without slip

Solution to Divergence at V=0

$$V \propto \exp\left(-\frac{E-\tau_c\Omega}{kT}\right) - \exp\left(-\frac{E+\tau_c\Omega}{kT}\right)$$

$$= \exp\left(-\frac{E}{kT}\right)\left[\exp\left(\frac{\tau_c\Omega}{kT}\right) - \exp\left(-\frac{\tau_c\Omega}{kT}\right)\right]$$

$$= \exp\left(-\frac{E}{kT}\right) 2\sinh\left(\frac{\tau_c\Omega}{kT}\right)$$

[Rice, Lapusta, Ranjith, J. Mech. Phys. Solids, 2001]

Friction in TPV101/102

$$\tau = a\sigma \operatorname{arcsinh}\left[\frac{V}{2V_0}\exp\left(\frac{f_0 + b\ln(V_0\theta/L)}{a}\right)\right]$$

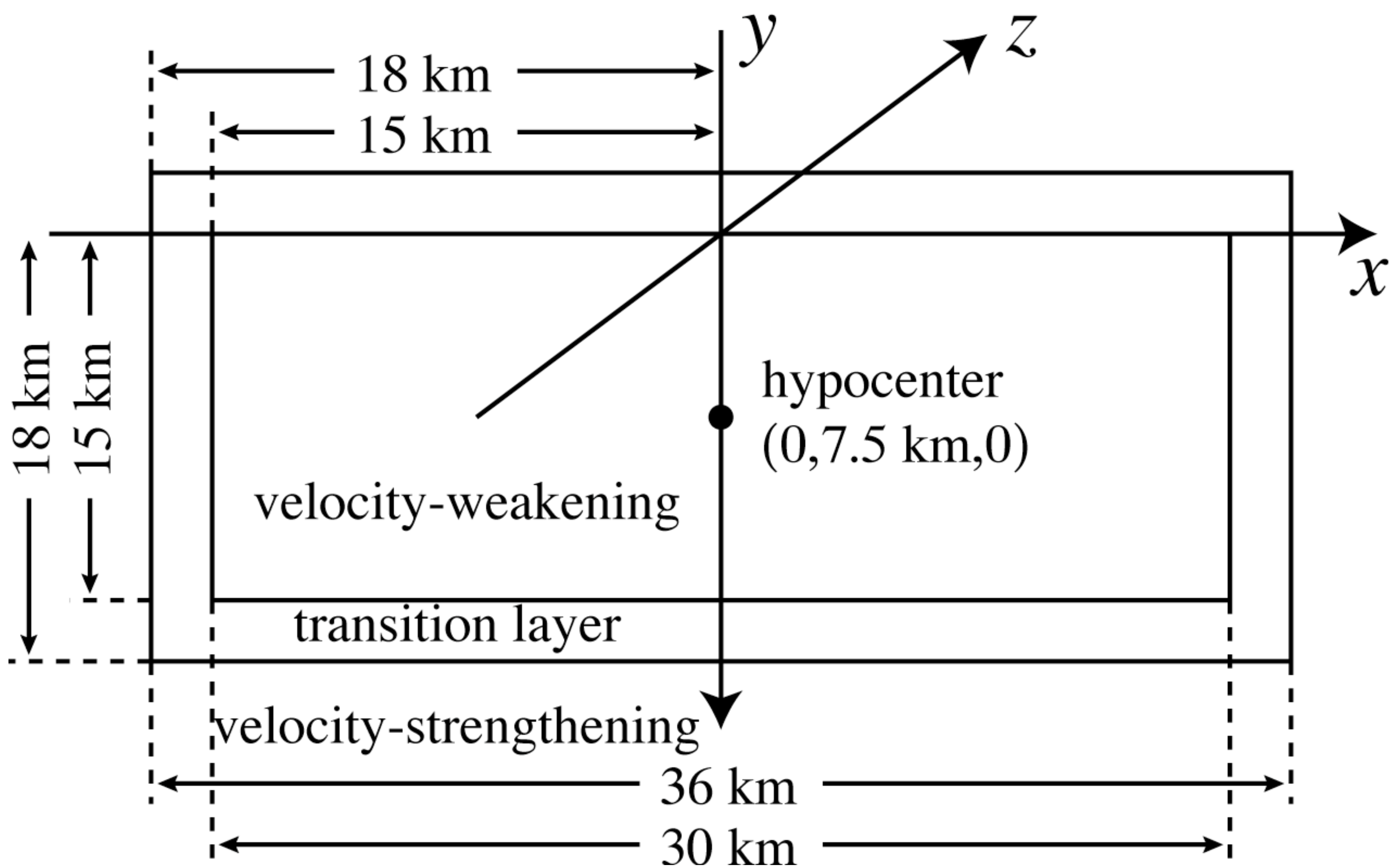
$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L} \qquad \text{(ageing law)}$$

so long as τ is nonzero, V is nonzero (fault is always sliding, but at $V \sim 10^{-12}$ m/s in interseismic period)

[Rice, Lapusta, Ranjith, J. Mech. Phys. Solids, 2001]

TPV101/102 Model Geometry

(TPV101=whole space, TPV102=half-space with free surface at y=0)



TPV101/102 Parameters

ρ	c_s	c_p
2670 kg/m^3	3.464 km/s	6 km/s

(uniform material properties, same as TPV5)

f_0	V_0	$a(x,y)$	b	L
0.6	10^{-6} m/s	0.008+$\Delta a(x,y)$	0.012	0.02 m

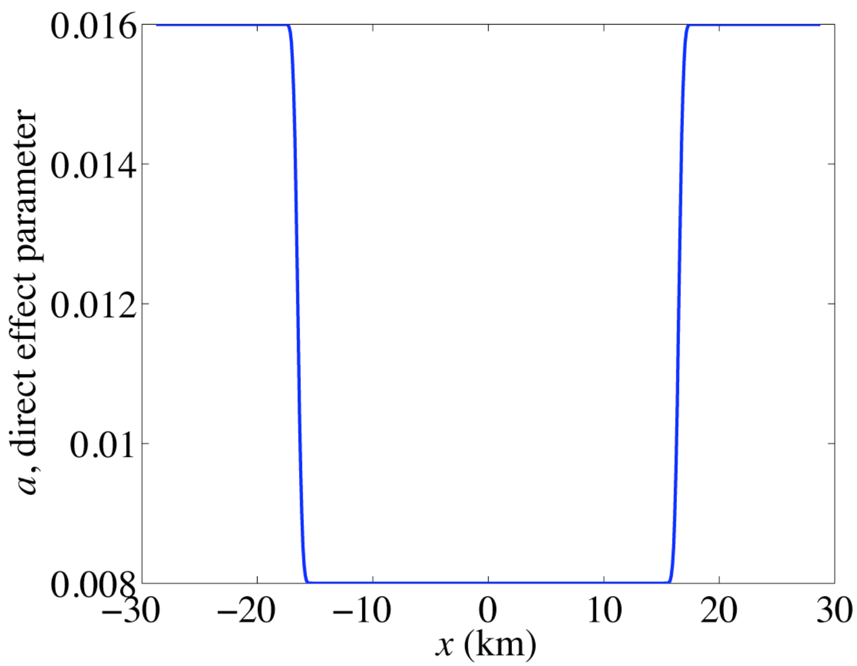
$$\Delta a(x,y) = \Delta a_0\left[1 - B(x;W,w)B(y-y_0;W/2,w)\right] \qquad \Delta a_0 = 0.008$$

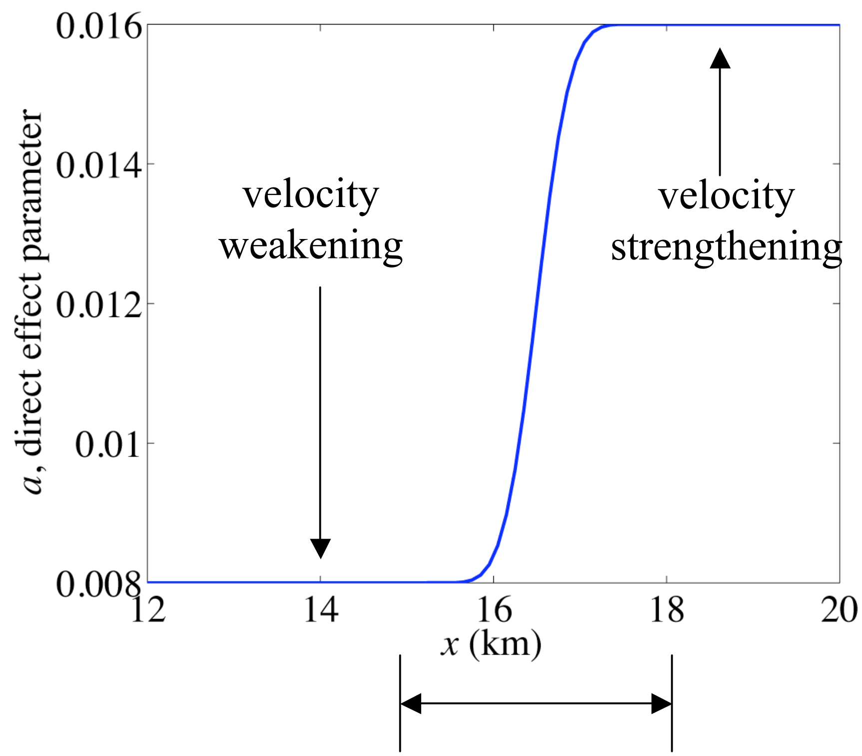
$$B(x;W,w) = \begin{cases} 1, & |x| \le W \\ \frac{1}{2}\left[1 + \tanh\left(\frac{w}{|x| - W - w} + \frac{w}{|x| - W}\right)\right], & W < |x| < W + w \\ 0, & |x| \ge W + w \end{cases}$$

W=15 km (fault width and half of fault length)

TPV101/102 Parameters

$$\Delta a(x,y) = \Delta a_0\left[1 - B(x;W,w)B(y-y_0;W/2,w)\right] \qquad \Delta a_0 = 0.008$$





class C^∞ with compact support for transition zone

(smooth function, all derivatives continuous, transition occurs over finite interval)

TPV101/102 Initial Conditions

V_{ini}	τ_{ini}	σ_{ini}	$\theta_{ini}(x,y)$
10^{-12} m/s	75 MPa	120 MPa	$1.606238999213454\times10^{9}$ s $+\Delta\theta(x,y)$ $= 50.899729562171359$ yr $+\Delta\theta(x,y)$

$$\sigma_{zx}(x,y,z)=\tau_{ini} \text{ and } \sigma_{zz}(x,y,z)=-\sigma_{ini} \text{ at } t=0$$

$$v_x(x,y,z)=\begin{cases} V_{ini}/2, & z>0 \\ -V_{ini}/2, & z<0 \end{cases} \text{ at } t=0$$

~duration of interseismic period

can only specify two of V_{ini}, τ_{ini}, θ_{ini}
(must be consistent with frictional strength)

TPV101/102 Nucleation Procedure

time-dependent loading in circular patch (with smooth edges)

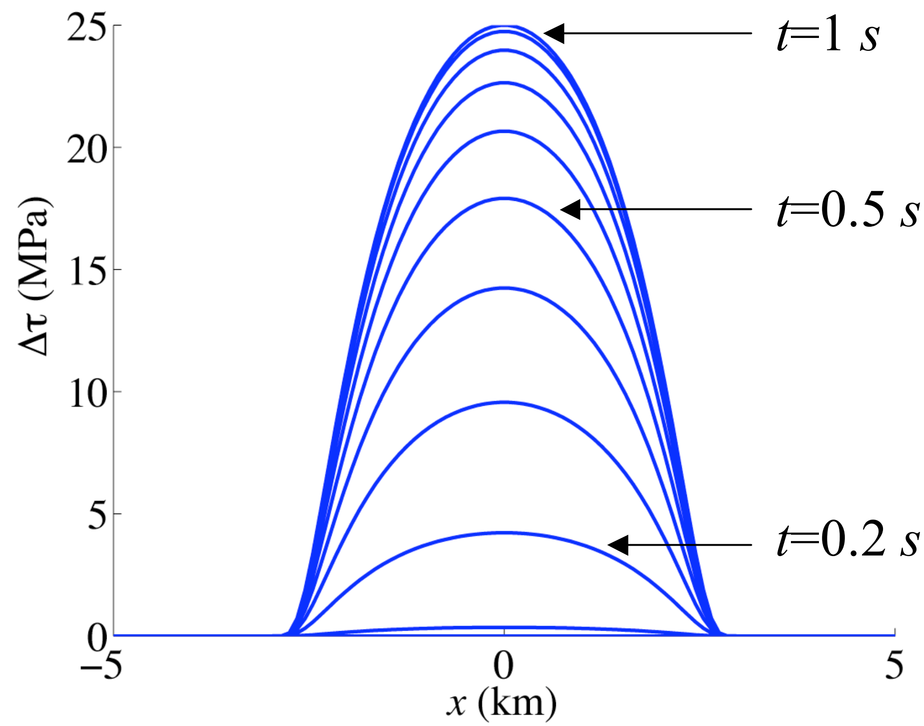
$$\Delta\tau(x,y,t) = \Delta\tau_0 F\left(\sqrt{(x-x_0)^2+(y-y_0)^2}\right)G(t)$$

$$F(r) = \begin{cases} \exp\left(\dfrac{r^2}{r^2-R^2}\right), & r < R \\ 0, & r \geq R \end{cases}$$

$(R = 3\ \text{km})$

$$G(t) = \begin{cases} \exp\left[\dfrac{(t-T)^2}{t(t-2T)}\right], & 0 < t < T \\ 1, & t \geq T \end{cases}$$

$(T = 1\ \text{s})$



class C^∞ with compact support in x and t

Connecting Rate-and-State and Slip-Weakening Parameters

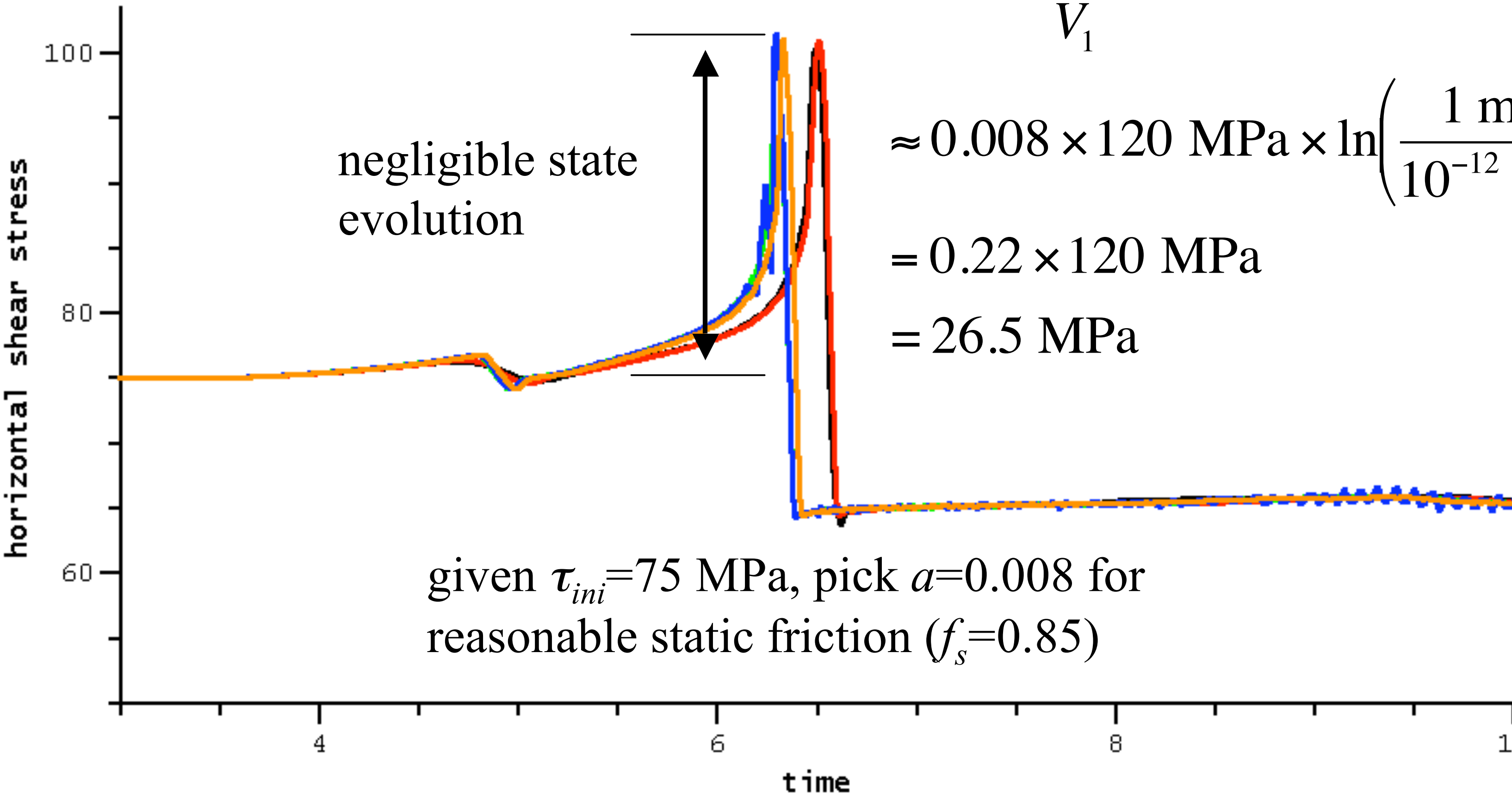
$$\tau \propto a\sigma \ln\frac{V}{V_0} \Rightarrow \quad \tau_2 - \tau_1 \propto a\sigma \ln\frac{V_2}{V_0} - a\sigma \ln\frac{V_1}{V_0}$$

$$= a\sigma \ln\frac{V_2}{V_1}$$

$$\approx 0.008 \times 120 \text{ MPa} \times \ln\left(\frac{1 \text{ m/s}}{10^{-12} \text{ m/s}}\right)$$

$$= 0.22 \times 120 \text{ MPa}$$

$$= 26.5 \text{ MPa}$$



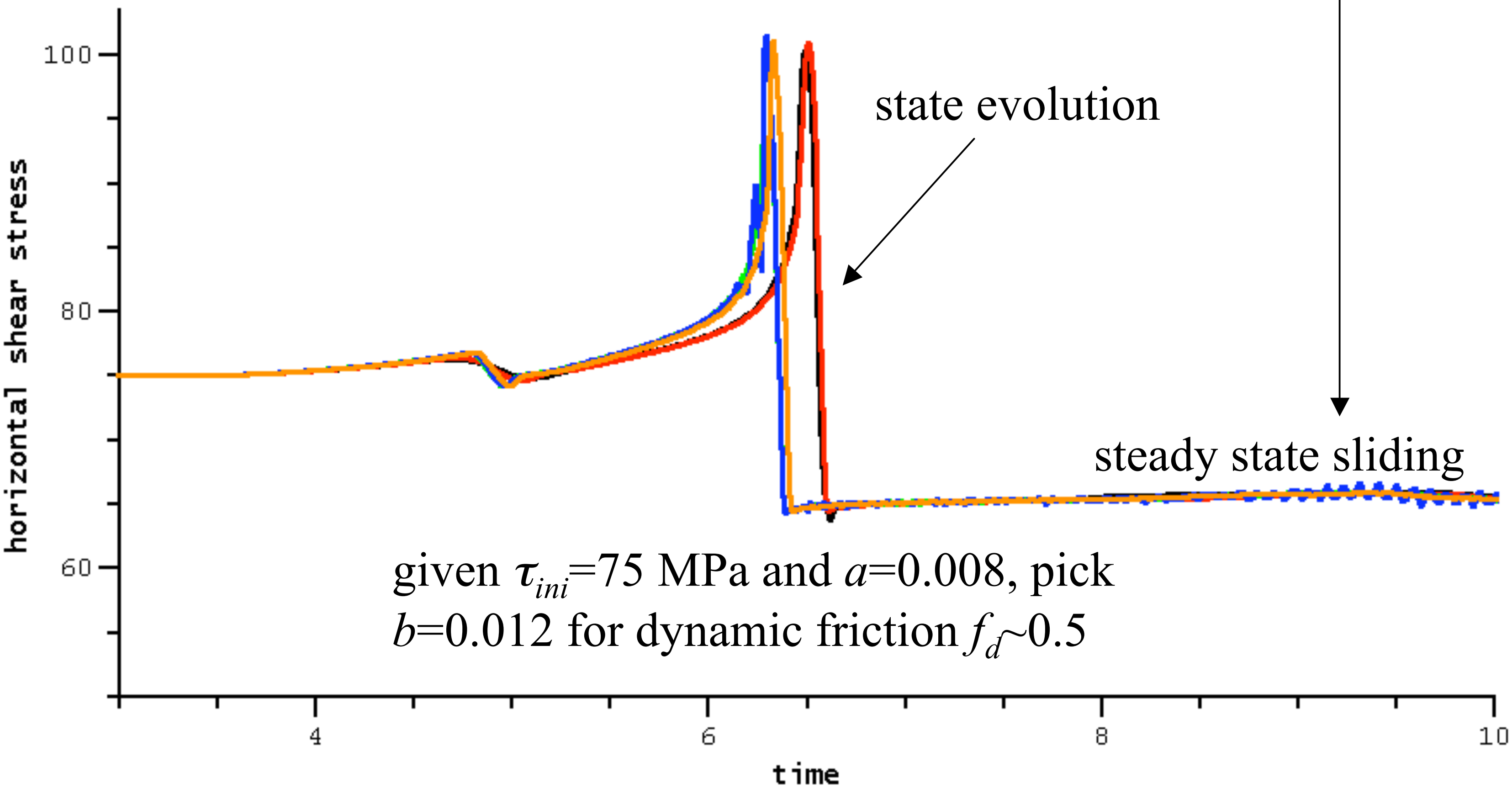
given τ_{ini}=75 MPa, pick a=0.008 for reasonable static friction (f_s=0.85)

Connecting Rate-and-State and Slip-Weakening Parameters

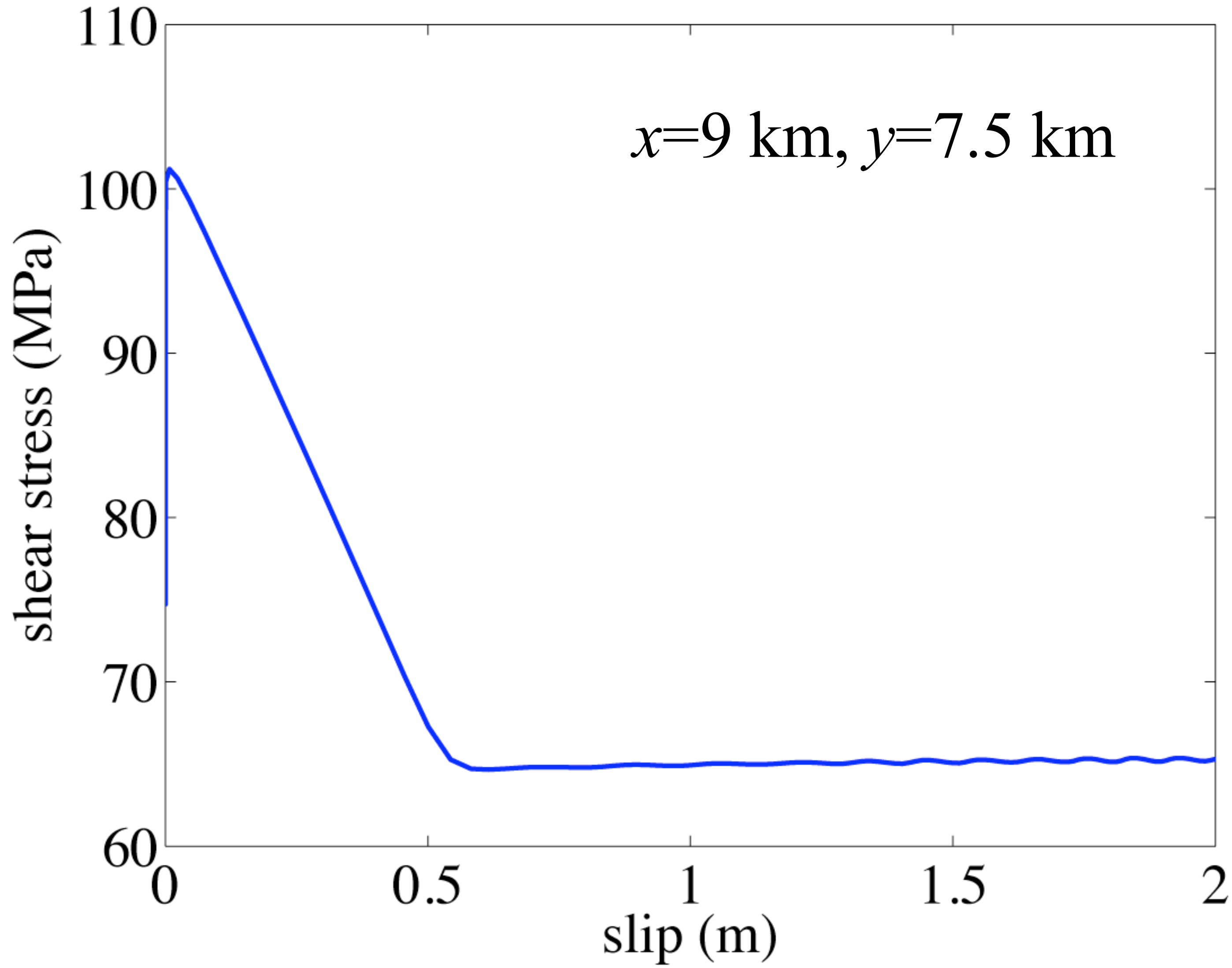
$$\tau = \sigma\left[f_0 - (b-a)\ln\frac{V}{V_0}\right] \approx \tau_{ini} - \frac{\mu}{2c_s}V \Rightarrow V \sim 2 \text{ m/s and } \tau/\sigma \sim 0.5$$

for $b \sim 0.012$

(friction) (elasticity)



Connecting Rate-and-State and Slip-Weakening Parameters



Numerical Resolution

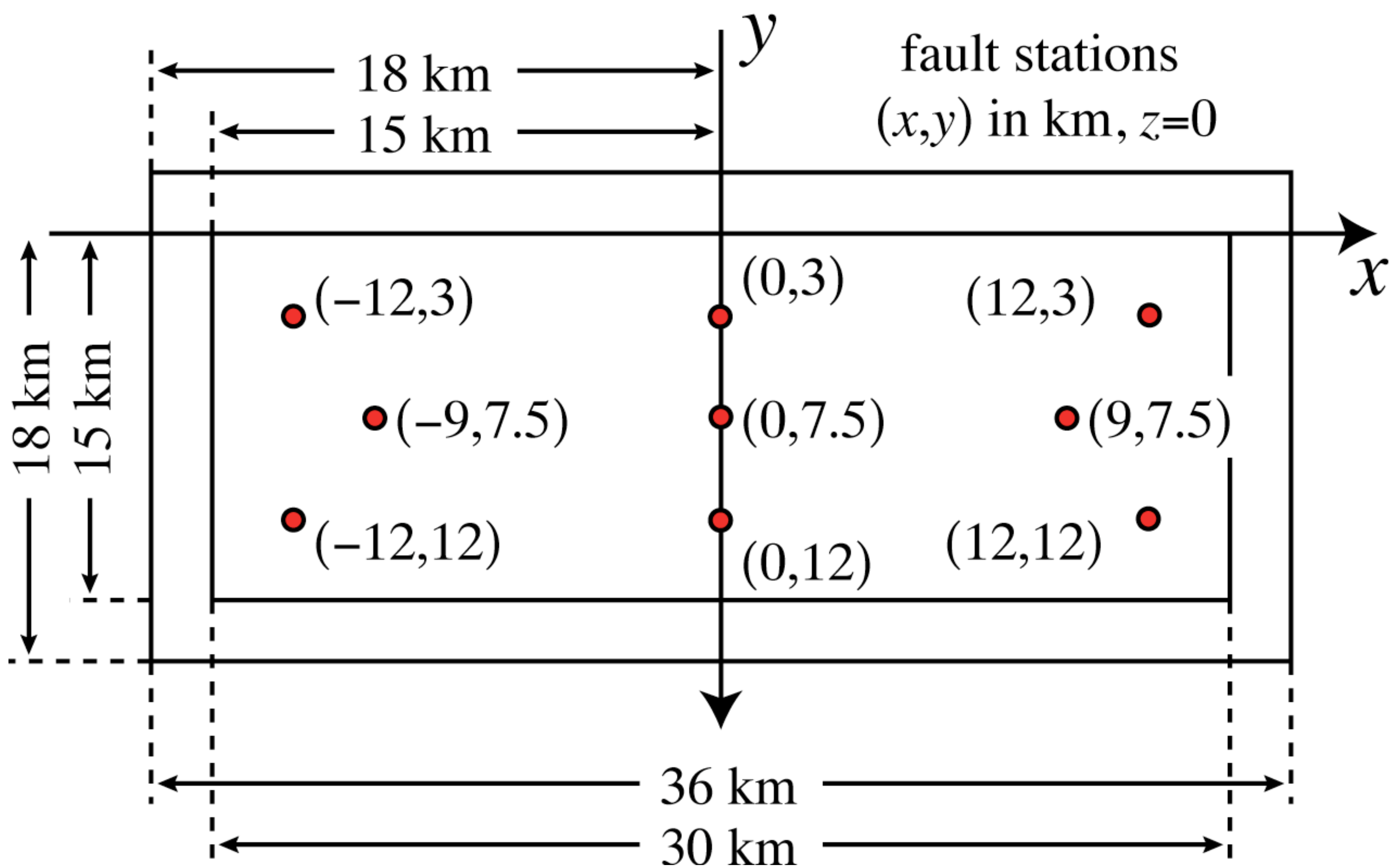
Steady sliding is

- unstable to large-wavelength perturbations
- stable to short-wavelength perturbations

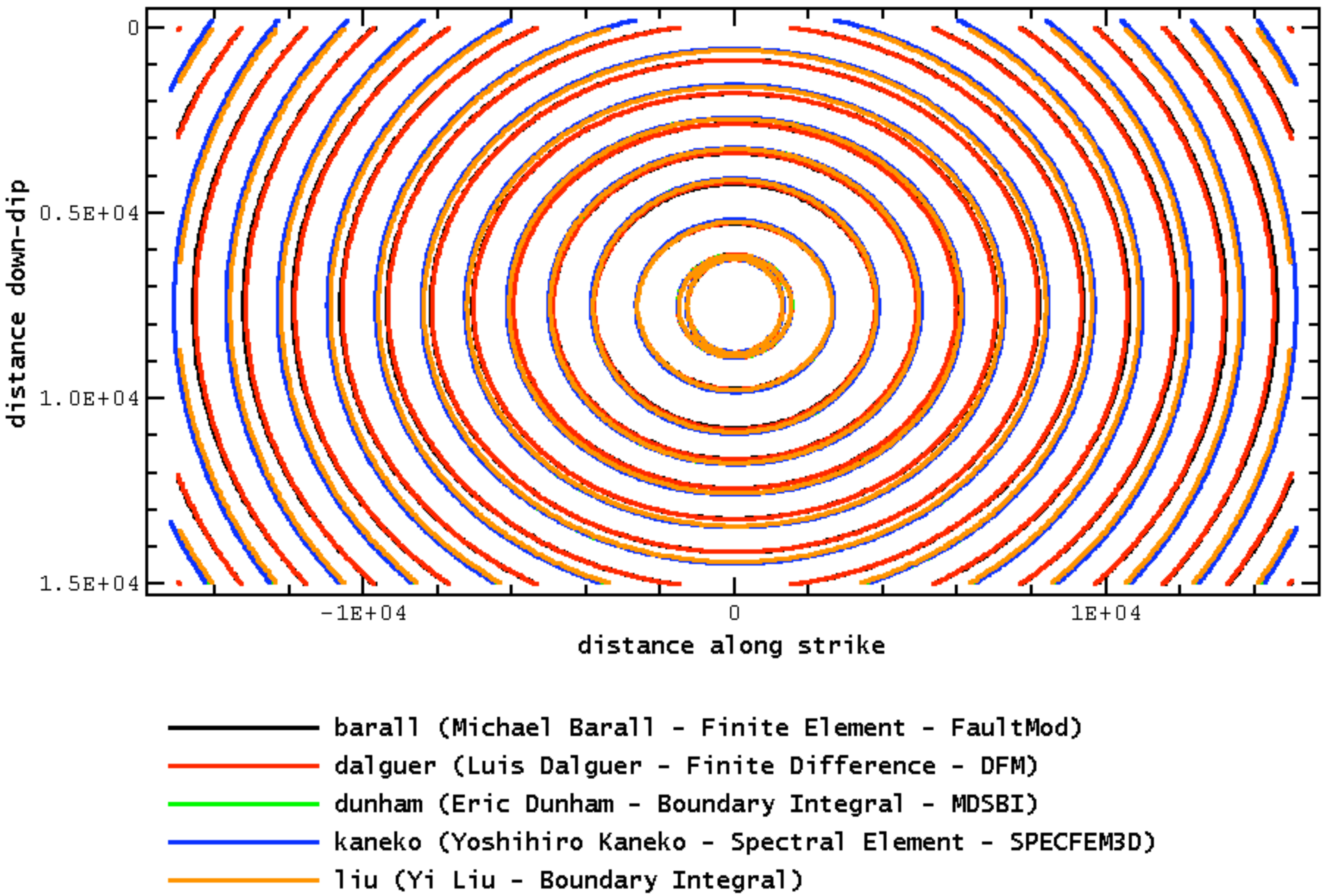
$$\lambda_{crit} \sim \frac{\mu L}{(b-a)\sigma} \sim 1.3 \text{ km for } L = 0.02 \text{ m}$$

use grid spacing $h \ll \lambda_{crit}$ for proper resolution

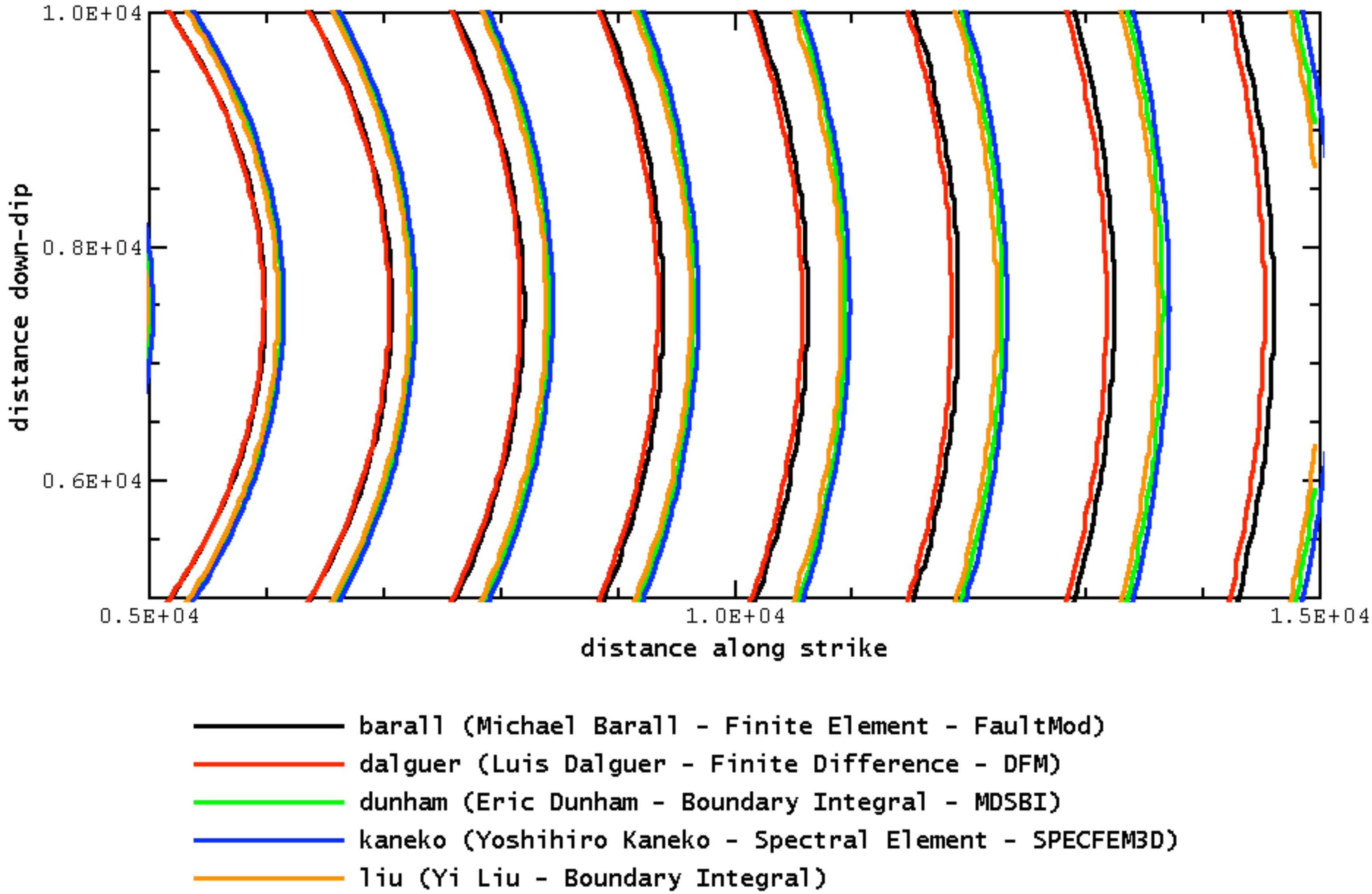
TPV101/102 Fault Stations



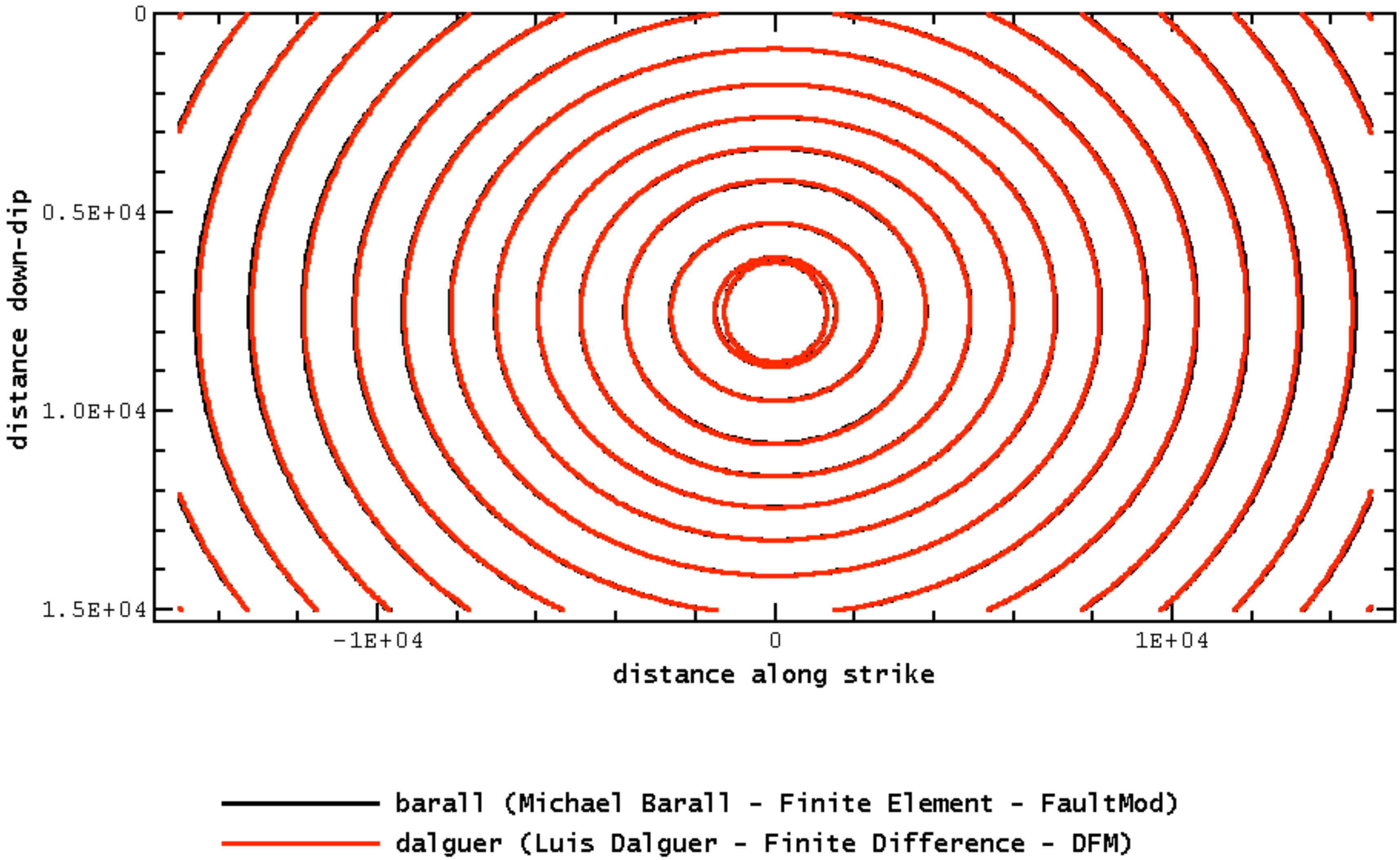
TPV101 Results



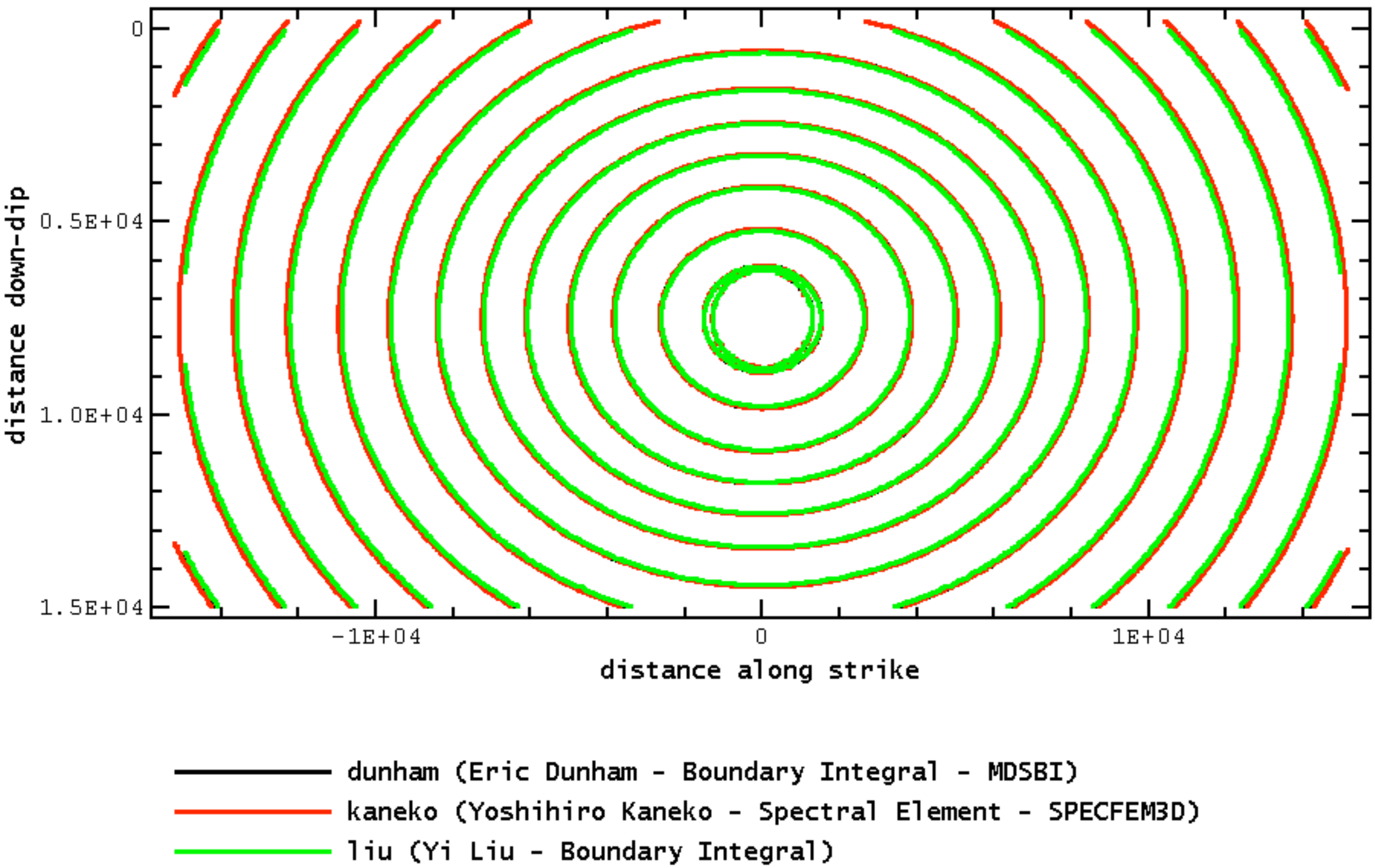
TPV101 Results



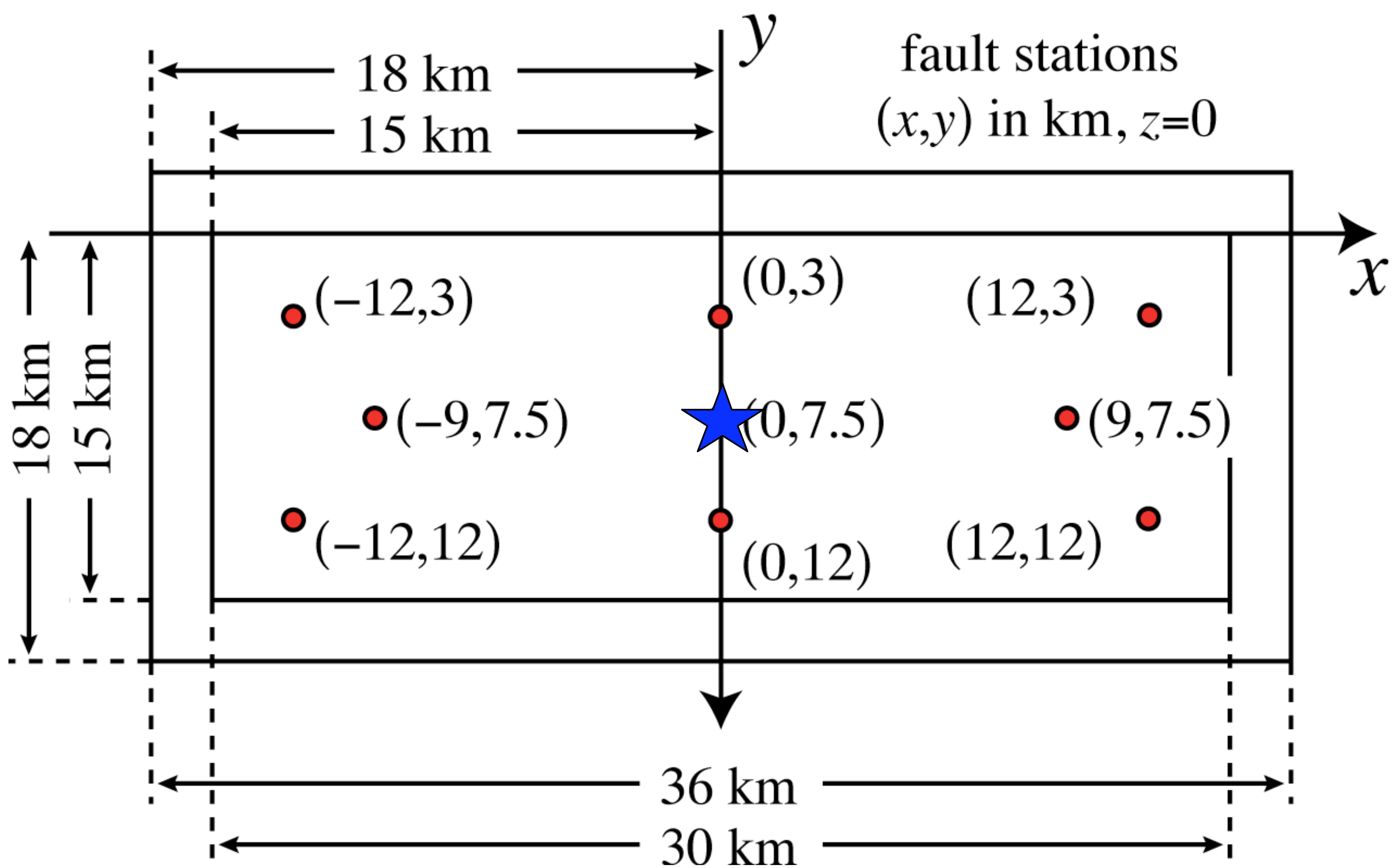
TPV101 Results



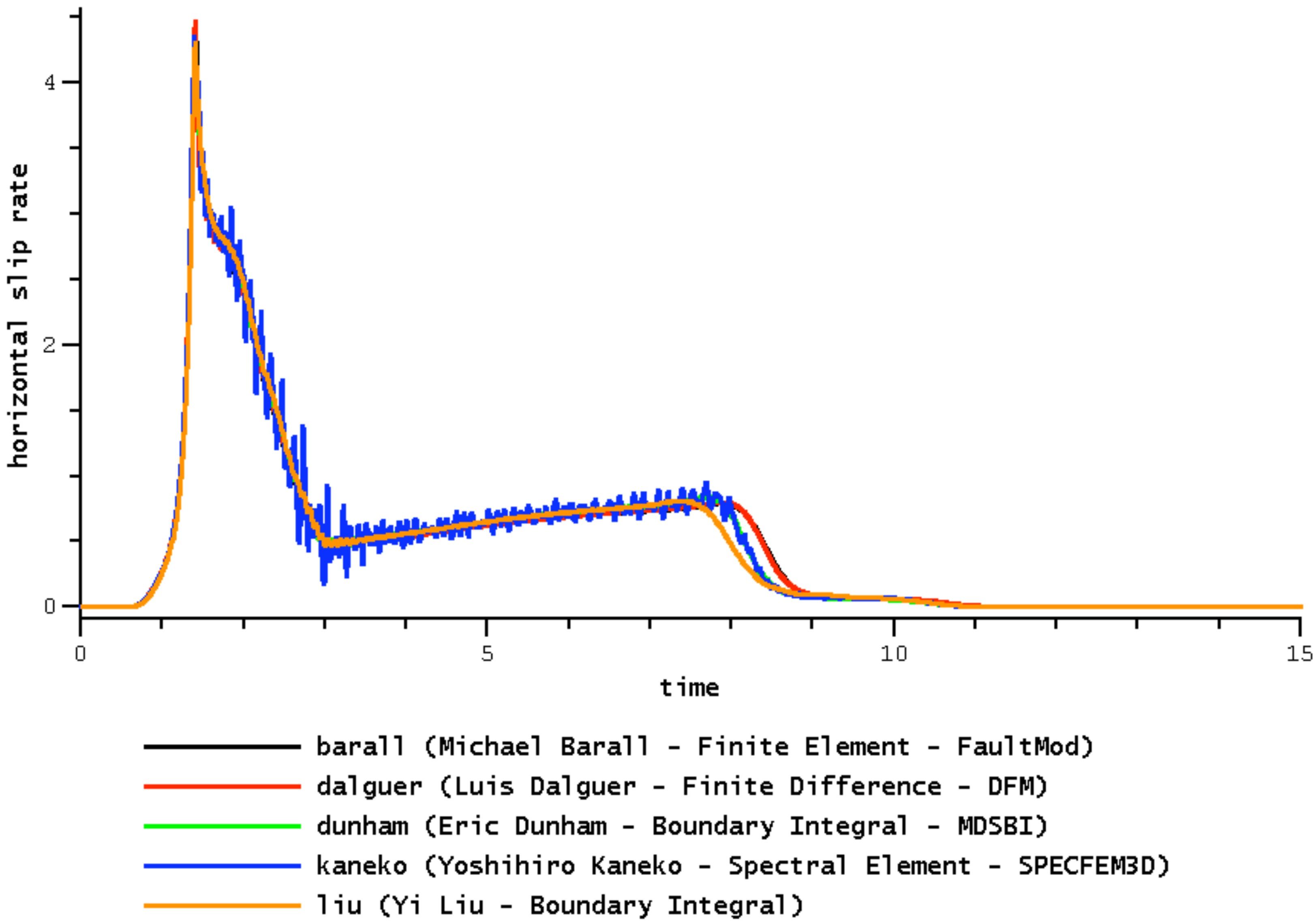
TPV101 Results



TPV101/102 Fault Stations



TPV101, x=0, y=7.5 km



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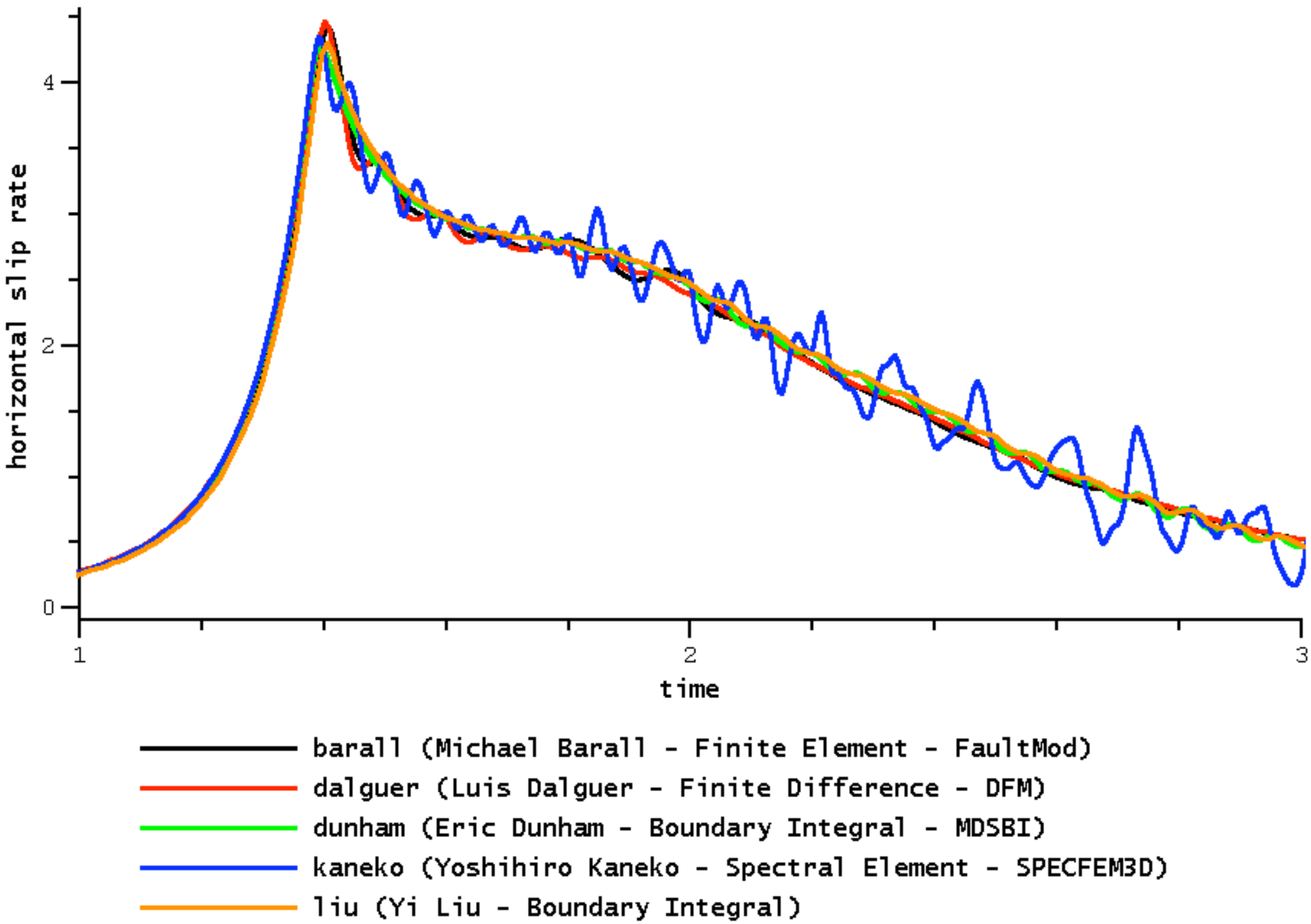
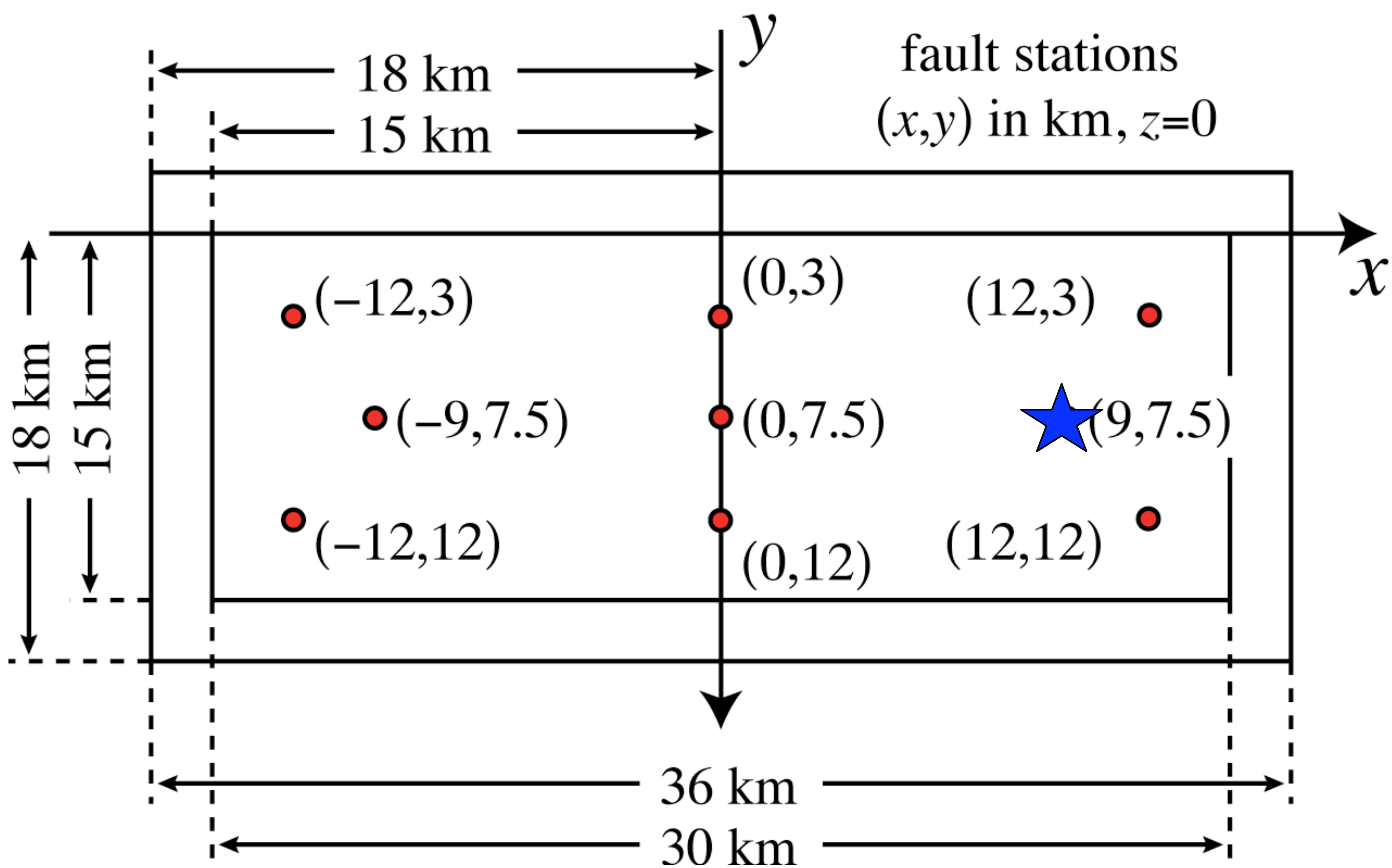
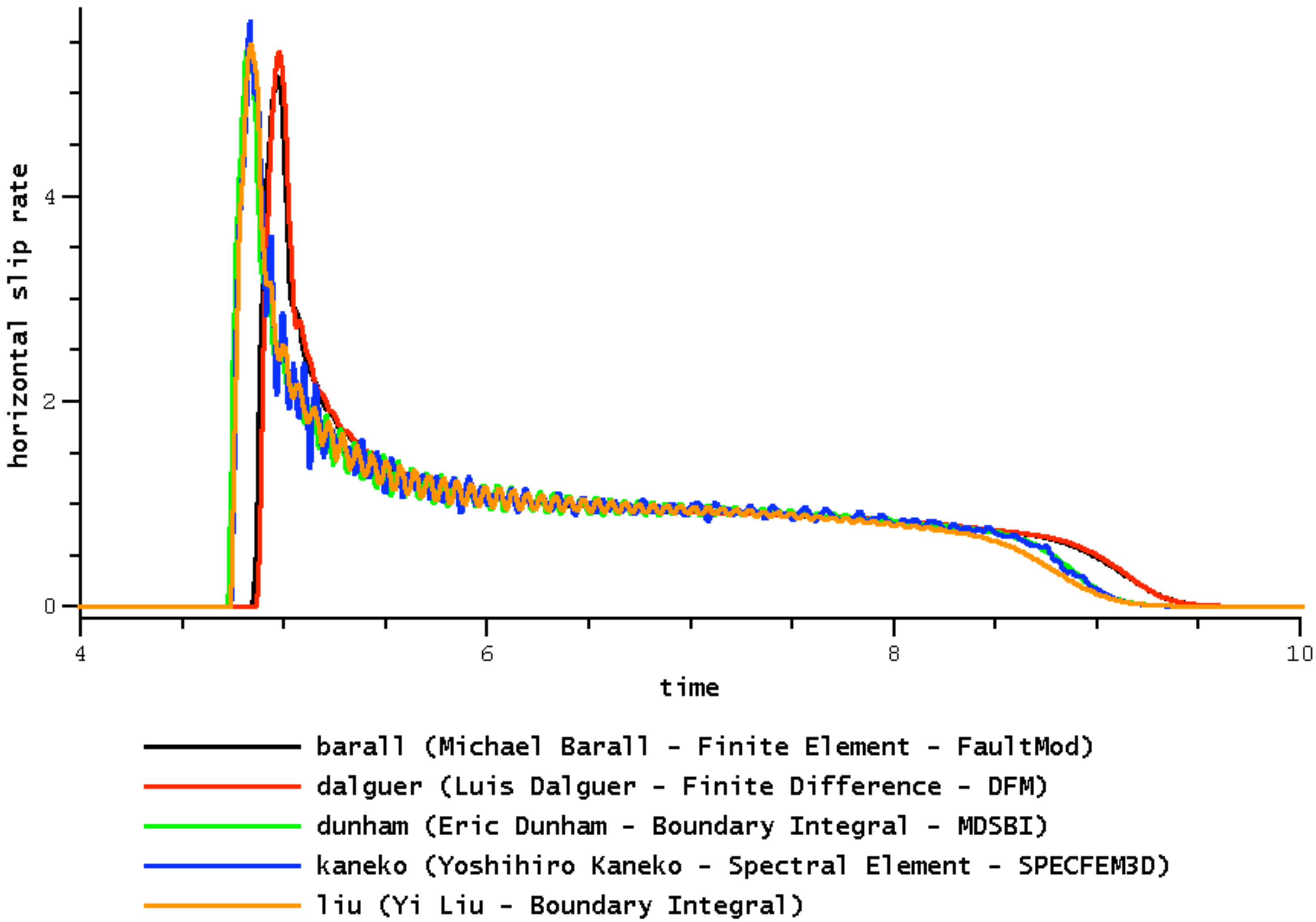


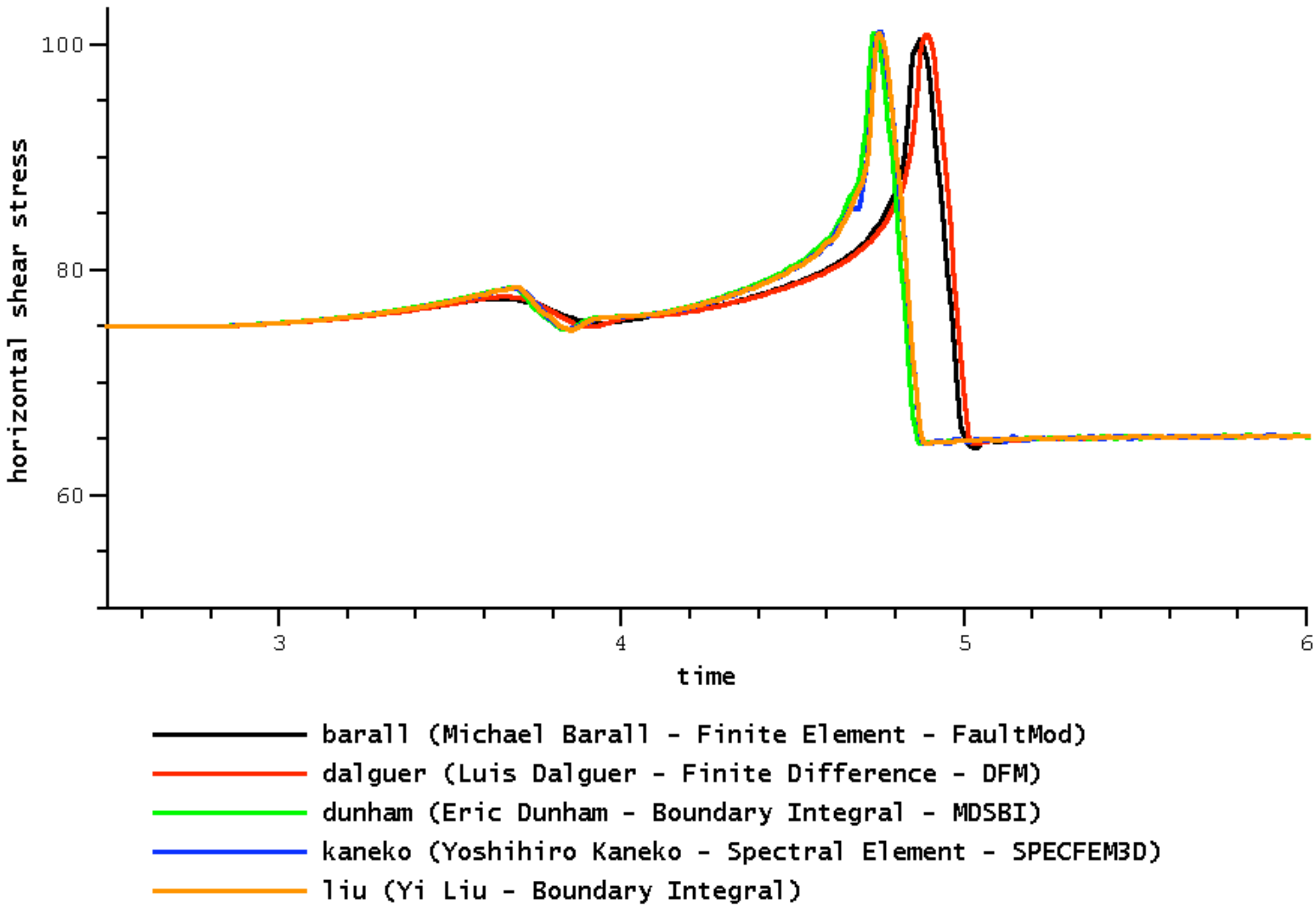
TPV101/102 Fault Stations



TPV101, x=9 km, y=7.5 km



TPV101, x=9 km, y=7.5 km



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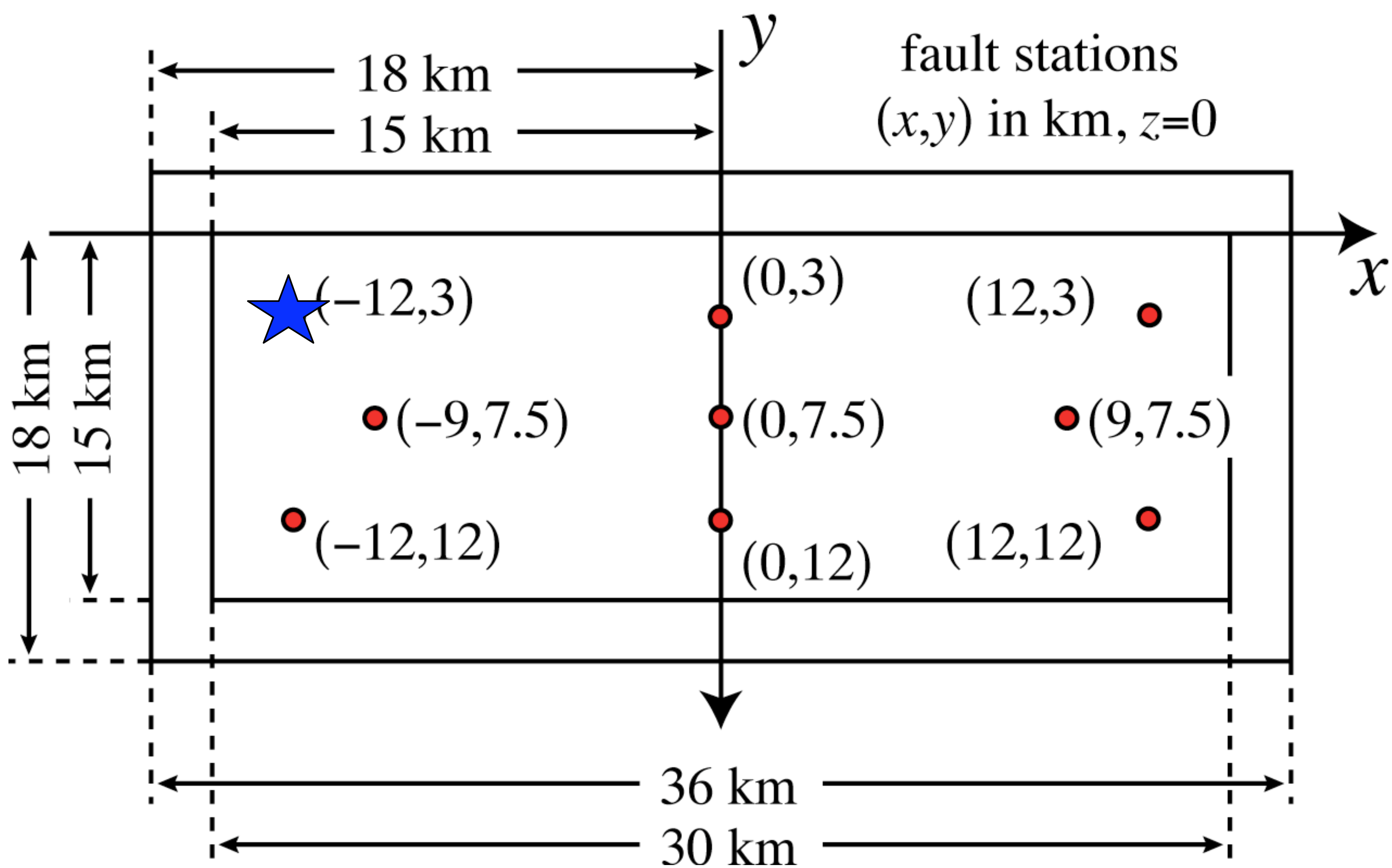
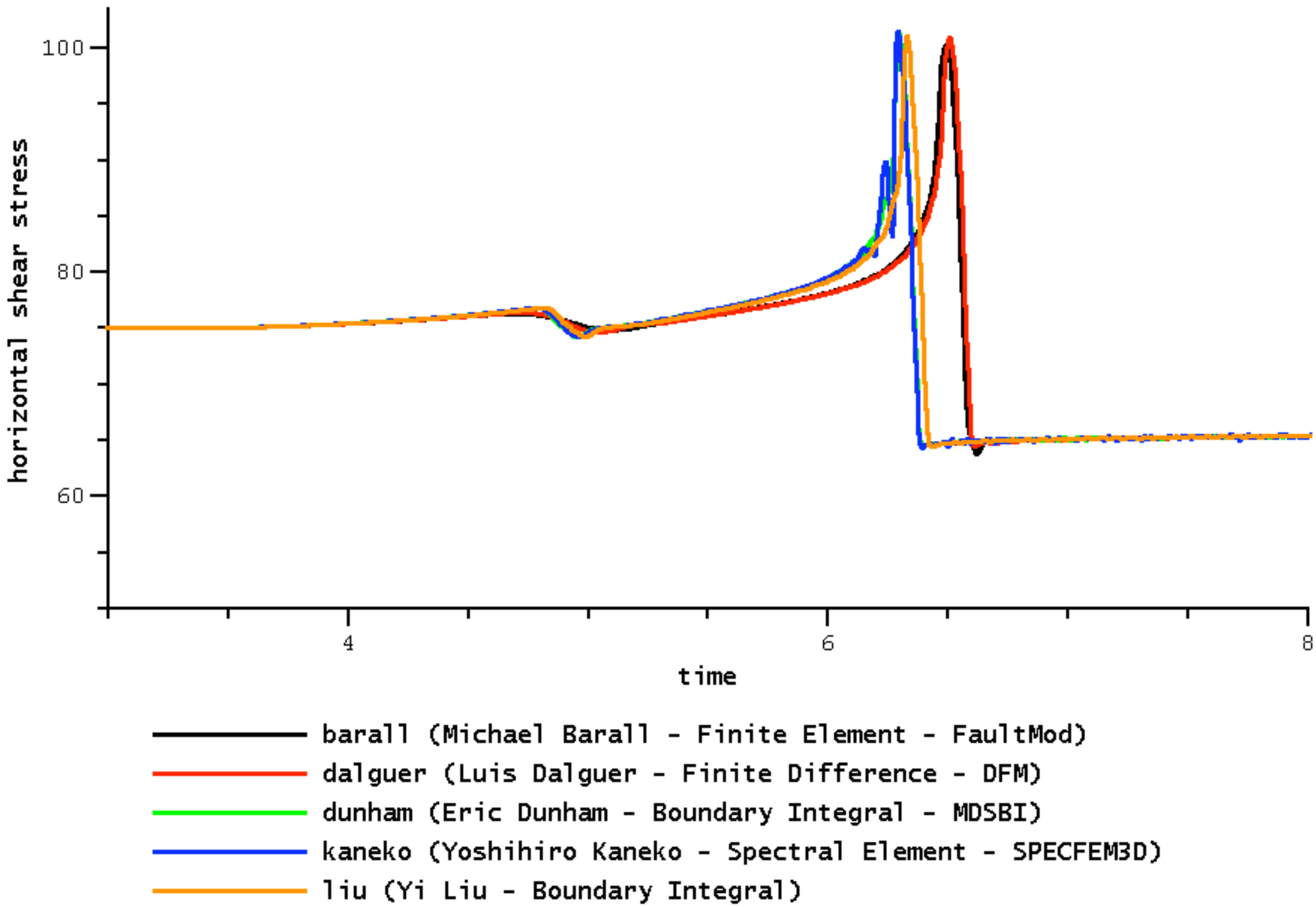
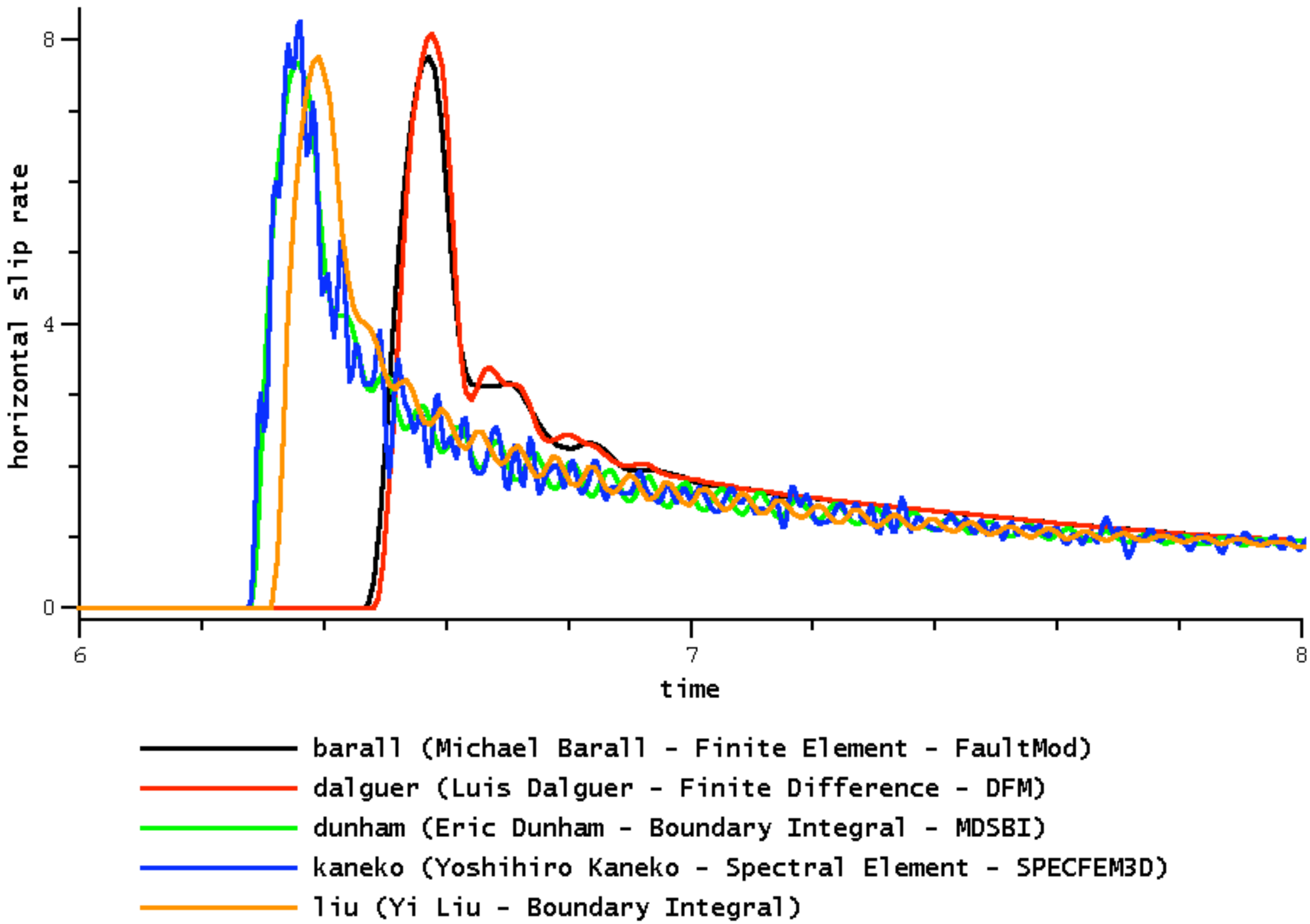


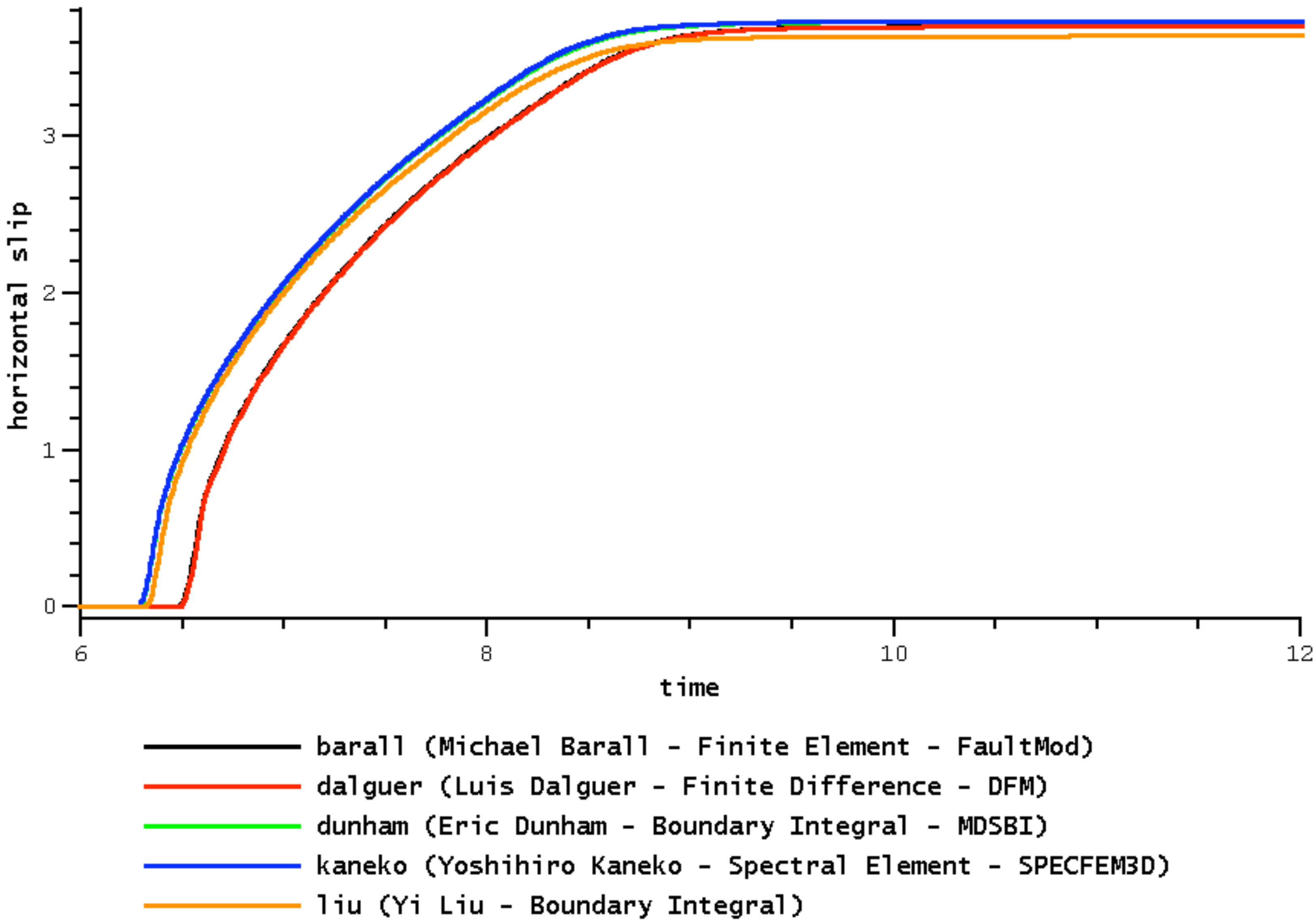
TPV101, x=-12 km, y=3 km



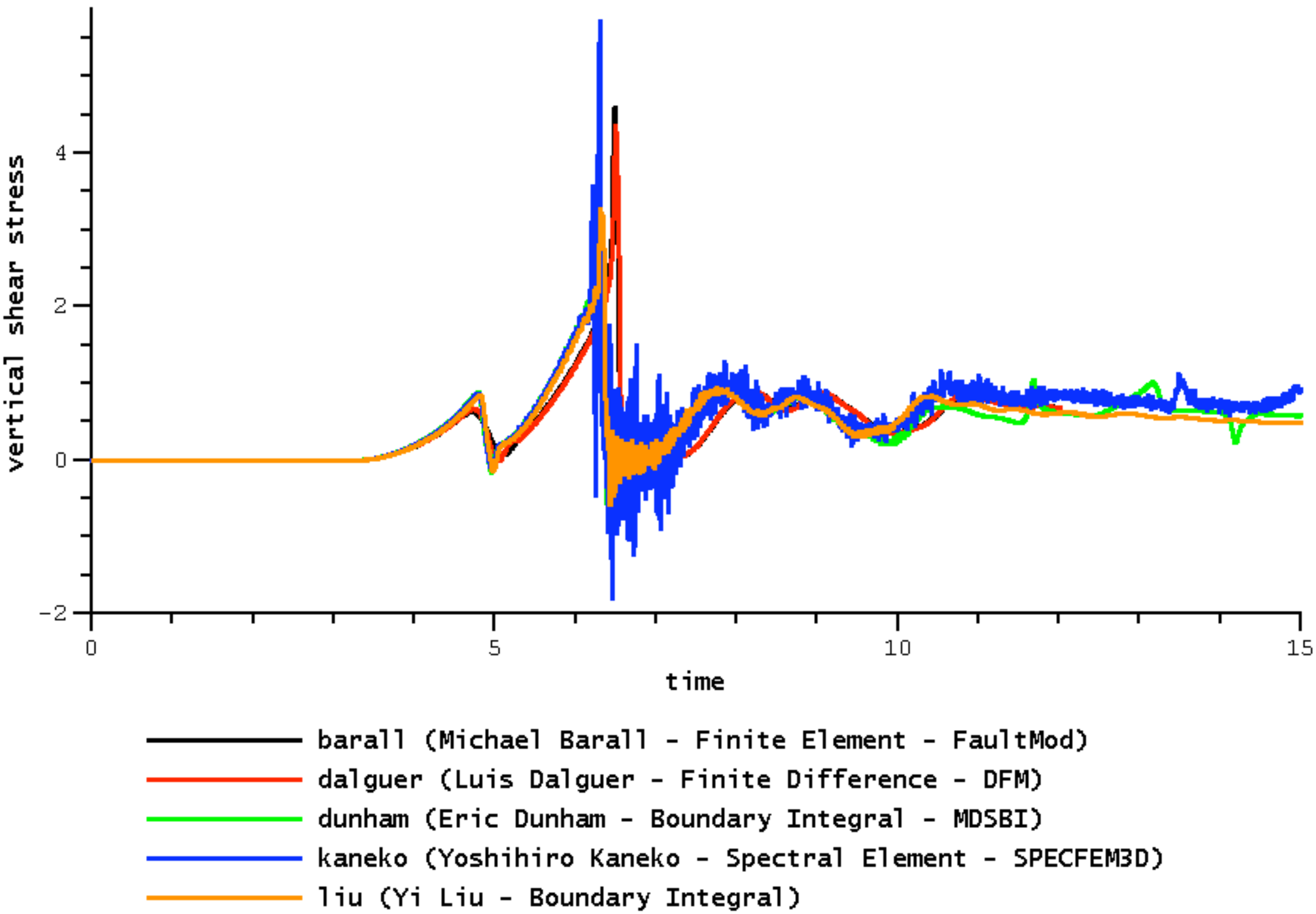
TPV101, x=-12 km, y=3 km



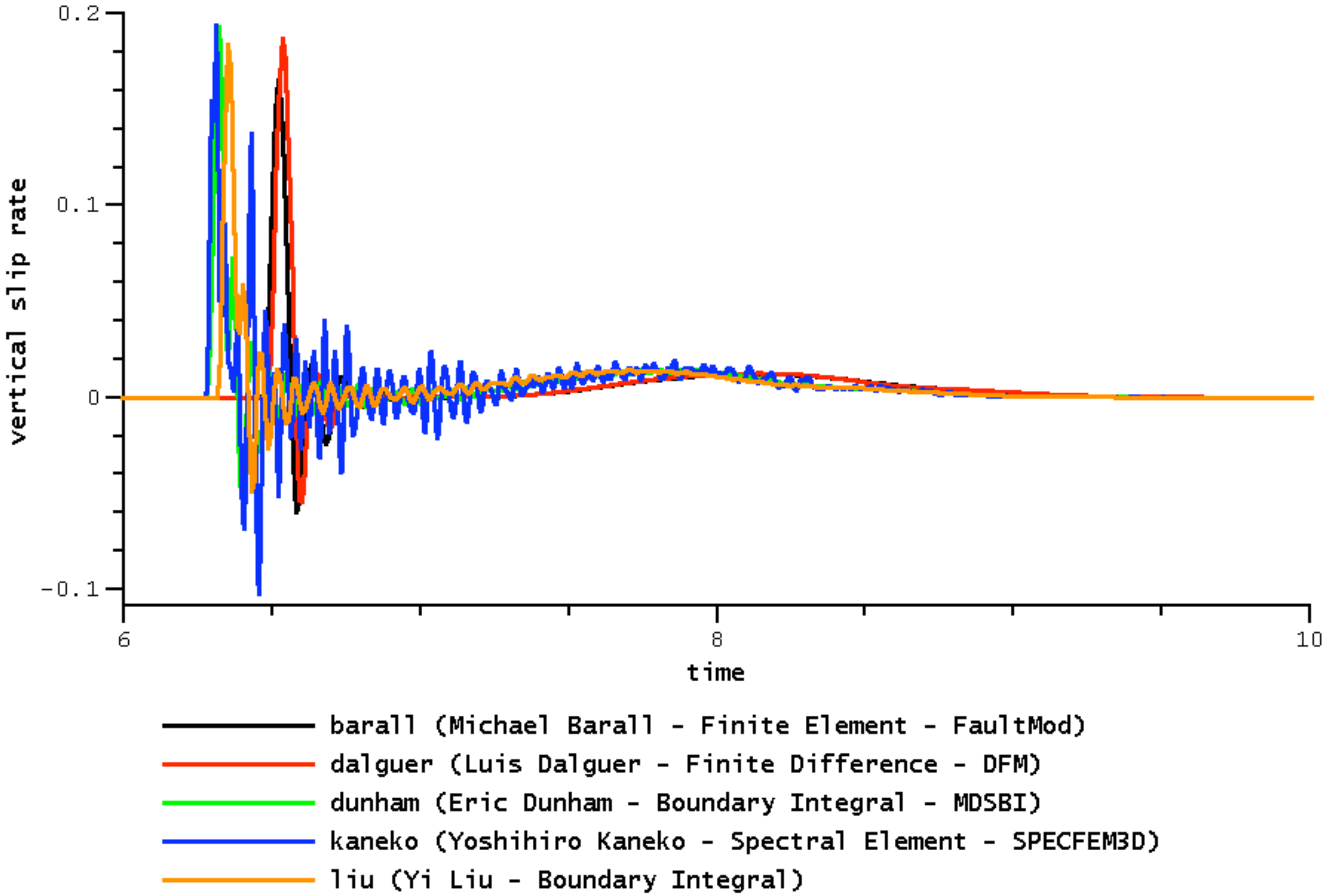
TPV101, x=-12 km, y=3 km



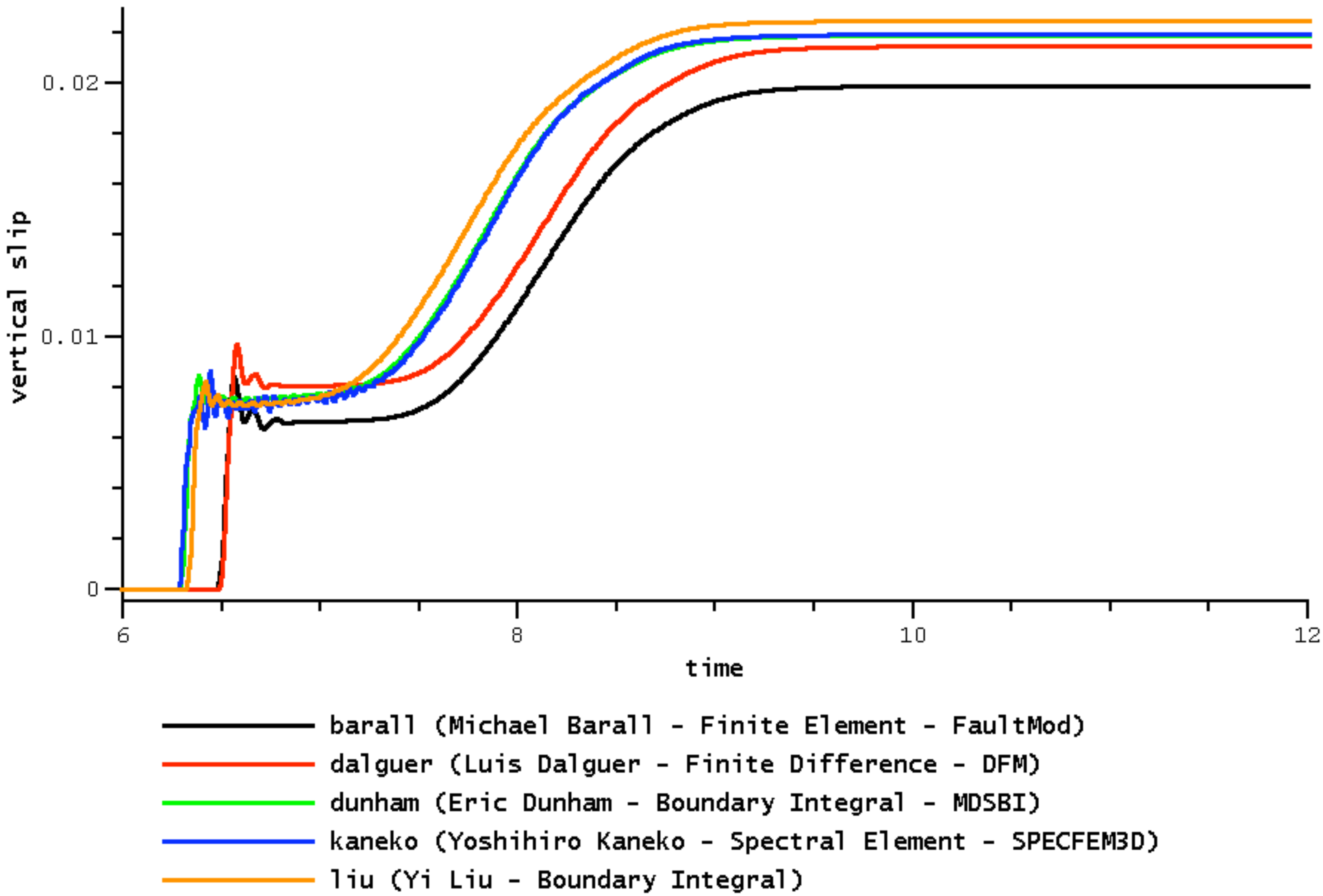
TPV101, x=-12 km, y=3 km



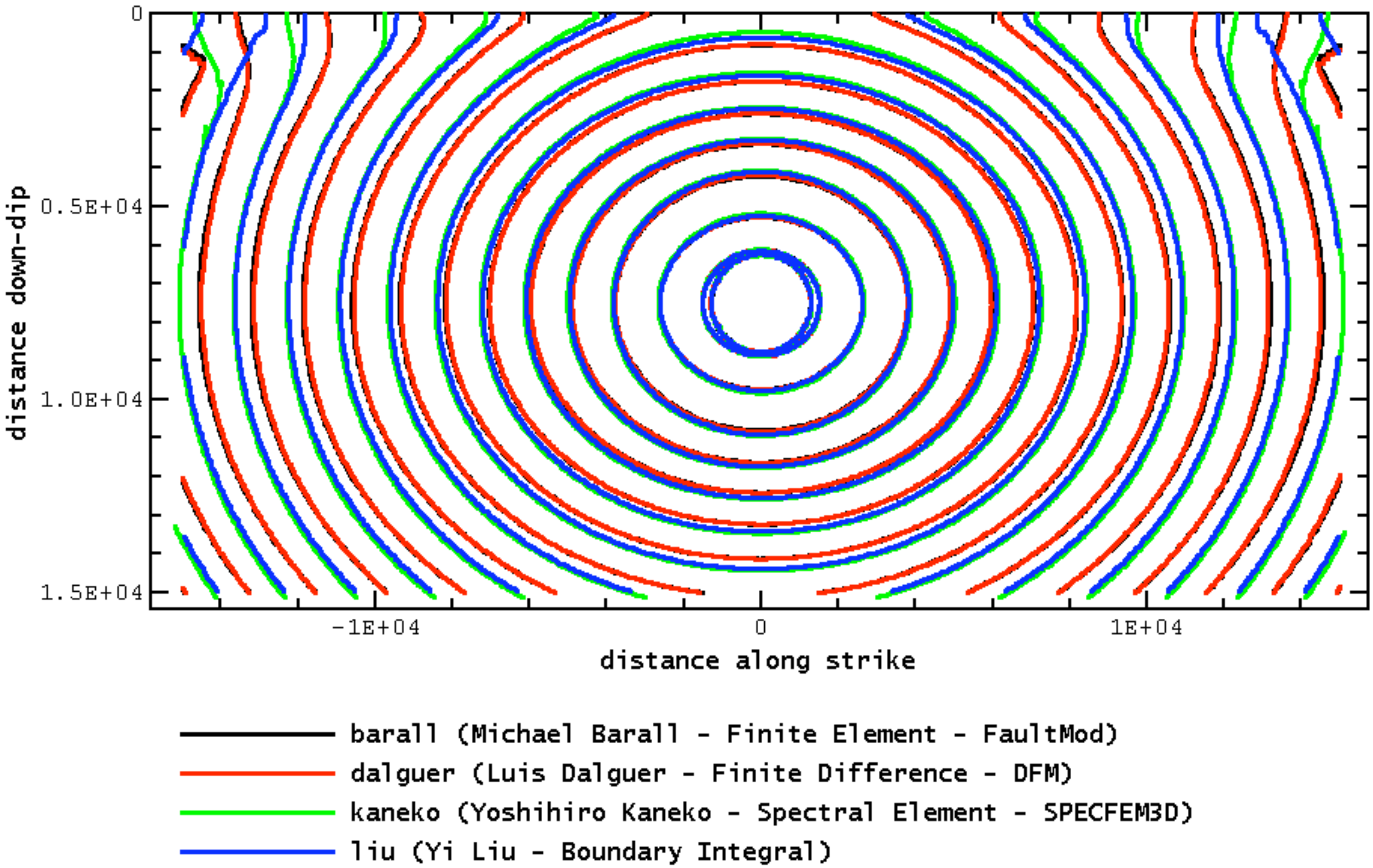
TPV101, *x*=-12 km, *y*=3 km



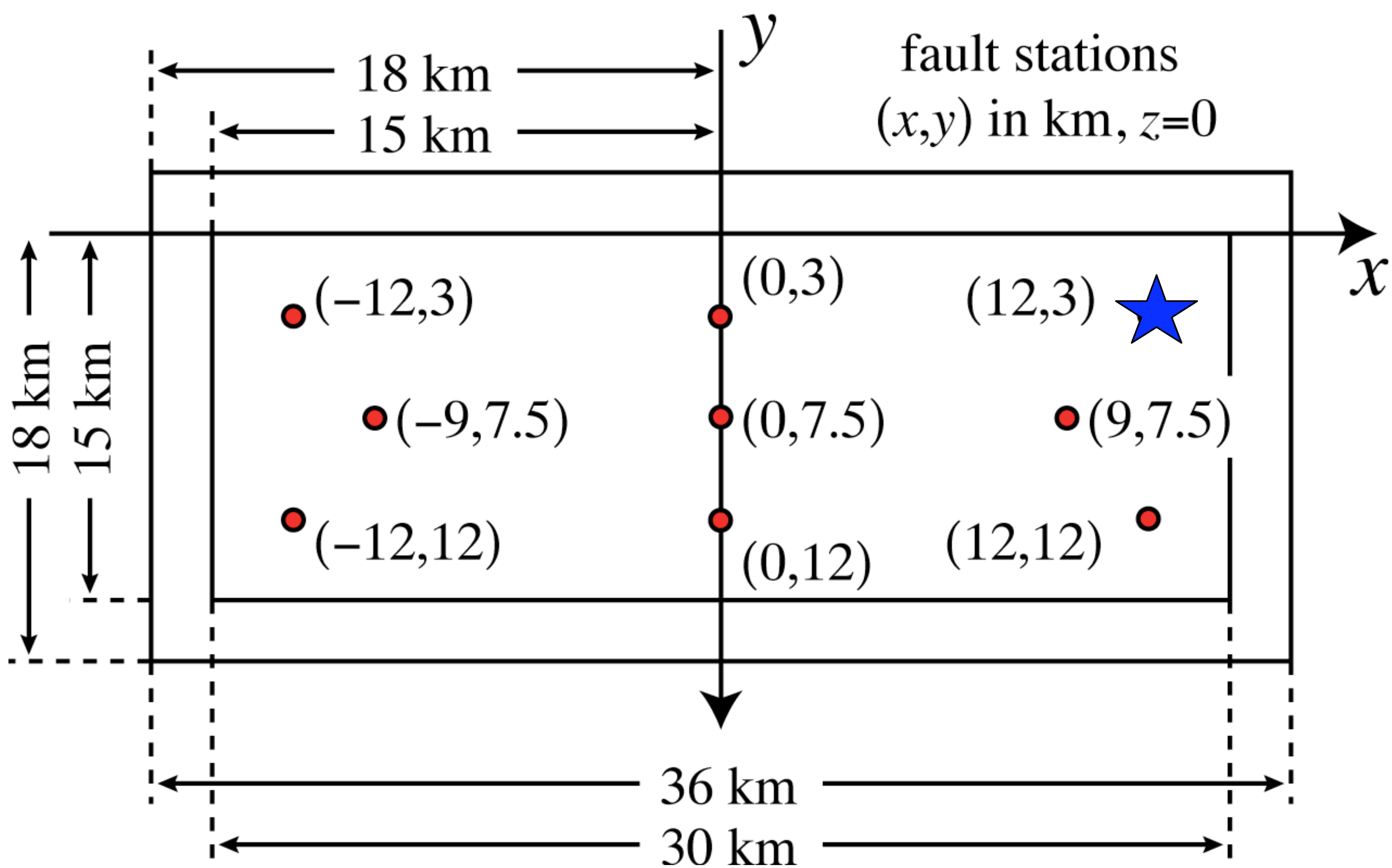
TPV101, x=-12 km, y=3 km



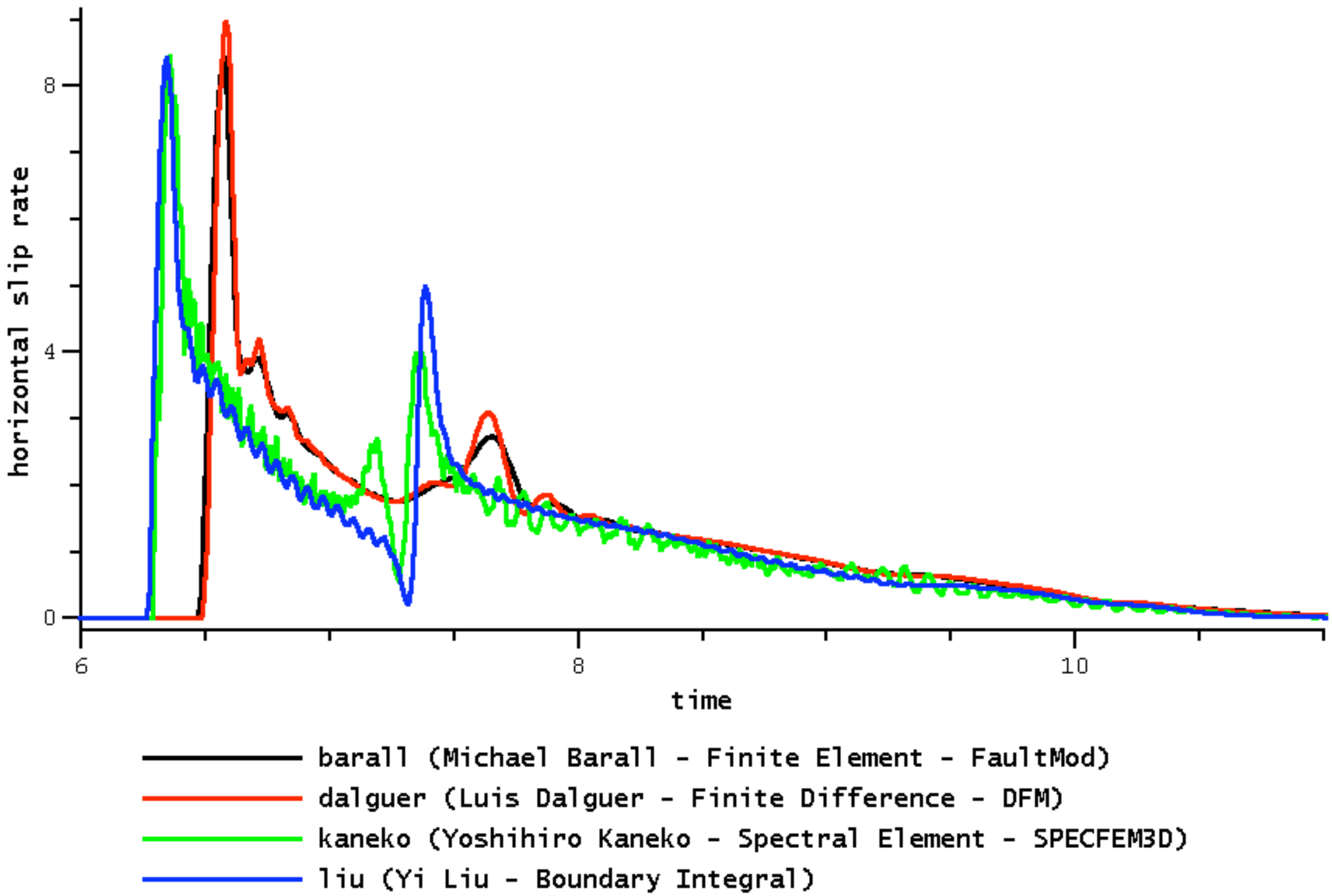
TPV102 Results



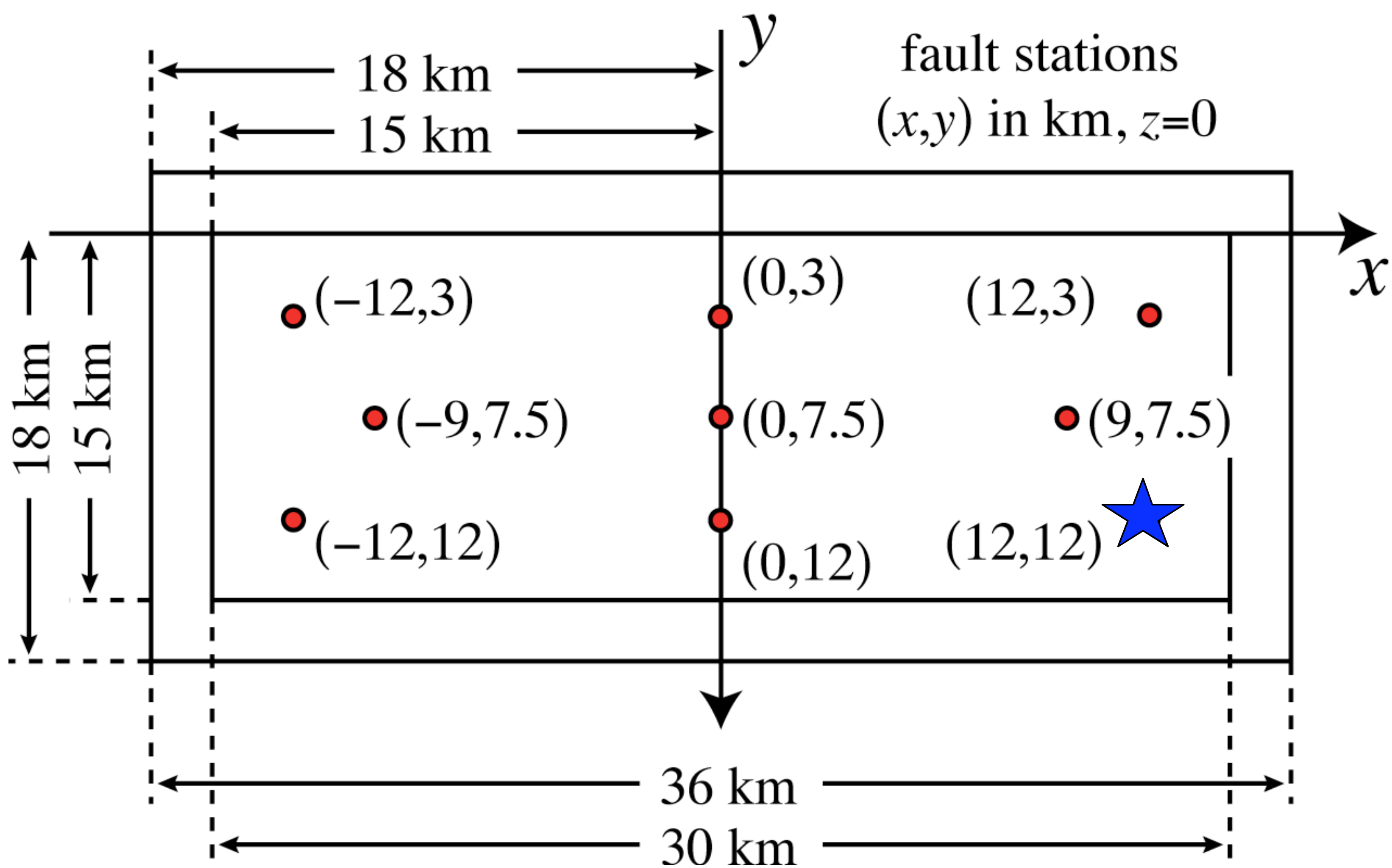
TPV101/102 Fault Stations



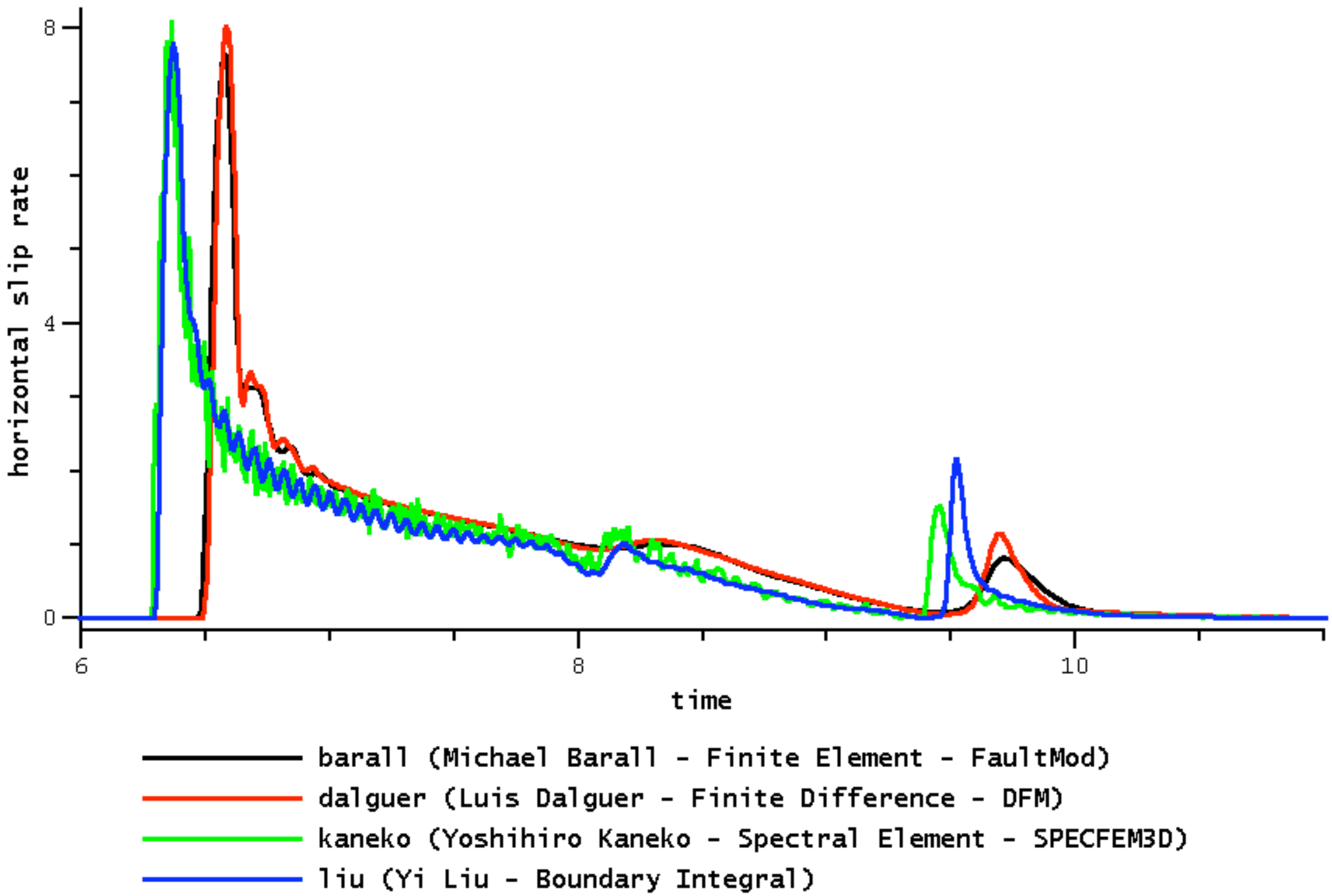
TPV102, x=12 km, y=3 km



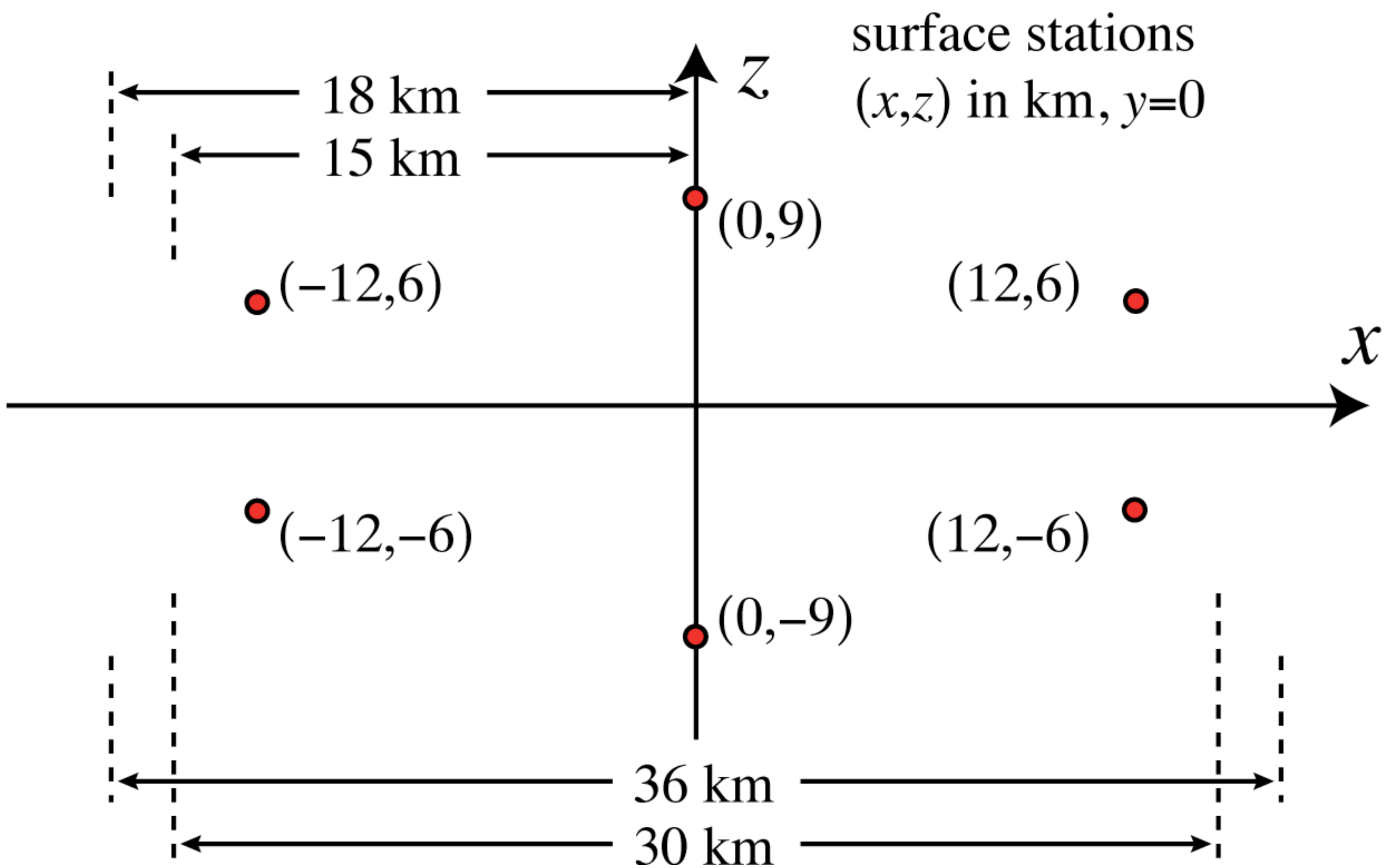
TPV101/102 Fault Stations



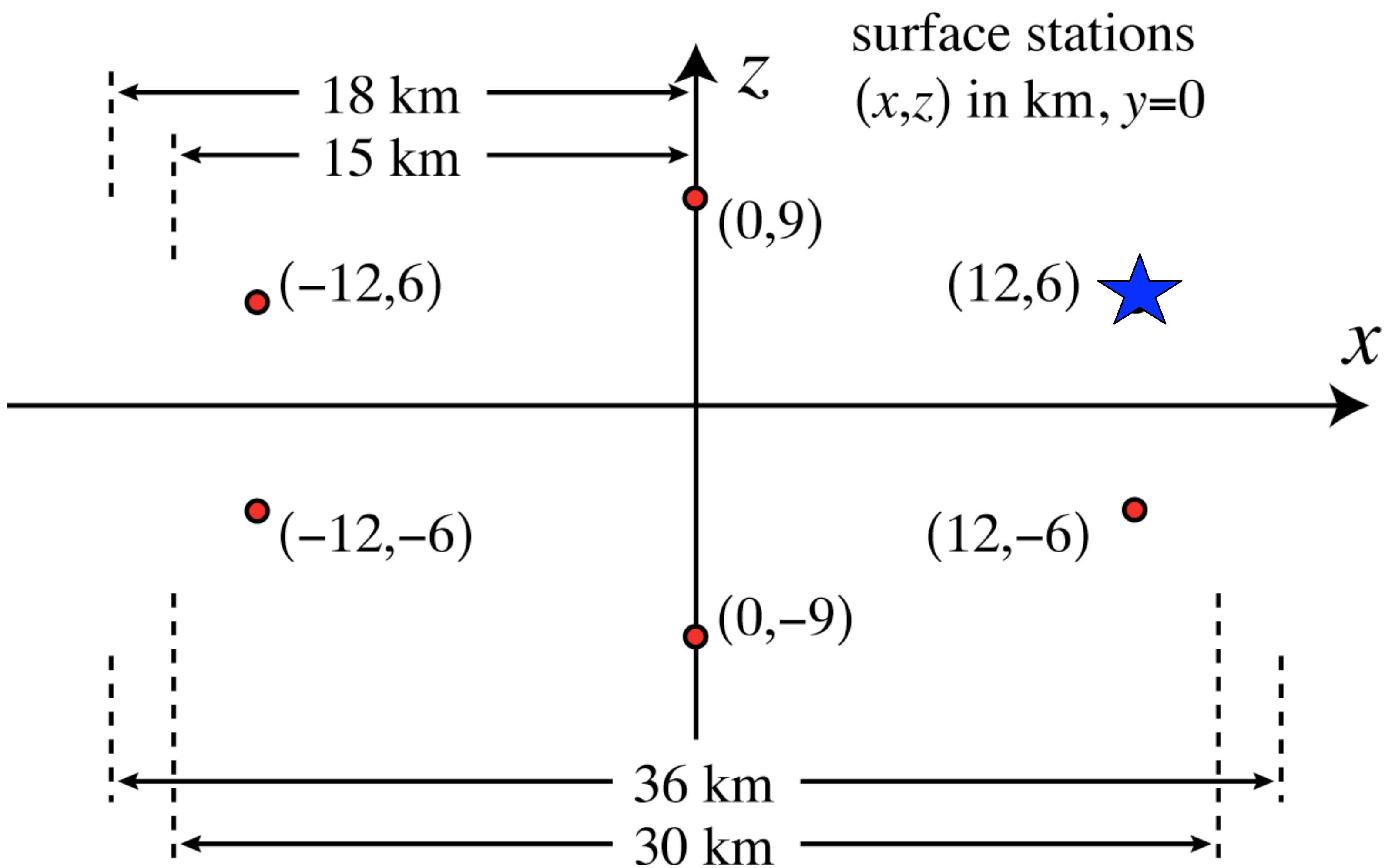
TPV102, x=12 km, y=12 km



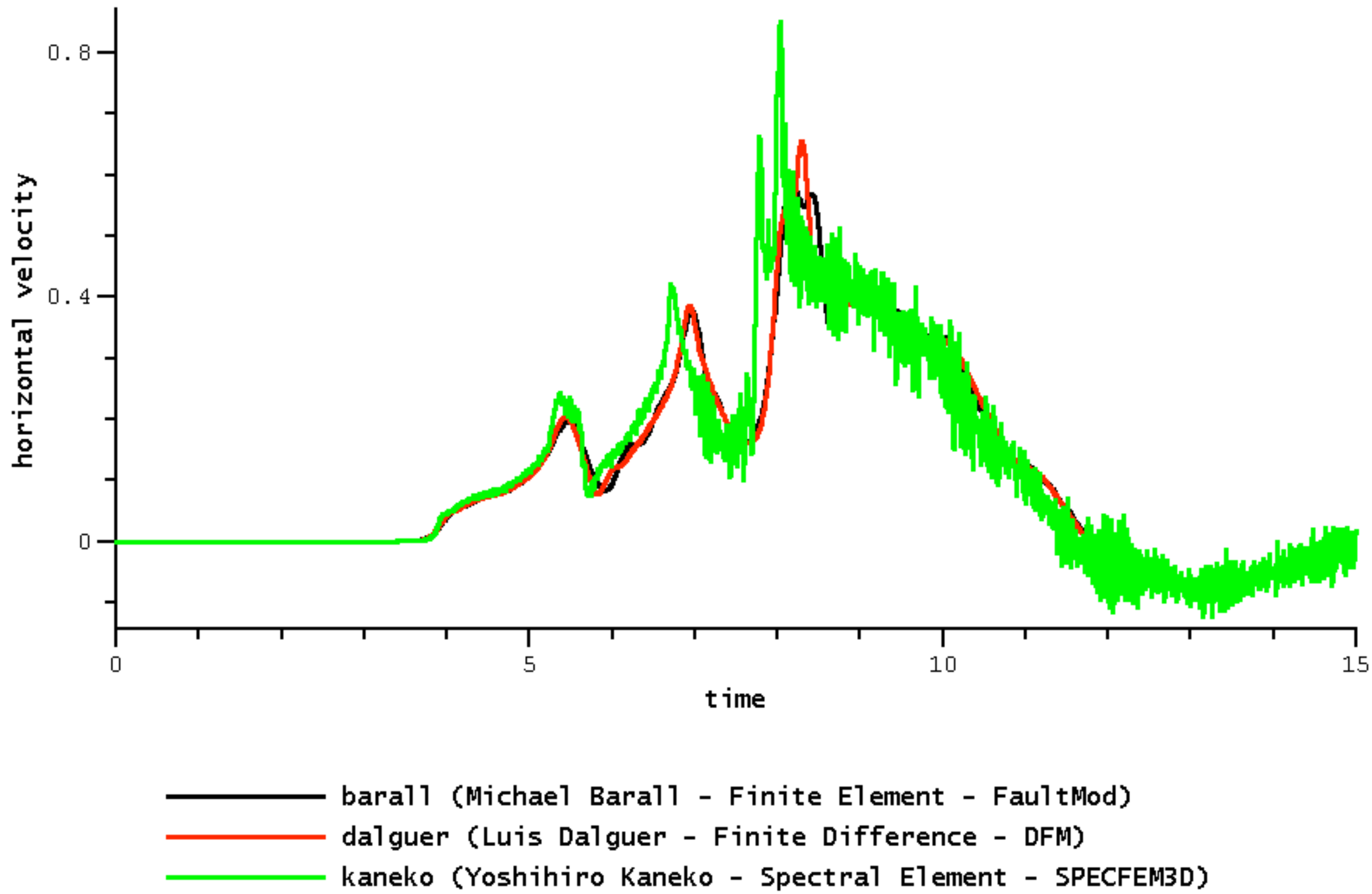
TPV102 Surface Stations



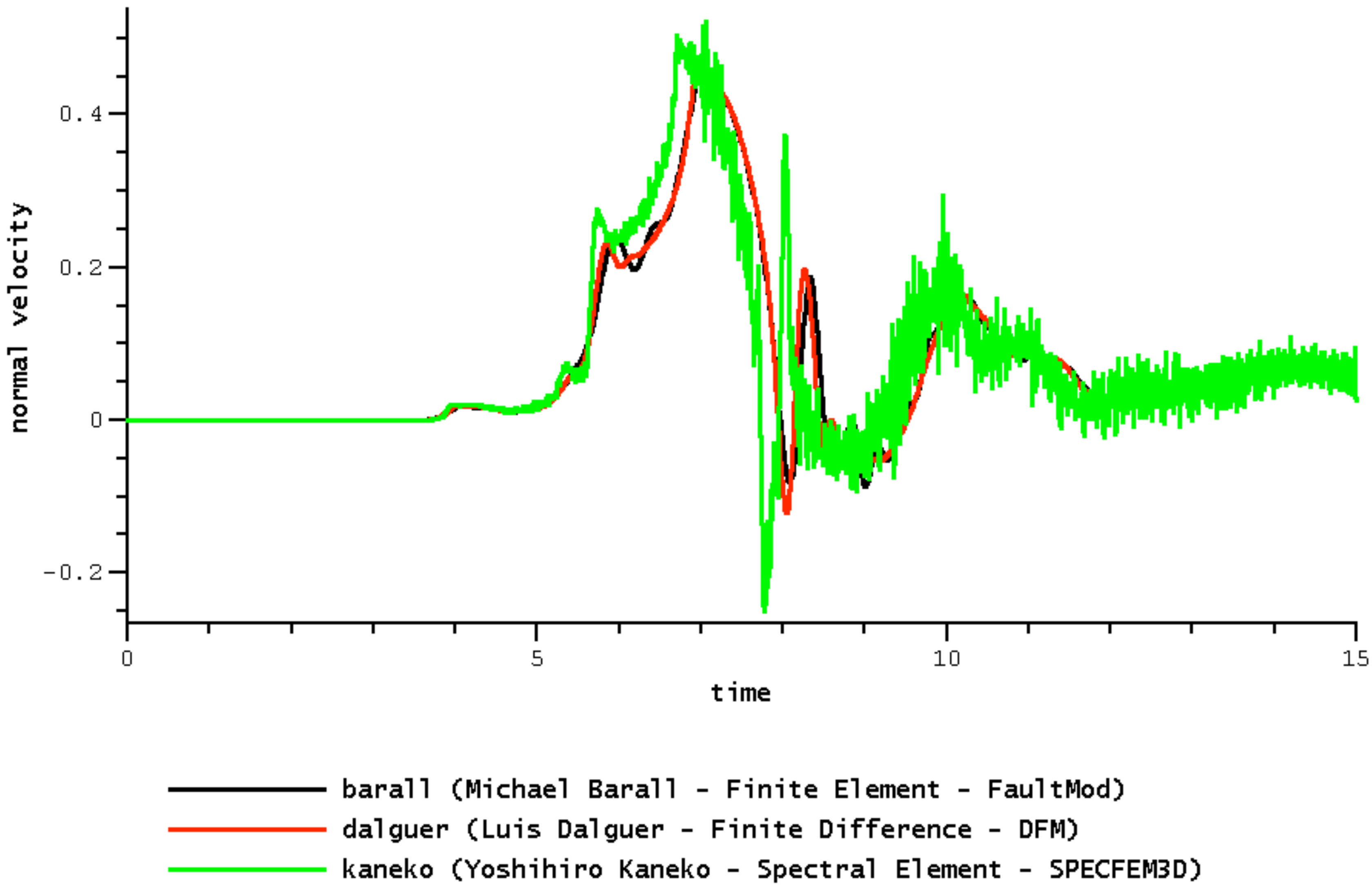
TPV102 Surface Stations



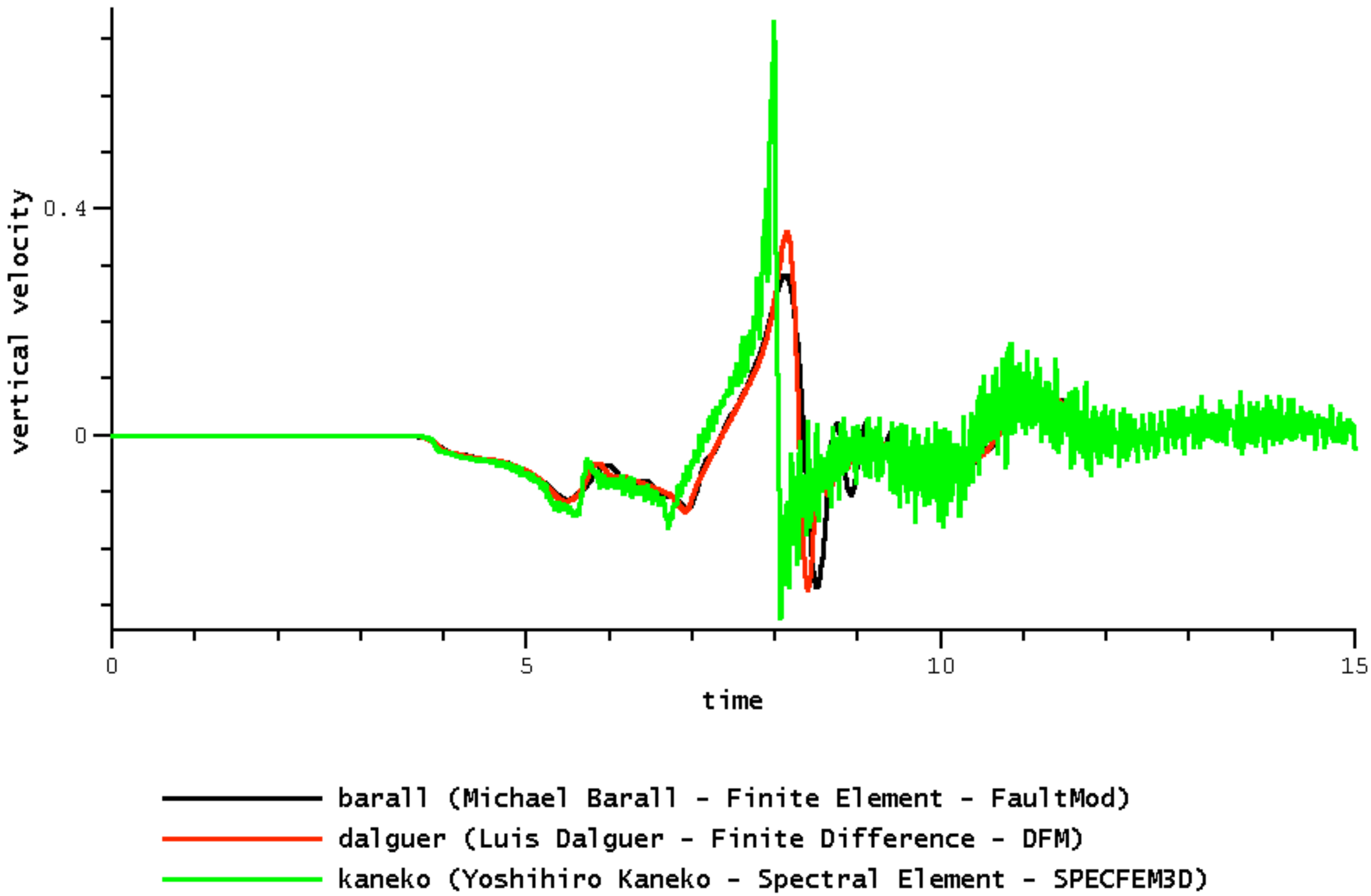
TPV102, x=12 km, z=6 km



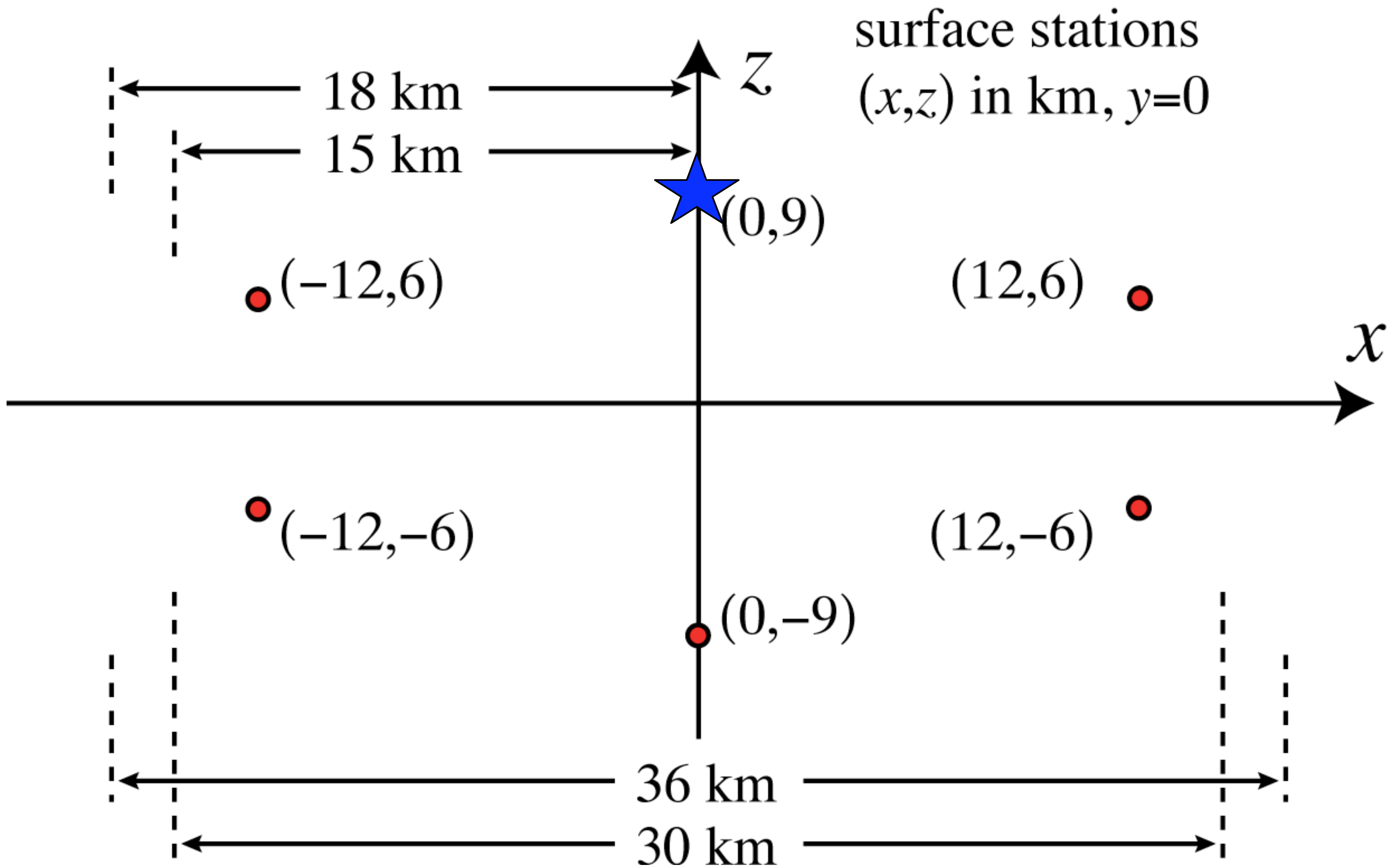
TPV102, x=12 km, z=6 km



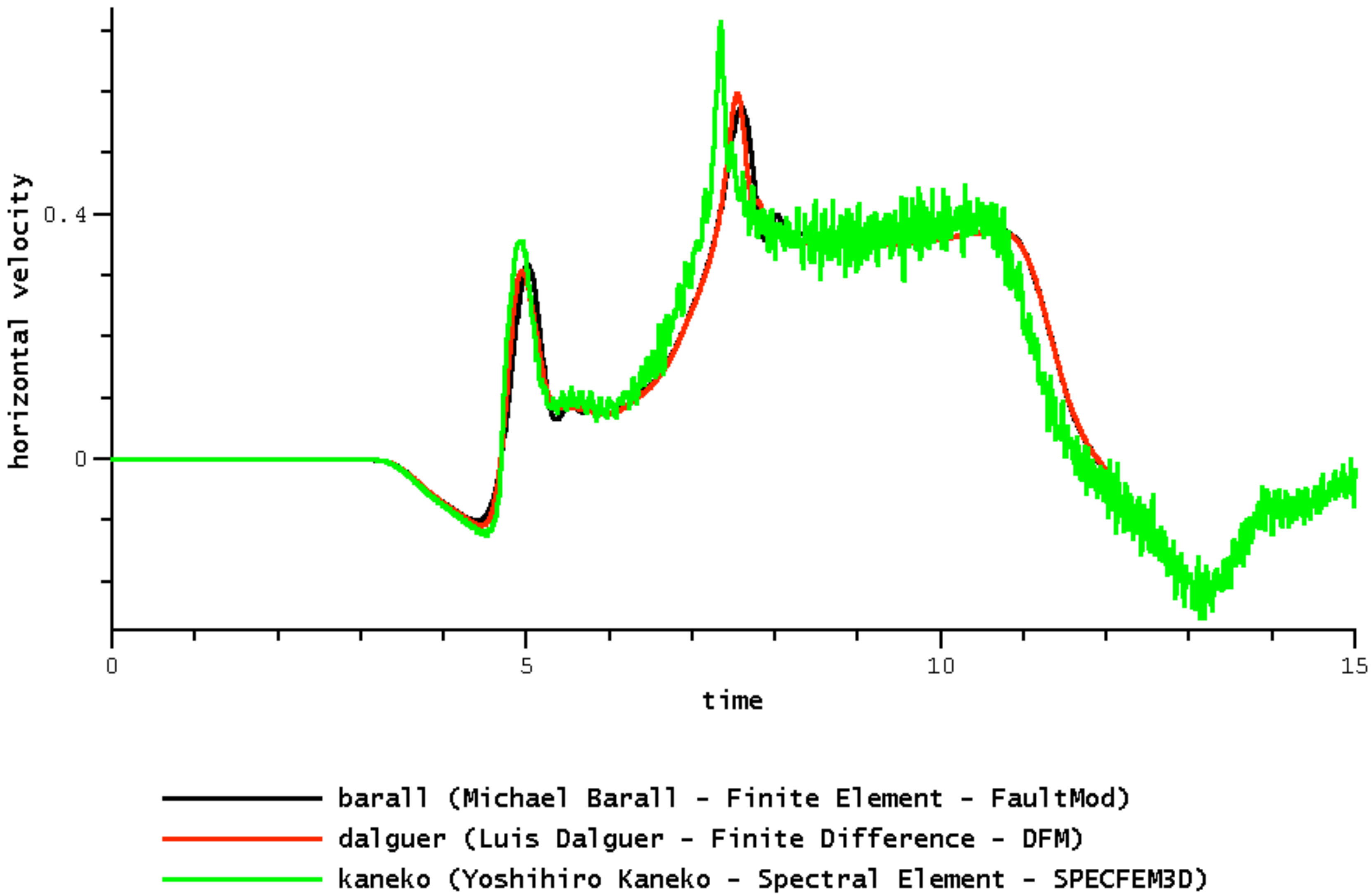
TPV102, x=12 km, z=6 km



TPV102 Surface Stations



TPV102, x=0, z=9 km



Conclusions

- good agreement between codes!
- minor differences between FD/FE codes and BIE / spectral FE codes—why?

What's Next?

- slip law
- extreme velocity weakening (slip pulses)

$$f = \frac{f_0 + a \ln \dfrac{V}{V_0}}{1 - \dfrac{b}{f_0} \ln \dfrac{V_0 \theta}{L} + \dfrac{L}{\theta V_w}}$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L} \qquad \text{(ageing law)}$$

$$f = f_0 + a \ln \frac{V}{V_0} + \psi$$

$$\frac{d\psi}{dt} = -\frac{V}{L}\left[\mathrm{f} - \mathrm{f}_{\mathrm{ss}}(V)\right] \qquad \text{(slip law)}$$