Numerical convergence: Implications for results

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3D Rupture Dynamics Code Validation Workshop
March 10, 2008

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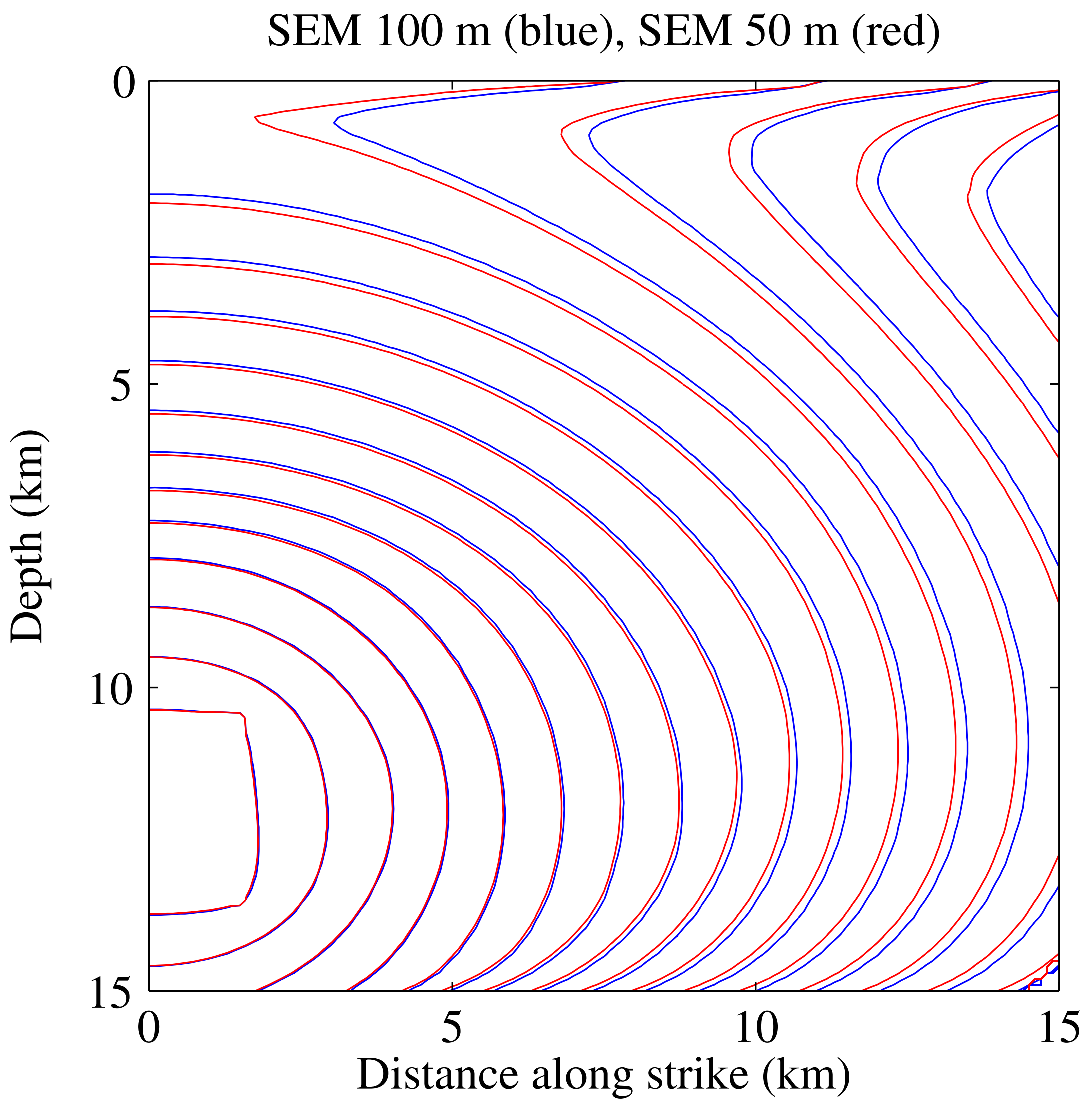
Outline:

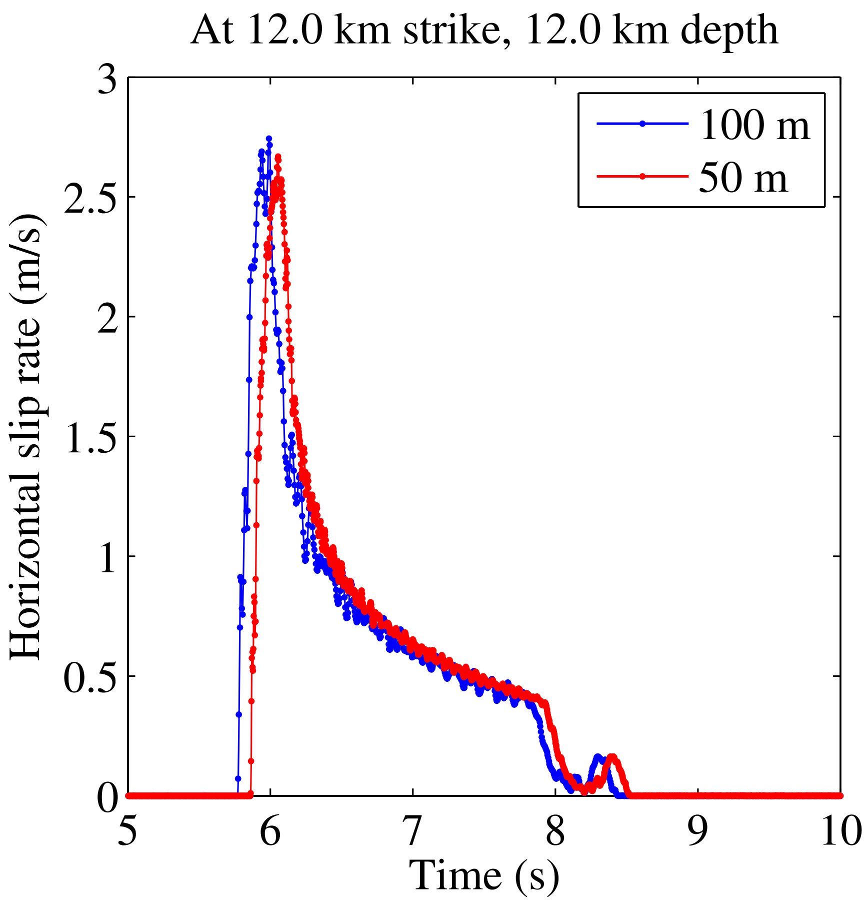
- Part I: Numerical convergence in TPV8
- Part II: Convergence of SEM and BIM solutions with grid reduction in an anti-plane (2D) rate-state problem

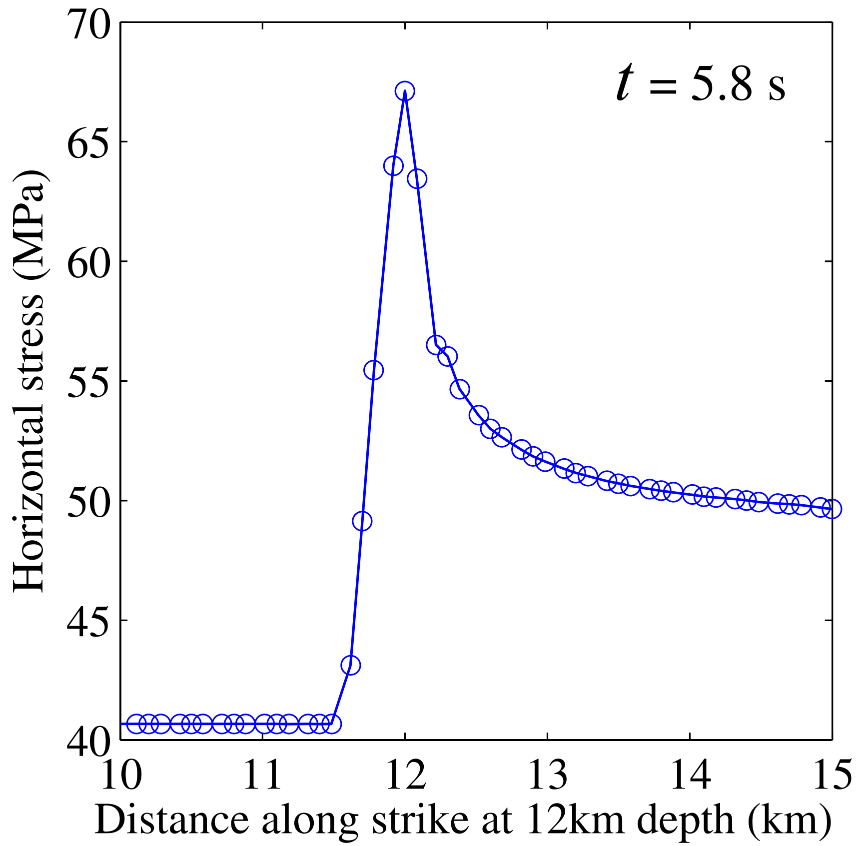
Comparison between 100-m and 50-m cases in TPV8



- Rupture contours of the 100-m and 50-m cases differ by $\lesssim$0.1 s.
- Rupture speed in the 50-m case is slower than the 100-m case.

Cohesive zone resolution





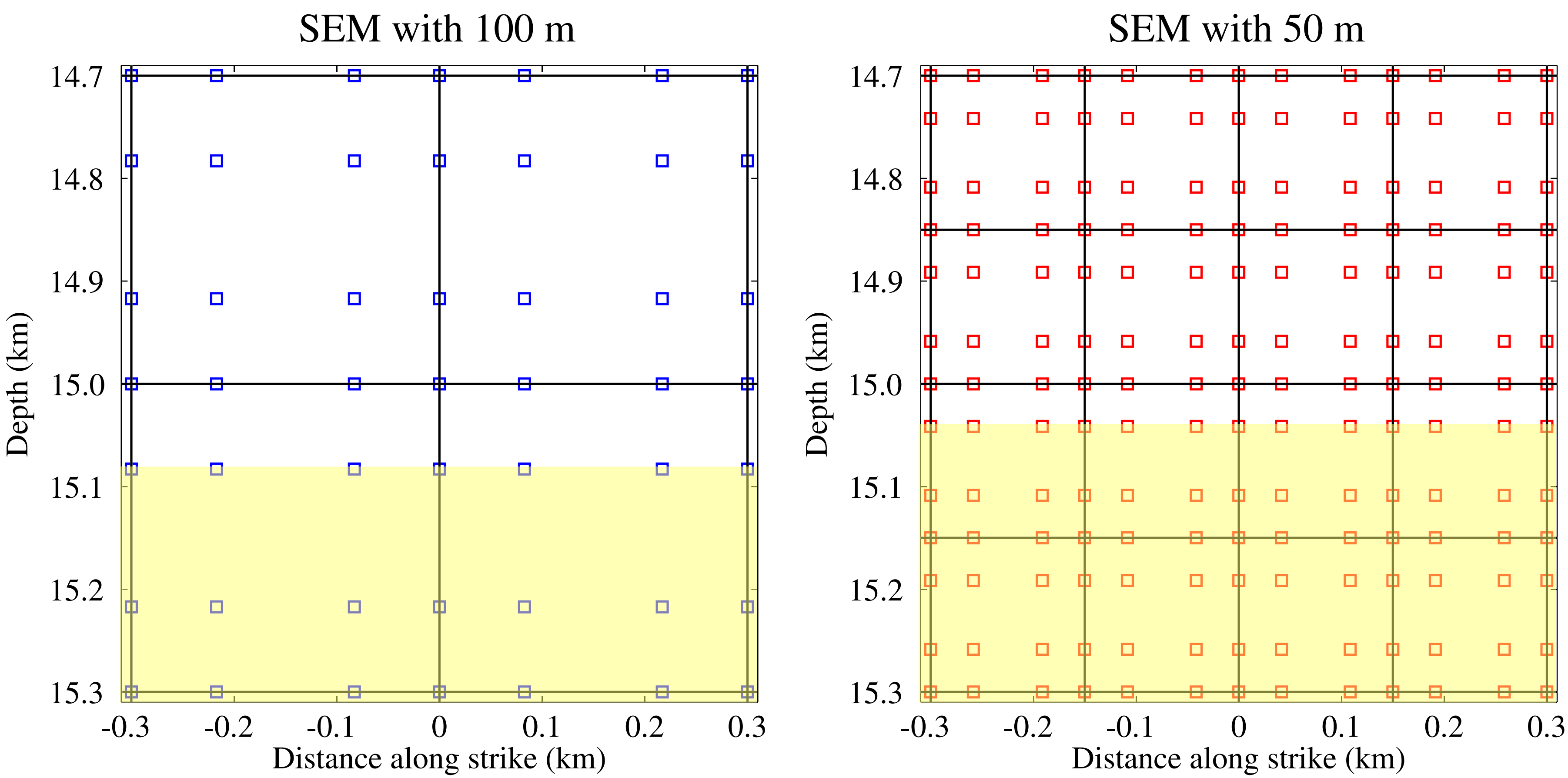
- The number of grid points in cohesive (slip-weakening) zone controls numerical resolutions (e.g. *Day et al.*, 2005).
- Cohesive zone is well resolved even for the 100-m case.

Implications: Numerical solutions are mesh-dependent

There are two potential sources:

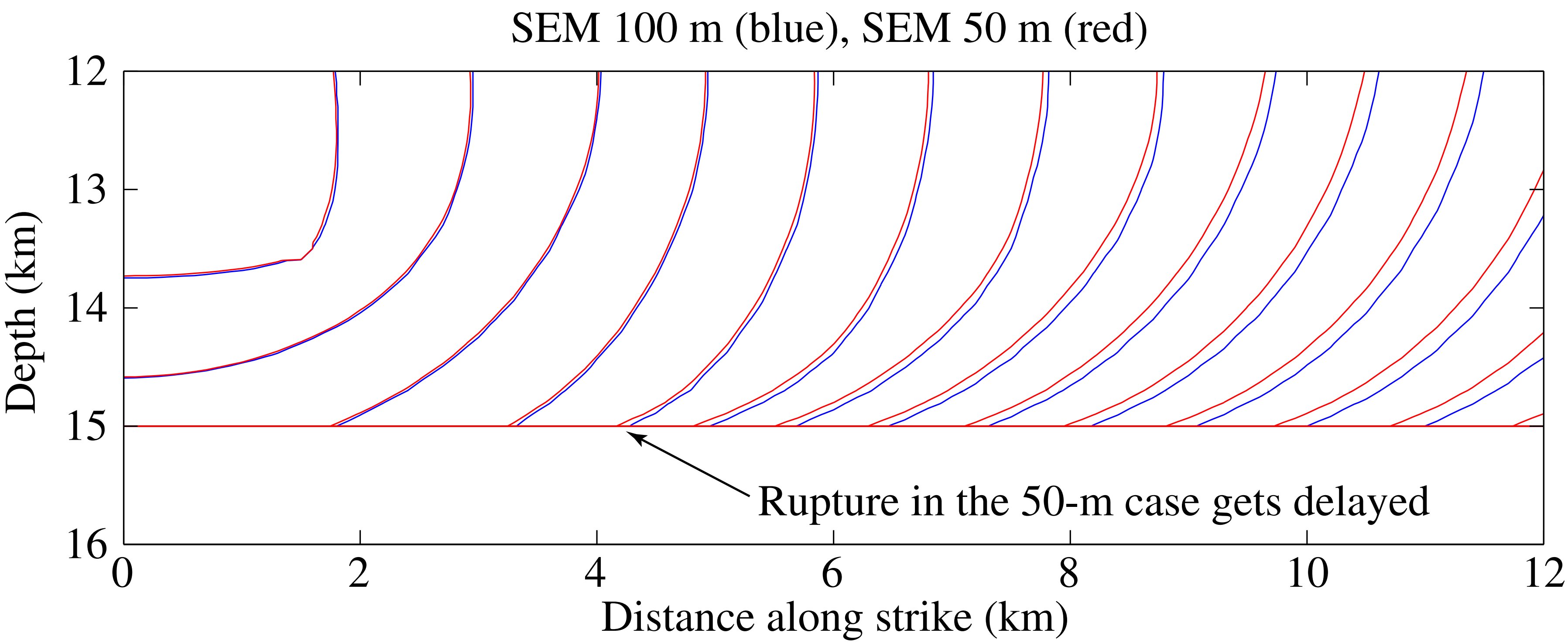
1. Discontinuous strength at the bottom boundary between the ruptured area and the strength barrier.
   $\rightarrow$ causing the change in an effective rupture area
2. Discontinuous stress at the boundary of the nucleation patch
   $\rightarrow$ causing the change in an effective nucleation-patch size

Illustration: SEM computational nodes at the bottom fault boundary



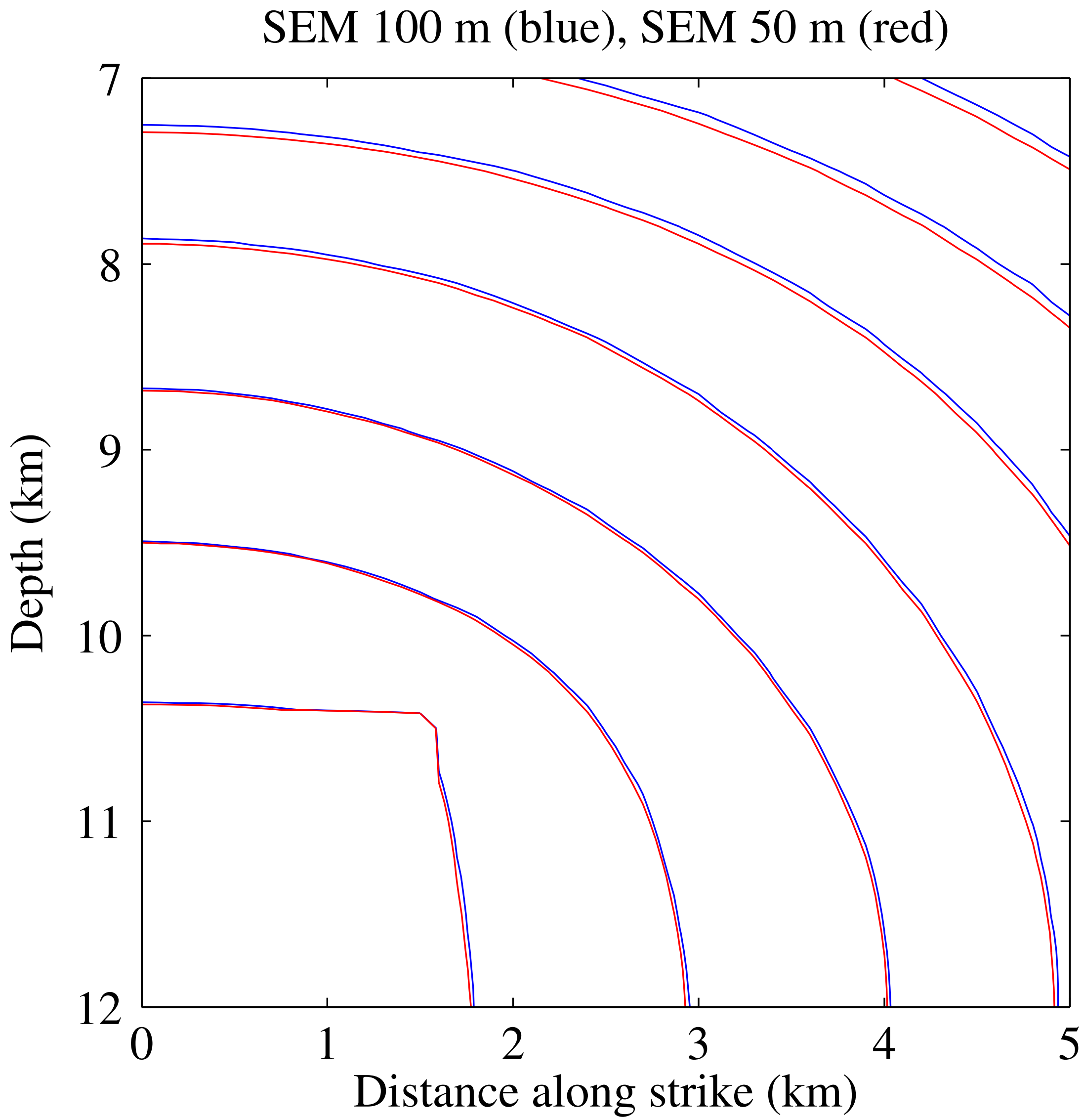
- The difference in node locations results in different sizes of an effective slip-weakening area.
- The effective area is larger in the 100-m case than in the 50-m case.

TPV8: Rupture contours near the bottom fault boundary



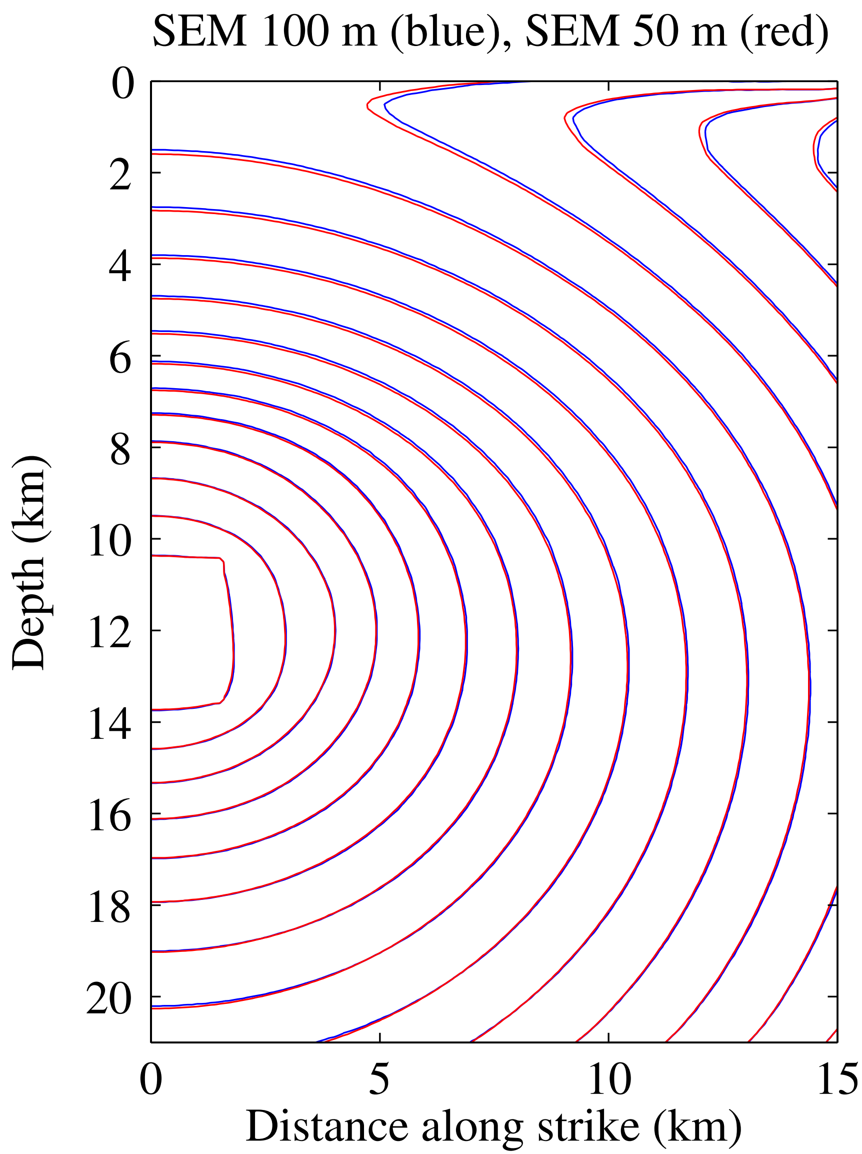
- The rupture speed at the bottom fault boundary is slower due to the smaller effective size of a slip-weakening area.
- The difference in rupture speed gets larger as the rupture propagates farther.

TPV8: Rupture contours near the nucleation patch



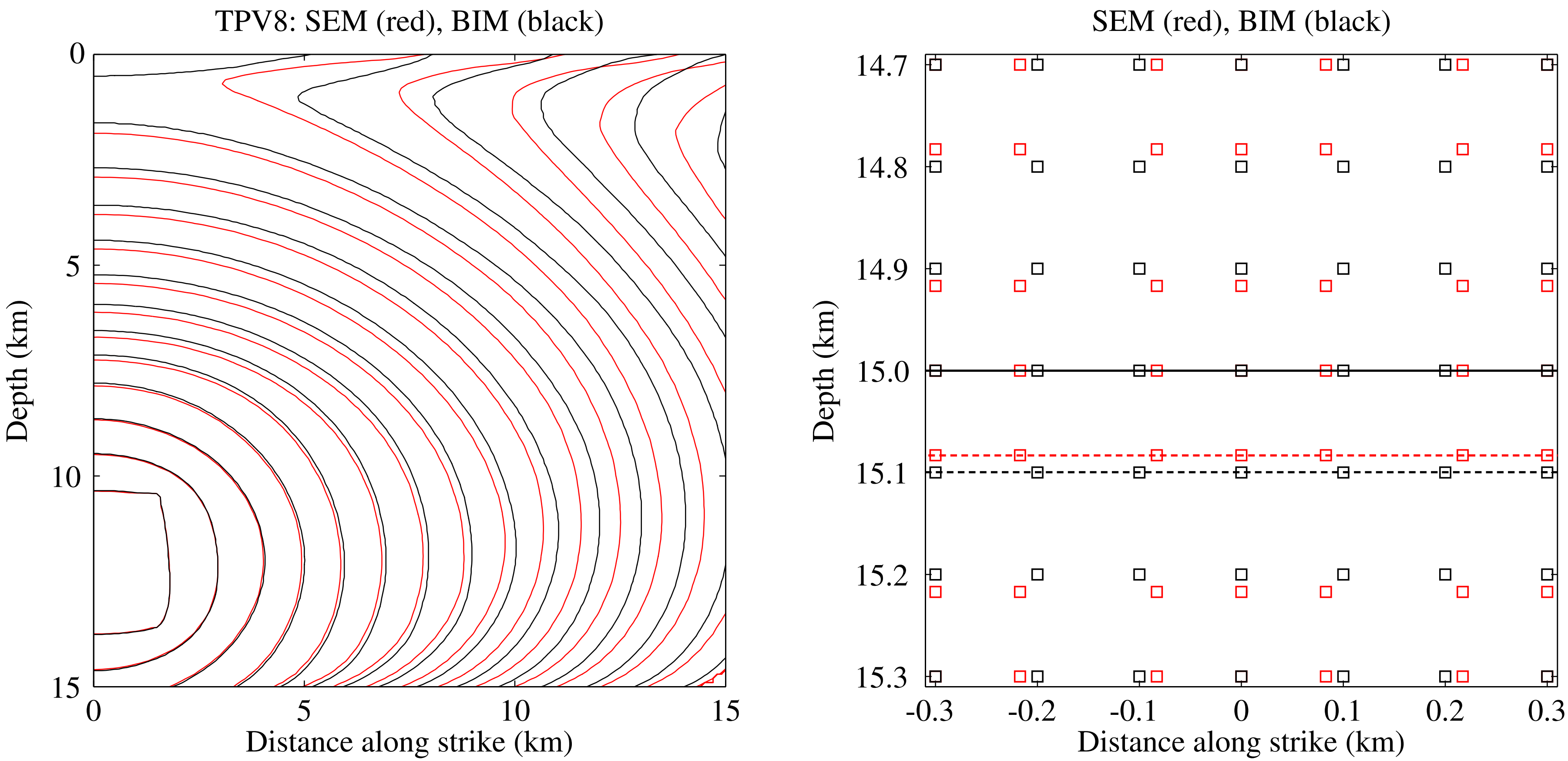
- The rupture speed in the 50-m case is slower from the very beginning.
- The difference in node locations also results in a different effective nucleation-patch size, and hence different rupture speed.

Comparison between 100-m and 50-m cases with a larger fault



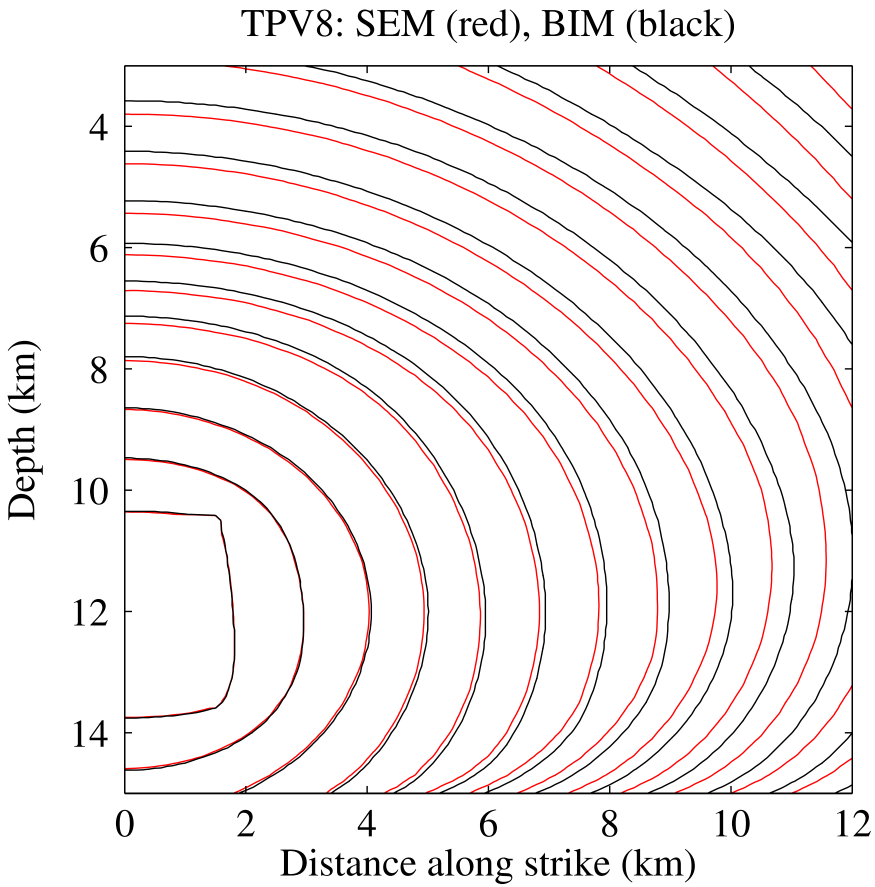
- The difference between 100-m and 50-m cases gets smaller.
- The mesh-dependence at the bottom boundary is the primary source of the difference in rupture speed.

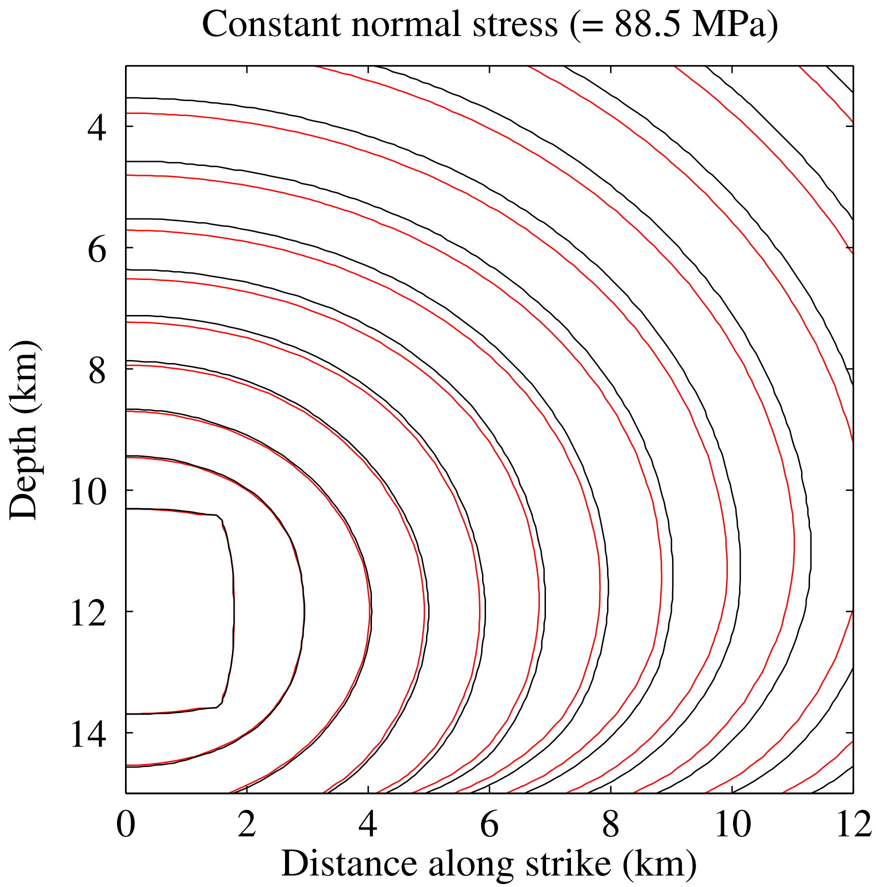
TPV8: Comparison of SEM with boundary integral method (BIM) [Liu/Lapusta]



- Differences in node locations in different codes also affect the results of the code validation (TPV8 and TPV9).
- The rupture speed is higher in BIM than in SEM due to the difference in the effective nucleation-patch size and slip-weakening area.

Effects of depth-variable normal stress on rupture speed



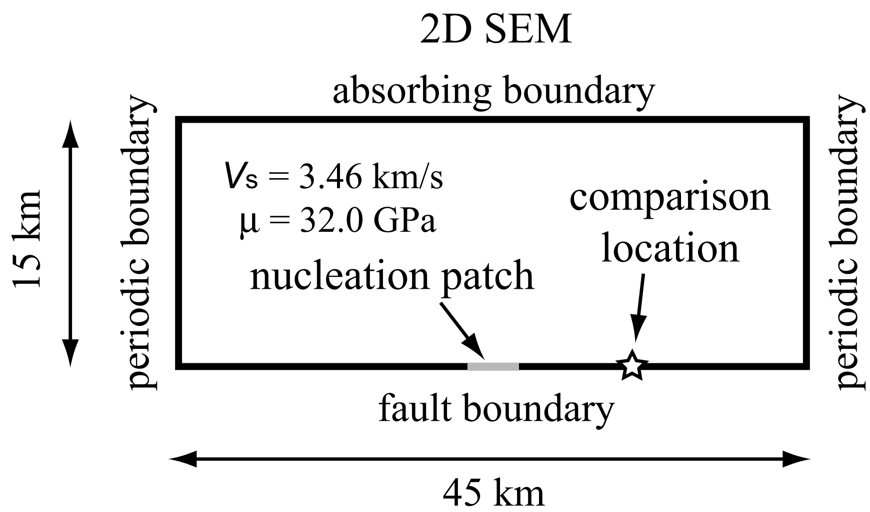


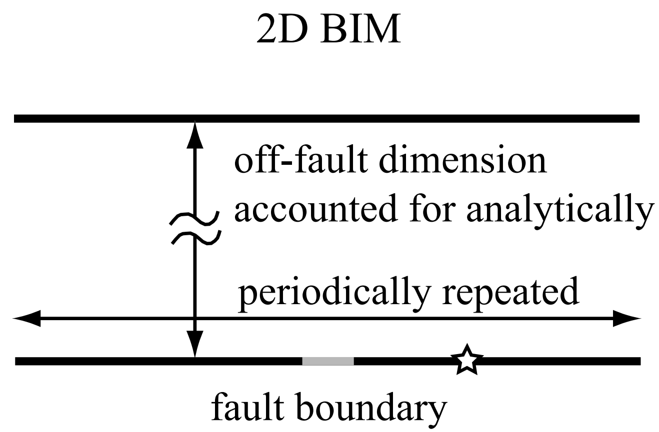
- Depth-variable normal stress does not seem to influence the difference in rupture speed.

Part I conclusions

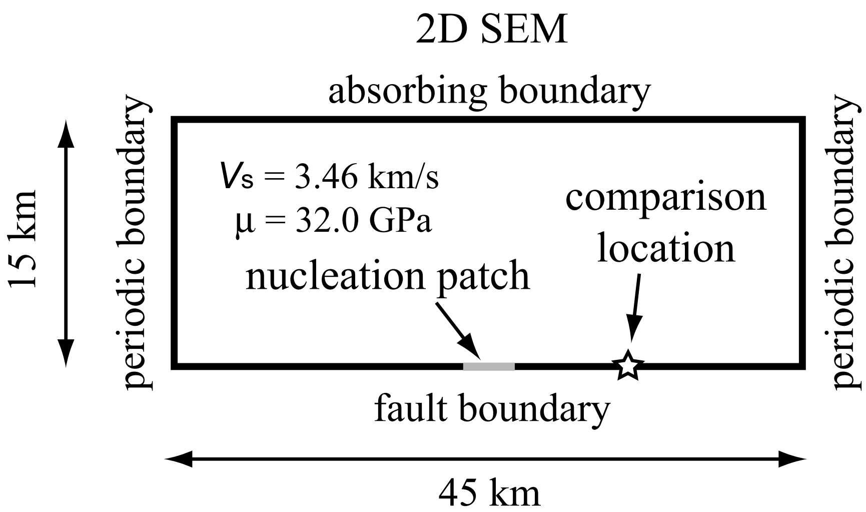
- Discontinuities in the initial conditions and fault properties result in mesh-dependent solutions.
- For TPV8 and TPV9, numerical convergence is controlled by the resolutions of those discontinuities.
- One solution is to formulate a benchmark problem with a smooth transition of stress/strength (e.g. a smooth transition to lower initial shear stress/higher static strength to arrest rupture).

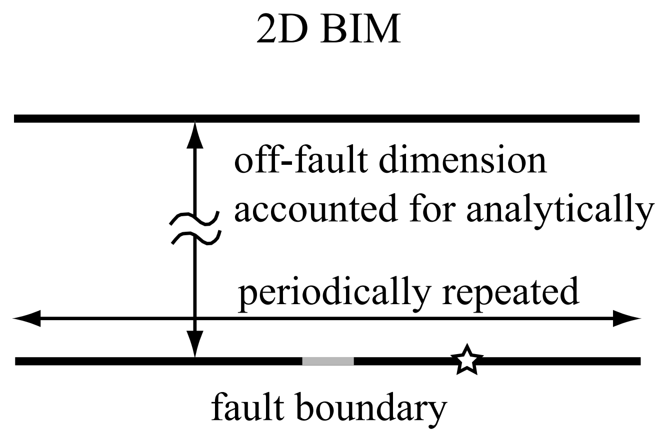
Part II: Convergence of SEM and BIM solutions with grid reduction in an anti-plane (2D) rate-state problem

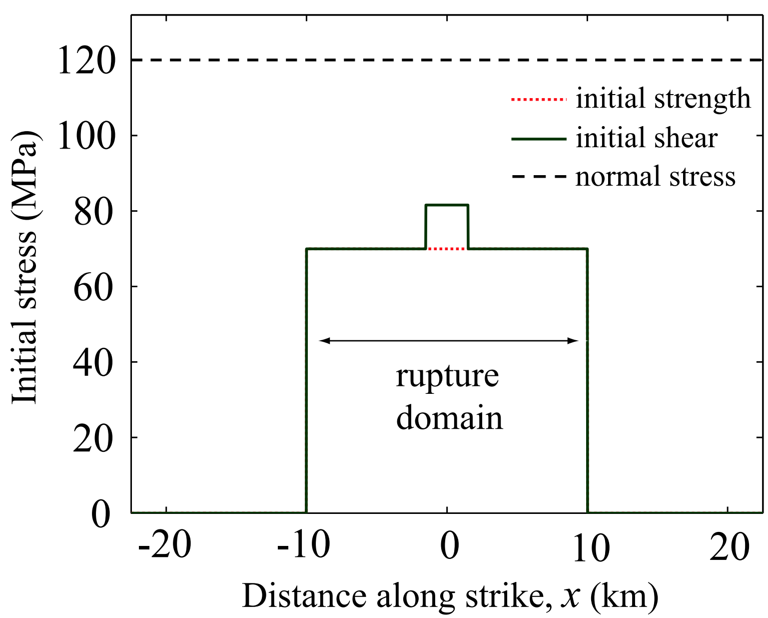


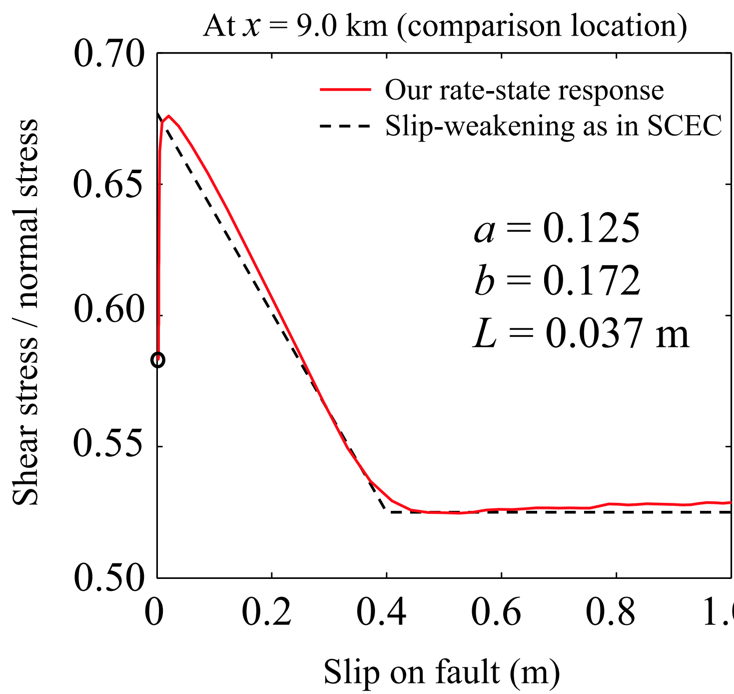


Part II: Convergence of SEM and BIM solutions with grid reduction in an anti-plane (2D) rate-state problem



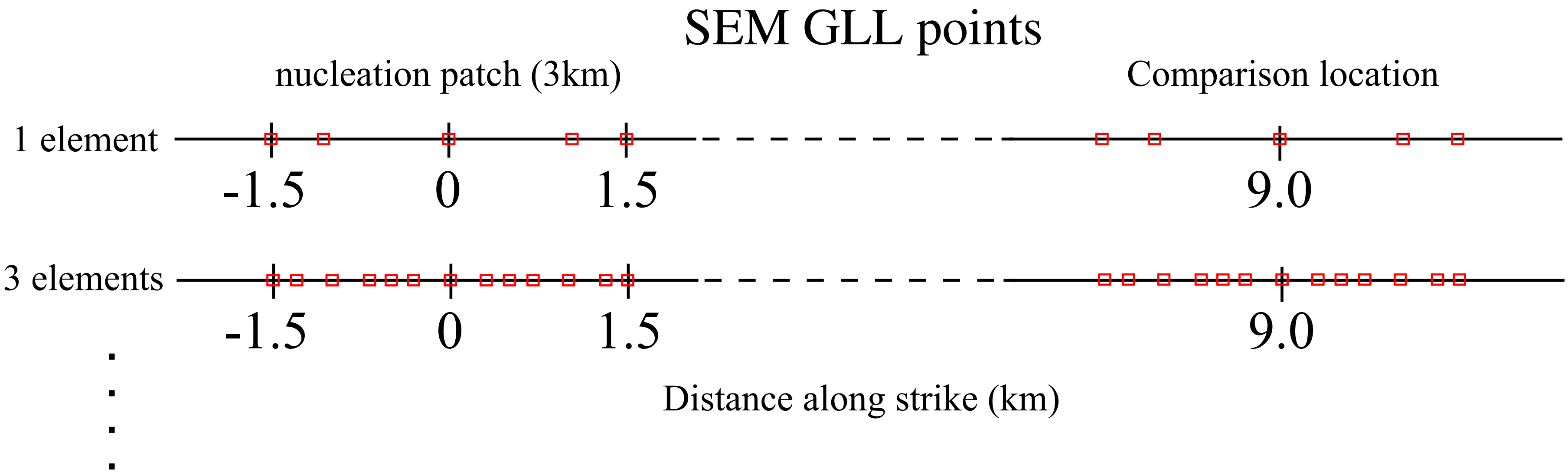


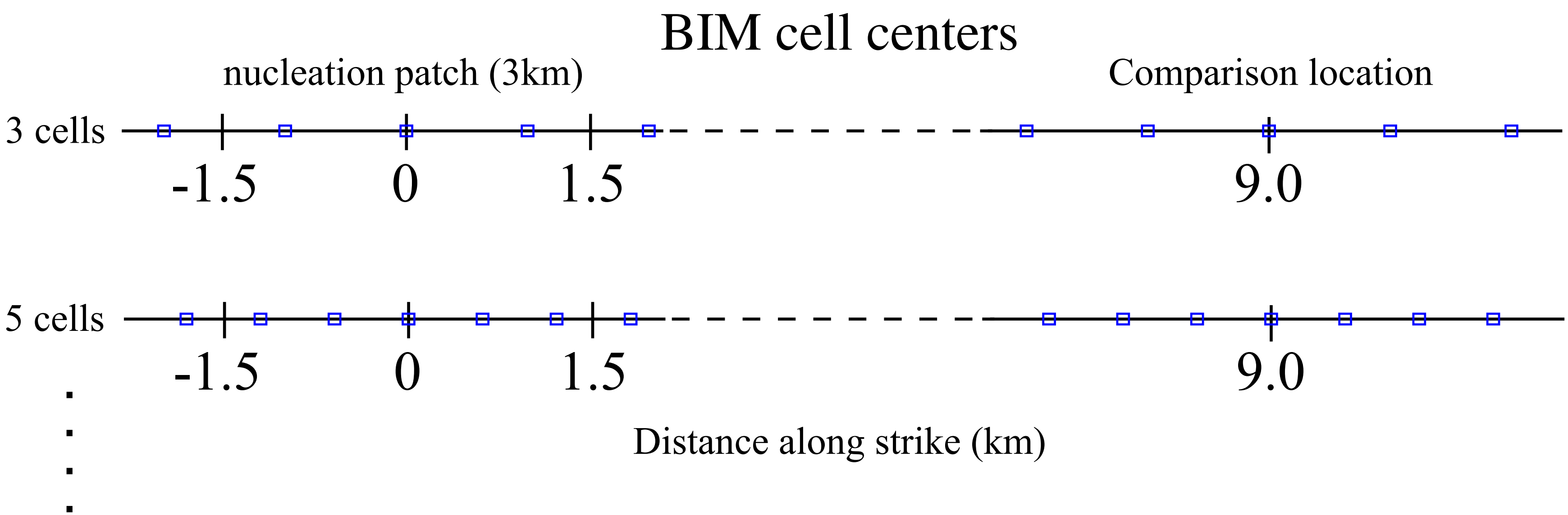




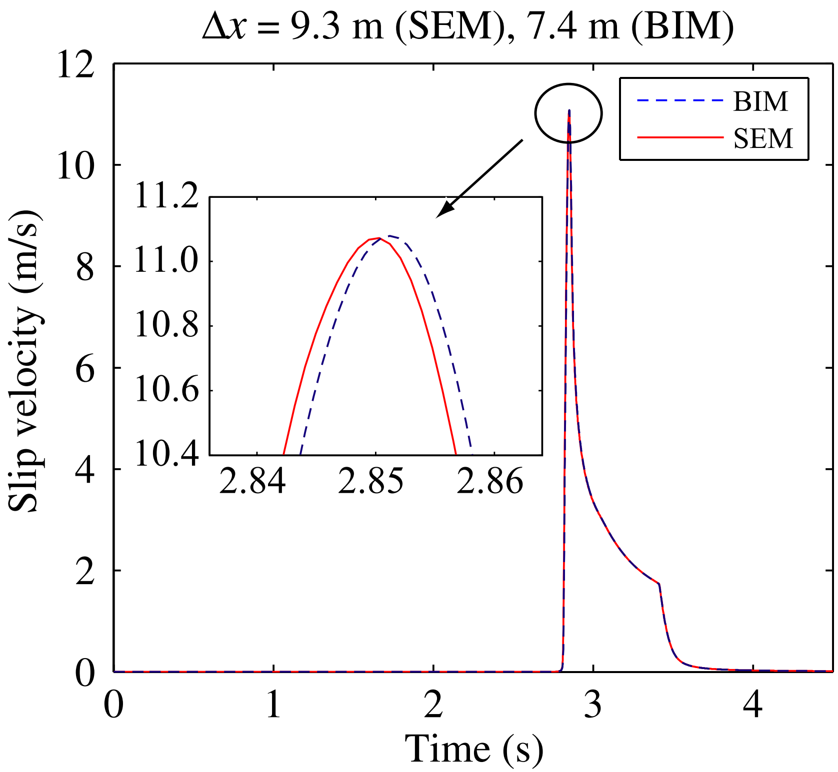
All comparisons are given in terms of slip velocity at an on-fault receiver at $x = 9.0$ km.

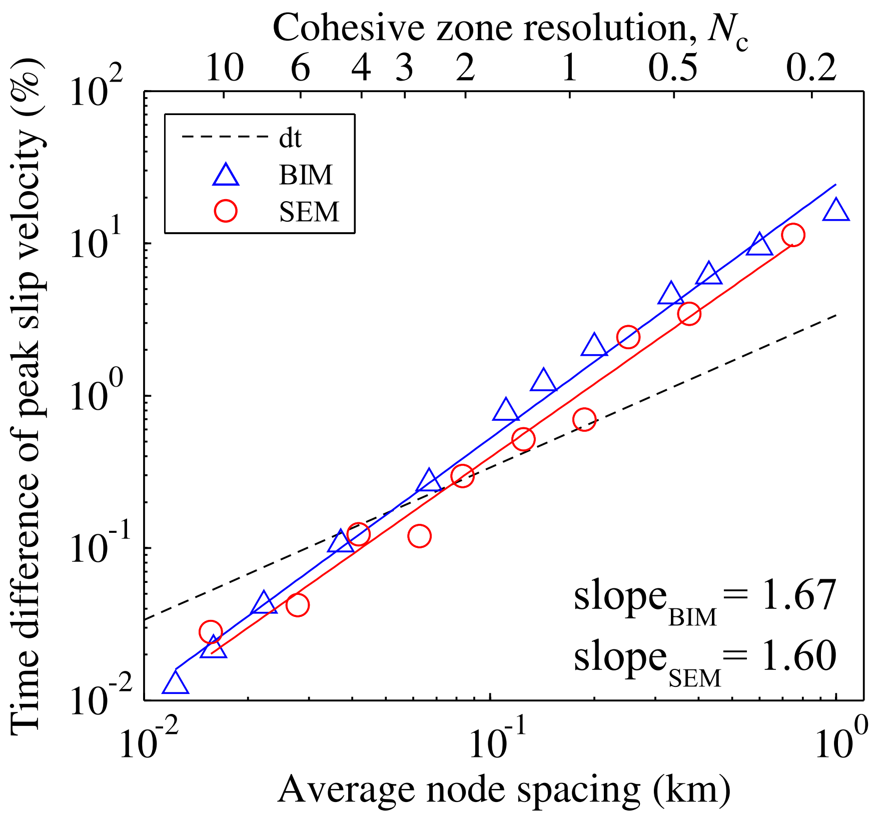
Design of computational nodes





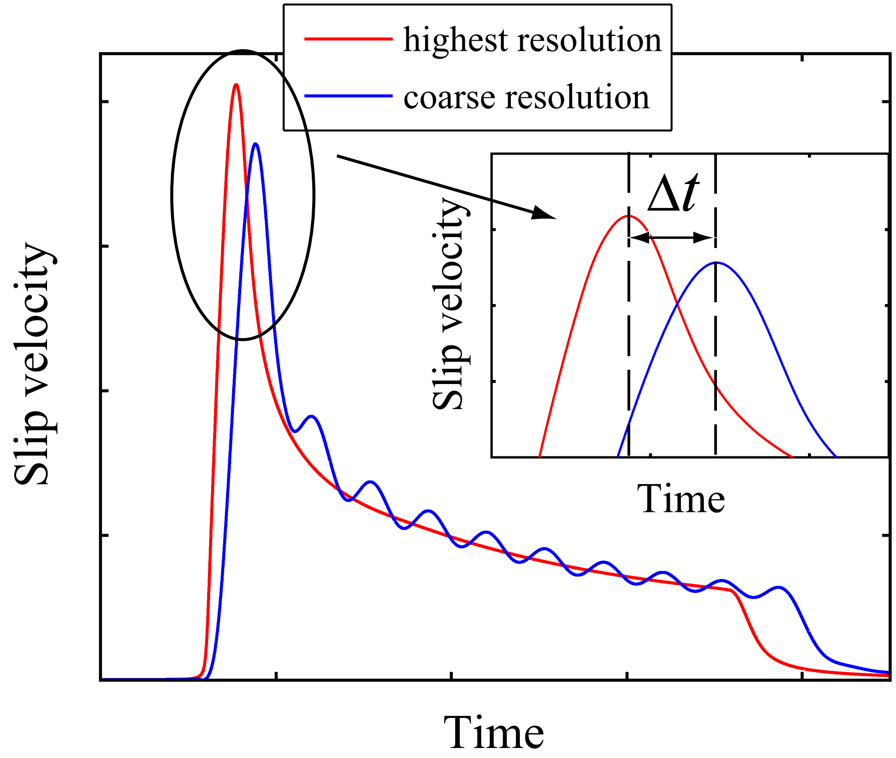
Slip velocity comparison





- Virtually identical time histories of BIM and SEM for the highest-resolution cases

Illustration: Time difference of peak slip velocity (PSV)

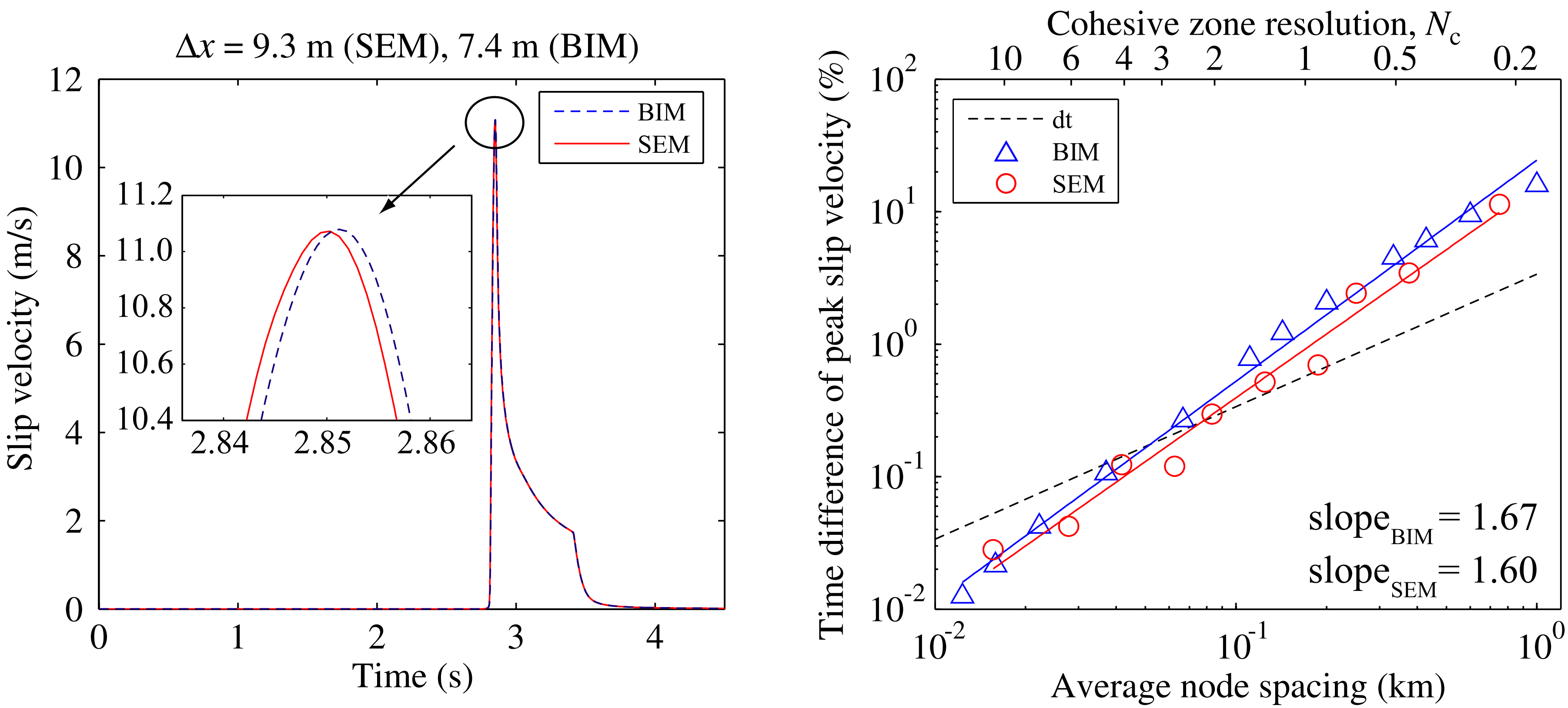


PSV arrival time instead of the arrival time of $V = 1$ mm/s

Procedure for computing a time difference of PSV:

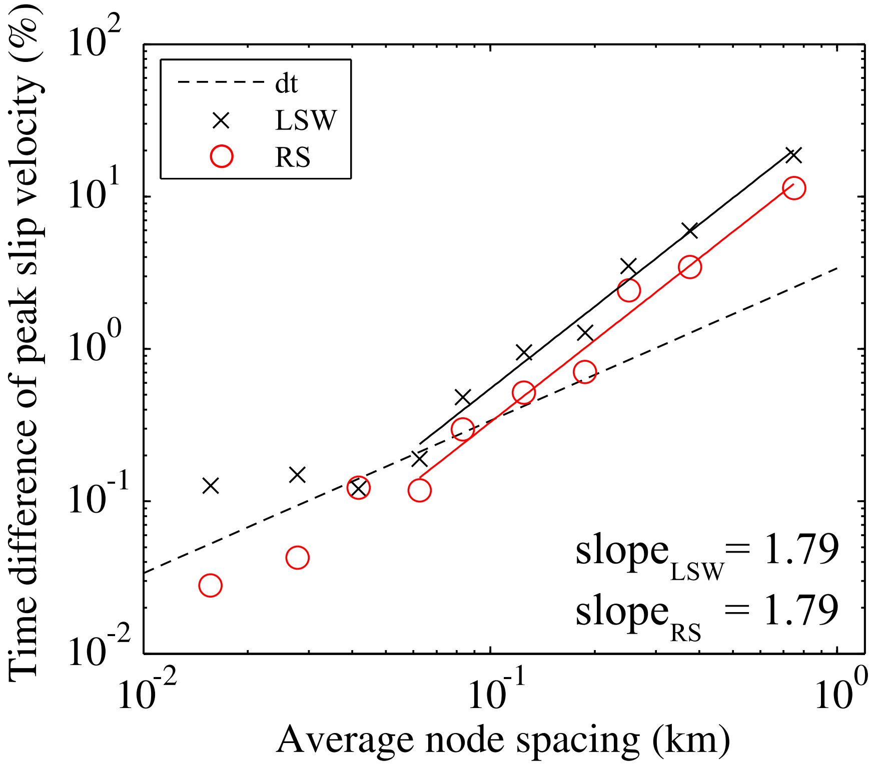
1. Compute PSV arrival time of a coarser simulation.
2. Compute the difference Δt.
3. Compute the percent difference Δt (%) using the PSV arrival time of the highest resolution.

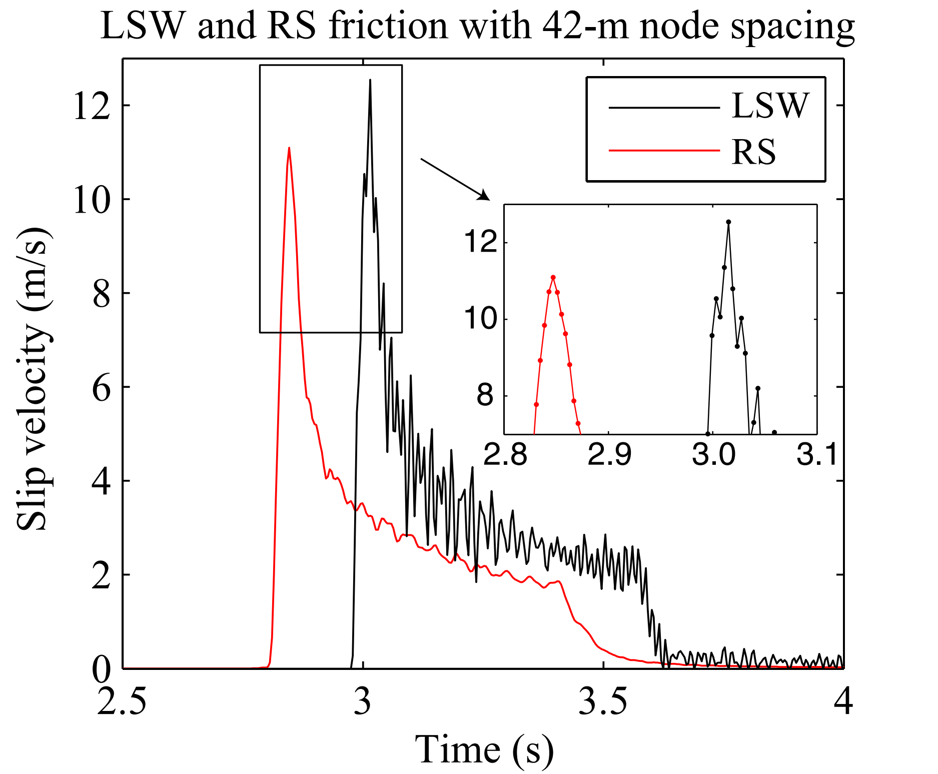
Similar convergence rates

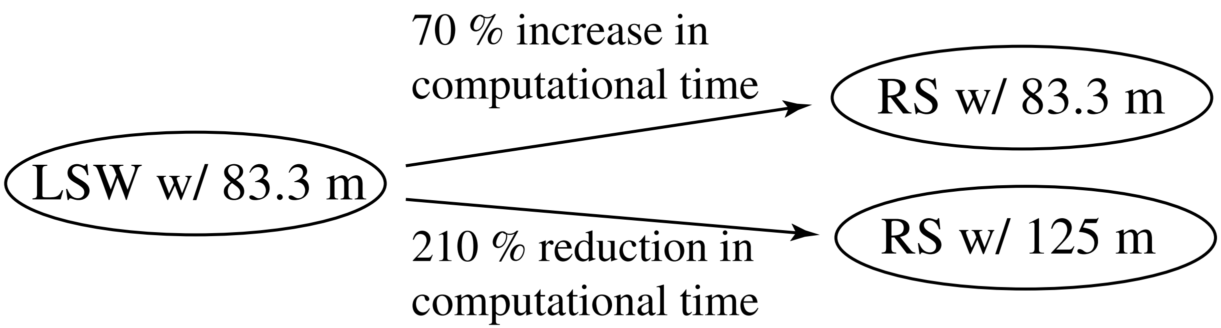


- Virtually identical time histories of BIM and SEM for the highest-resolution cases
- Similar convergence rates for BIM and SEM

Comparison of errors for SEM with LSW and RS friction

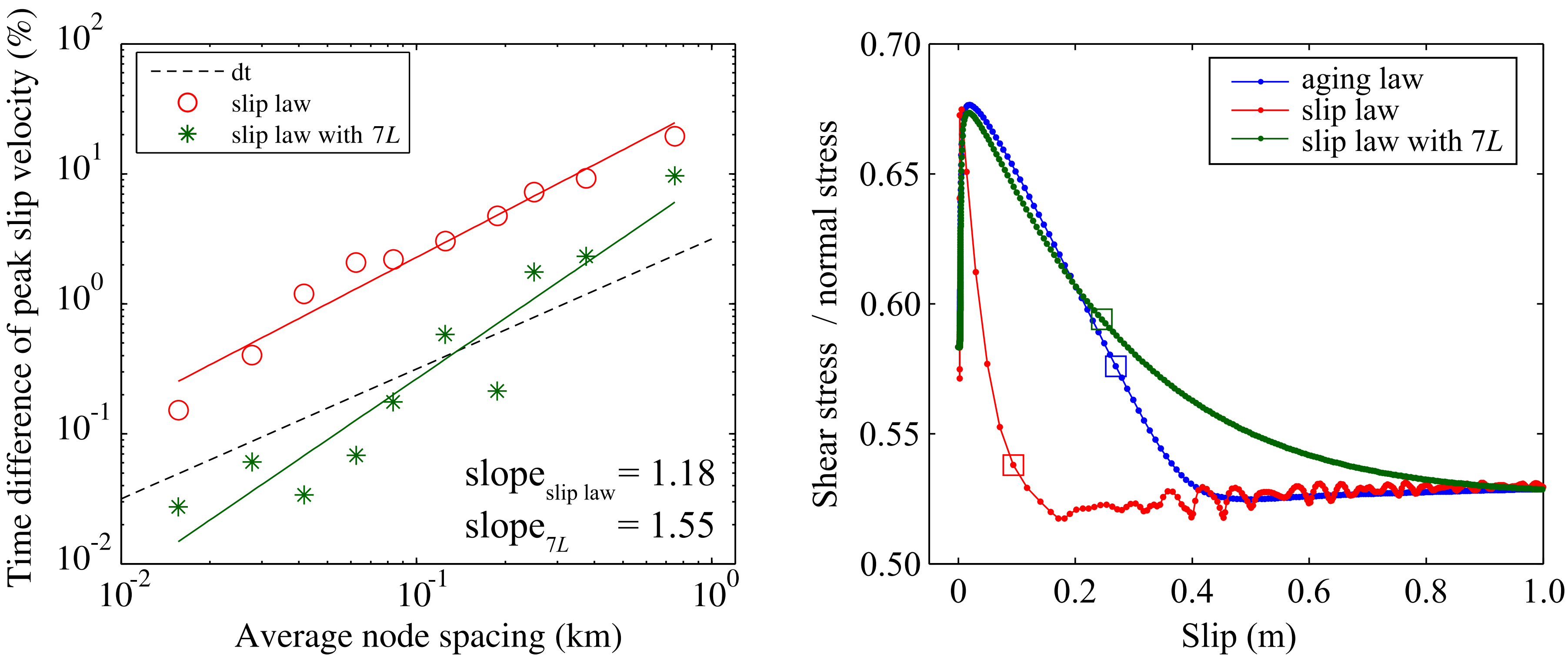






- The computational cost is smaller in calculations with RS friction than with LSW friction.
- The aging form of RS law leads to more stable and accurate simulations than LSW law.

Errors for SEM with slip law of state-variable evolution



- The slip law of RS friction requires much higher numerical resolution to establish the same order of accuracy.
- Errors are controlled by the resolution of maximum effective slip-weakening rate.

Part II conclusions

- The highest-resolution solutions, convergence rates, and errors for both SEM and BIM are nearly identical.
- For the same computational time, the aging form of RS law leads to more stable and accurate simulations than LSW law.
- In comparison to the aging law, the slip law of RS friction requires much higher numerical resolution to establish the same order of accuracy.
- Errors are controlled by the resolution of maximum effective slip-weakening rate.
- More details are in *Kaneko et al.*, *submitted to JGR*.