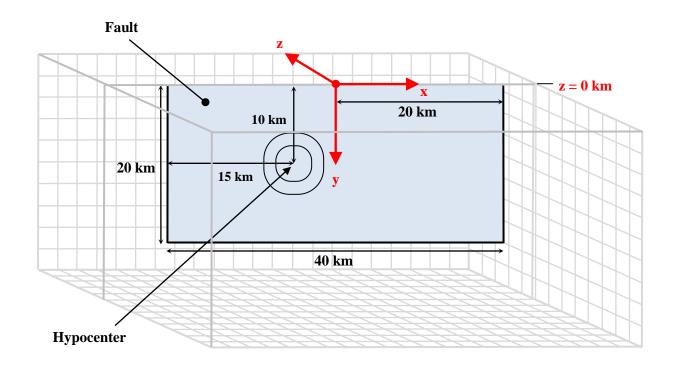
Vertical Planar Fault with Viscoplasticity Benchmarks TPV26 — Elastic Case TPV27 — Viscoplastic Case

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SCEC Rupture Dynamics Code Verification Workshop

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TPV26 (Elastic) and TPV27 (Viscoplastic) — Vertical Planar Fault with Viscoplasticity



Right-lateral strike-slip fault in a uniform half-space.

TPV26 — Elastic material properties.

TPV27 — Viscoplastic material properties.

Issues for a Viscoplasticity Benchmark

What is Viscoplasticity?

(Not mathematically rigorous)

A plastic or viscoplastic material has a *yield stress Y* .

- So long as the magnitude of the stress tensor σ remains less than Y, the material behaves like a linear elastic material.
- If the magnitude of the stress tensor σ exceeds Y, then the material *yields*.

Yielding means that the stress tensor changes spontaneously (i.e., without application of stain), so that the magnitude of the stress tensor is reduced to the yield stress. Yielding is accompanied by the release of energy (heat).

Plasticity and viscoplasticity differ in how quickly the magnitude of the stress tensor is reduced to the yield stress.

- In plasticity, the magnitude of the stress tensor drops instantaneously to the yield stress.
- In viscoplasticity, the magnitude of the stress tensor decays gradually to the yield stress. The decay is exponential, with characteristic time T_v .

The time constant T_v is called the *relaxation time* of the viscoplastic material.

Return Map Algorithm

Given:

 σ_{ij}^0 = Stress tensor at the start of a time step.

 ϵ_{ij} = Strain tensor applied during the time step.

 Δt = Duration of the time step.

Find:

 σ_{ij} = Stress tensor at the end of the time step.

1. First, calculate a *trial stress tensor* by assuming the material behaves elastically:

$$\sigma_{ij}^{\text{trial}} = \sigma_{ij}^0 + \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33})\delta_{ij} + 2\mu\epsilon_{ij}$$

- 2. If the magnitude of $\sigma_{ij}^{\text{trial}}$ is less than or equal to the yield stress Y, then $\sigma_{ij} = \sigma_{ij}^{\text{trial}}$.
- 3. Otherwise, the material yields. For a plastic material, take

$$\sigma_{ij} = \frac{1}{3} \left(\sigma_{11}^{\text{trial}} + \sigma_{22}^{\text{trial}} + \sigma_{3}^{\text{trial}} \right) \delta_{ij} + r_p s_{ij}^{\text{trial}}$$

where s_{ij}^{trial} is the deviatoric part of $\sigma_{ij}^{\text{trial}}$, and where r_p is chosen so that the magnitude of σ_{ij} is Y.

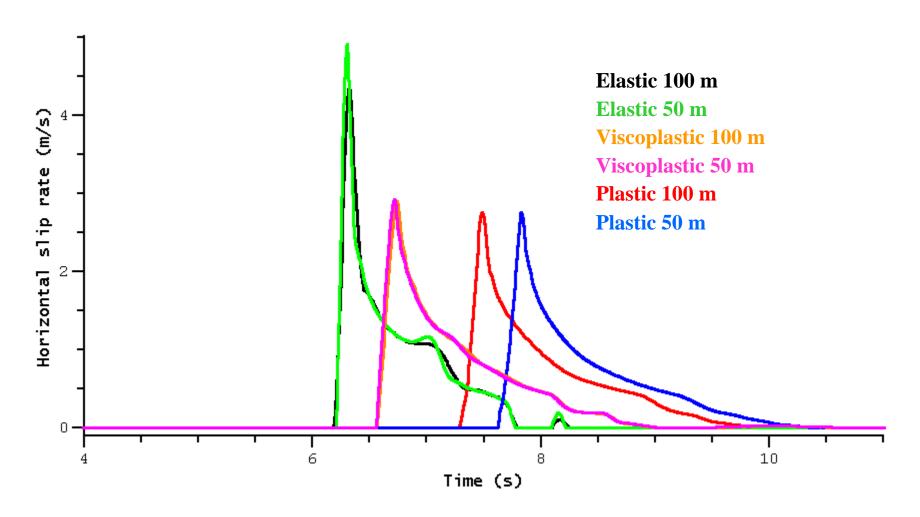
4. For a viscoplastic material, take

$$\sigma_{ij} = \frac{1}{3} \left(\sigma_{11}^{\text{trial}} + \sigma_{22}^{\text{trial}} + \sigma_{3}^{\text{trial}} \right) \delta_{ij} + r_{vp} s_{ij}^{\text{trial}}$$

where

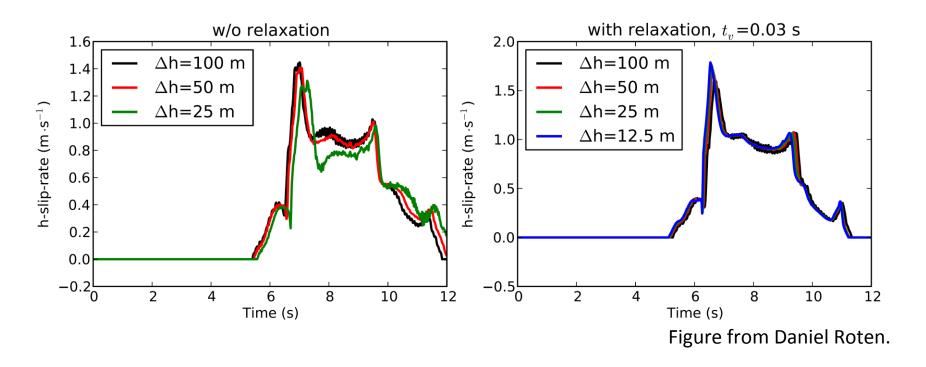
$$r_{vp} = 1 - (1 - r_p)(1 - e^{-\Delta t/T_v})$$

Viscoplasticity is Needed for Numerical Convergence



Pure plasticity does not give good agreement between 100 m and 50 m resolutions.

Viscoplasticity is Needed for Numerical Convergence



Pure plasticity (left panel) shows reasonable agreement between 100 m and 50 m element sizes. But results change when the element size is reduced to 25 m.

Signatures of Viscoplasticity

We chose parameters so that several effects of viscoplasticity would be clearly visible in the benchmark results, when compared to an elastic benchmark.

- Viscoplastic benchmark has slower rupture propagation, producing distinctly different rupture contours.
- Viscoplastic benchmark has time-varying normal stress on the fault surface.
- Elastic benchmark has high peak slip rate that increases with distance from the hypocenter. Viscoplastic benchmark has lower peak slip rate that levels off with distance.
- Others....

TPV26-27 Design

Material Properties are the Only Difference Between TPV26 and TPV27

TPV26 uses linear elastic material:

Density $\rho = 2670 \text{ kg/m}^3$

Shear-wave velocity $V_S = 3464 \text{ m/s}$

Pressure-wave velocity $V_P = 6000 \text{ m/s}$

TPV27 uses viscoplastic material:

Density $\rho = 2670 \text{ kg/m}^3$

Shear-wave velocity $V_S = 3464 \text{ m/s}$

Pressure-wave velocity $V_P = 6000 \text{ m/s}$

Cohesion c = 1.36 MPa

Bulk friction $\nu = 0.1934$

Relaxation time $T_v = 0.03 \text{ s}$

Initial Stress Tensor

In a calculation with plasticity:

- The initial stress tensor must be specified through the model volume.
- The shear and normal stress on the fault are obtained by resolving the stress tensor.

The initial stress tensor and fluid pressure are:

Fluid pressure
$$P_f = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

Vertical stress
$$\sigma_{22} = -(2670 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

Fault-parallel stress
$$\sigma_{11} = \Omega(\text{depth}) \left(b_{11} \left(\sigma_{22} + P_f \right) - P_f \right) + (1 - \Omega(\text{depth})) \sigma_{22}$$

Fault-normal stress
$$\sigma_{33} = \Omega(\text{depth}) \left(b_{33} \left(\sigma_{22} + P_f \right) - P_f \right) + (1 - \Omega(\text{depth})) \sigma_{22}$$

Horizontal shear stress
$$\sigma_{13} = \Omega(\text{depth}) \left(b_{13} \left(\sigma_{22} + P_f \right) \right)$$

Vertical shear stress
$$\sigma_{23} = 0$$

Fault-parallel shear stress
$$\sigma_{12} = 0$$

Initial Stress Tensor

The tapering coefficient $\Omega(depth)$ causes the deviatoric component of stress to taper down to zero at depths between 15000 m and 20000 m.

$$\Omega(\text{depth}) = \begin{cases} 1 \,, & \text{if depth} \leq 15000 \, \text{m} \\ (20000 \, \text{m} - \text{depth})/(5000 \, \text{m}) \,, & \text{if } 15000 \, \text{m} \leq \text{depth} \leq 20000 \, \text{m} \\ 0 \,, & \text{if depth} \geq 20000 \, \text{m} \end{cases}$$

The coefficients b_{11} , b_{33} , and b_{13} are the ratios of effective stress components.

Coefficient	Value for TPV26 and TPV27
b_{11}	0.926793
b_{33}	1.073206
b_{13}	-0.169029

effective stress tensor =
$$\begin{pmatrix} \sigma_{11} + P_f & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} + P_f & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} + P_f \end{pmatrix}$$

Slip-Weakening Friction Parameters (With Time-Weakening Nucleation)

Static coefficient of friction $\mu_s = 0.18$

Dynamic coefficient of friction $\mu_d = 0.12$

Slip-weakening critical distance $d_0 = 0.30 \text{ m}$

Cohesion
$$C_0 = \begin{cases} 0.40 \text{ MPa} + (0.00072 \text{ MPa/m}) (5000 \text{ m} - \text{depth}), & \text{if depth} \leq 5000 \text{ m} \\ 0.40 \text{ MPa}, & \text{if depth} \geq 5000 \text{ m} \end{cases}$$

Radius of nucleation zone $r_{\rm crit} = 4000 \, {\rm m}$

$$\text{Time of forced rupture} \quad T = \begin{cases} \frac{r}{0.7 \, V_S} + \frac{0.081 \, r_{\rm crit}}{0.7 \, V_S} \Big(\frac{1}{1 - (r/r_{\rm crit})^2} - 1 \Big) \; , \; \text{if} \; r < r_{\rm crit} \\ 1.0 \text{E} + 9 \; , \; \text{if} \; r \geq r_{\rm crit} \end{cases}$$

Time-weakening decay time $t_0 = 0.50 \text{ s}$

(r = distance from the hypocenter.)

Slip-Weakening Friction Law (With Time-Weakening Nucleation)

When the fault is sliding, the shear stress τ is:

$$\tau = C_0 + \mu \max(0, \sigma_n - P_f)$$

The time-varying friction coefficient μ is:

$$\mu = \mu_s + (\mu_d - \mu_s) \max(f_1, f_2)$$

$$f_1 = \begin{cases} D/d_0, & \text{if } D < d_0 \\ 1, & \text{if } D \ge d_0 \end{cases}$$

$$f_2 = \begin{cases} 0, & \text{if } t < T \\ (t - T)/t_0, & \text{if } T \le t < T + t_0 \\ 1, & \text{if } t \ge T + t_0 \end{cases}$$

Where:

 σ_n = normal stress

 P_f = fluid pressure

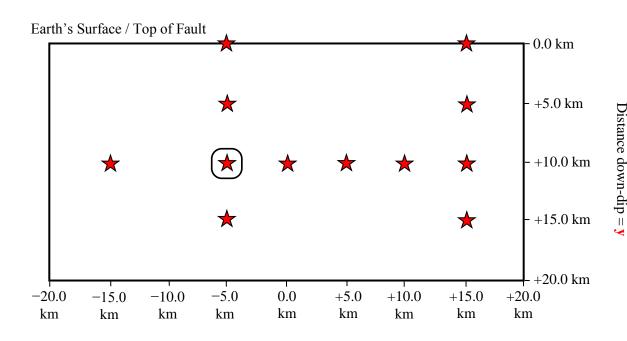
D = total distance the fault has slipped

t =time since the start of the simulation

On-Fault Stations.

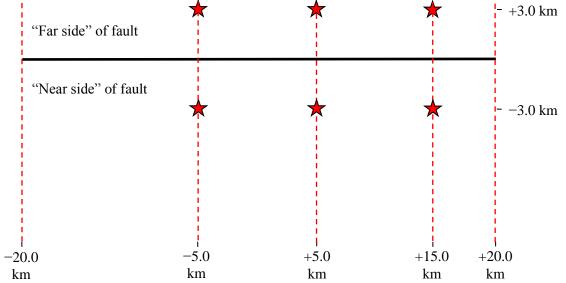
Modelers are asked to submit slip, slip rate, and stress as a function of time, for 12 stations on the fault.

In addition, modelers are asked to submit the time at which each point on the fault begins to slip, from which we construct rupture contour plots.



Distance along-strike = \mathbf{x}

Modele submit



Distance along-strike = \mathbf{x}

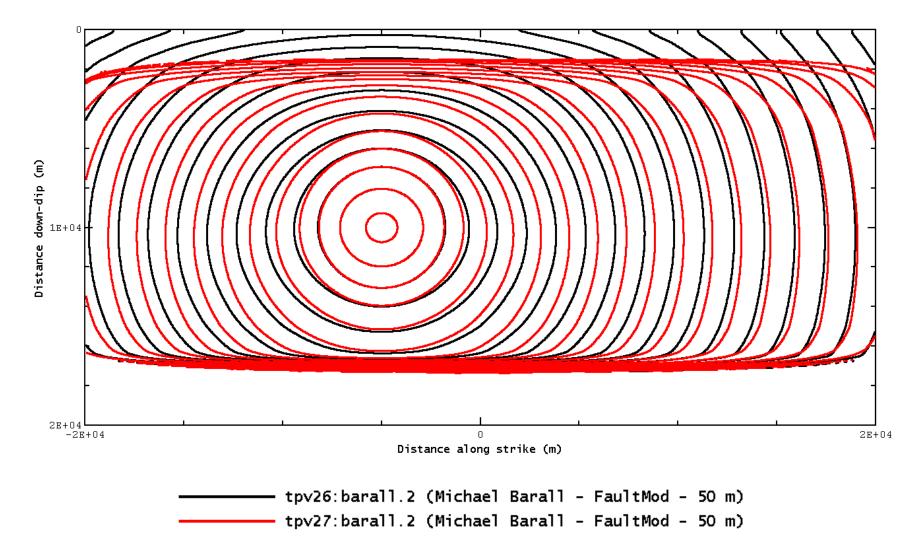
Off-Fault Stations.

Modelers are asked to submit displacement and velocity as a function of time, for 6 stations on the earth's surface.

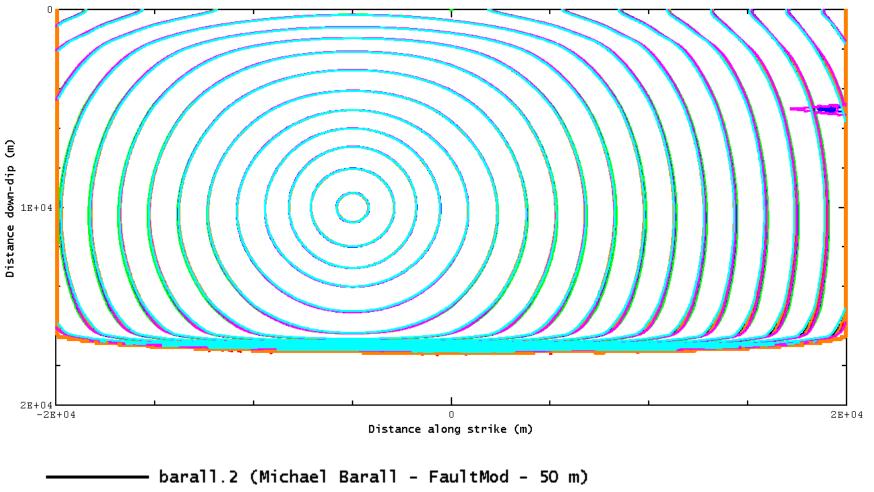
Distance perpendicular to fault =

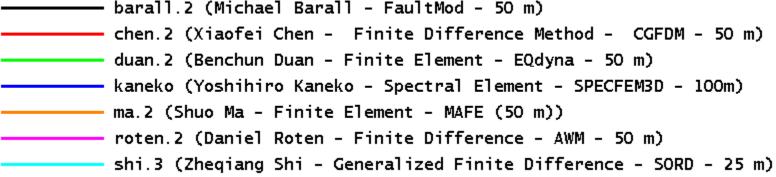
TPV26-27 Rupture Contours

TPV26 (Elastic) Versus TPV27 (Viscoplastic) Rupture Contours

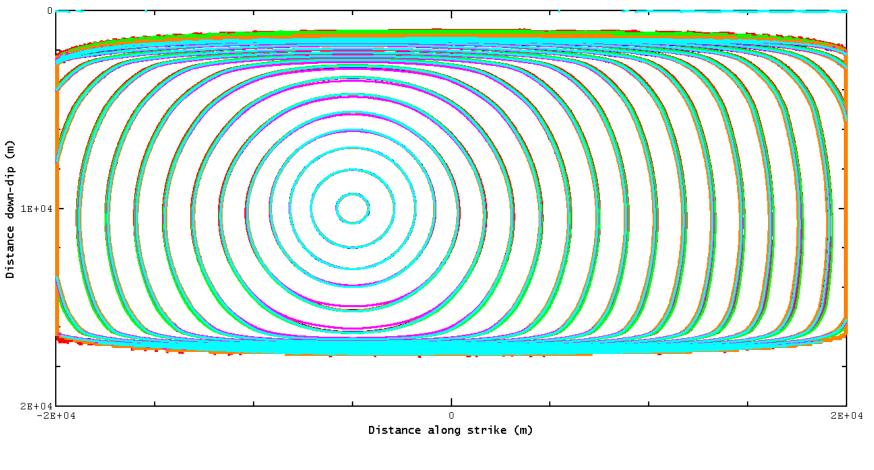


TPV26 (Elastic) Rupture Contours





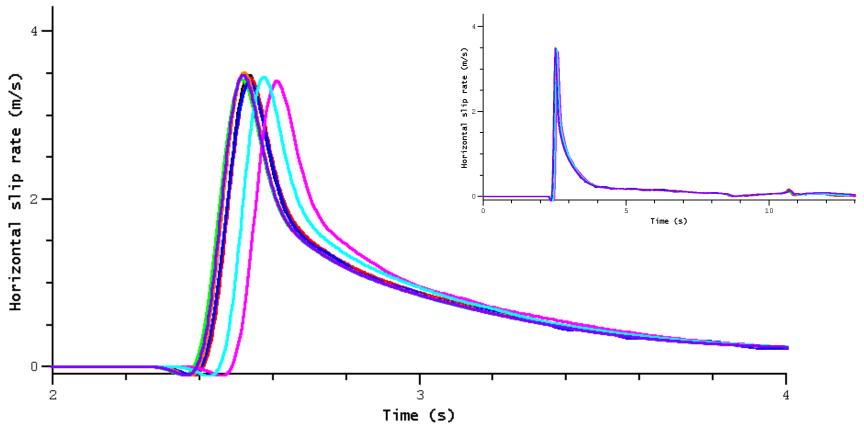
TPV27 (Viscoplastic) Rupture Contours

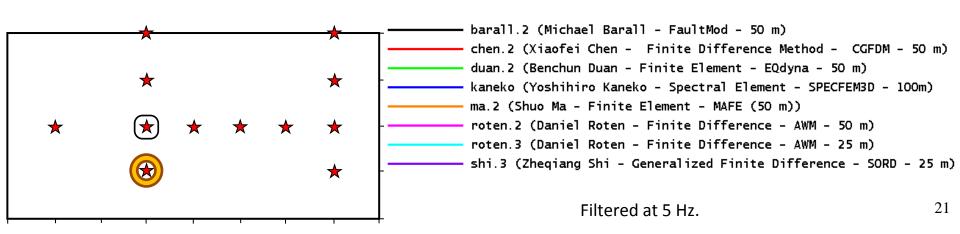




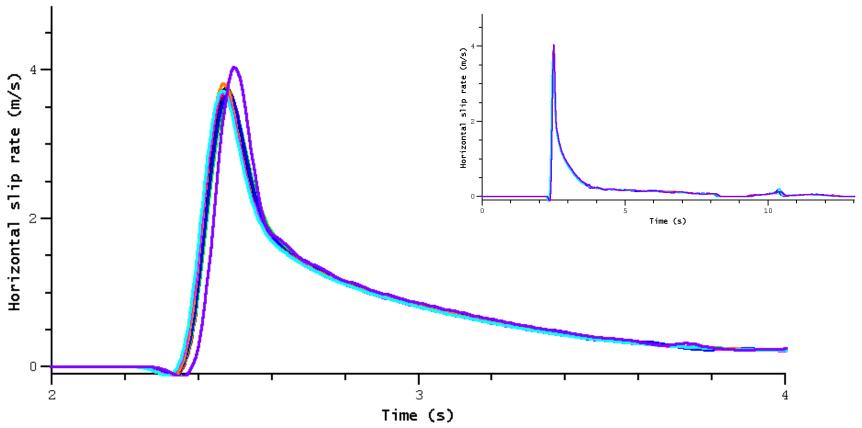
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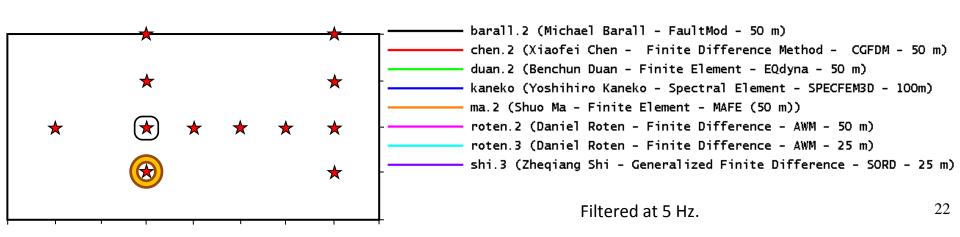
TPV27 (Viscoplastic) Horizontal Slip Rate at faultst-050dp150



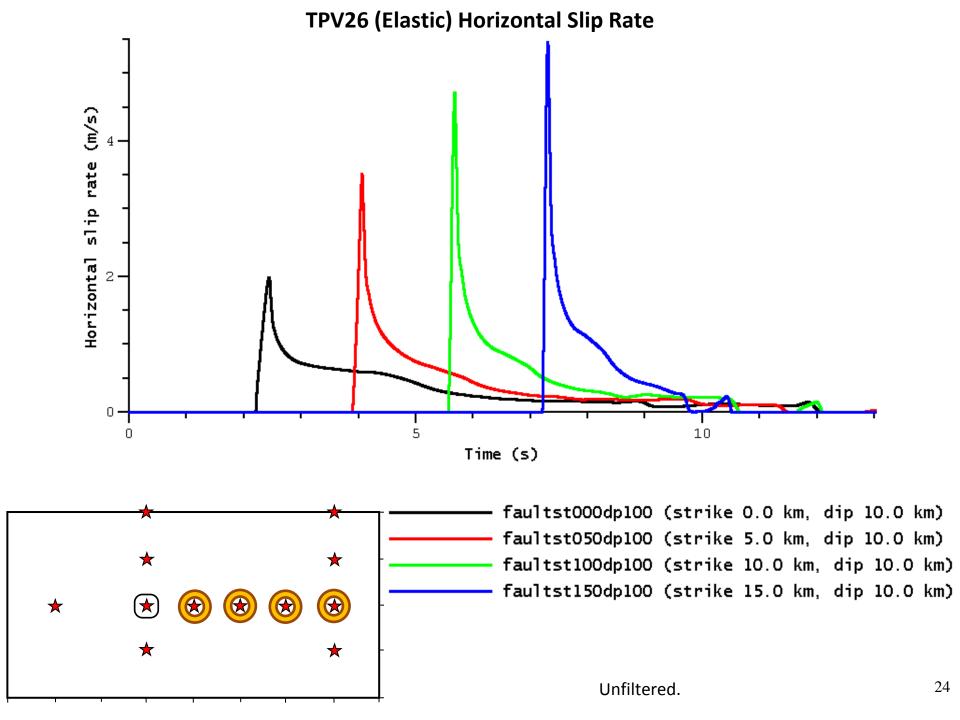


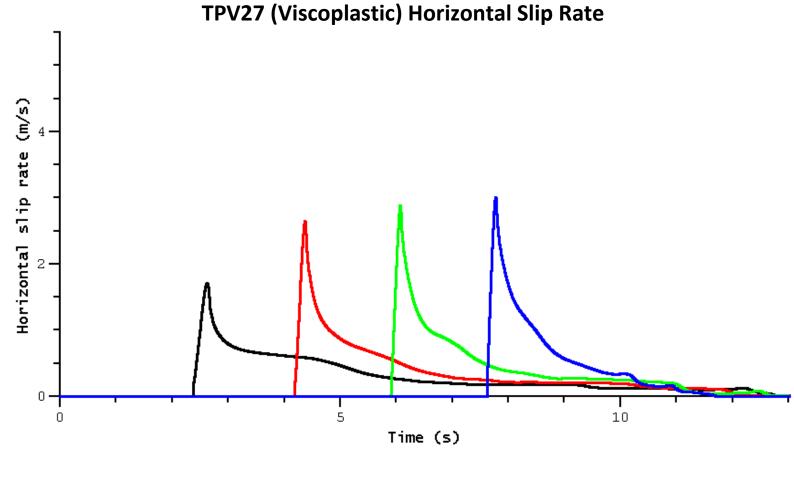
TPV26 (Elastic) Horizontal Slip Rate at faultst-050dp150



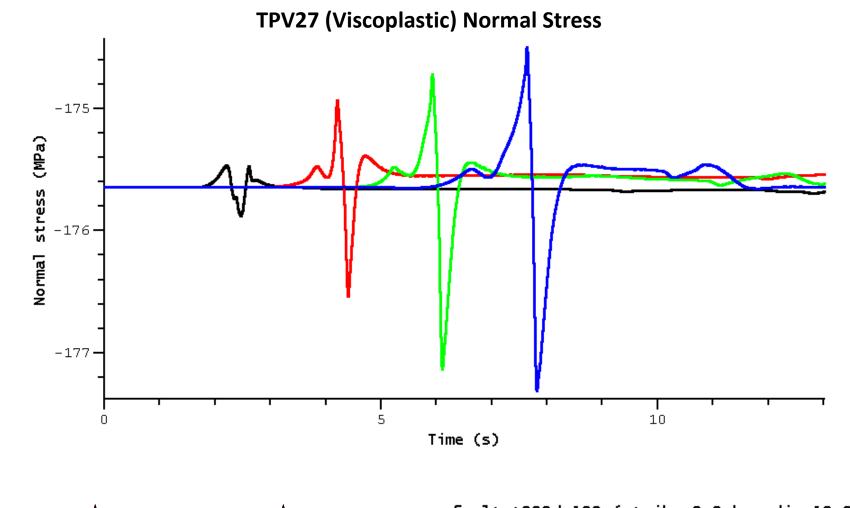


Time Series: Viscoplasticity versus Elasticity



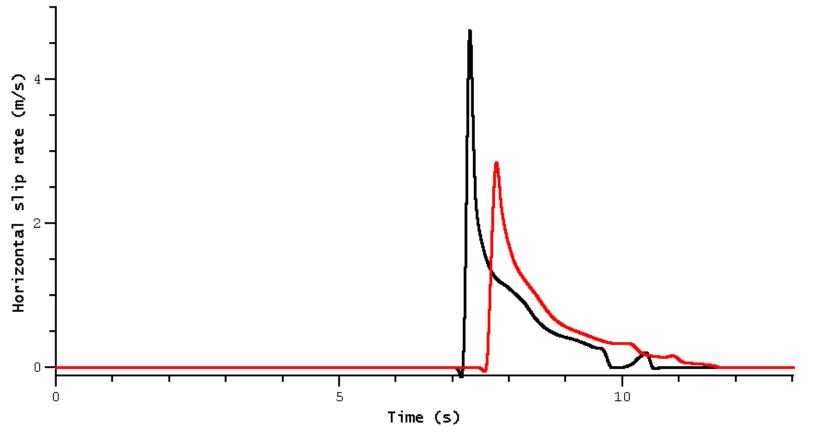


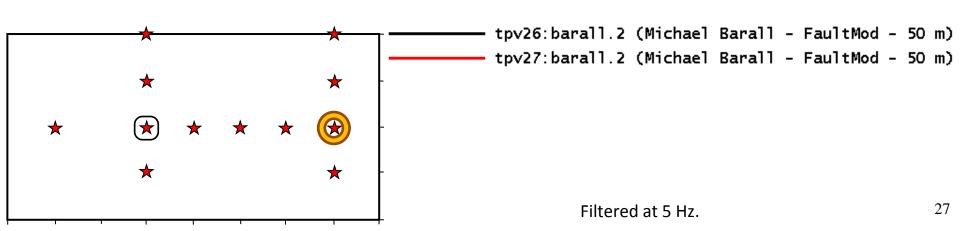


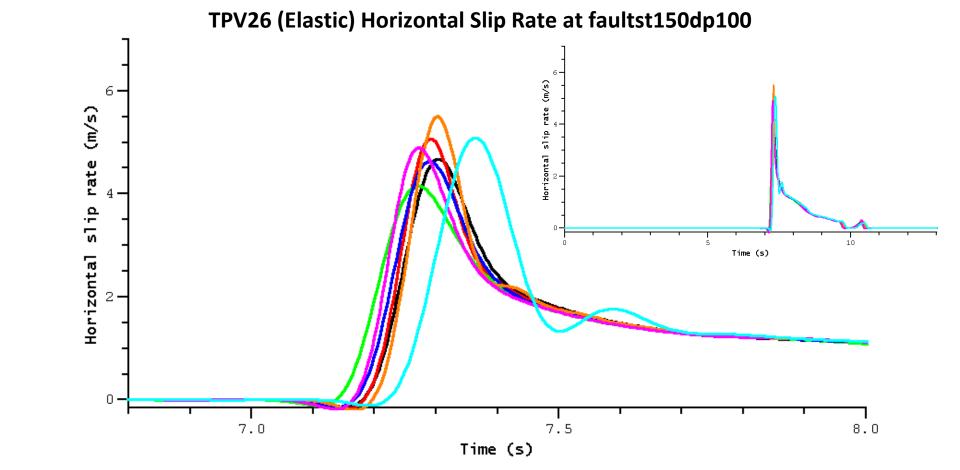


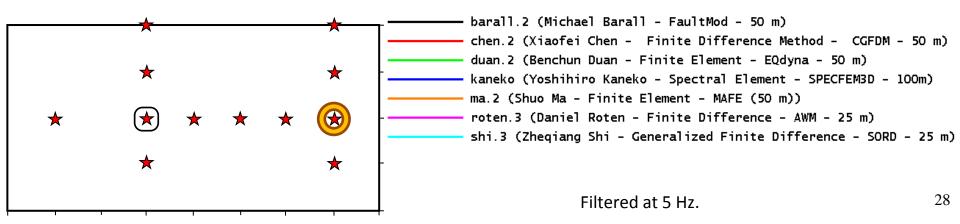


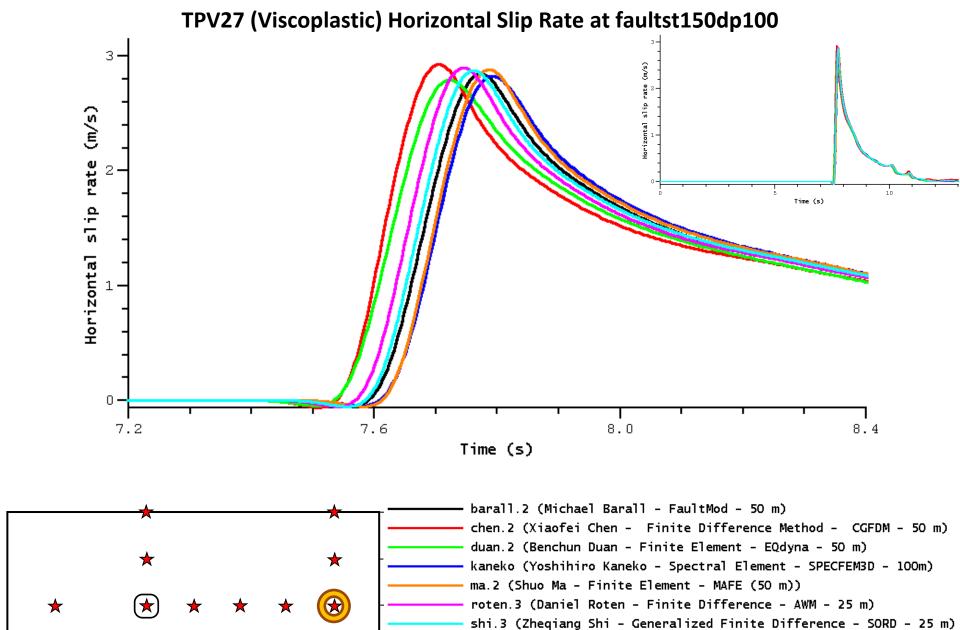
TPV26 (Elastic) Versus TPV27 (Viscoplastic) Horizontal Slip Rate at faultst150dp100











Filtered at 5 Hz.

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TPV27 (Viscoplastic) 2D Slip Rate at faultst150dp100

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
(1) barall		1.5	5.0	4.7	4.5	2.2	2.2	2.7	2.7	3.7	3.0	2.7	2.2	2.6	2.7
(2) barall.2	1.5		4.4	3.8	4.7	1.8	1.3	2.2	1.5	2.7	1.7	1.4	1.9	1.5	1.5
(3) chen	5.0	4.4		2.1	8.3	5.5	4.4	3.8	3.9	4.4	3.8	3.6	3.6	3.6	3.7
(4) chen.2	4.7	3.8	2.1		7.4	4.8	3.7	3.7	3.3	3.9	3.3	2.9	3.5	3.0	3.0
(5) duan	4.5	4.7	8.3	7.4		3.2	4.8	6.5	5.8	5.9	5.7	5.6	6.0	5.6	5.5
(6) duan.2	2.2	1.8	5.5	4.8	3.2		1.8	3.5	2.7	3.1	2.6	2.5	3.1	2.6	2.4
(7) kaneko	2.2	1.3	4.4	3.7	4.8	1.8		2.2	1.4	2.9	1.9	1.6	2.0	1.4	1.4
(8) ma	2.7	2.2	3.8	3.7	6.5	3.5	2.2		1.6	3.2	2.4	2.1	1.2	1.9	2.2
(9) ma.2	2.7	1.5	3.9	3.3	5.8	2.7	1.4	1.6		2.5	1.4	1.1	1.7	0.8	1.1
(10) roten	3.7	2.7	4.4	3.9	5.9	3.1	2.9	3.2	2.5		1.6	1.9	3.1	2.4	2.4
(11) roten.2	3.0	1.7	3.8	3.3	5.7	2.6	1.9	2.4	1.4	1.6		0.7	2.3	1.5	1.4
(12) roten.3	2.7	1.4	3.6	2.9	5.6	2.5	1.6	2.1	1.1	1.9	0.7		2.1	1.1	0.9
(13) shi	2.2	1.9	3.6	3.5	6.0	3.1	2.0	1.2	1.7	3.1	2.3	2.1		1.7	2.0
(14) shi.2	2.6	1.5	3.6	3.0	5.6	2.6	1.4	1.9	0.8	2.4	1.5	1.1	1.7		0.7
(15) shi.3	2.7	1.5	3.7	3.0	5.5	2.4	1.4	2.2	1.1	2.4	1.4	0.9	2.0	0.7	

Metric values are very small, indicating excellent agreement.

TPV26 (Elastic) 2D Slip Rate at faultst150dp100

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
(1) barall		6.6	10.5	11.5	13.0	5.5	6.4	17.7	15.9	8.3	135.6	10.2	11.6	13.4	11.6
(2) barall.2	6.6		9.3	5.3	16.3	6.8	2.0	16.1	9.9	4.4	135.6	4.1	11.6	7.6	11.6
(3) chen	10.5	9.3		10.5	22.0	13.8	9.7	8.0	12.3	12.7	136.1	10.3	5.5	9.9	5.5
(4) chen.2	11.5	5.3	10.5		20.6	11.3	5.9	15.4	5.3	6.9	135.7	2.4	12.7	3.7	12.7
(5) duan	13.0	16.3	22.0	20.6		9.9	16.1	29.5	25.1	14.6	133.8	19.3	24.1	23.2	24.1
(6) duan.2	5.5	6.8	13.8	11.3	9.9		6.3	21.3	16.2	6.0	134.9	9.8	15.8	14.1	15.8
(7) kaneko	6.4	2.0	9.7	5.9	16.1	6.3		16.4	10.7	4.5	135.4	4.3	11.6	8.4	11.6
(8) ma	17.7	16.1	8.0	15.4	29.5	21.3	16.4		14.8	19.4	136.8	15.8	7.3	13.4	7.3
(9) ma.2	15.9	9.9	12.3	5.3	25.1	16.2	10.7	14.8		11.3	136.1	7.4	14.1	2.9	14.1
(10) roten	8.3	4.4	12.7	6.9	14.6	6.0	4.5	19.4	11.3		135.0	6.0	15.0	9.5	15.0
(11) roten.2	135.6	135.6	136.1	135.7	133.8	134.9	135.4	136.8	136.1	135.0		135.5	136.5	136.1	136.5
(12) roten.3	10.2	4.1	10.3	2.4	19.3	9.8	4.3	15.8	7.4	6.0	135.5		12.4	5.6	12.4
(13) shi	11.6	11.6	5.5	12.7	24.1	15.8	11.6	7.3	14.1	15.0	136.5	12.4		12.0	0.0
(14) shi.2	13.4	7.6	9.9	3.7	23.2	14.1	8.4	13.4	2.9	9.5	136.1	5.6	12.0		12.0
(15) shi.3	11.6	11.6	5.5	12.7	24.1	15.8	11.6	7.3	14.1	15.0	136.5	12.4	0.0	12.0	

Larger metric values indicate agreement is not as good as in the viscoplastic case.

TPV26 (Elastic) Process Zone Width

Process zone width (m)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	
faultst-050dp000	1744	1625	1676	1607	1560	1565	1636	1778	1600	1389	1464	1670	1584	1670	
faultst-050dp050	871	867	869	864	881	872	875	849	862	866	866	860	865	860	
faultst-050dp100		1	1	1	-		1				1		1		
faultst-050dp150	198	216	222	228	249	243	235	201	214	233	231	204	216	204	
faultst-150dp100	455	405	409	367	499	417	363	390	363	402	368	412	373	412	
faultst000dp100	609	605	605	605	659	618	601	571	600	625	609	593	597	593	•
faultst050dp100	454	405	411	366	499	417	365	393	363	402	368	409	373	409	•
faultst100dp100	332	274	297	225	380	296	256	274	219	272	229	302	236	302	•
faultst150dp000	2775	2814	2758	2827	2640	2692	2756	2748	2783	2528	2684	2684	2780	2684	
faultst150dp050	464	427	414	398	575	485	432	379	361	458	422	415	380	415	
faultst150dp100	297	233	267	192	343	268	232	244	179	243	385	264	195	264	•
faultst150dp150	212	168	206	143	250	201	174	190	135	184	138	203	150	203	

Red arrows identify stations at distances of 5, 10, 15, and 20 km from the hypocenter.

Process zone width decreases with increasing distance from the hypocenter.

TPV27 (Viscoplastic) Process Zone Width

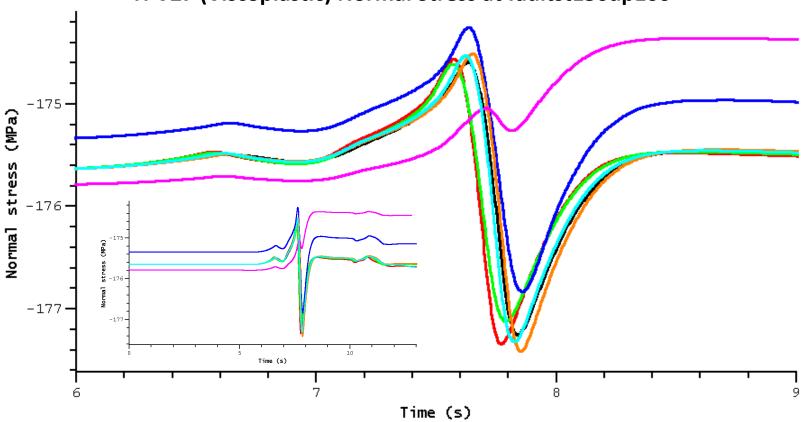
Process zone width (m)

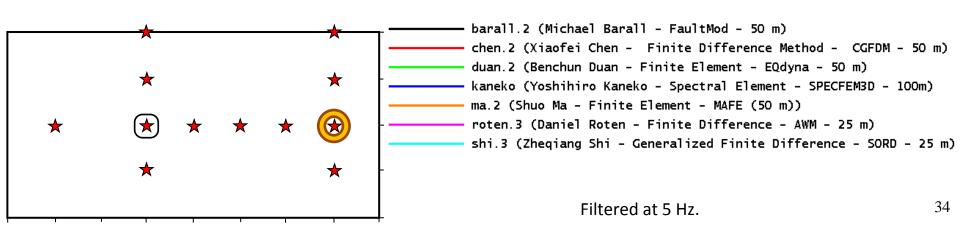
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
faultst-050dp000			1	1	1	1	1	1		-	1		-	
faultst-050dp050	932	920	904	910	942	929	936	929	925	846	857	908	921	922
faultst-050dp100			1	1	1	-	1	1			1		1	
faultst-050dp150	223	235	246	244	255	256	249	220	234	226	215	219	233	245
faultst-150dp100	487	476	449	447	512	480	467	467	460	478	466	477	463	462
faultst000dp100	594	598	593	594	615	608	602	588	596	598	597	594	594	596
faultst050dp100	486	476	451	445	510	481	473	471	461	481	469	476	464	463
faultst100dp100	437	421	394	390	470	431	426	414	405	424	414	422	408	409
faultst150dp000	-		1	1	ł	1	1	1		1	ł		1	
faultst150dp050	784	788	725	765	810	797	791	778	795	766	773	779	785	788
faultst150dp100	420	401	379	370	448	413	406	395	385	391	388	401	387	390
faultst150dp150	270	259	258	233	316	283	272	257	243	261	250	266	247	245

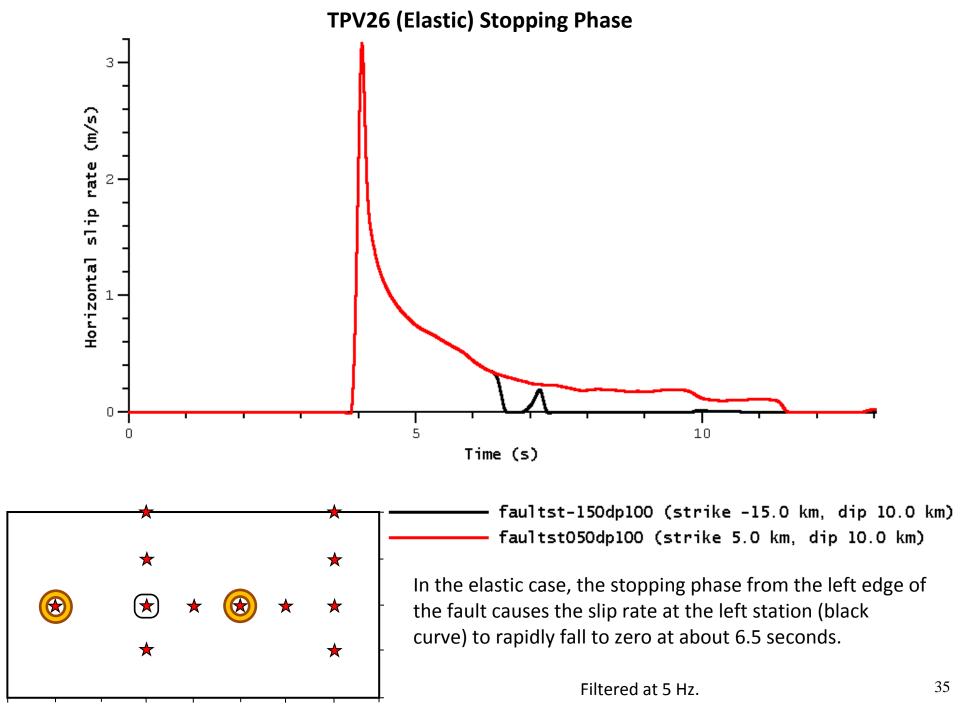
Red arrows identify stations at distances of 5, 10, 15, and 20 km from the hypocenter.

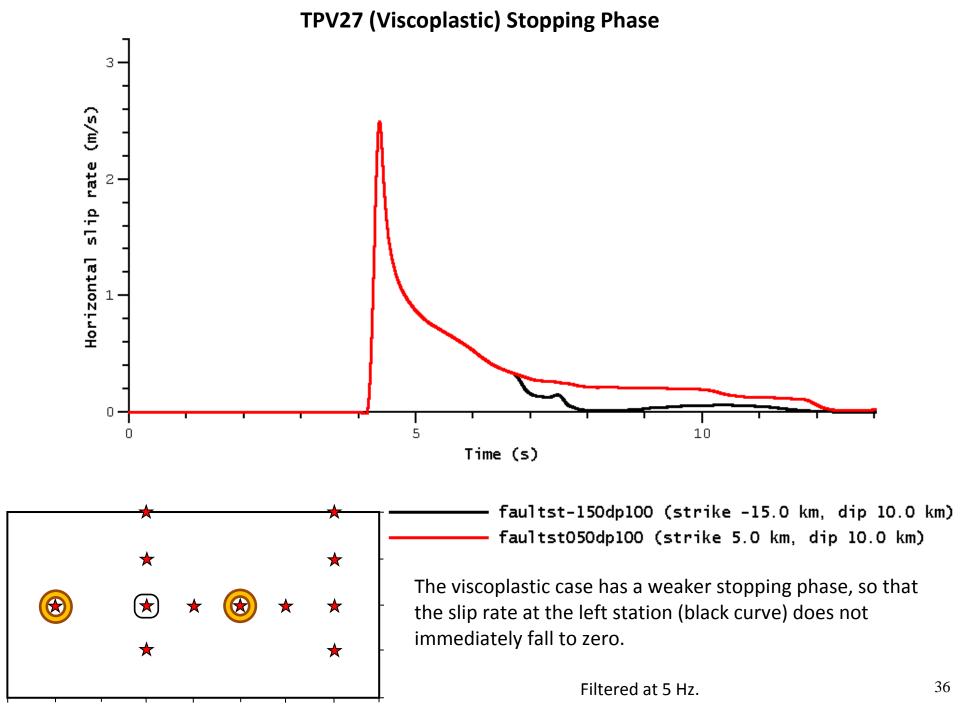
Process zone width decreases more slowly than in the elastic case, and may be leveling off.

TPV27 (Viscoplastic) Normal Stress at faultst150dp100

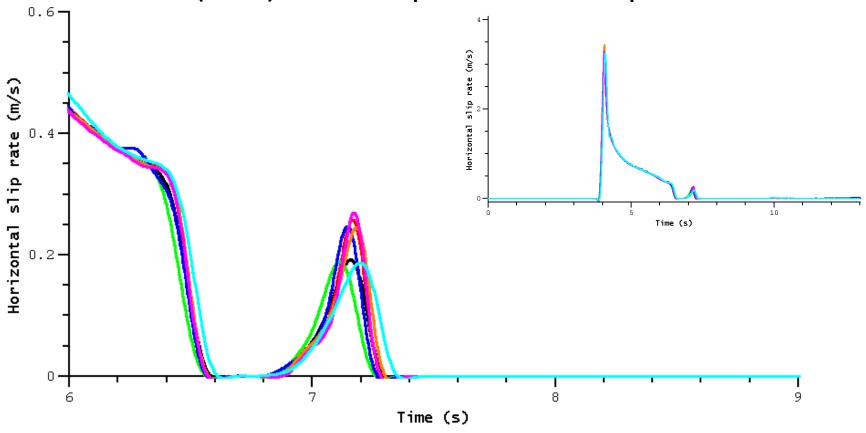


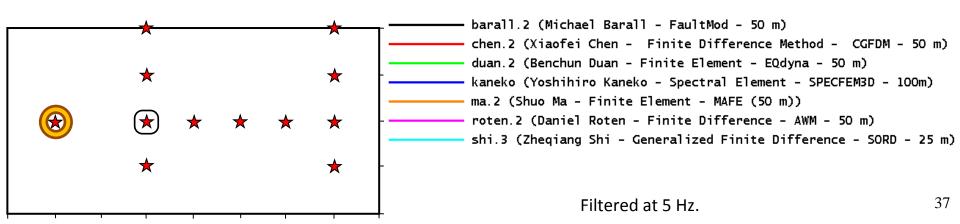




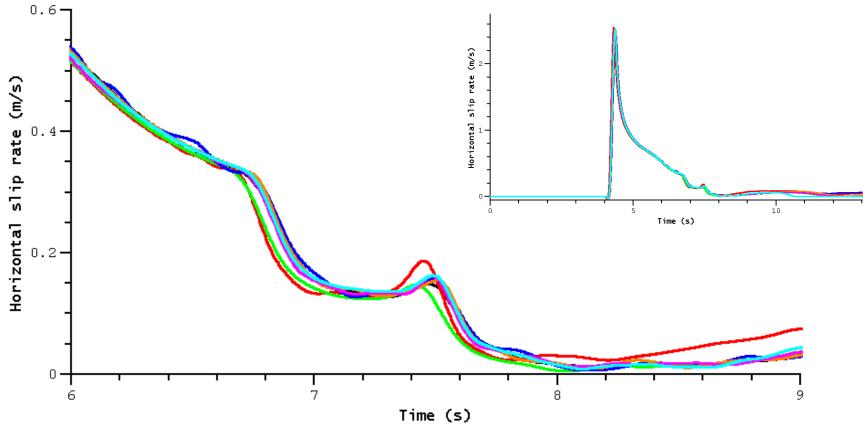


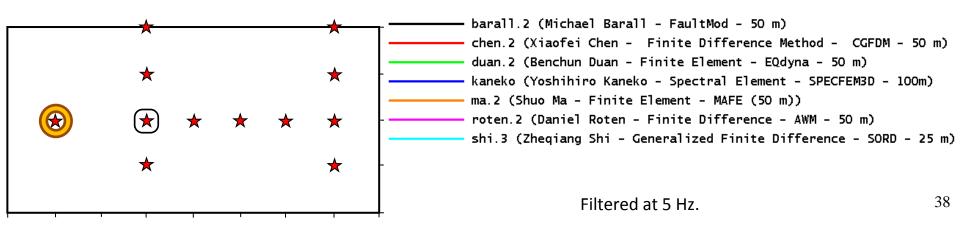
TPV26 (Elastic) Horizontal Slip Rate at faultst-150dp100



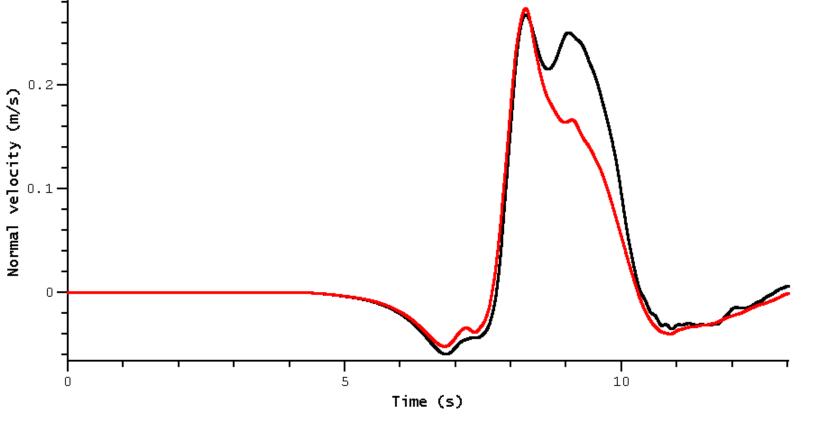


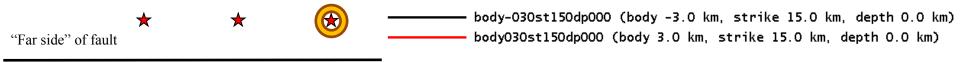
TPV27 (Viscoplastic) Horizontal Slip Rate at faultst-150dp100







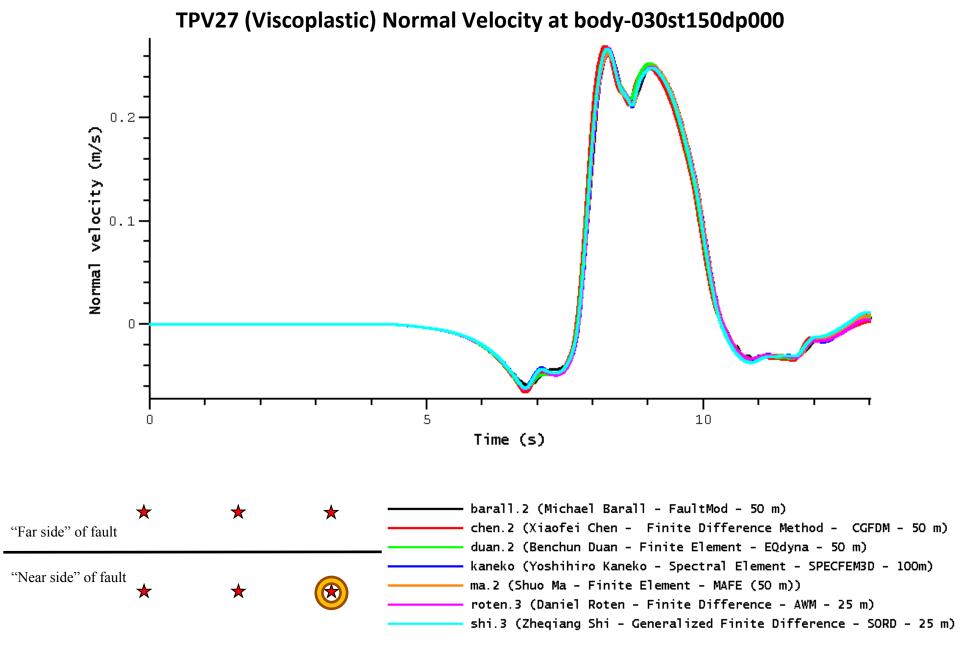


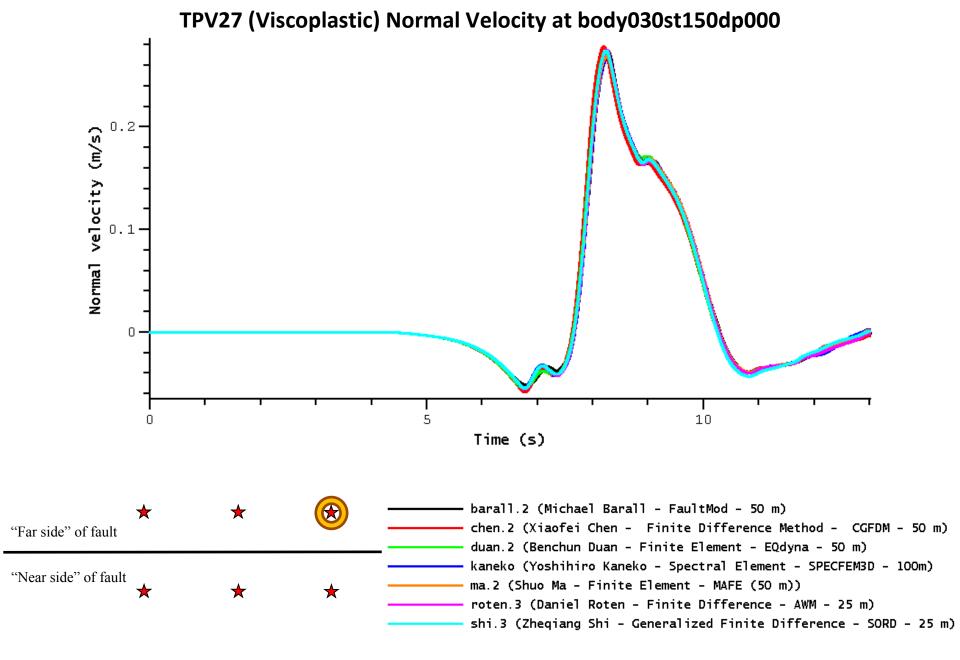


"Near side" of fault

*

In the elastic case, the two curves have to be identical due to symmetry. So the difference between the curves is a direct result of viscoplastic yielding in the bulk material.





Metrics for Off-Fault Stations

TPV26 (Elastic)

Summary across all users (click to drill down)

	3d-disp	3d-vel	t-shift
body-030st-050dp000	0.6	2.9	0.016
body-030st050dp000	0.8	2.9	0.013
body-030st150dp000	0.9	3.9	0.017
body030st-050dp000	<u>28.9</u>	<u>29.2</u>	0.157
body030st050dp000	0.7	2.9	0.013
body030st150dp000	<u>0.9</u>	3.8	0.017

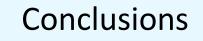
TPV27 (Viscoplastic)

Summary across all users (click to drill down)

	3d-disp	3d-vel	t-shift
body-030st-050dp000	<u>1.0</u>	<u>5.0</u>	0.040
body-030st050dp000	<u>0.9</u>	<u>2.9</u>	0.035
body-030st150dp000	<u>1.1</u>	<u>3.5</u>	0.044
body030st-050dp000	<u>1.0</u>	<u>5.4</u>	0.039
body030st050dp000	0.8	3.0	0.032
body030st150dp000	<u>1.0</u>	3.6	0.040

Off-Fault Stations								
body-030st-050dp000	body -3.0 km, strike -5.0 km, depth 0.0 km							
body-030st050dp000	body -3.0 km, strike 5.0 km, depth 0.0 km							
body-030st150dp000	body -3.0 km, strike 15.0 km, depth 0.0 km							
body030st-050dp000	body 3.0 km, strike -5.0 km, depth 0.0 km							
body030st050dp000	body 3.0 km, strike 5.0 km, depth 0.0 km							
body030st150dp000	body 3.0 km, strike 15.0 km, depth 0.0 km							

Fields								
3d-disp	Displacement vector							
3d-vel	Velocity vector							
t-shift	Time shift							



Conclusions

Benchmarks TPV26 and TPV27 demonstrate a variety of ways that viscoelasticity affects modeling results:

- Rupture propagation velocity and rupture front contours.
- Ability of the rupture to propagate into unfavorable areas of the fault.
- Variation of normal stress on the fault during the rupture.
- Dependence of peak slip rate on distance from the hypocenter.
- Dependence of process zone width on distance from the hypocenter.
- Strength of stopping phases.
- Difference in ground motion on opposite sides of the fault due to viscoplastic yielding.

The participating codes were in very good agreement on all these effects.