

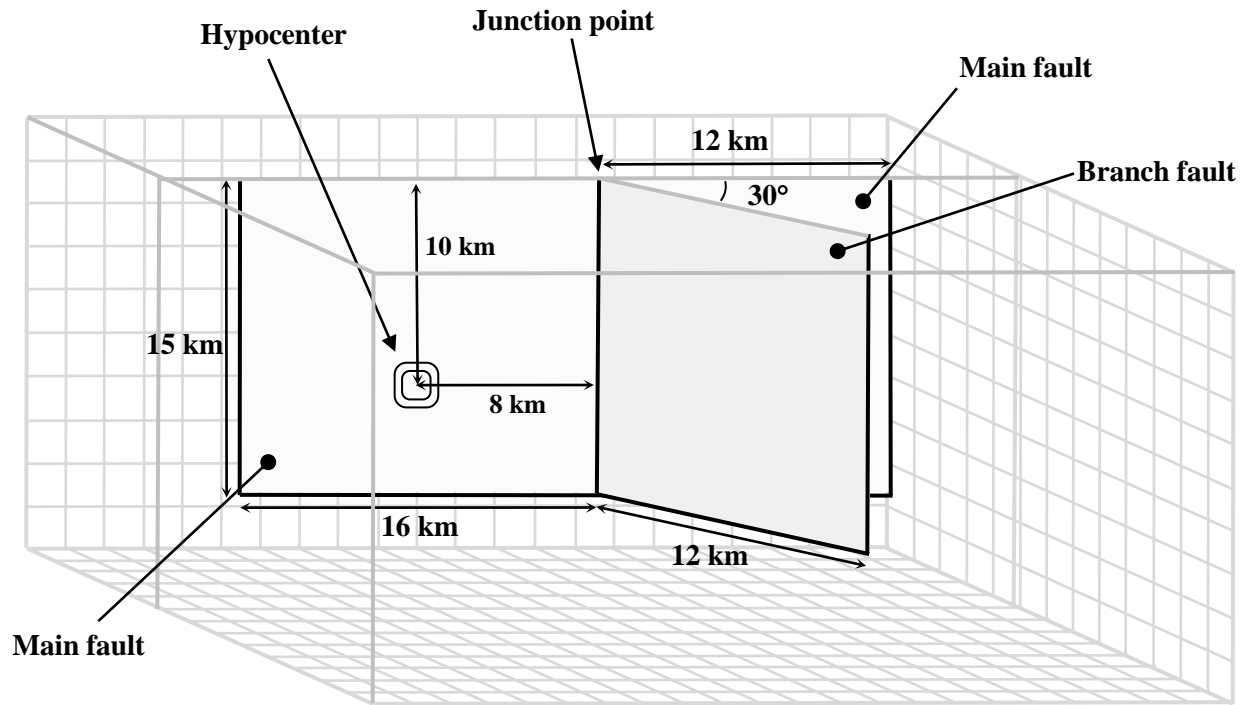
Branched Fault Benchmarks TPV24 and TPV25

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SCEC Dynamic Rupture Code Validation Workshop

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TPV24-25 Summary.

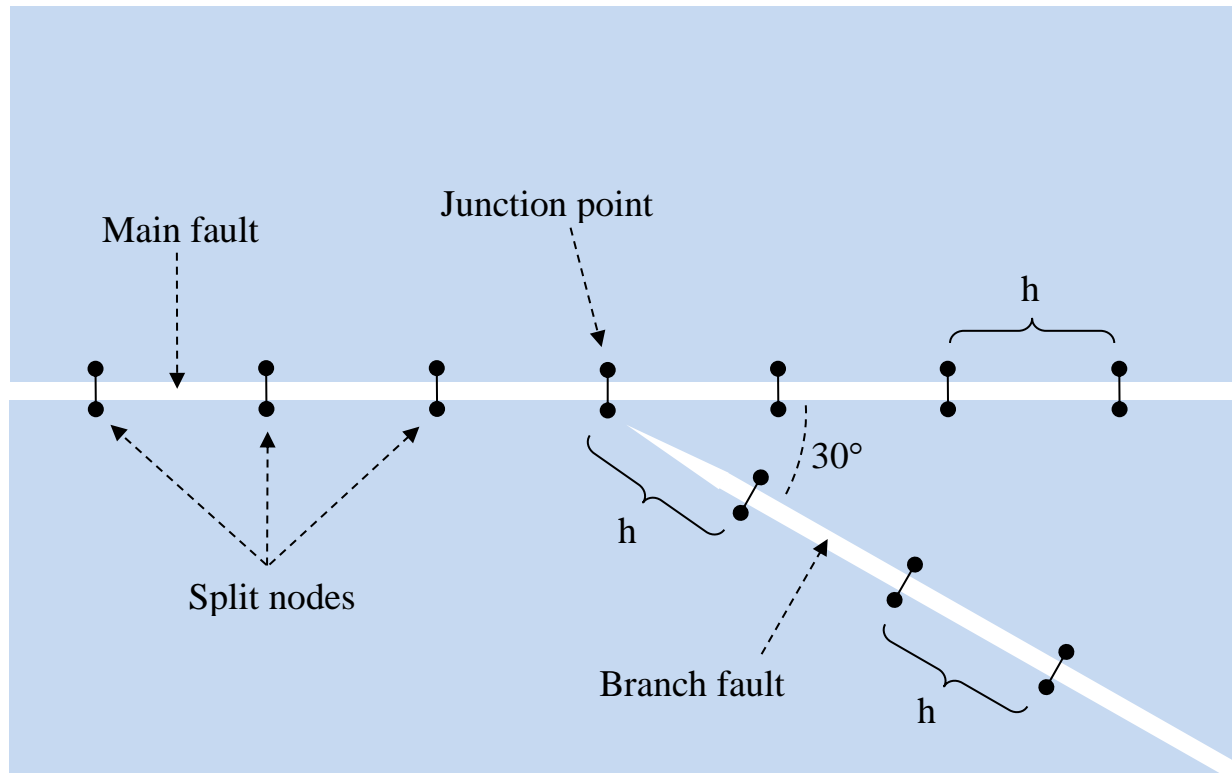


<i>Benchmark</i>	<i>Dimension</i>	<i>Rupture Type</i>	<i>Material Properties</i>
TPV24	3D	Right-lateral, releasing branch.	Linear elastic.
TPV25	3D	Left-lateral, restraining branch.	Linear elastic.

Requested resolutions: 100 m and 50 m.

Although these are linear elastic benchmarks, they are constructed like plastic benchmarks.

Junction Point Behavior

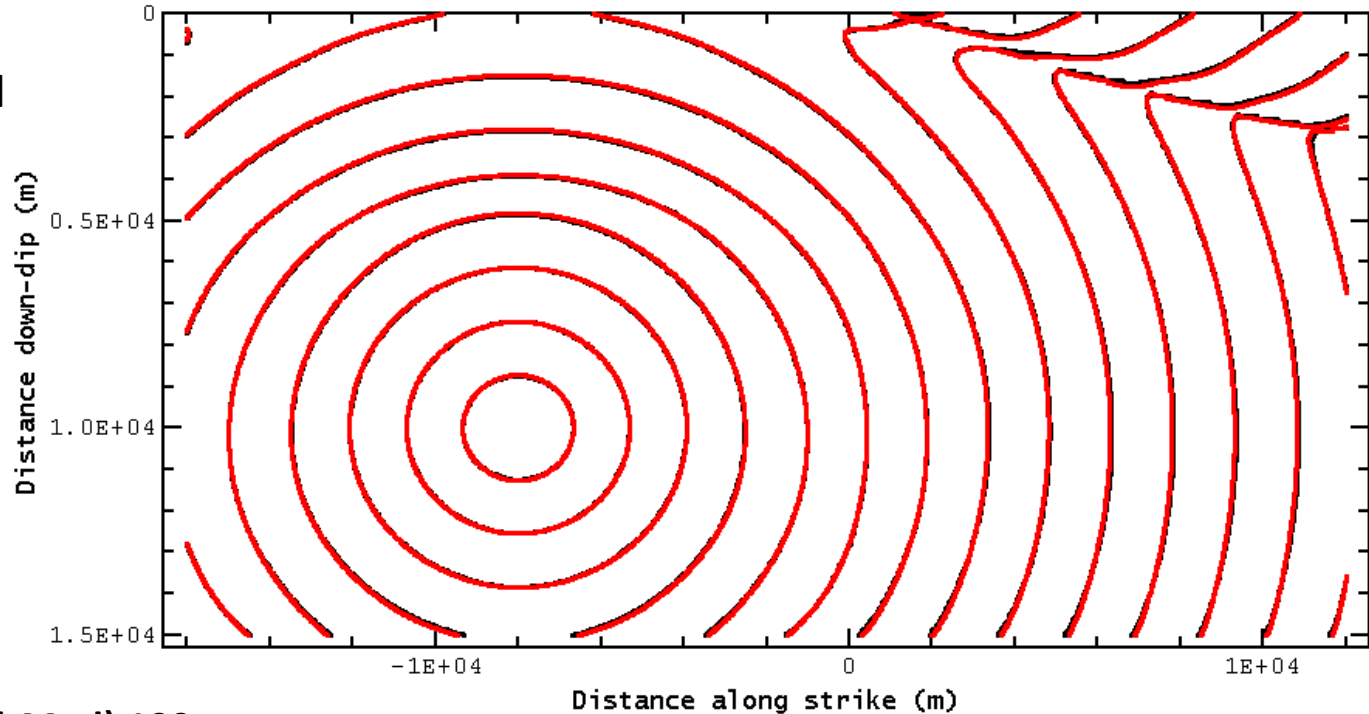


The boundary condition is that slip on the branch fault goes to zero at the junction point.

The picture shows a possible implementation for a finite-element code that uses split nodes. The junction point behaves as an ordinary split-node on the main fault. Other types of code may implement the junction point in different ways.

Issues in the Design of Branched Fault Benchmarks

Loss of Numerical Precision.

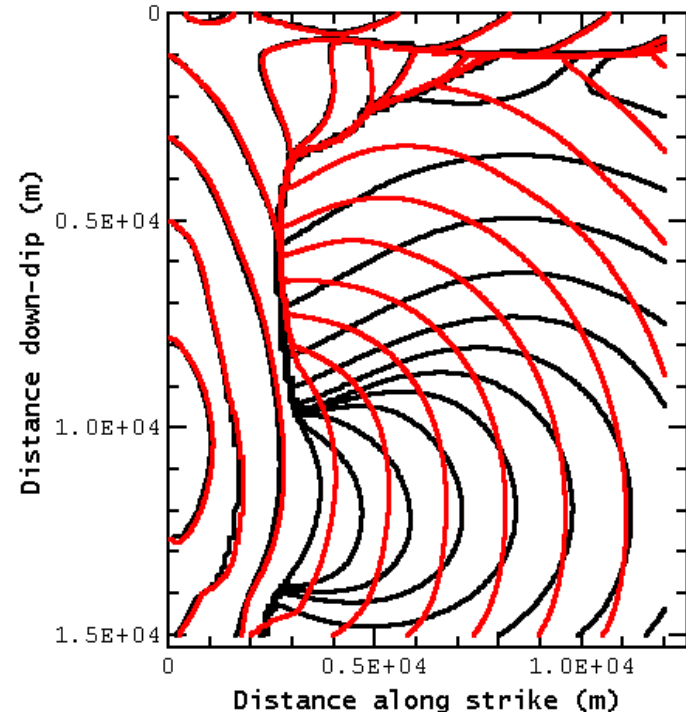


Finite Element (FaultMod) 100 m
Finite Element (FaultMod) 50 m

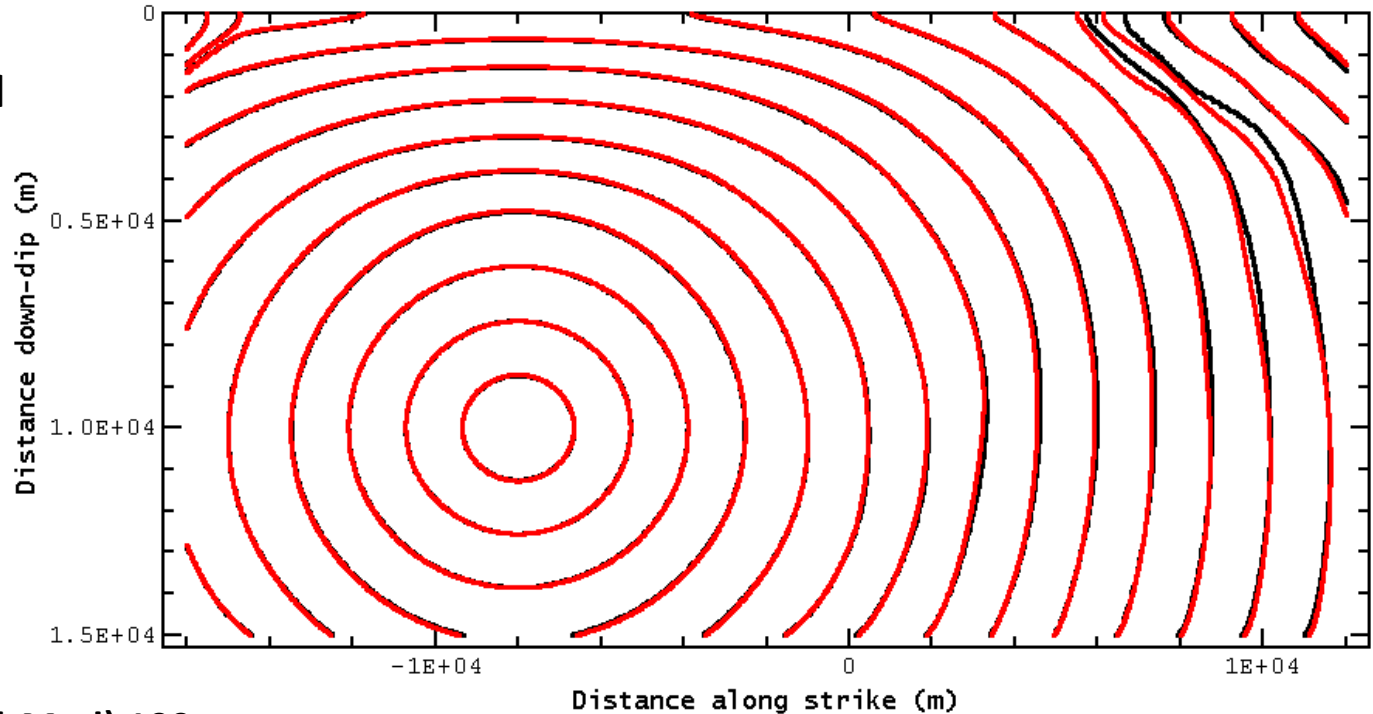
This is an extreme example of loss of numerical precision on the branch fault.

The 50 m and 100 m results begin to diverge in the part of the branch fault where stresses are close to the minimum needed for the rupture to propagate.

(Figure shows a right-lateral case that differs from TPV24 in initial stress tensor and other ways.)



Loss of Numerical Precision.

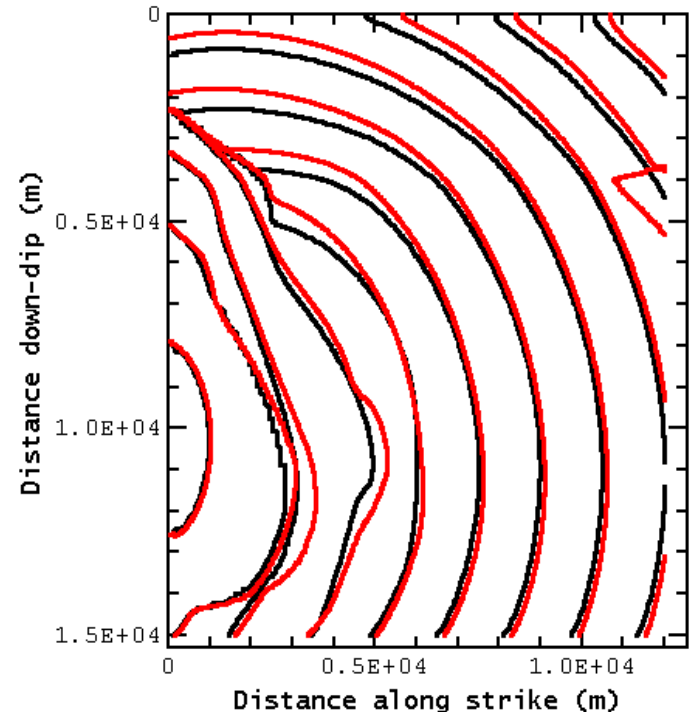


Finite Element (FaultMod) 100 m
Finite Element (FaultMod) 50 m

This is another example of loss of numerical precision on the branch fault.

Divergence occurs where there is just barely enough stress, or just barely not enough stress, to sustain the rupture.

(Figure shows a left-lateral case that differs from TPV25 in initial stress tensor and other ways.)



Nucleation

Day Radius and the Problem of Nucleation.

Day (1982) obtained the following formula, which gives the minimum radius R_D that a circular rupture must have, such that it is energetically favorable for the rupture to expand.

The diagram shows the formula for the Day radius R_D with arrows indicating the contribution of each parameter:

$$R_D = \frac{7\pi\mu(\tau_s - \tau_d)D_c}{24(\tau - \tau_d)^2}$$

Labels and their connections to the formula:

- Shear modulus μ (top left) points to μ in the numerator.
- Static yield stress τ_s (top middle) points to τ_s in the numerator.
- Slip-weakening critical distance D_c (top right) points to D_c in the numerator.
- Day radius R_D (bottom left) points to R_D on the left side of the equation.
- Initial stress τ (bottom middle) points to τ in the denominator.
- Dynamic sliding stress τ_d (bottom right) points to τ_d in the denominator.

For typical parameter values used in spontaneous rupture simulations, the Day radius is about 3 to 4 km.

The nucleation problem is that, somehow, we must impose an artificial mechanism to get the size of the rupture up to the Day radius, at which point the rupture can be self-sustaining.

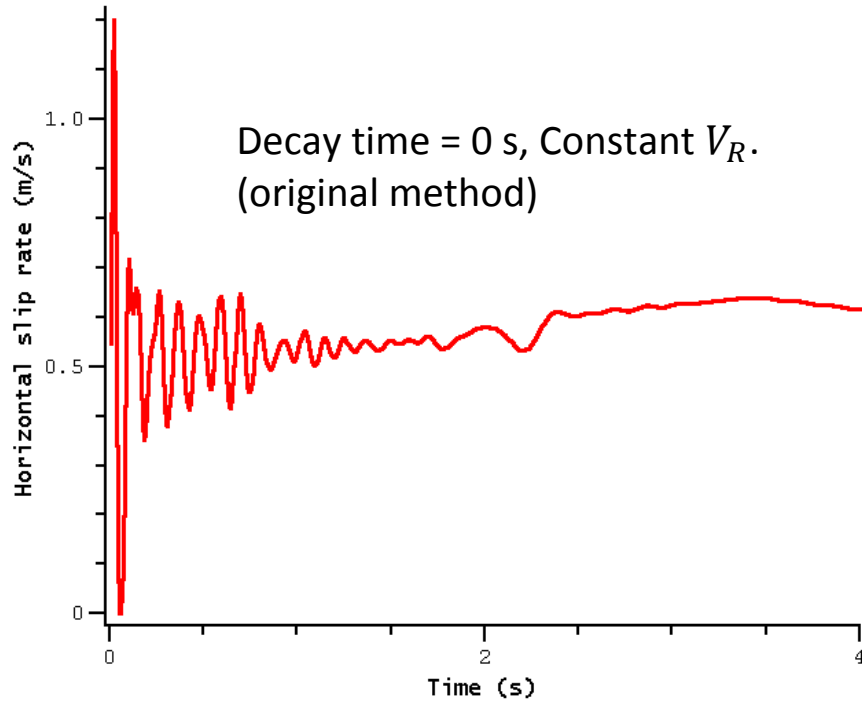
Pros and Cons of Two Nucleation Methods.

Overstress Method: Apply high initial shear stress in the nucleation zone.

- Pro: Simple to implement.
- Pro: Nucleation zone can be small, by making the initial stress high enough.
- Con: Small changes in the nucleation process affect the entire fault.
- Con: Injects a lot of excess energy into the rupture.
- Con: Much higher slip in nucleation zone than elsewhere on the fault.
- Con: Not compatible with a regional stress tensor (and so not usable with plasticity).

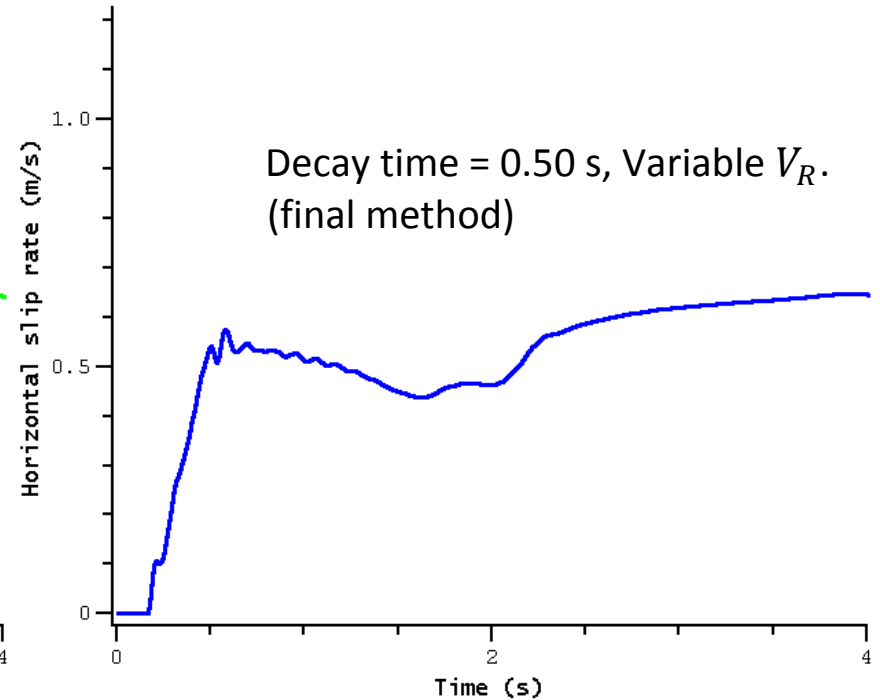
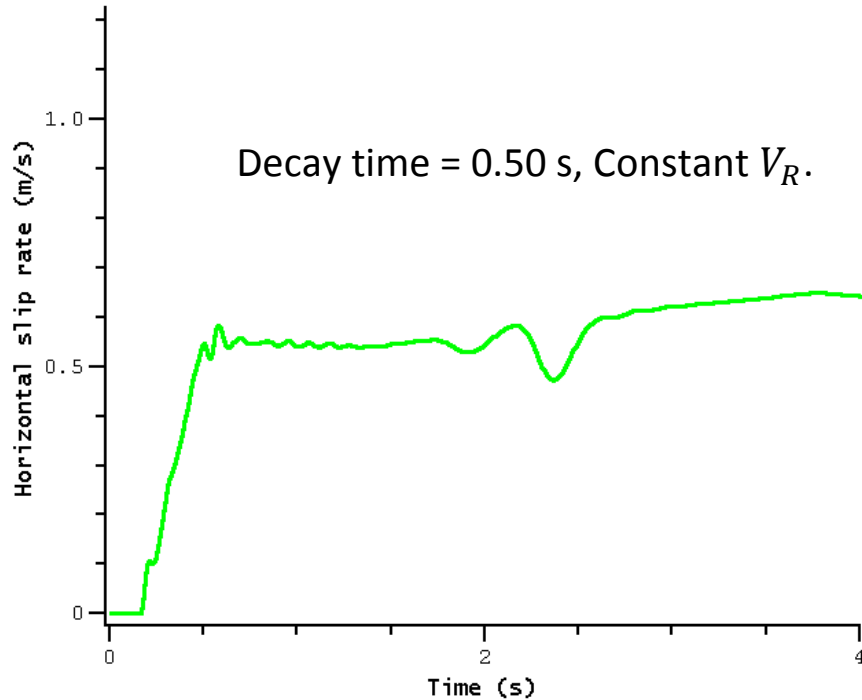
Forced-Rupture Method: Reduce the friction in the nucleation zone.

- Pro: Small changes in the nucleation process tend not to affect the entire fault.
- Pro: Does not produce higher slip in the nucleation zone.
- Pro: Does not require alteration of stress (and so compatible with a regional stress tensor and with plasticity).
- Con: More complicated to implement.
- Con: Requires large nucleation zone, at least the size of the Day radius.



Nucleation Methods Considered For Benchmarks TPV22-25.

Plots show horizontal slip rate at
the hypocenter.



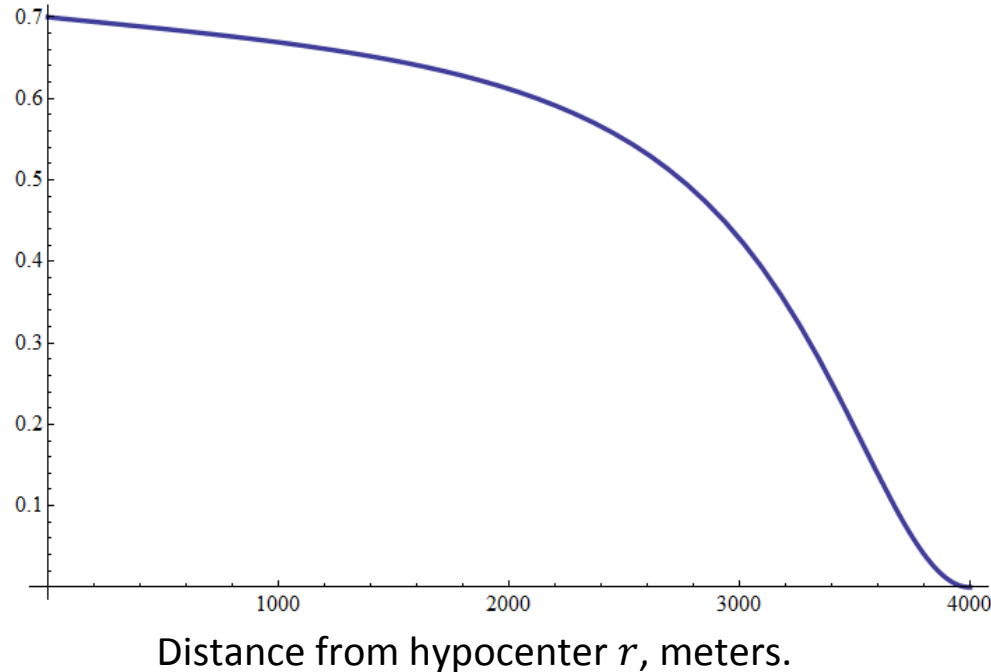
Nucleation Parameters.

Radius of nucleation zone $r_{\text{crit}} = 4000 \text{ m}$

$$\text{Time of forced rupture } T = \begin{cases} \frac{r}{0.7 V_S} + \frac{0.081 r_{\text{crit}}}{0.7 V_S} \left(\frac{1}{1-(r/r_{\text{crit}})^2} - 1 \right), & \text{if } r < r_{\text{crit}} \\ 1.0\text{E}+9, & \text{if } r \geq r_{\text{crit}} \end{cases}$$

Forced rupture decay time $t_0 = 0.50 \text{ s}$

Speed of forced rupture V_R/V_S .



TPV24-25 Design

Friction Parameters.

$$\mu_s = 0.18$$

$$\mu_d = 0.12$$

$$d_0 = 0.30 \text{ m}$$

$$C_0 = \begin{cases} 0.30 \text{ MPa} + (0.000675 \text{ MPa/m})(4000 \text{ m} - \text{depth}), & \text{if depth} \leq 4000 \text{ m} \\ 0.30 \text{ MPa}, & \text{if depth} \geq 4000 \text{ m} \end{cases}$$

Friction coefficients are low because of the high initial normal stress, which is lithostatic.

Cohesion tapers from 3.0 MPa at the earth's surface, to 0.3 MPa at depths of 4000 m or greater.

Cohesion in the upper 4 km suppresses free surface effects.

Initial Stress Tensor and Fluid Pressure.

$$P_f = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

$$\sigma_{11} = -(2670 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

$$\sigma_{22} = \begin{cases} b_{22} (\sigma_{11} + P_f) - P_f, & \text{if depth} \leq 15600 \text{ m} \\ \sigma_{11}, & \text{if depth} > 15600 \text{ m} \end{cases}$$

$$\sigma_{33} = \begin{cases} b_{33} (\sigma_{11} + P_f) - P_f, & \text{if depth} \leq 15600 \text{ m} \\ \sigma_{11}, & \text{if depth} > 15600 \text{ m} \end{cases}$$

$$\sigma_{23} = \begin{cases} b_{23} (\sigma_{11} + P_f), & \text{if depth} \leq 15600 \text{ m} \\ 0, & \text{if depth} > 15600 \text{ m} \end{cases}$$

$$\sigma_{13} = \sigma_{12} = 0$$

Initial Stress Tensor Coefficients		
<i>Coefficient</i>	<i>Value for TPV24</i>	<i>Value for TPV25</i>
b_{22}	0.926793	1.119338
b_{33}	1.073206	0.880661
b_{23}	-0.169029	0.138704

On-Fault Stations.

Modelers are asked to submit slip, slip rate, and stress as a function of time, for 8 stations on the main fault (top) and 6 stations on the branch fault (bottom).

In addition, modelers are asked to submit the time at which each point on the fault begins to slip, from which we construct rupture contour plots.

Earth's Surface / Top of Fault

This diagram shows a fault system with a main fault and a branch fault. The main fault is a horizontal line from -16.0 km to +12.0 km along-strike, with a depth from 0.0 km to +15.0 km down-dip. A vertical dashed line at 0.0 km along-strike marks the 'Junction point'. Eight red stars represent stations: three on the surface (0.0 km depth) at -8.0 km, -2.0 km, and +9.0 km; one at -8.0 km, +5.0 km depth; and four at +10.0 km depth at -8.0 km, -2.0 km, +2.0 km, and +9.0 km. The station at -8.0 km, +10.0 km is circled.

Distance down-dip

Distance along-strike

Distance along-strike (km)	Distance down-dip (km)	Station Type
-8.0	0.0	Surface
-2.0	0.0	Surface
+9.0	0.0	Surface
-8.0	+5.0	Intermediate
-8.0	+10.0	Branch Fault
-2.0	+10.0	Branch Fault
+2.0	+10.0	Branch Fault
+9.0	+10.0	Branch Fault

Earth's Surface / Top of Fault

This diagram shows a branch fault system. The main fault is a horizontal line from 0.0 km to +12.0 km along-strike, with a depth from 0.0 km to +15.0 km down-dip. A vertical dashed line at 0.0 km along-strike marks the 'Junction point'. Six red stars represent stations: two on the surface (0.0 km depth) at 0.0 km and +9.0 km; one at 0.0 km, +5.0 km depth; and three at +10.0 km depth at 0.0 km, +2.0 km, and +9.0 km.

Distance down-dip

Distance along-strike

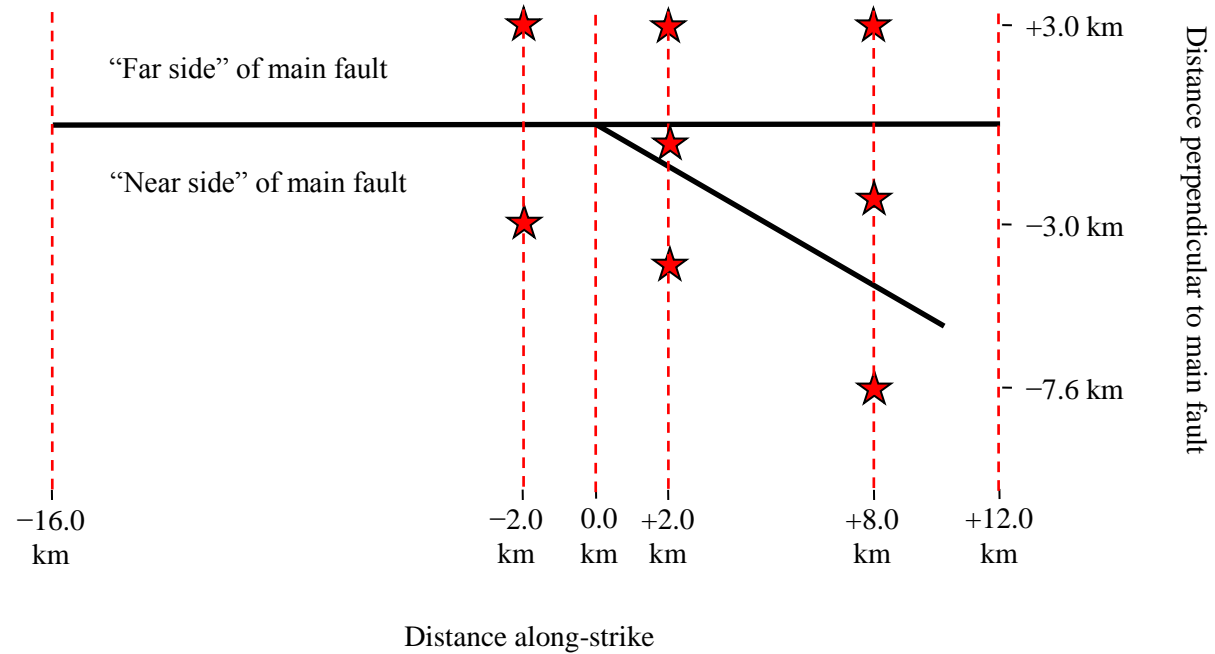
Distance along-strike (km)	Distance down-dip (km)	Station Type
0.0	0.0	Surface
+9.0	0.0	Surface
0.0	+5.0	Intermediate
0.0	+10.0	Branch Fault
+2.0	+10.0	Branch Fault
+9.0	+10.0	Branch Fault

Distance along-strike

15

Off-Fault Stations

Modelers are asked to submit displacement and velocity as a function of time, for 8 stations on the earth's surface.

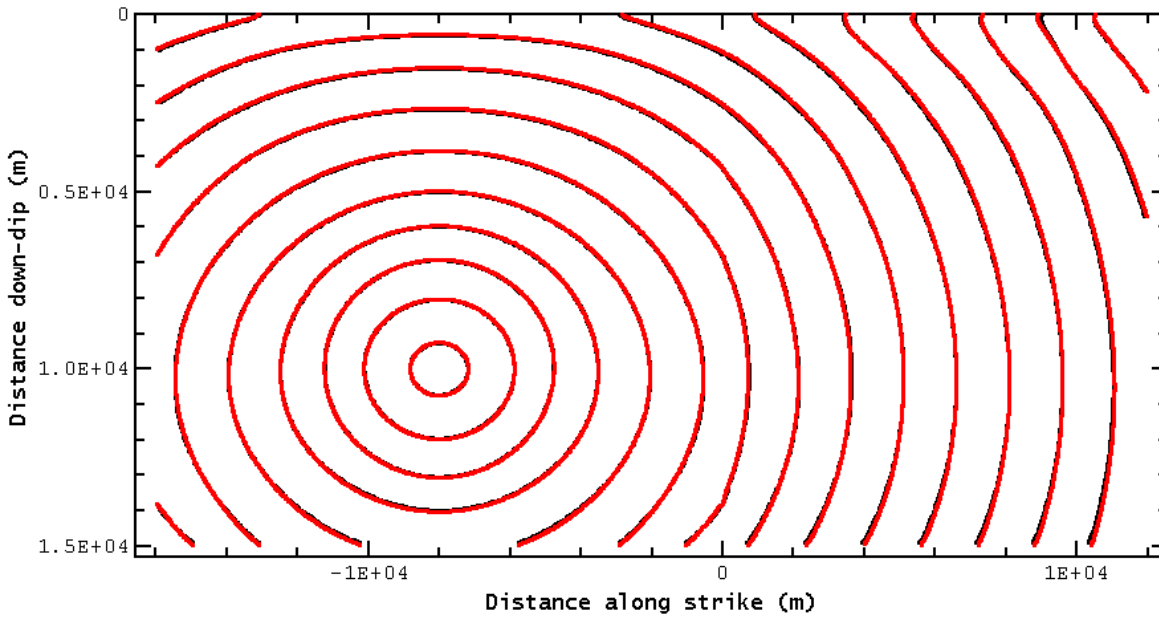


Changes in Branched-Fault Benchmarks from 2012 to 2013

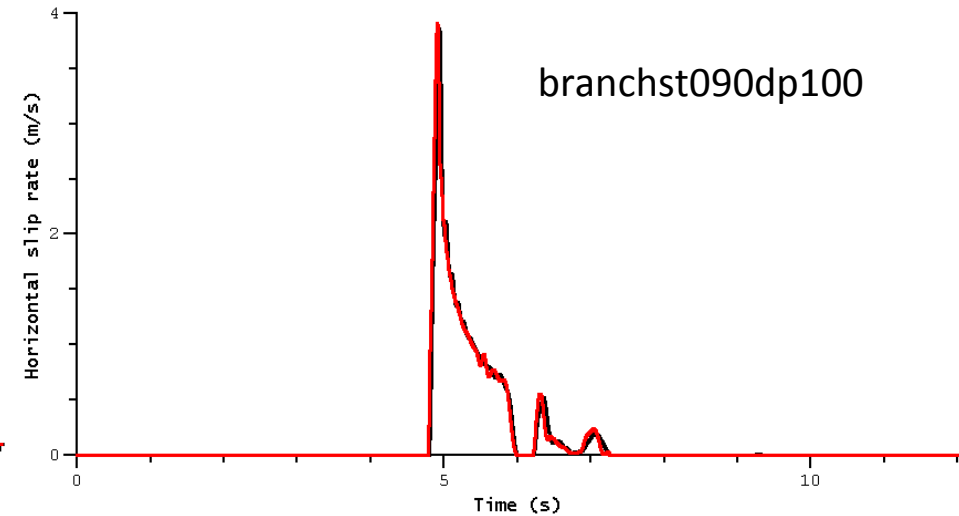
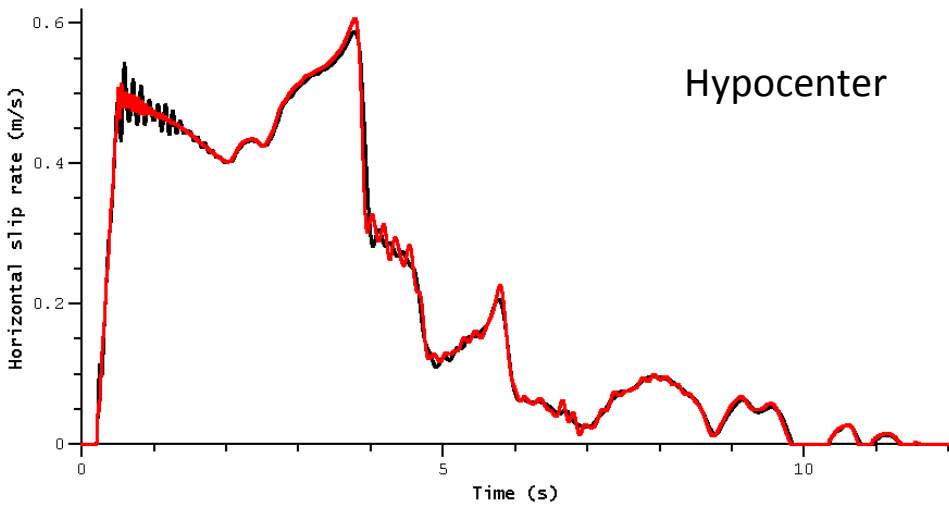
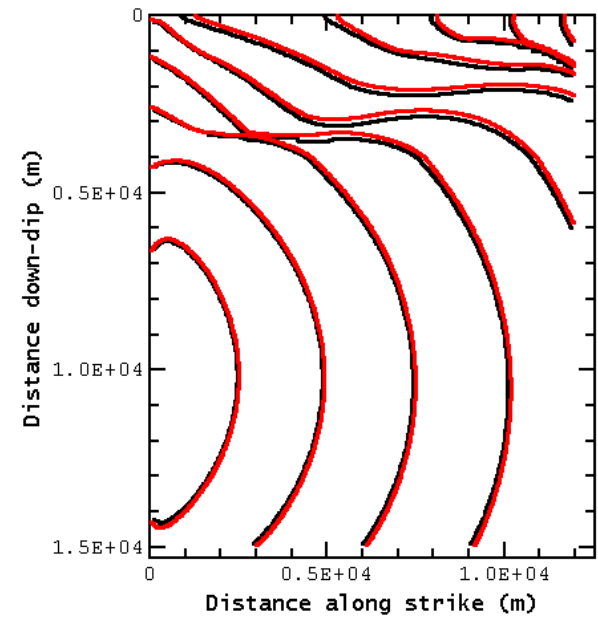
- The branch intersection definition changed, to say that slip on the branch goes to zero at the intersection, instead of saying that the branch ends one element away from the main fault. (This gives modelers more freedom to implement the branch in a way that is best for their individual codes.)
- The nucleation method is changed, to have a smoother nucleation with less unwanted oscillations.
- The regional stress field is changed, to being neutral instead of being strongly extensional. (This year the average horizontal stress equals the lithostatic stress, while last year it was about half the lithostatic stress.)
- The difference between static and dynamic coefficients is reduced by a factor of 8, which greatly increases the size of the cohesive zone, making it easier to resolve. (This year $\mu_s = 0.18$ and $\mu_d = 0.12$, while last year we had $\mu_s = 0.60$ and $\mu_d = 0.12$.)
- The slip-weakening critical distance is reduced, from 0.40 m to 0.30 m.
- This year, the convergence of each benchmark was tested by running 50 m and 100 m cases prior to publishing the benchmark. Then, modelers were asked to run both 50 m and 100 m cases to test the convergence properties of each code. Last year's benchmarks were run only at 100 m resolution.

TPV24 Results — 50 vs. 100 Meters

Main Fault

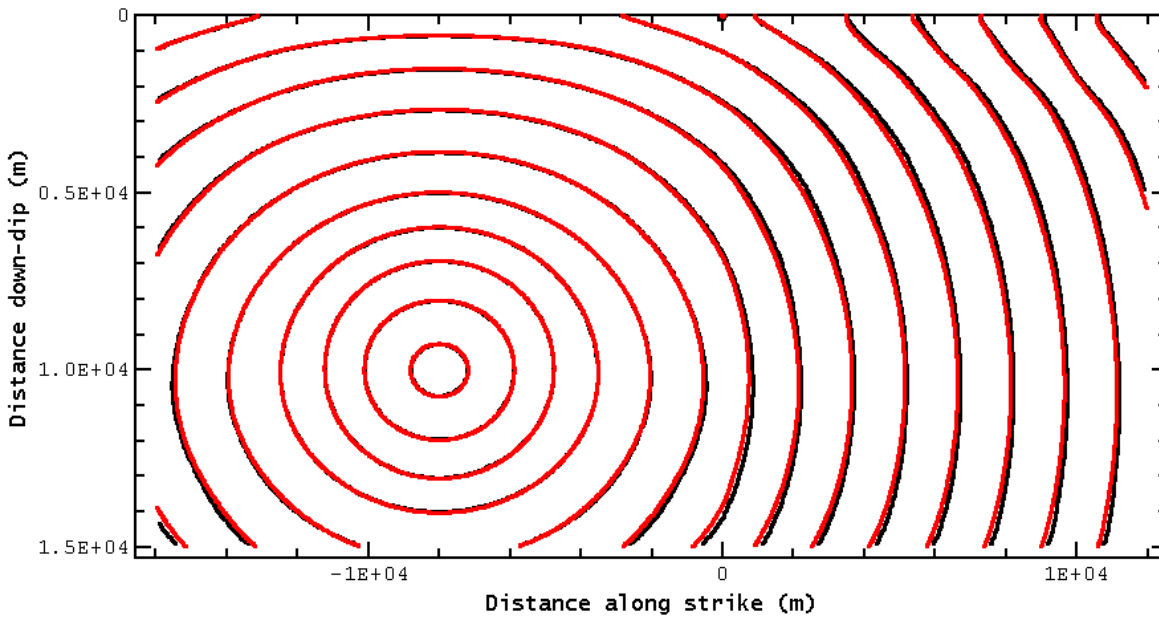


Branch Fault

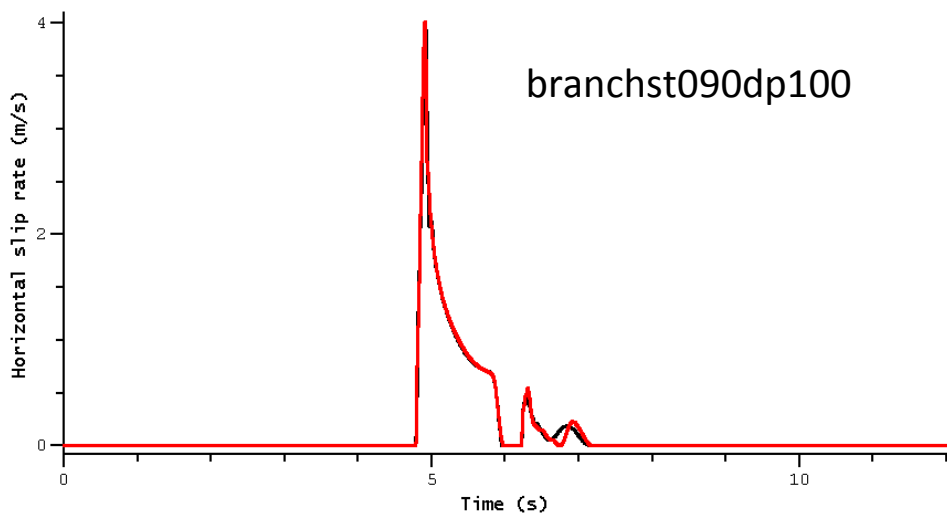
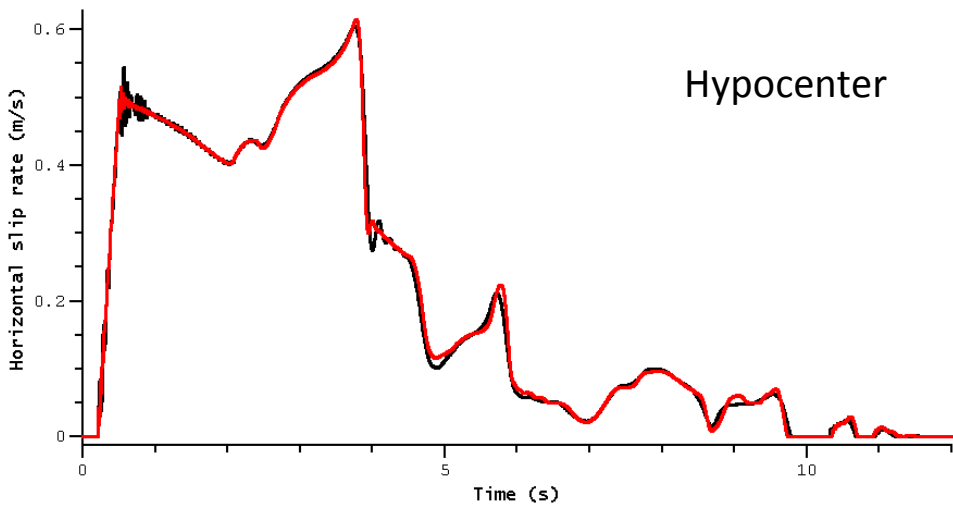
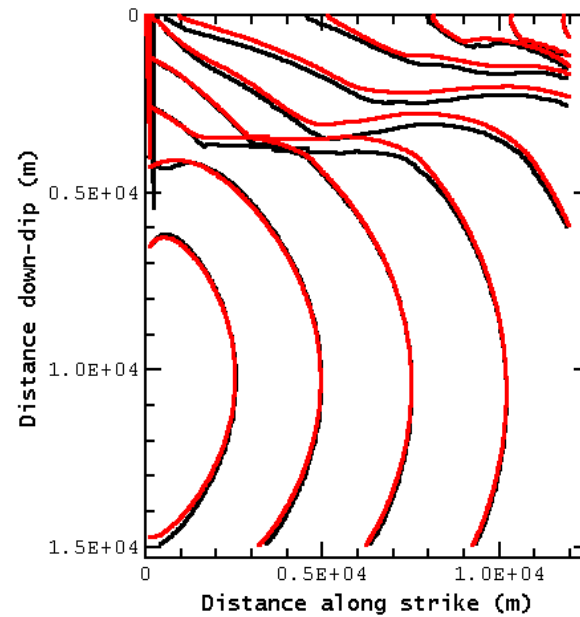


- barall (Michael Barall - Finite Element - FaultMod - 100 m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)

Main Fault

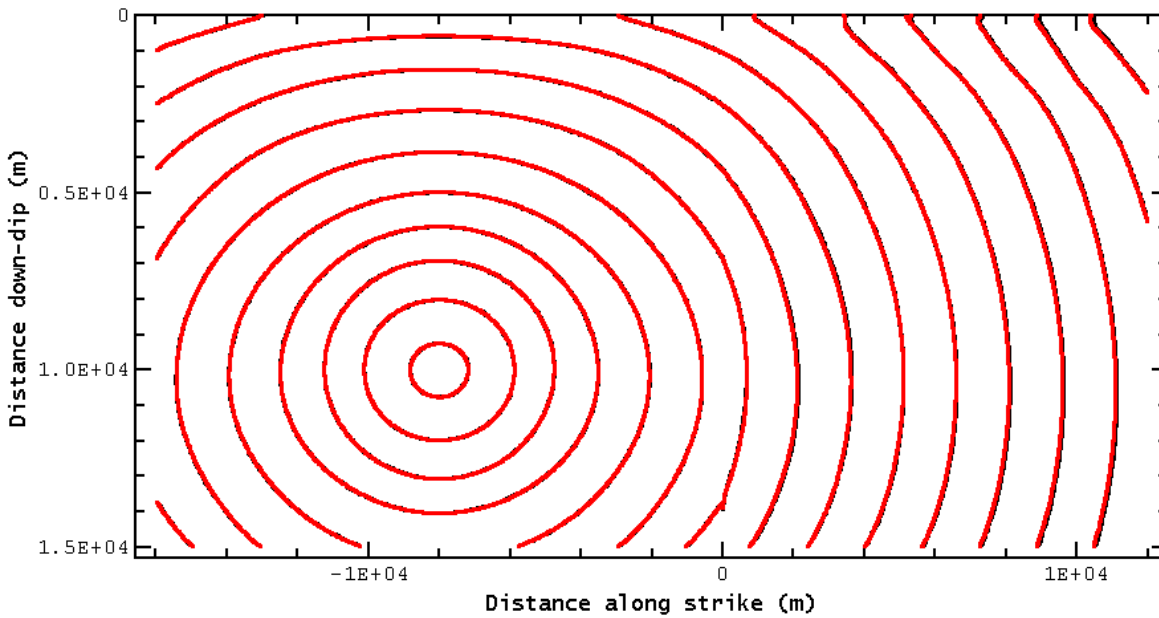


Branch Fault

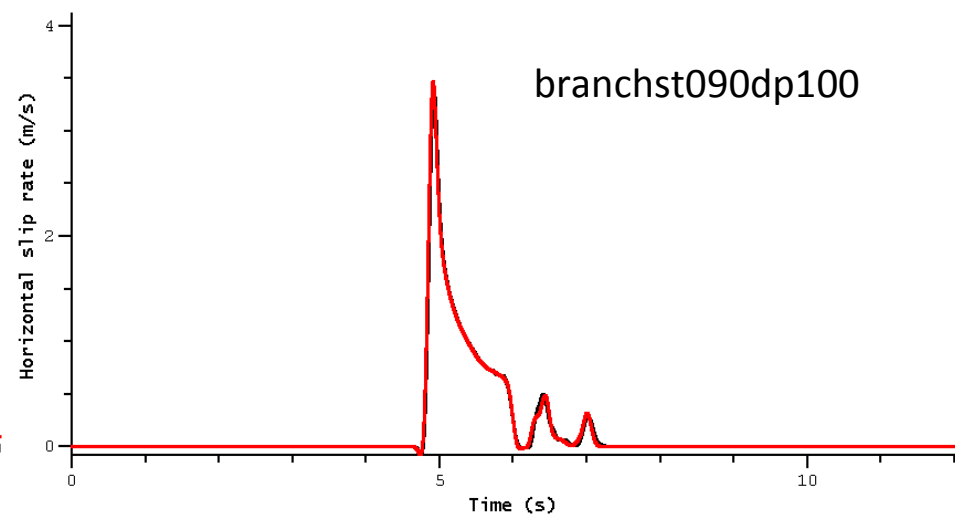
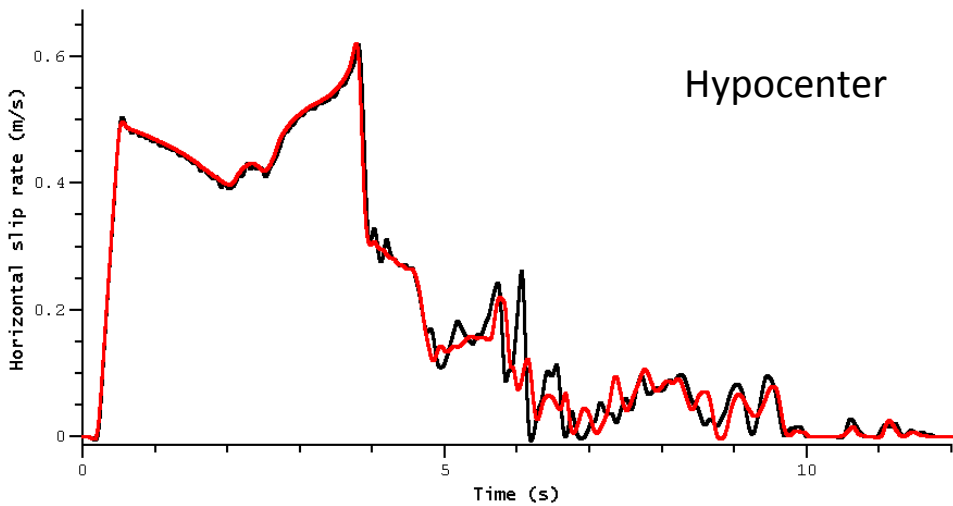
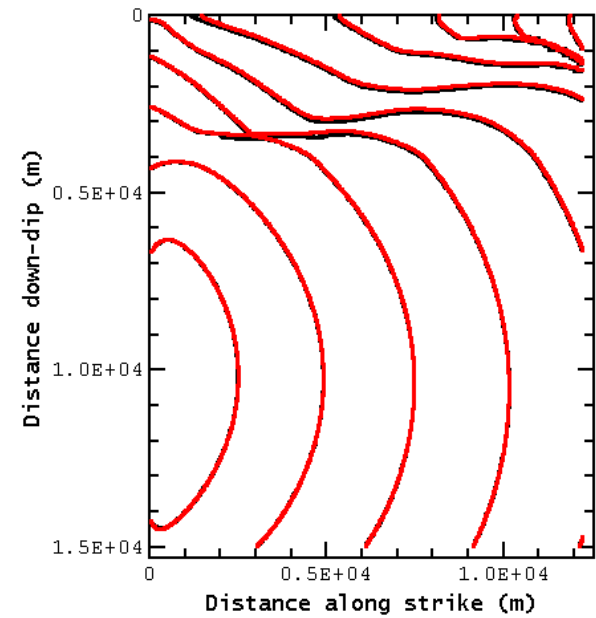


- duan (Benchun Duan - Finite Element - EQdyna - 100 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)

Main Fault

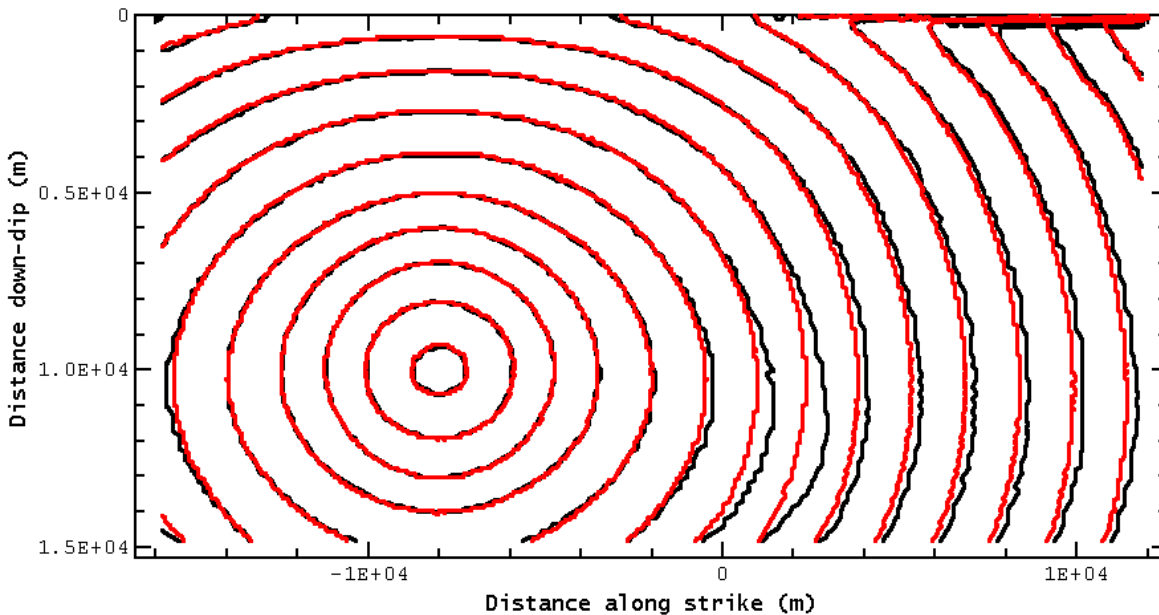


Branch Fault

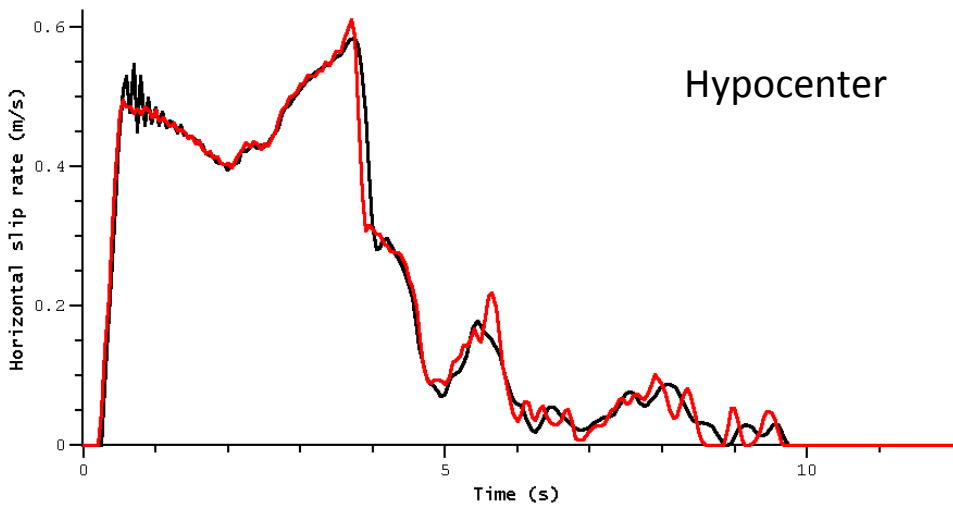
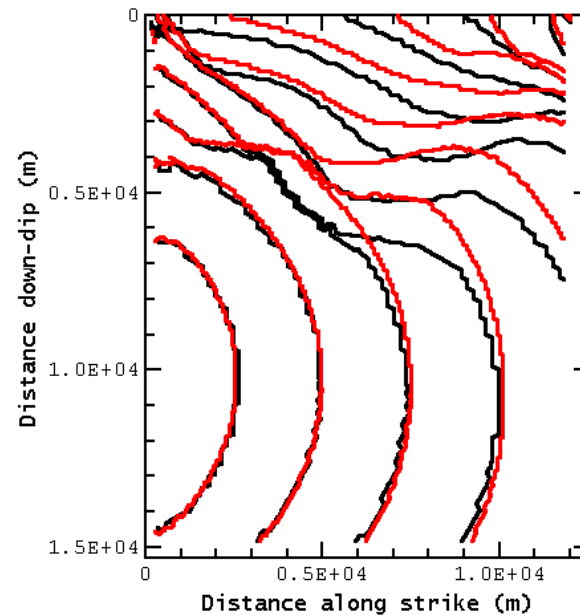


- somala (Surendra Somala - Spectral Element - SESAME (100m))
- somala.2 (Surendra Somala - Spectral Element - SESAME (50m))

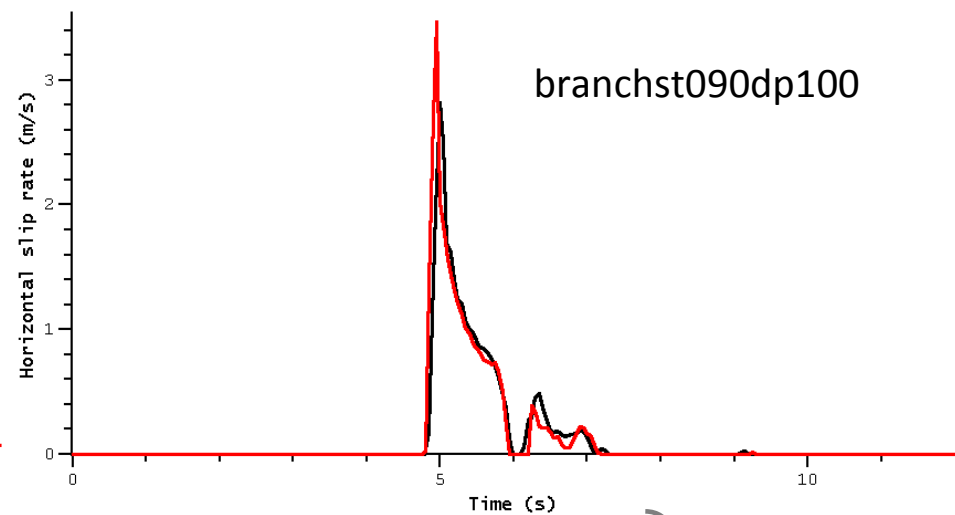
Main Fault



Branch Fault



Hypocenter



branchst090dp100

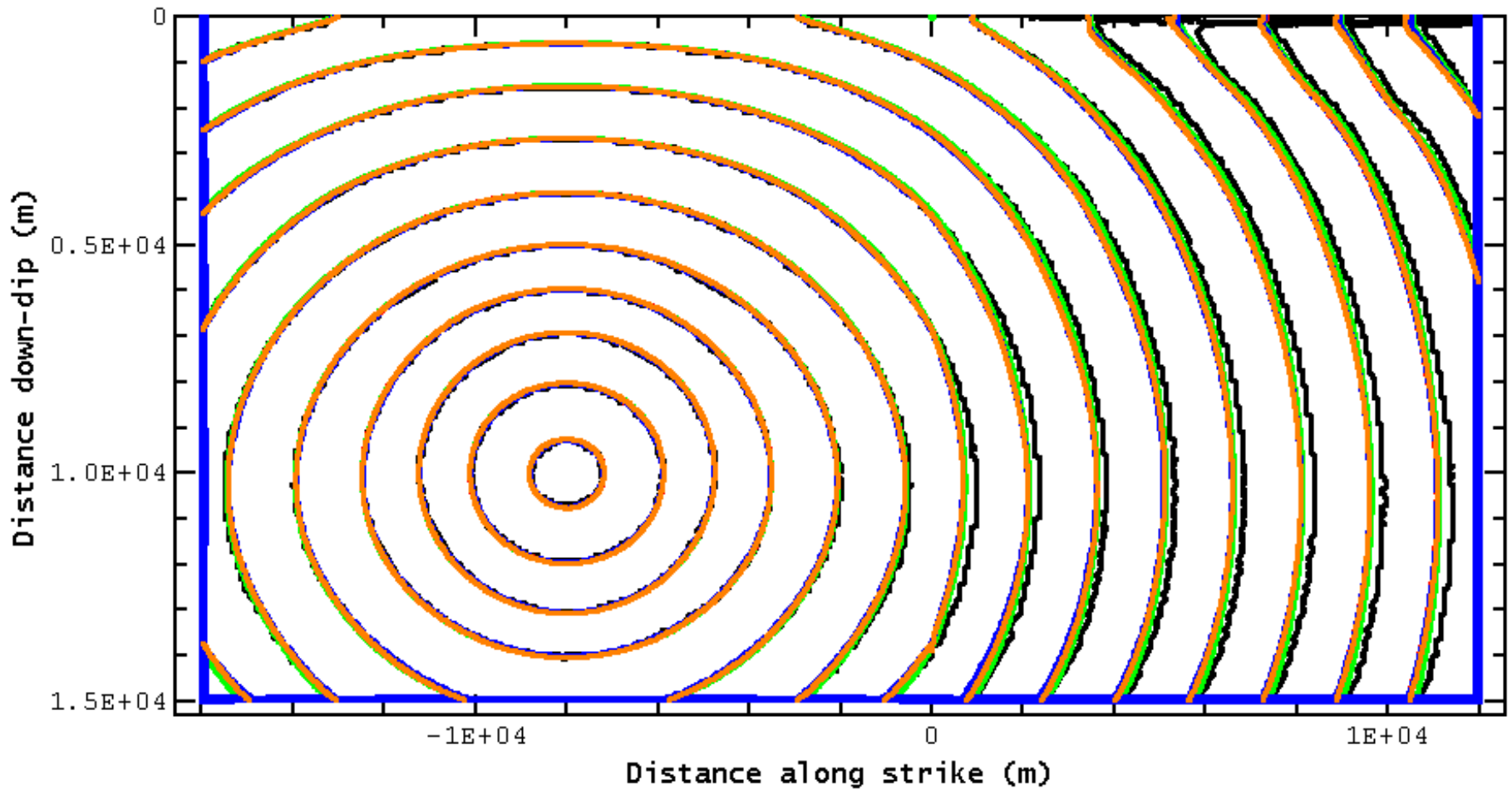
aagaard (Brad Aagaard - PyLith v1.9.0a - Tet4 200m)

aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)

} 200 vs. 100 m comparison

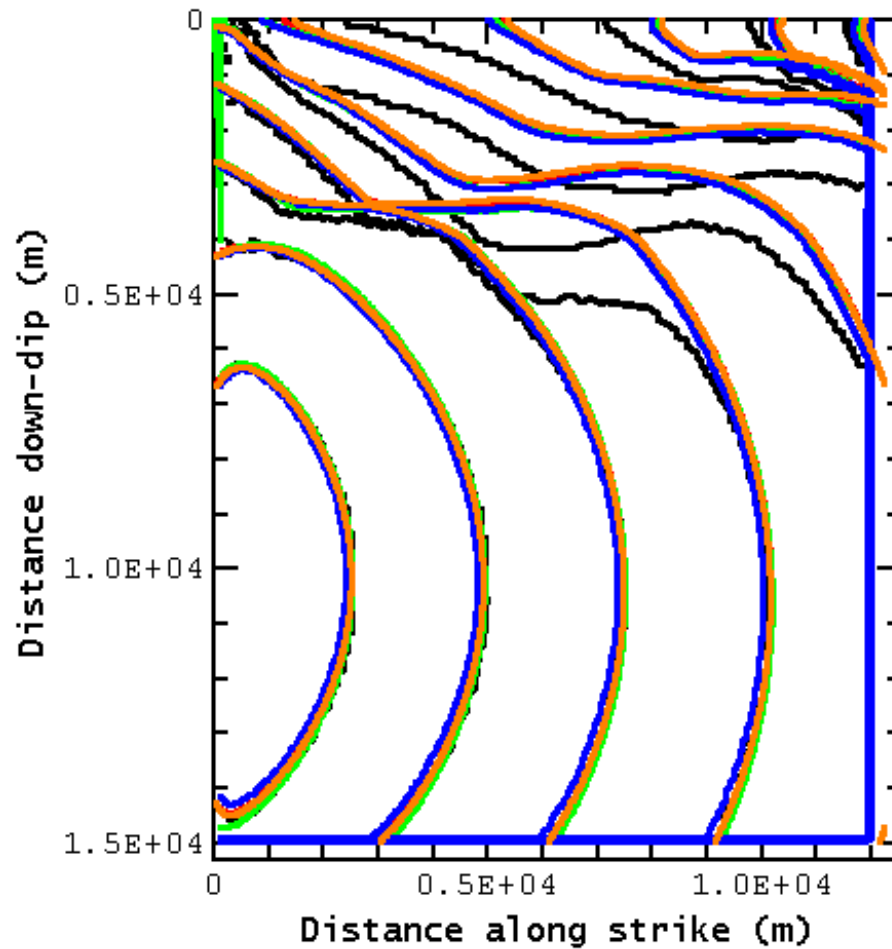
TPV24 Comparisons
(Right-Lateral, Releasing Branch)

Main Fault



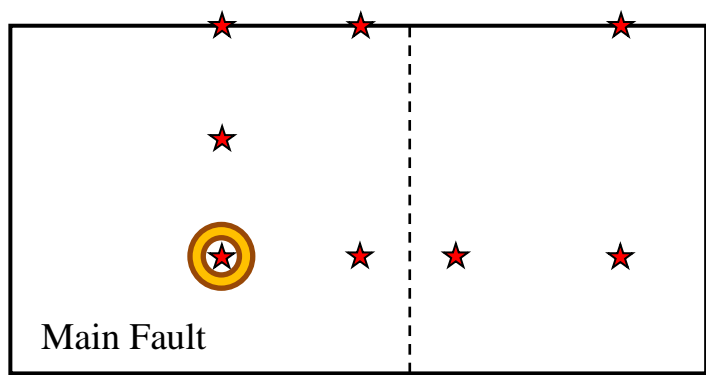
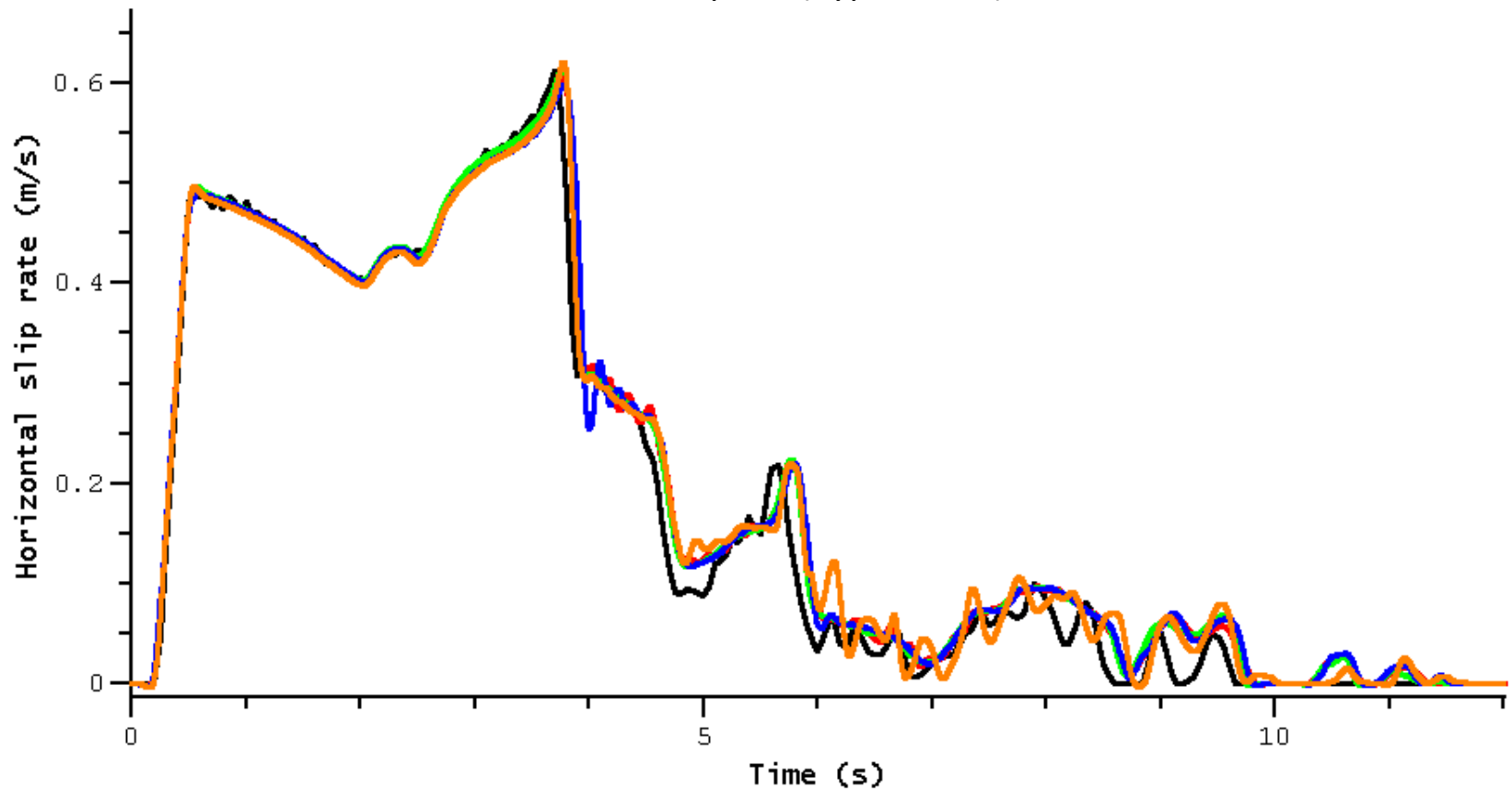
- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))
- somala.2 (Surendra Somala - Spectral Element - SESAME (50m))

Branch Fault



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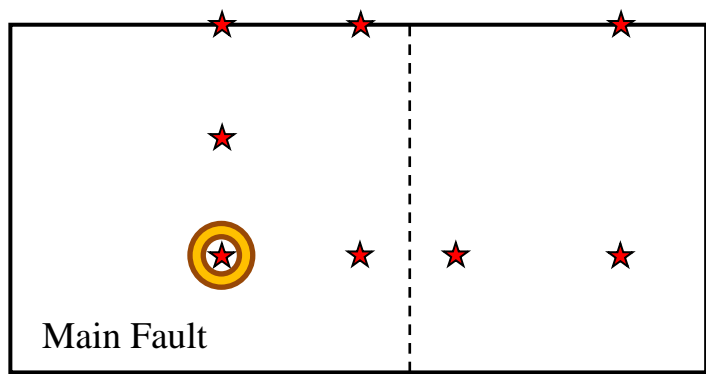
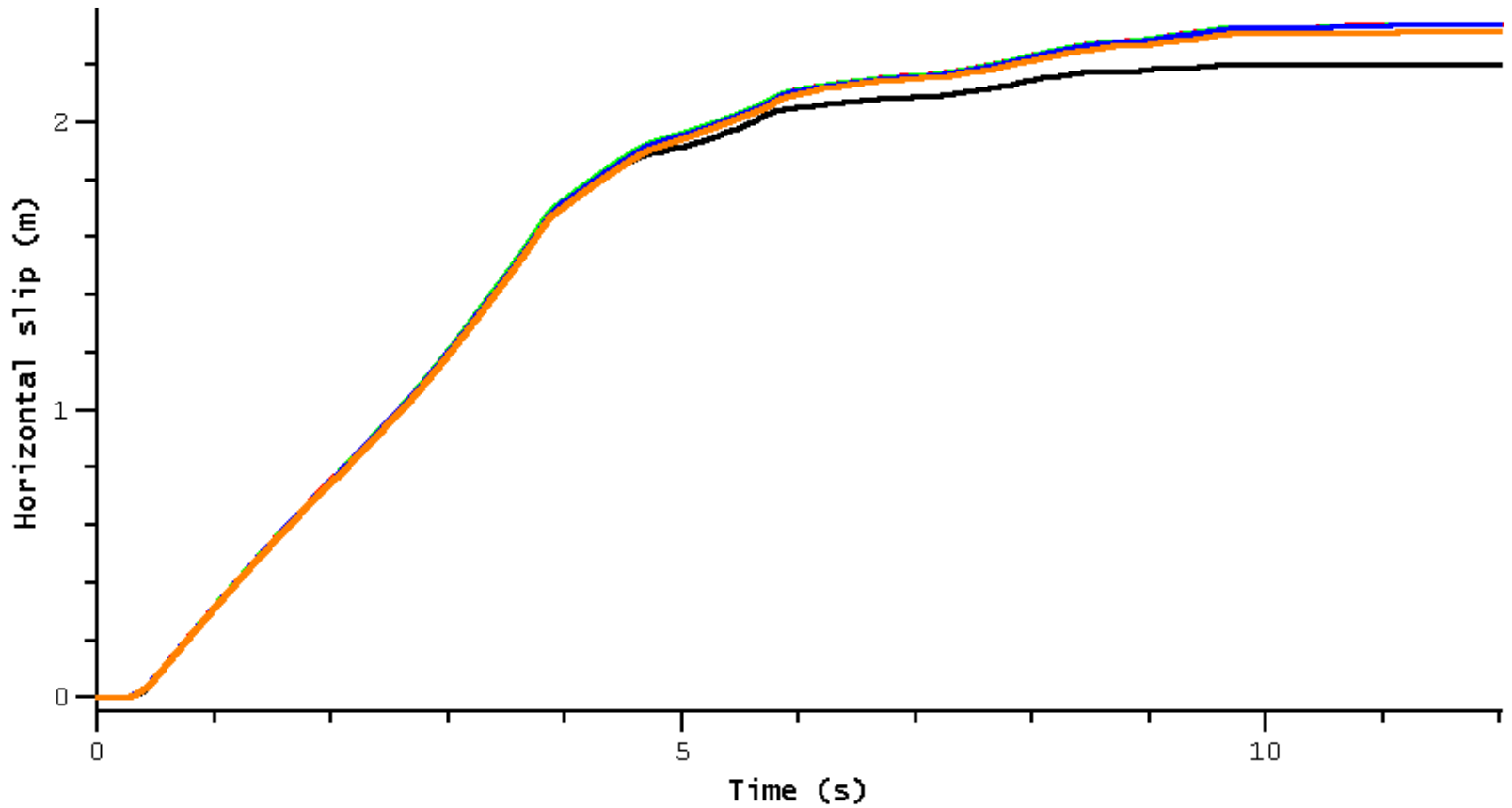
faultst-080dp100 (hypocenter)



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5 Hz low-pass filter applied to all time series.

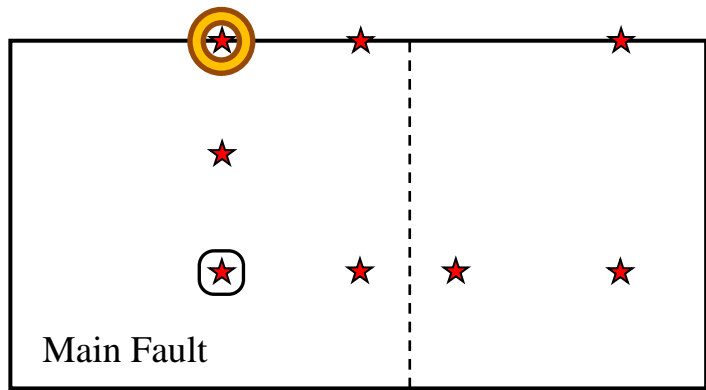
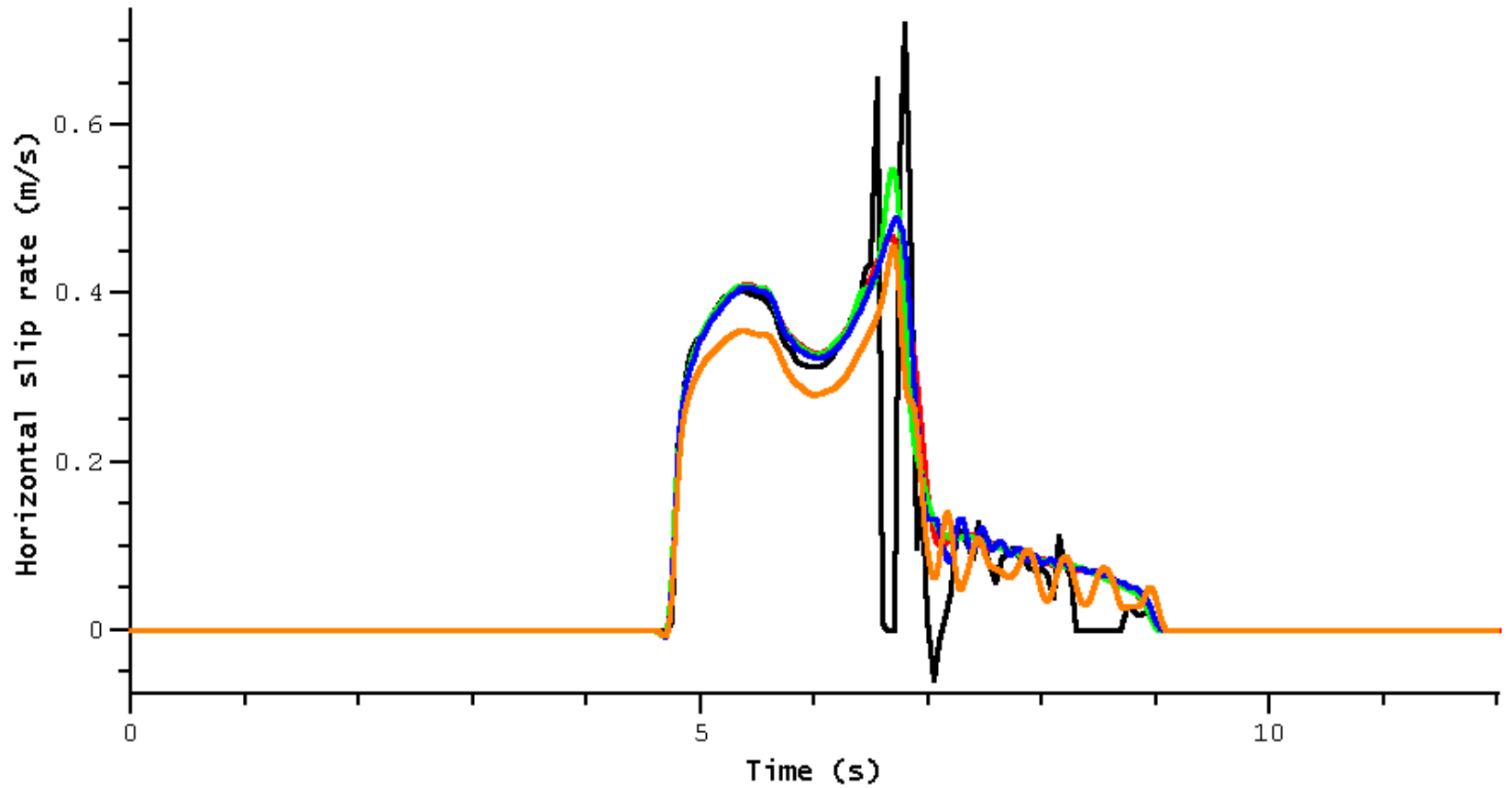
faultst-080dp100 (hypocenter)



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
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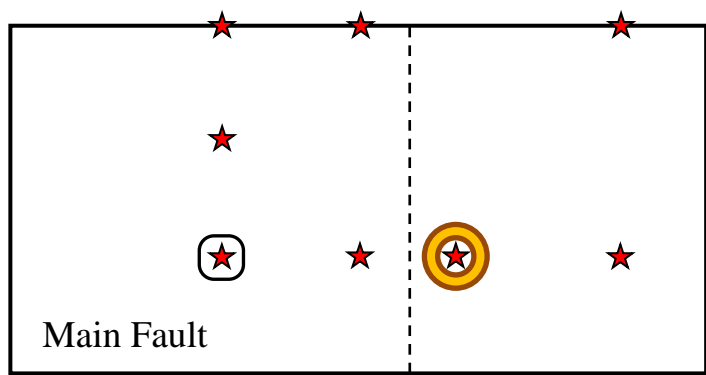
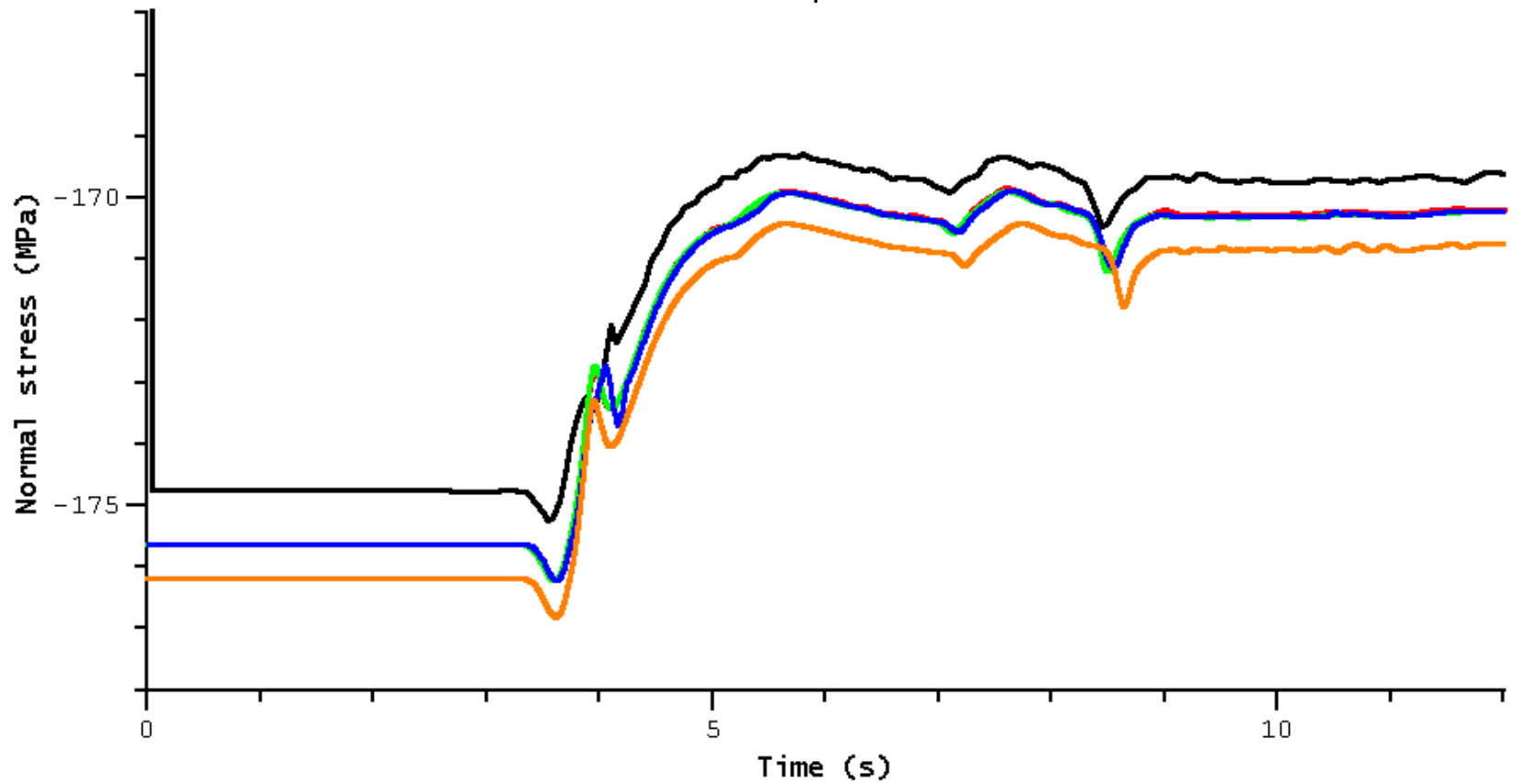
faultst-080dp000



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
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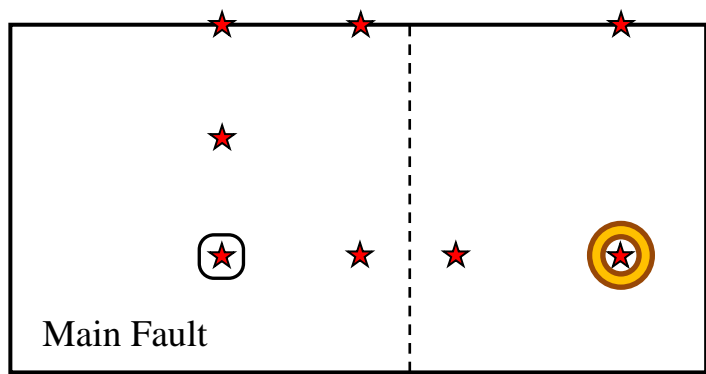
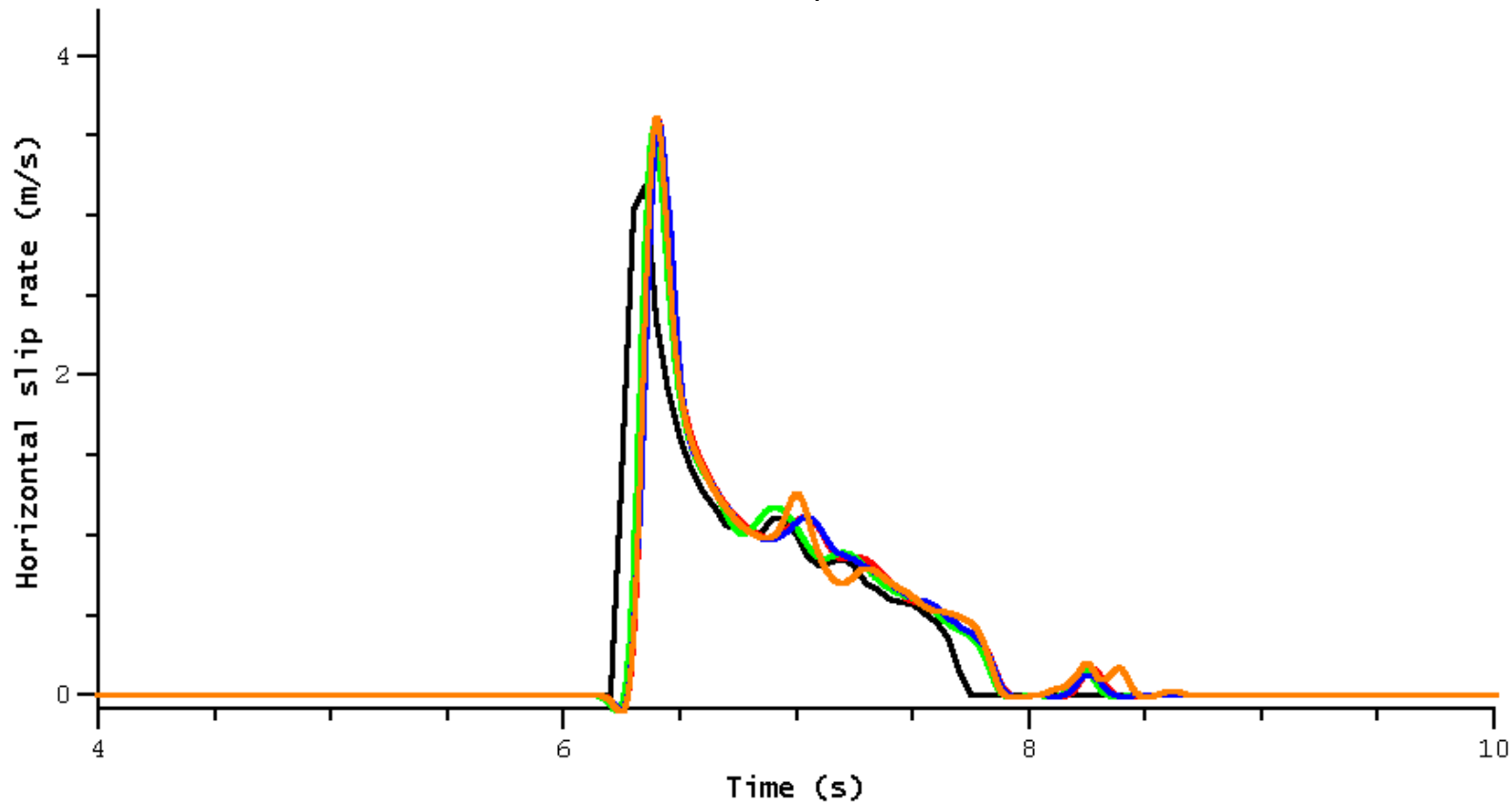
faultst020dp100



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
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- ma (Shuo Ma - Finite Element - MAFE (100 m))
- somala.2 (Surendra Somala - Spectral Element - SESAME (50m))

5 Hz low-pass filter applied to all time series.

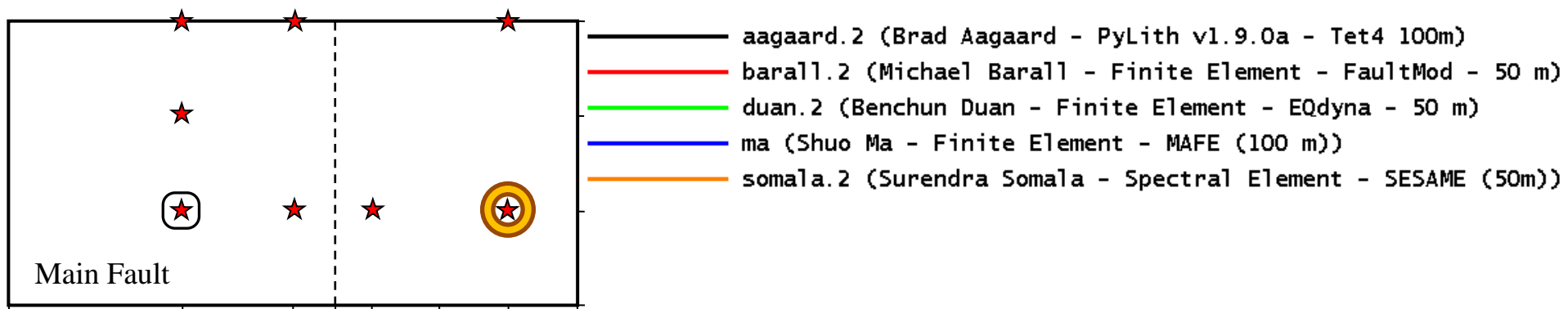
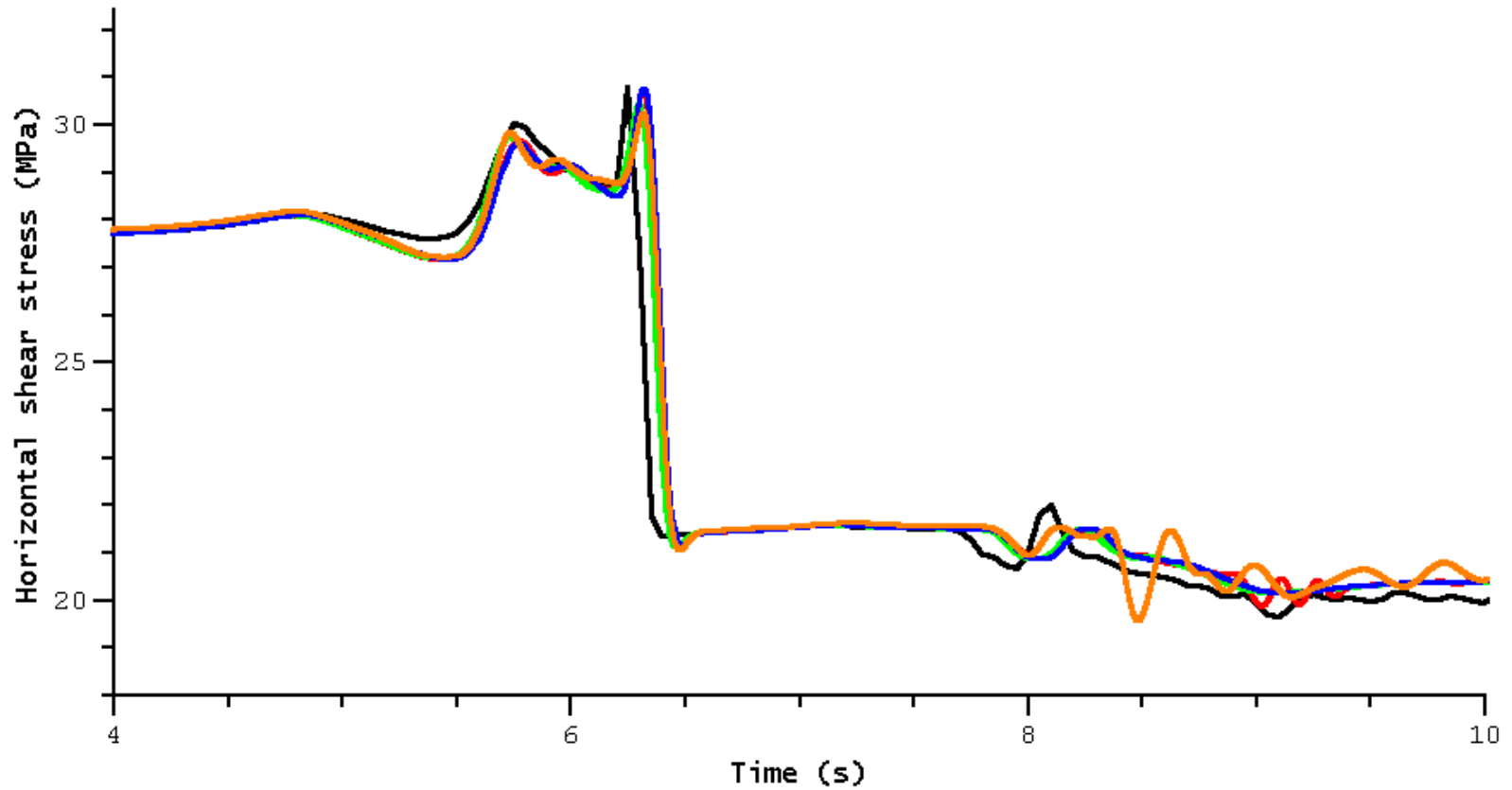
faultst090dp100



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
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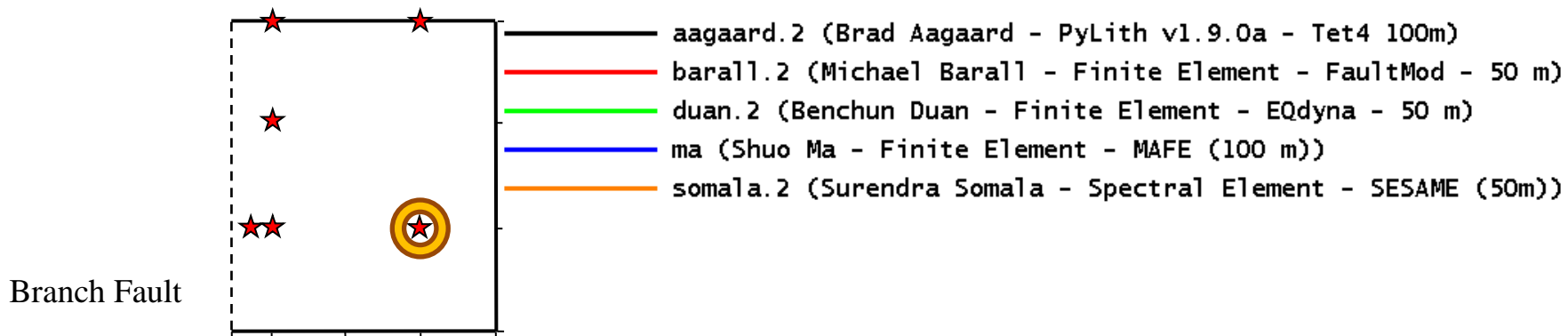
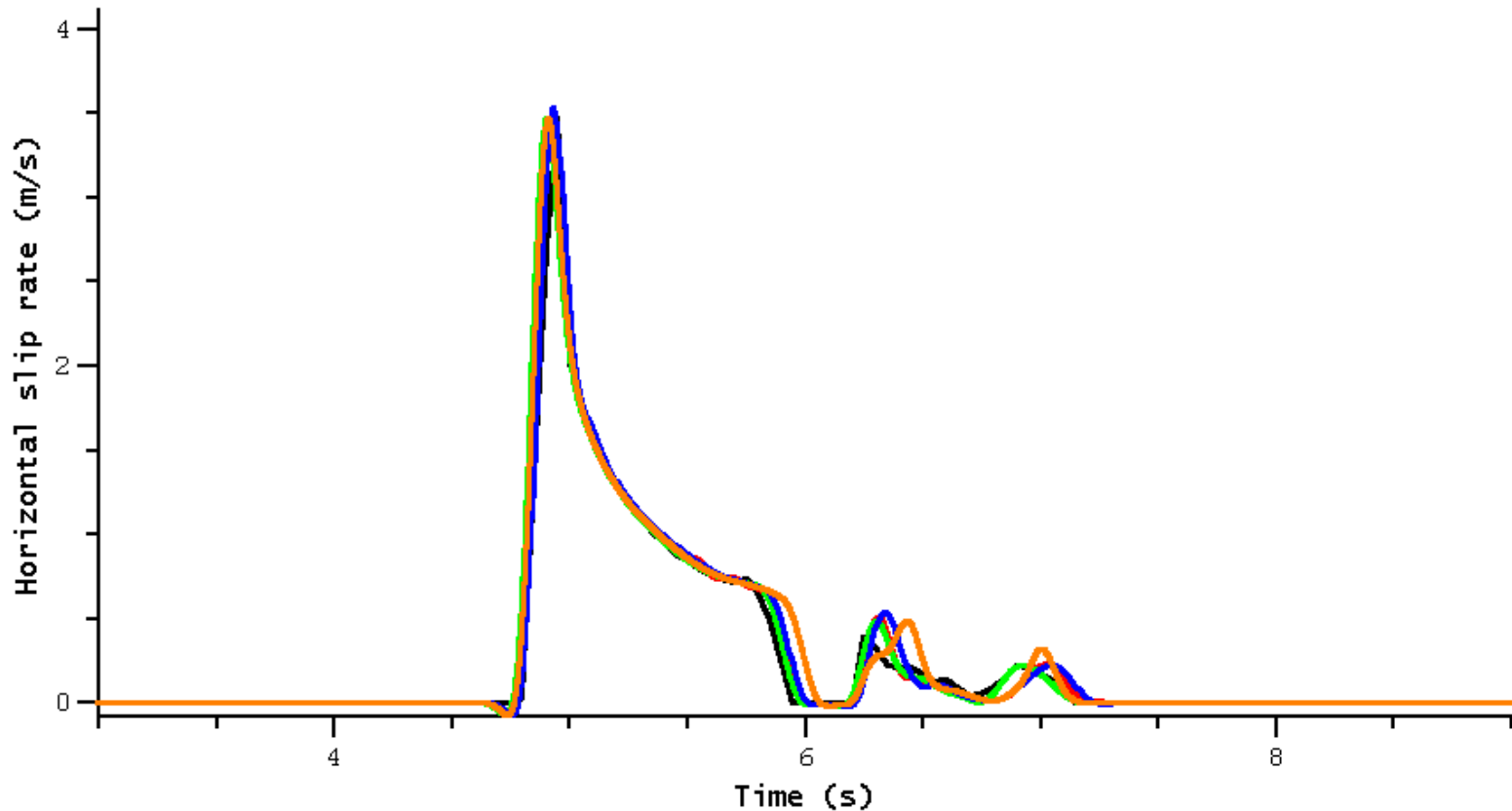
5 Hz low-pass filter applied to all time series.

faultst090dp100



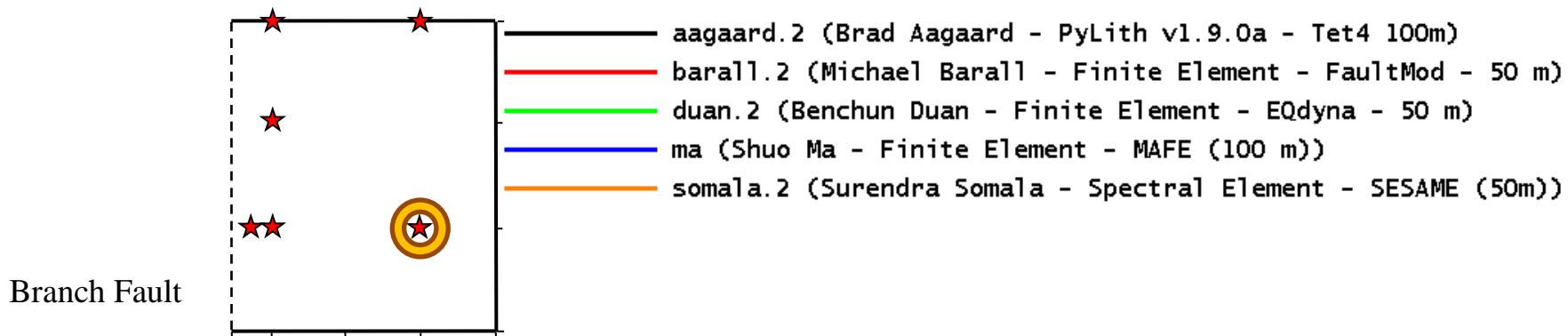
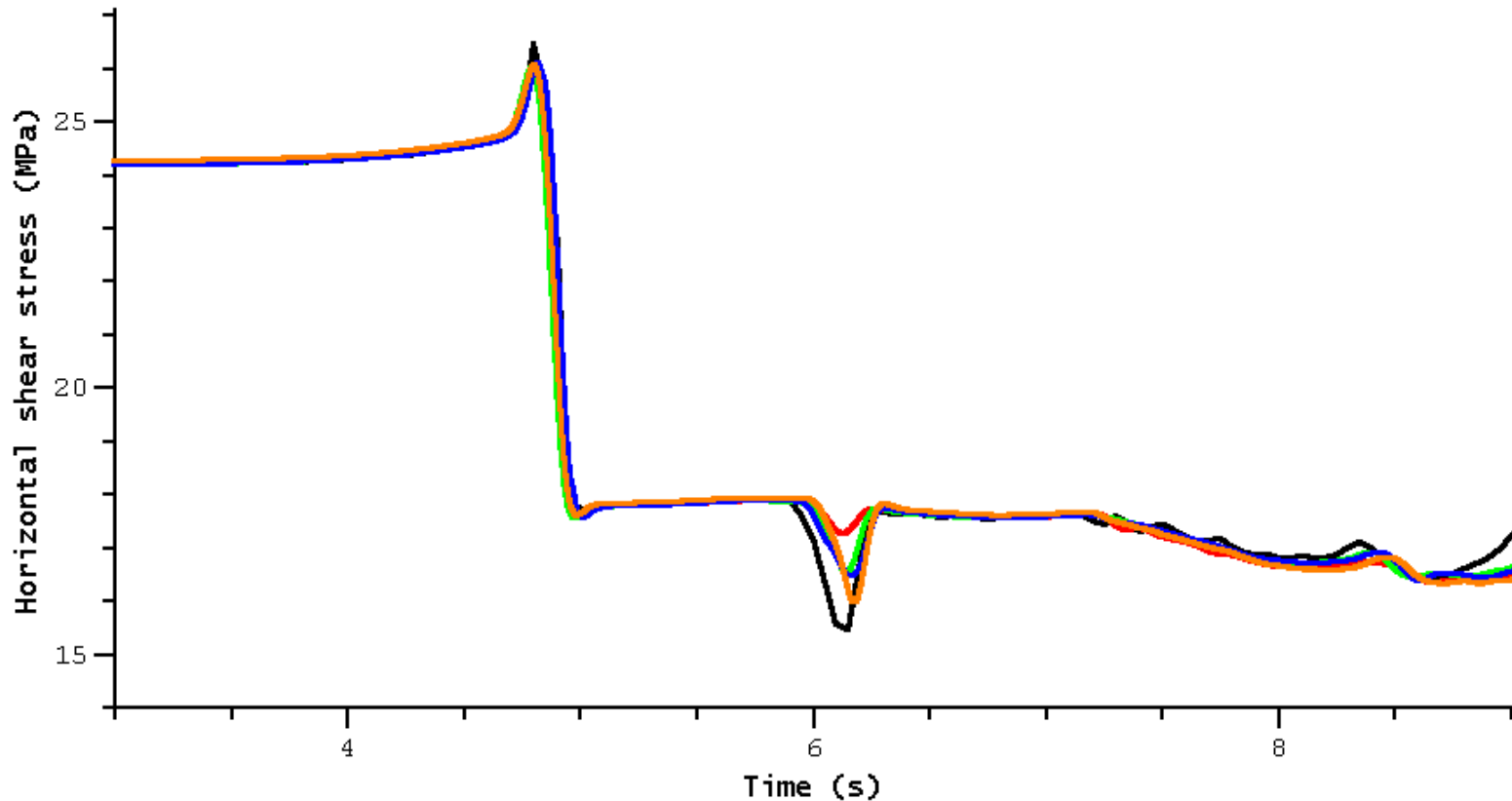
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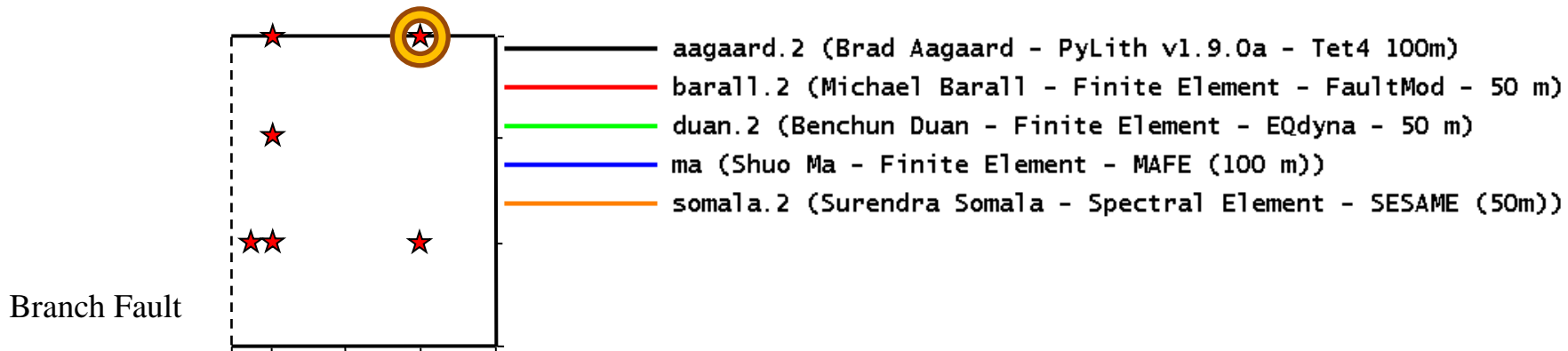
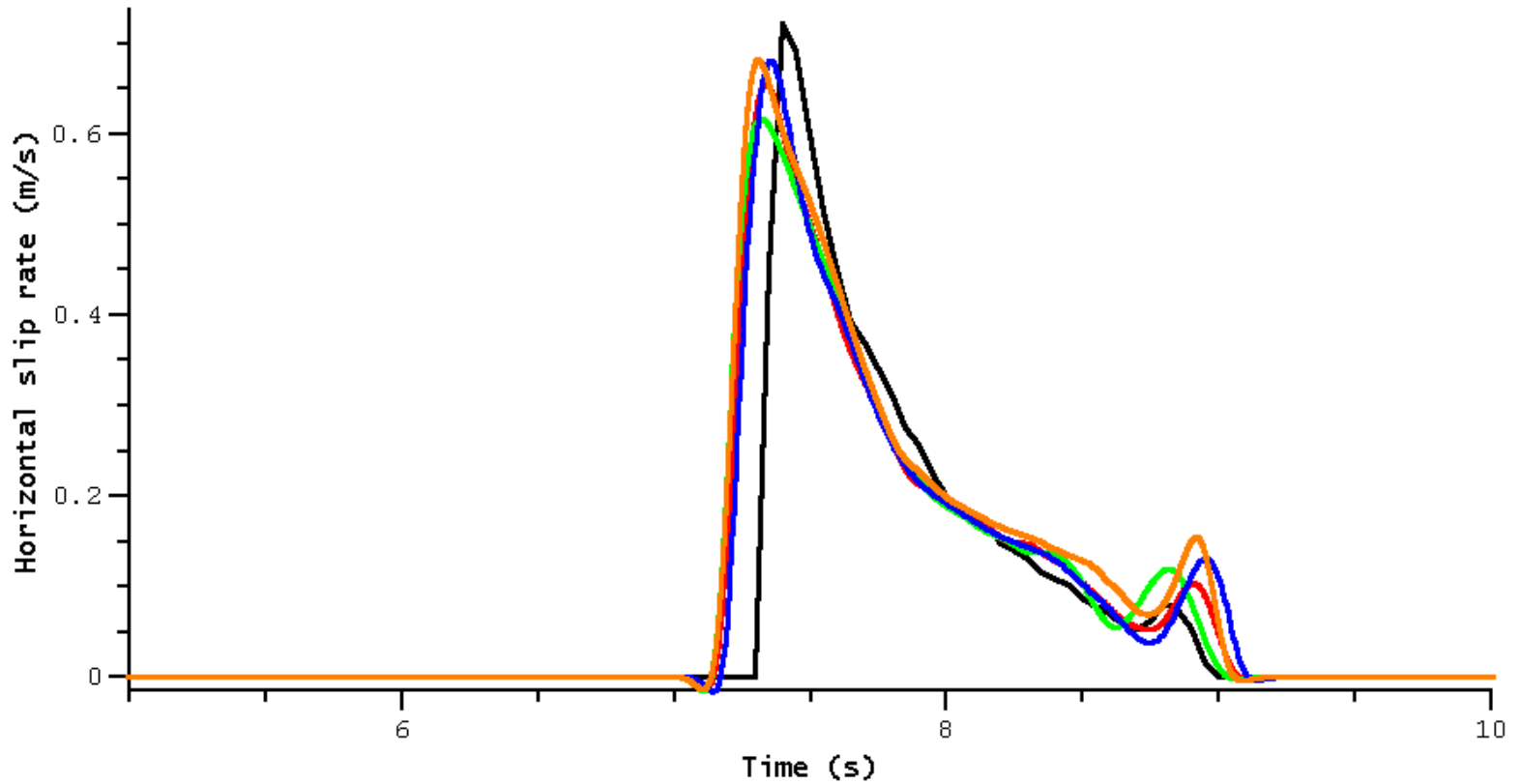
5 Hz low-pass filter applied to all time series.

branchst090dp100



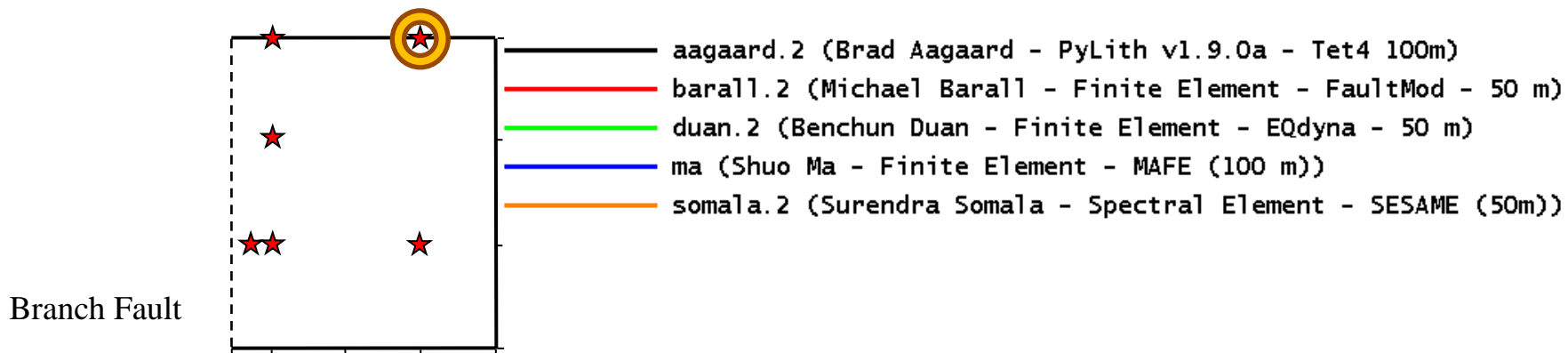
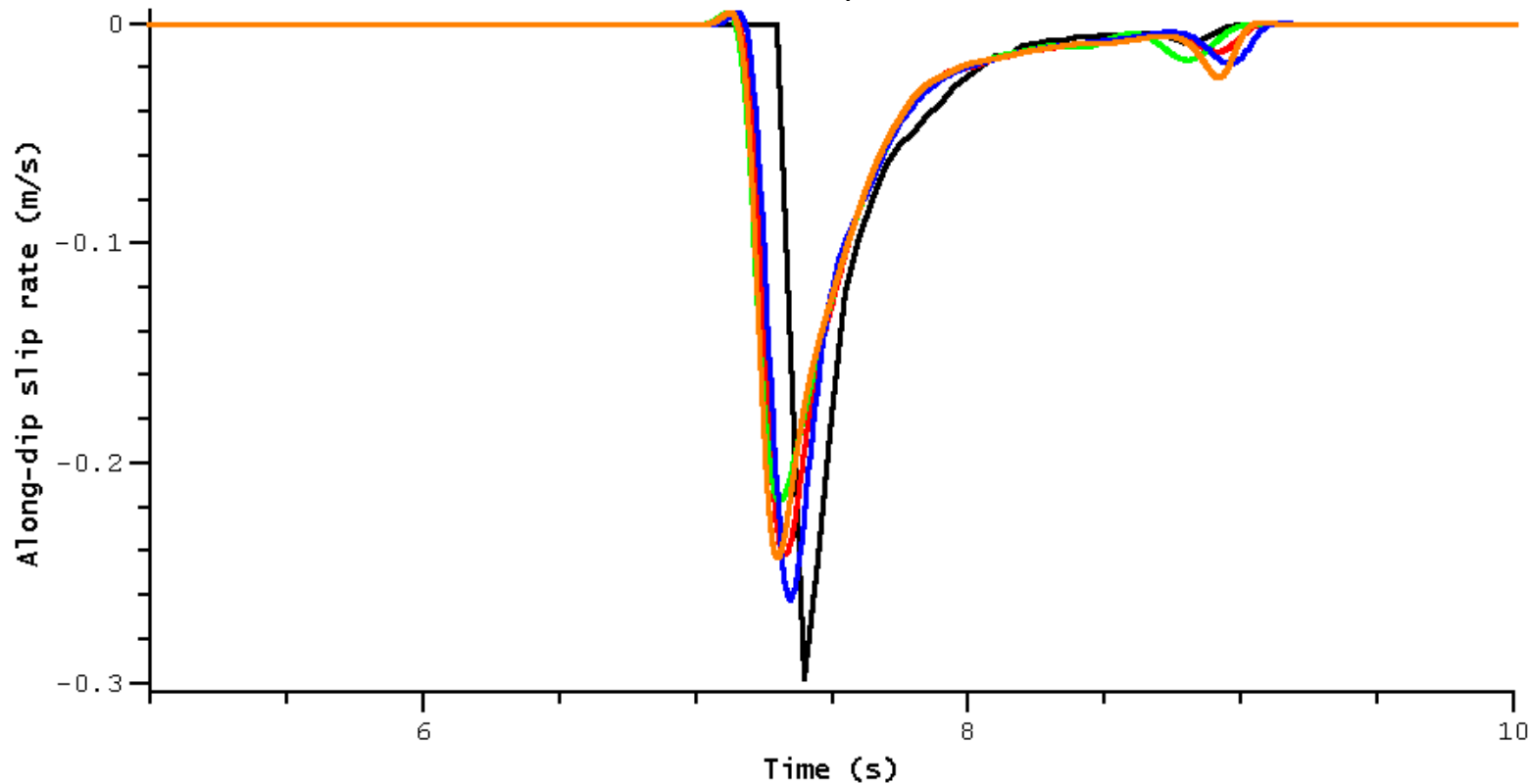
5 Hz low-pass filter applied to all time series.

branchst090dp000



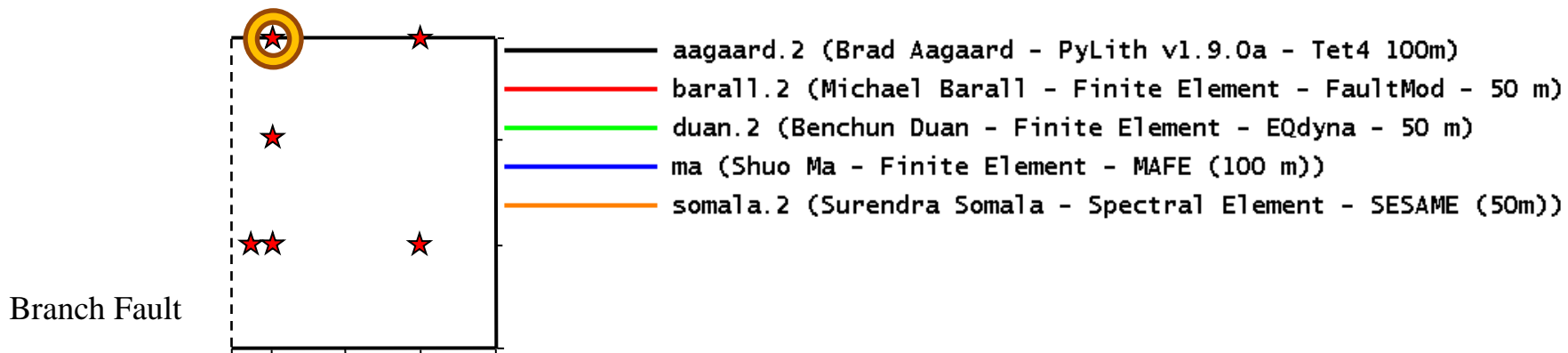
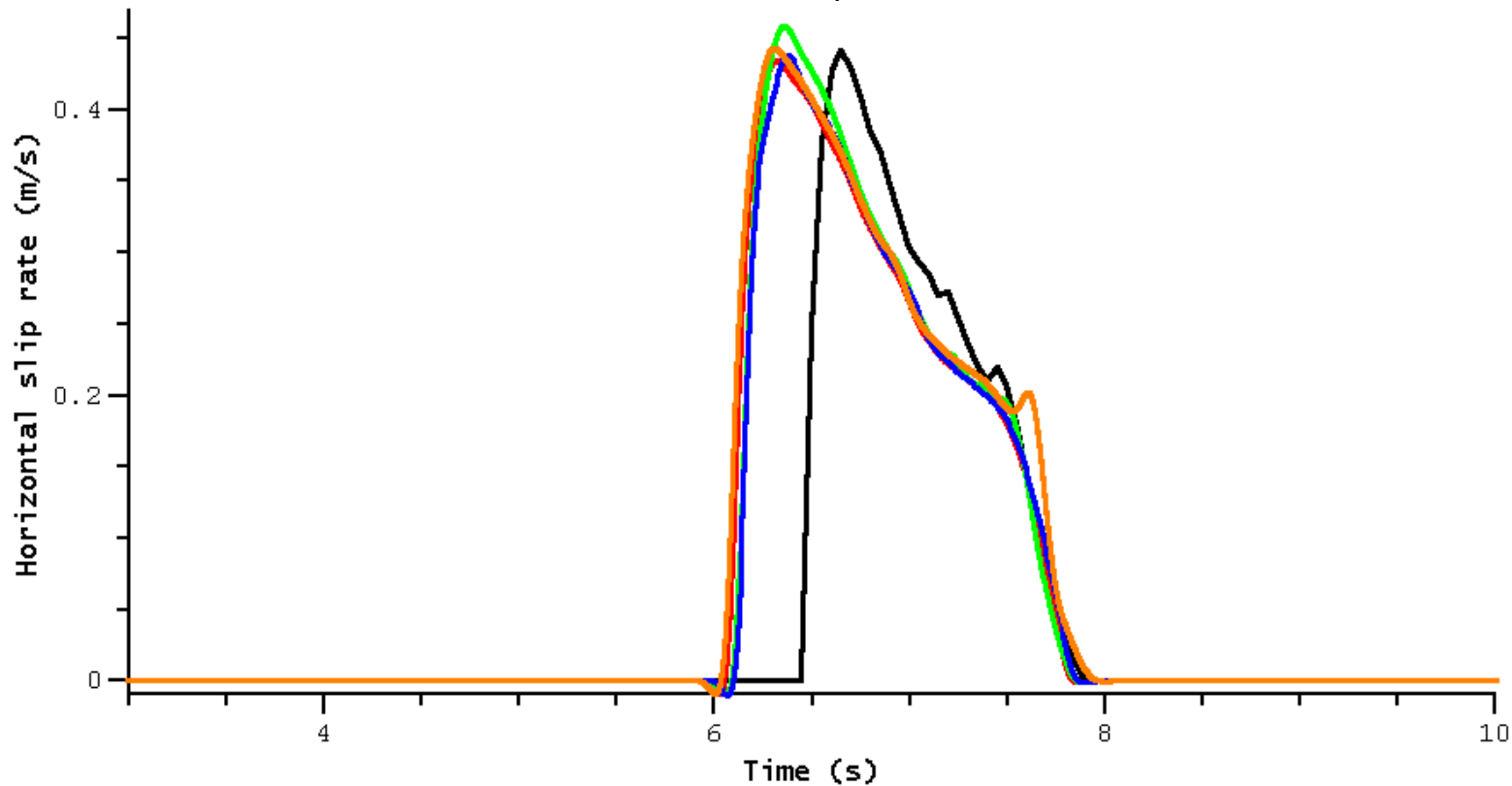
5 Hz low-pass filter applied to all time series.

branchst090dp000



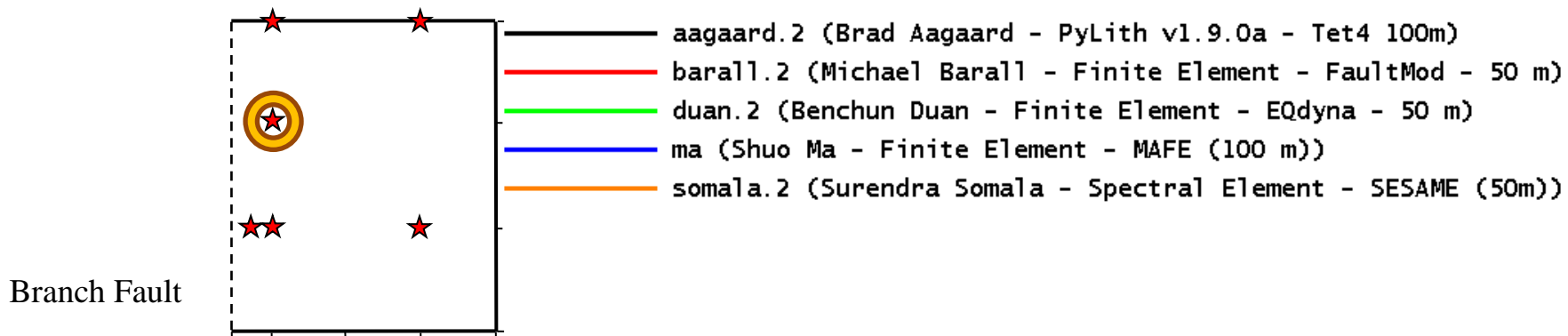
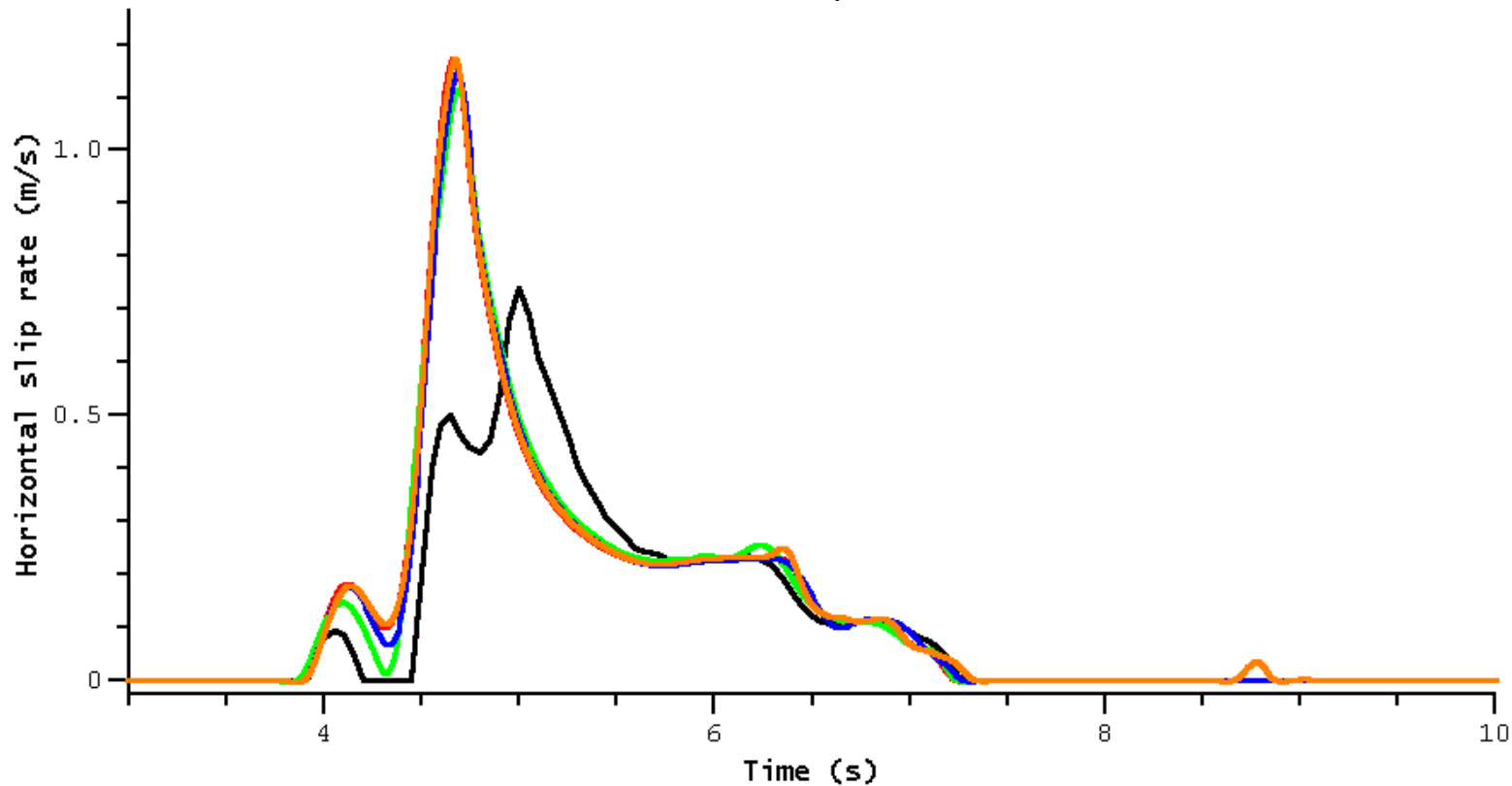
5 Hz low-pass filter applied to all time series.

branchst020dp000



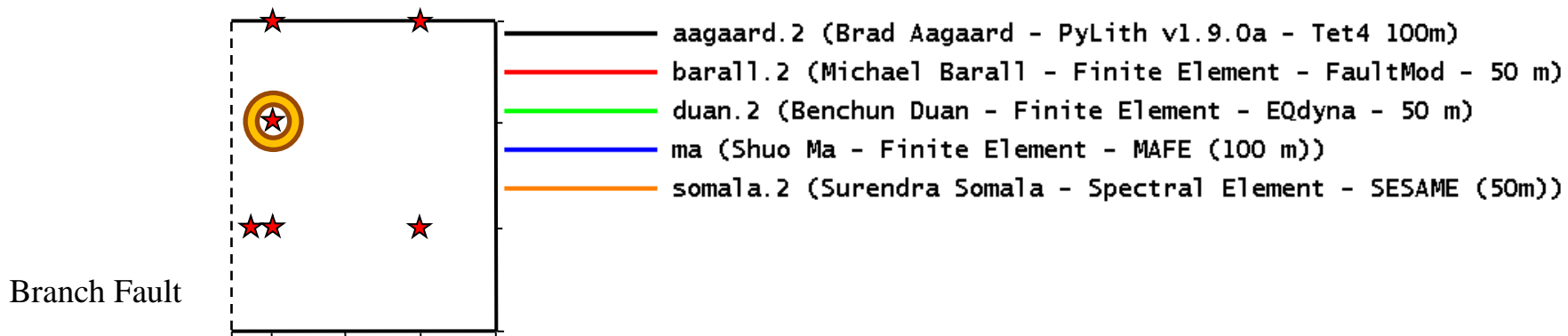
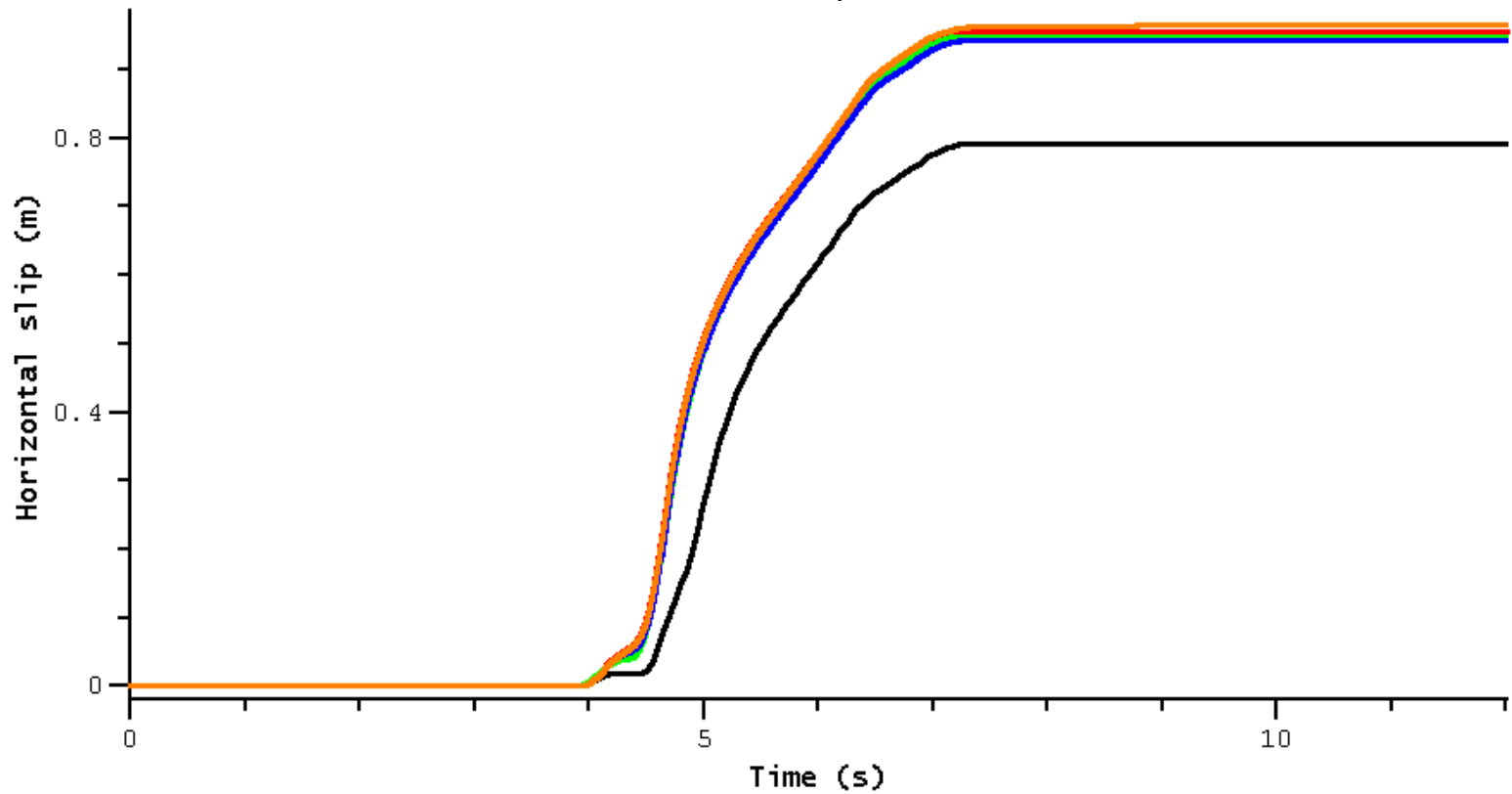
5 Hz low-pass filter applied to all time series.

branchst020dp050



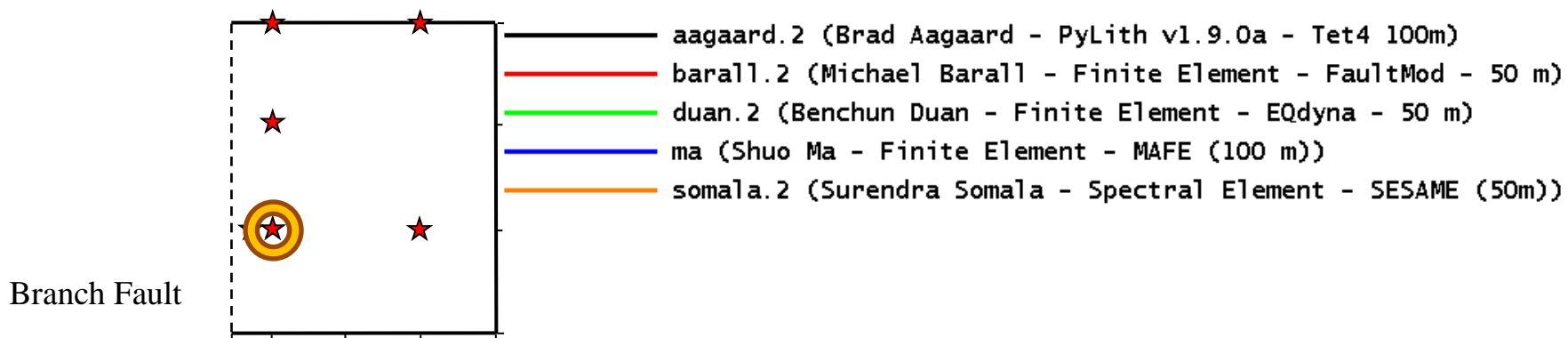
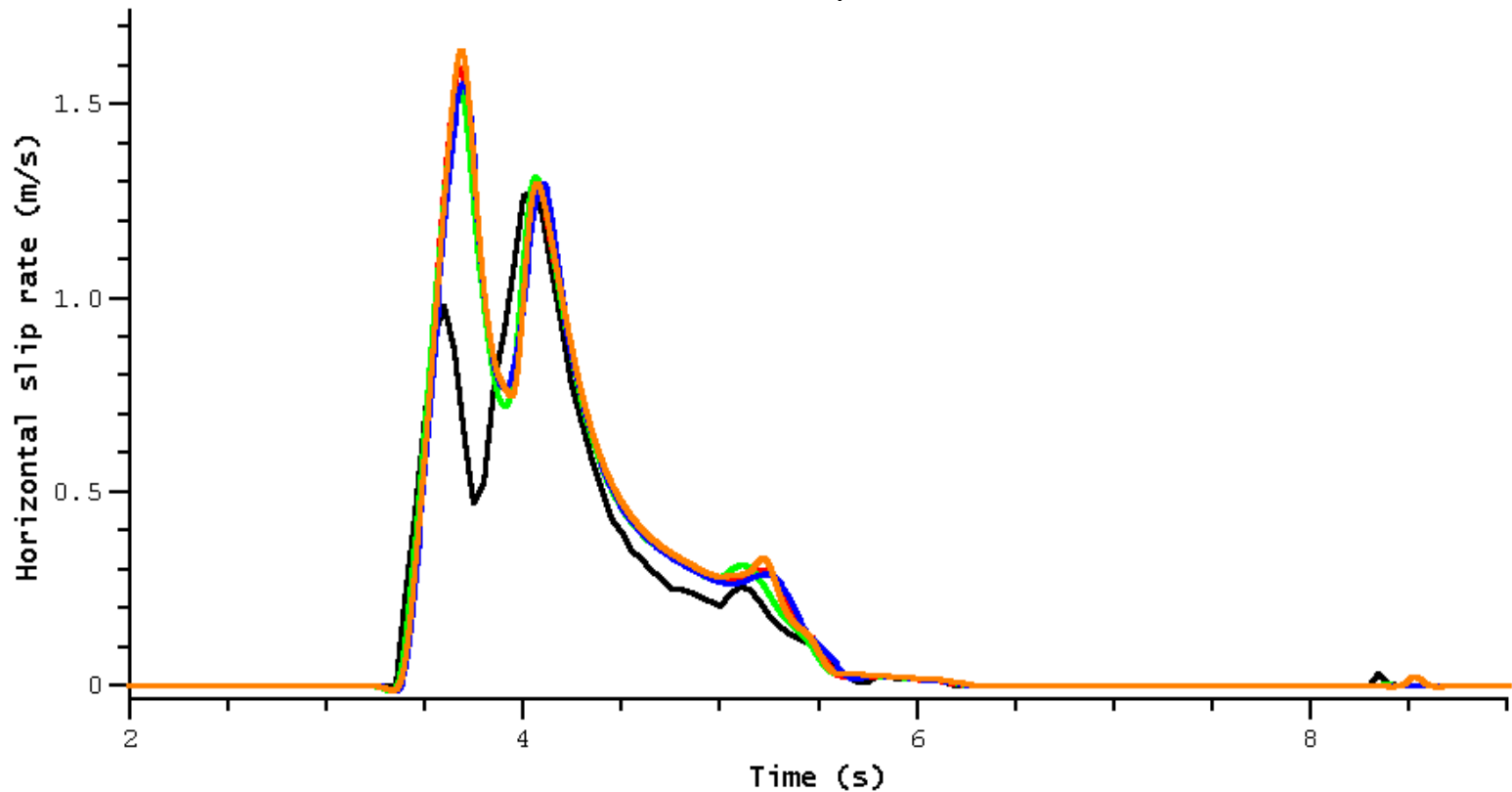
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branchst020dp050



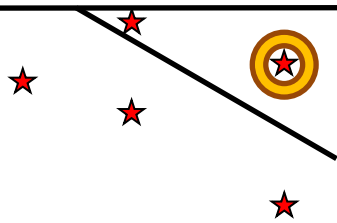
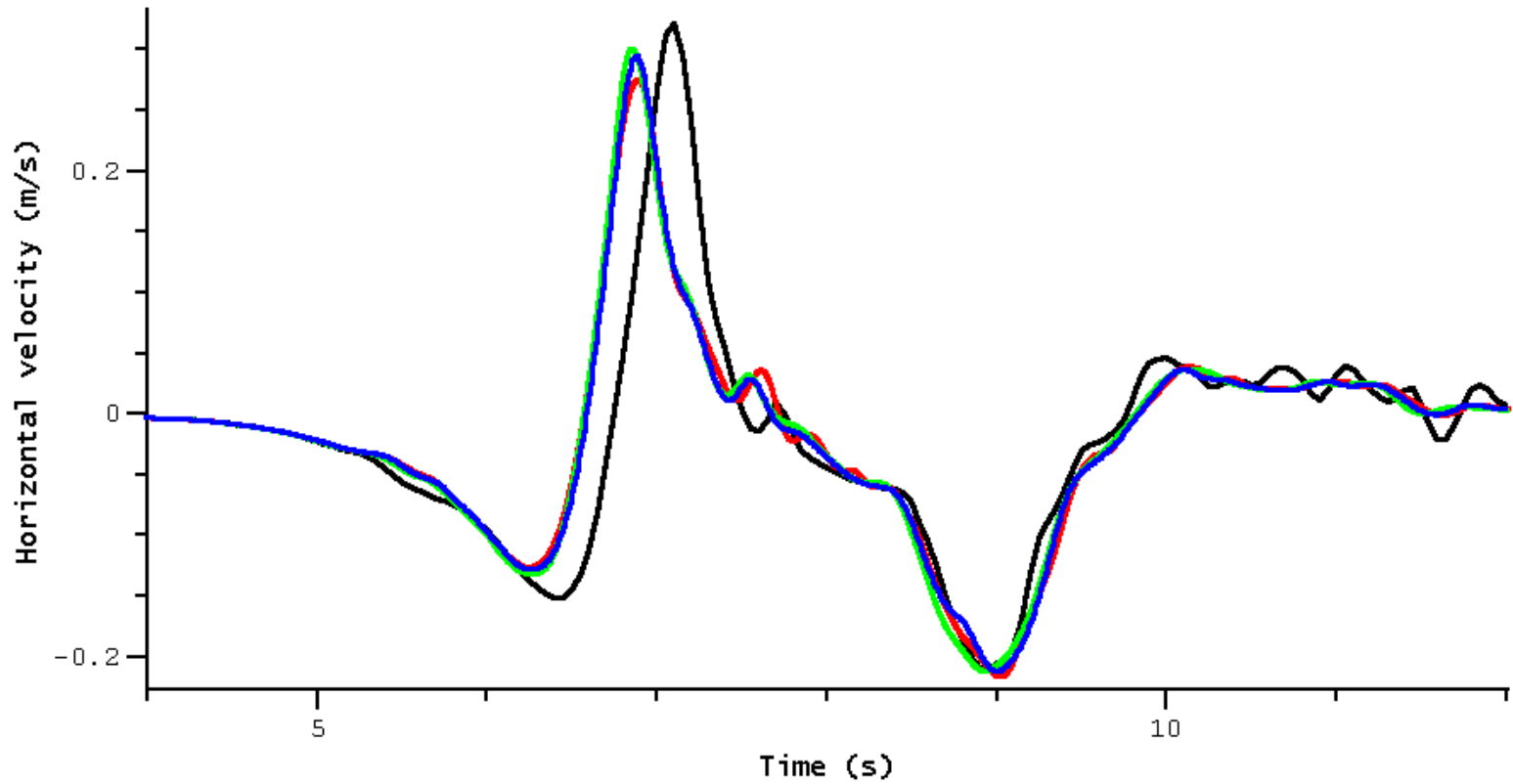
5 Hz low-pass filter applied to all time series.

branchst020dp100



5 Hz low-pass filter applied to all time series.

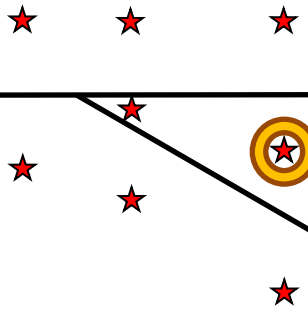
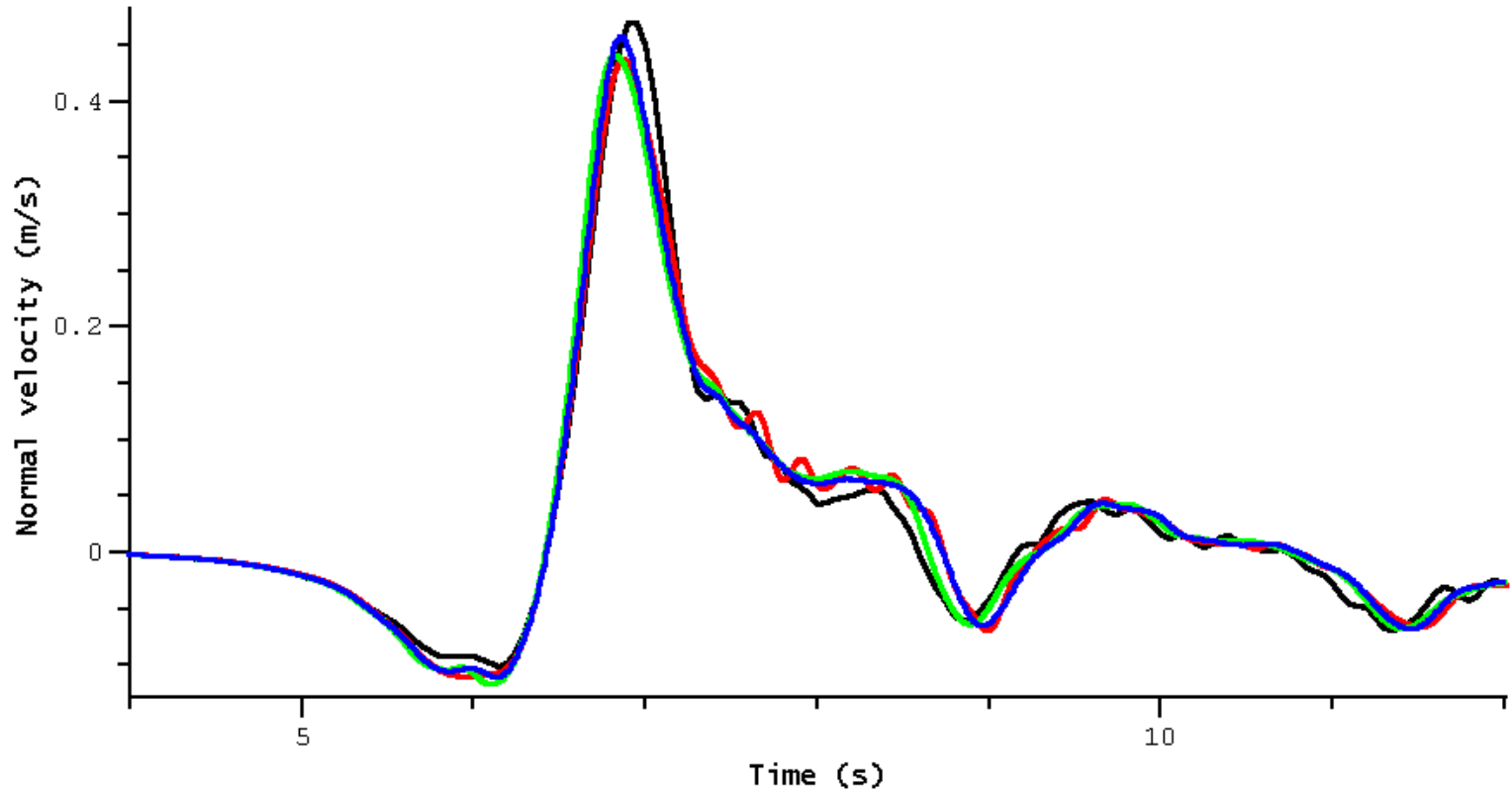
body-023st080dp000



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))

5 Hz low-pass filter applied to all time series.

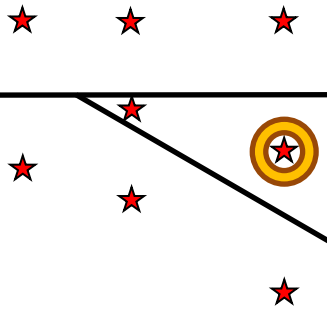
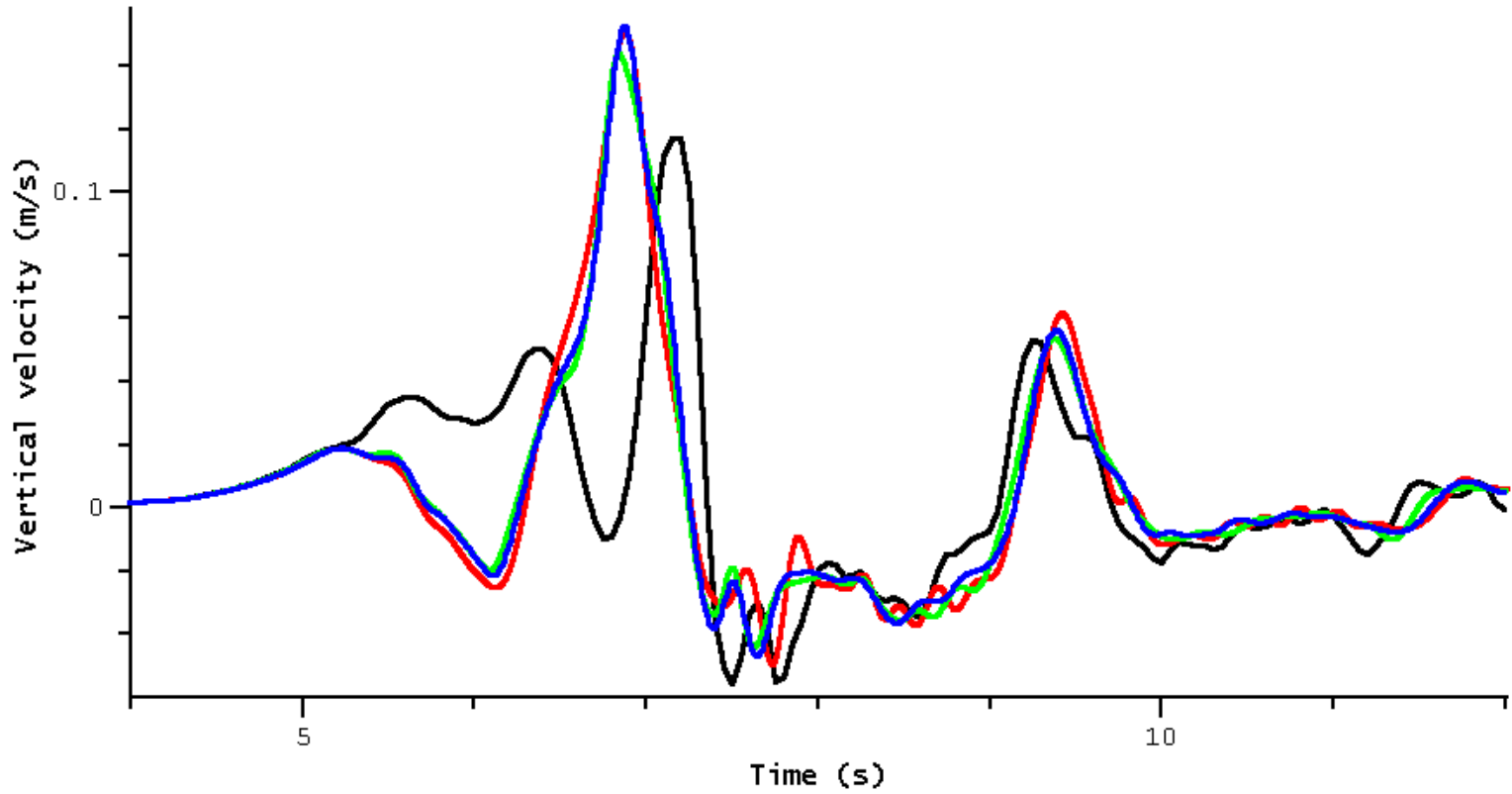
body-023st080dp000



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))

5 Hz low-pass filter applied to all time series.

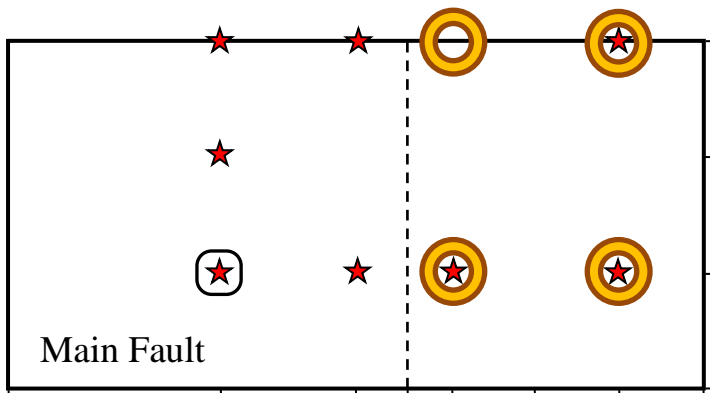
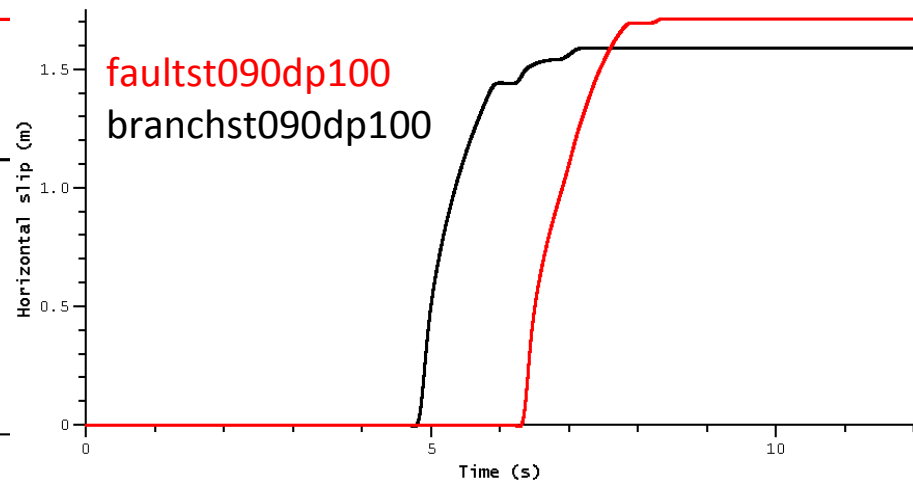
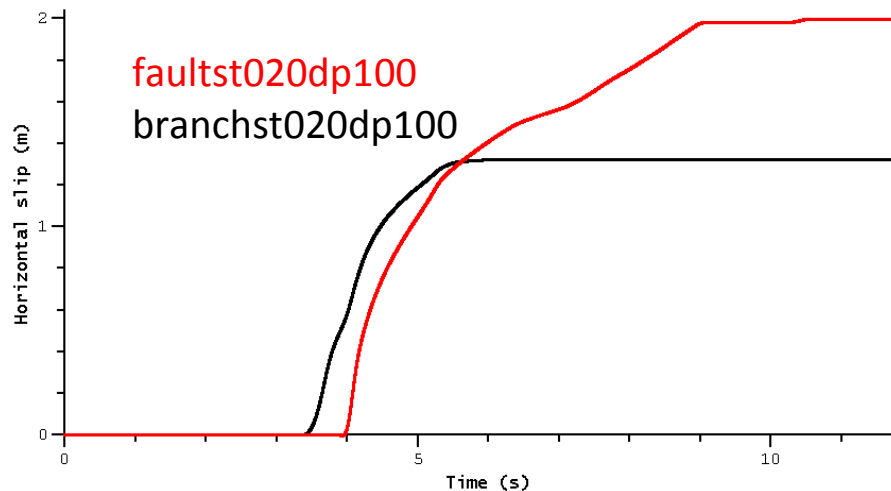
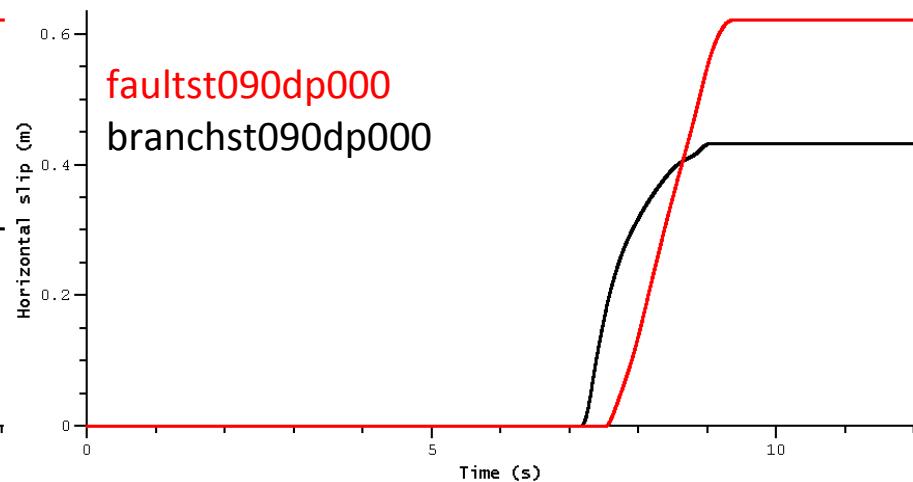
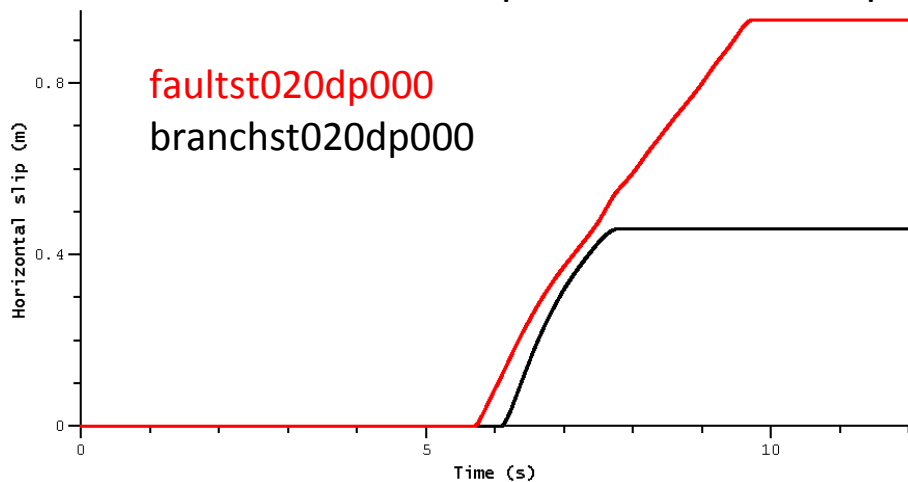
body-023st080dp000



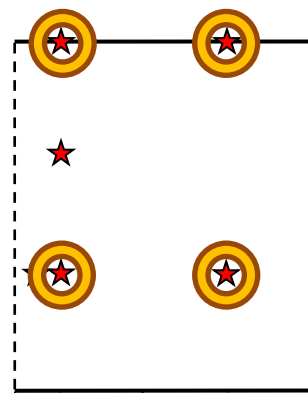
- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))

5 Hz low-pass filter applied to all time series.

Comparison of Final Slip on Main and Branch Faults

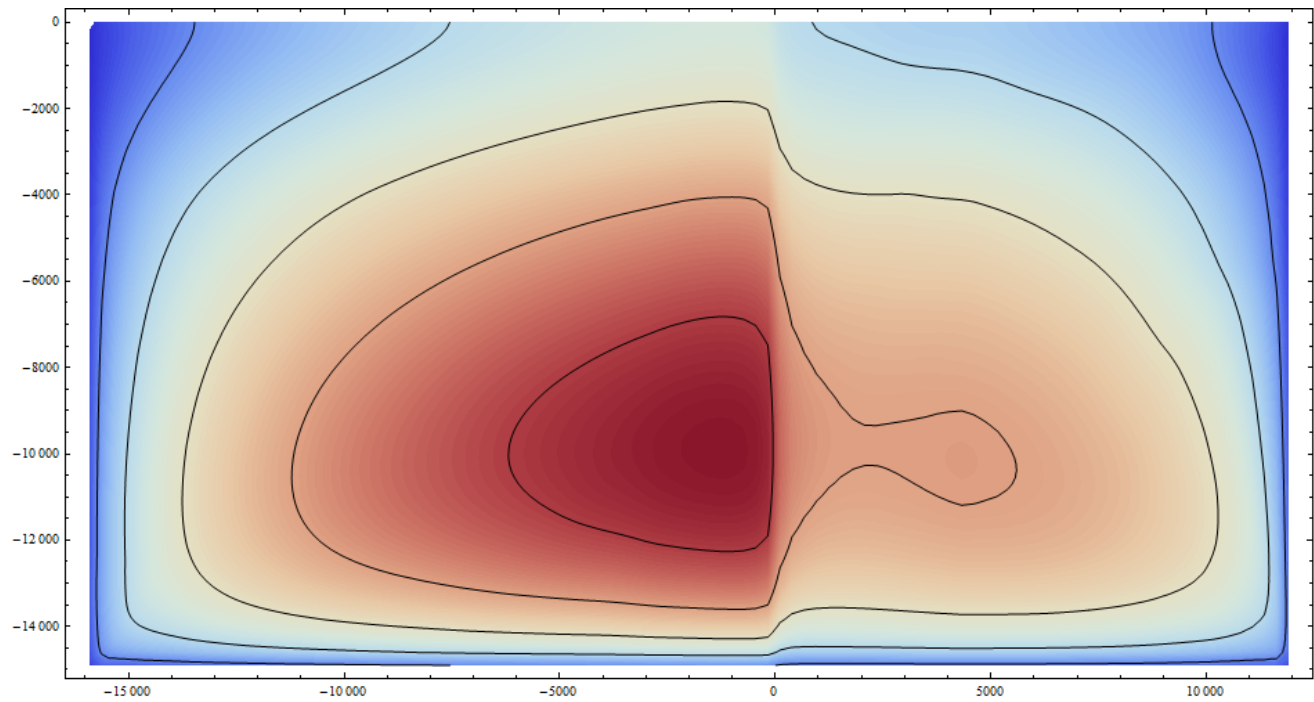


Main Fault

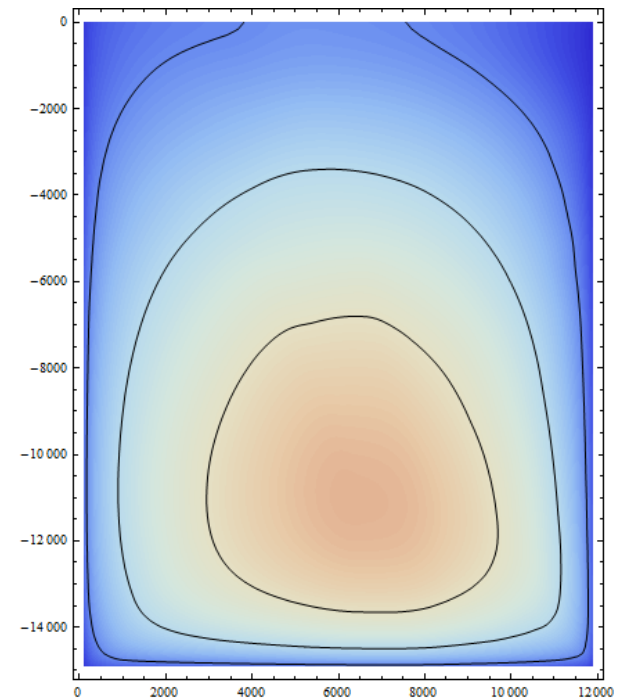
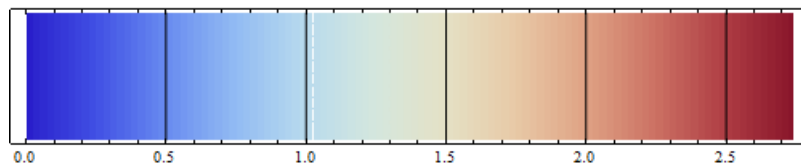


Branch Fault

Slip on Main Fault
Slip on Branch Fault

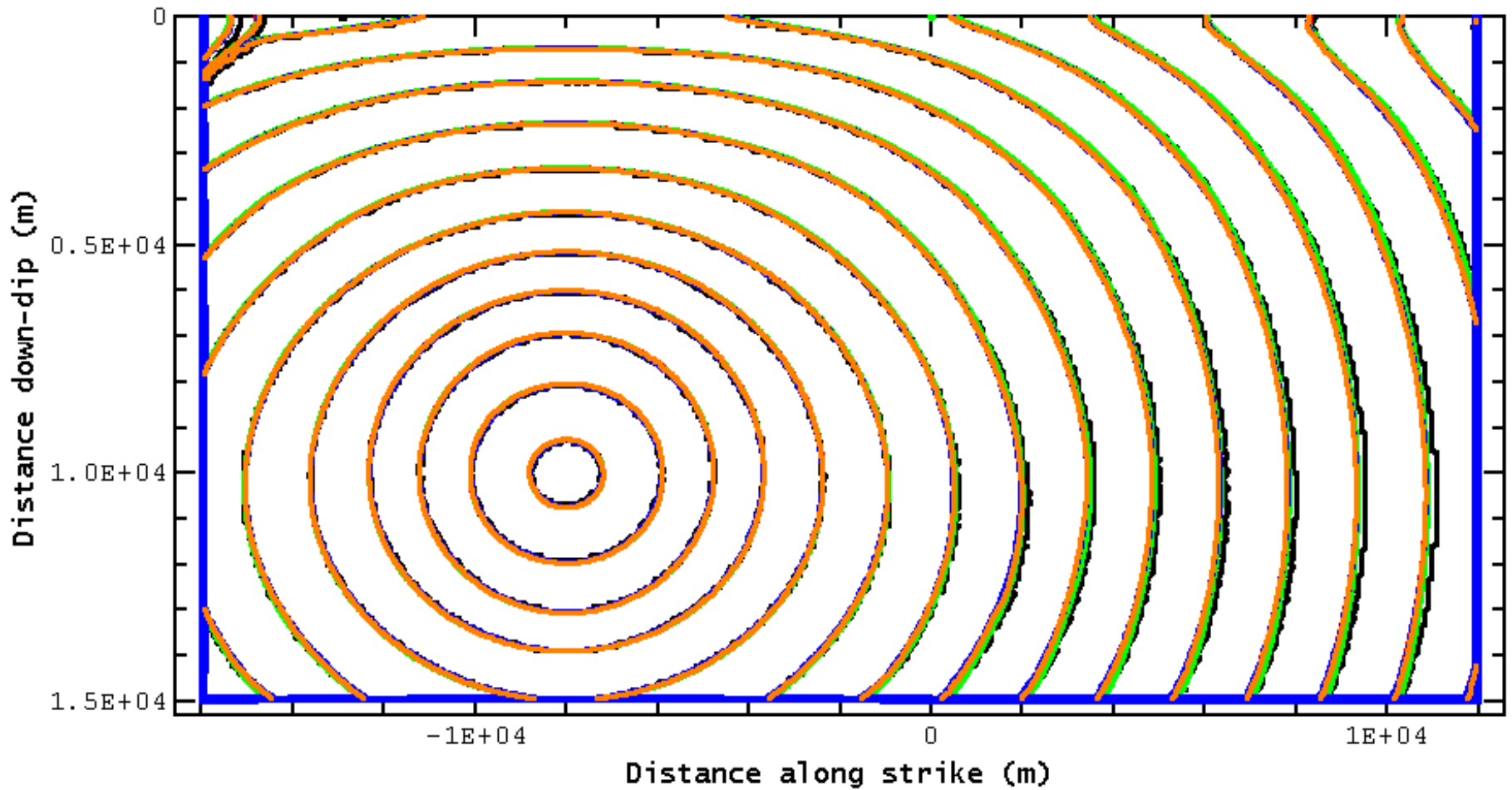


Final slip on main and branch faults for TPV24



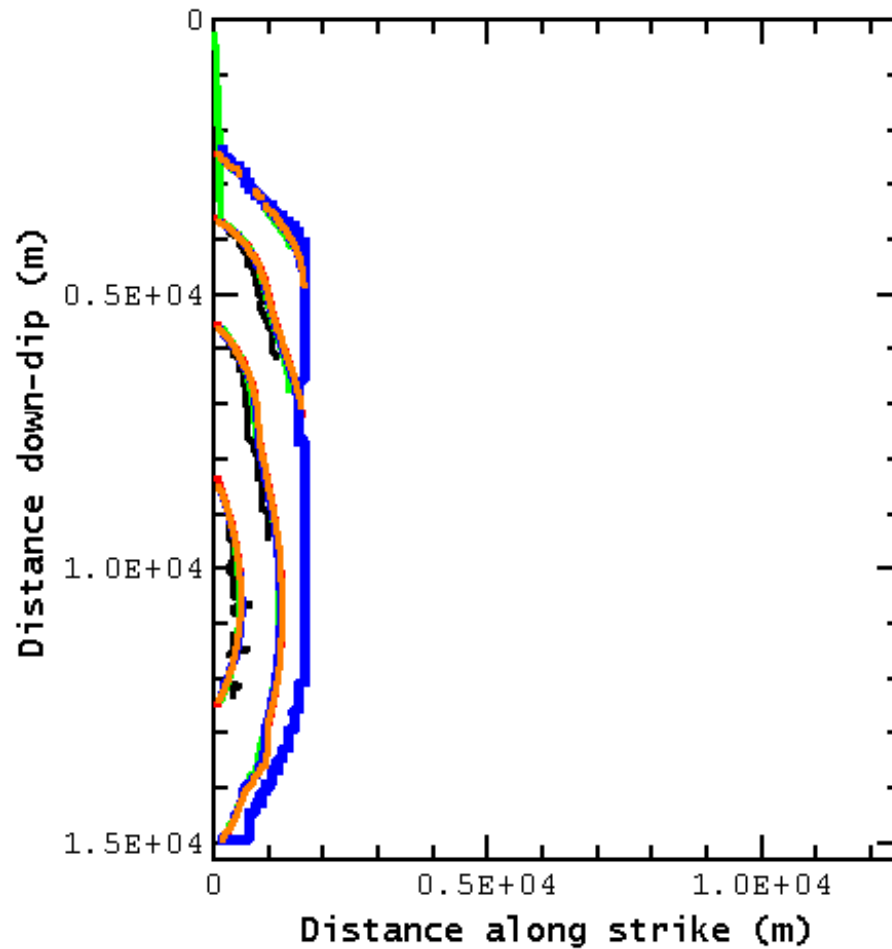
TPV25 Comparisons
(Left-Lateral, Restraining Branch)

Main Fault



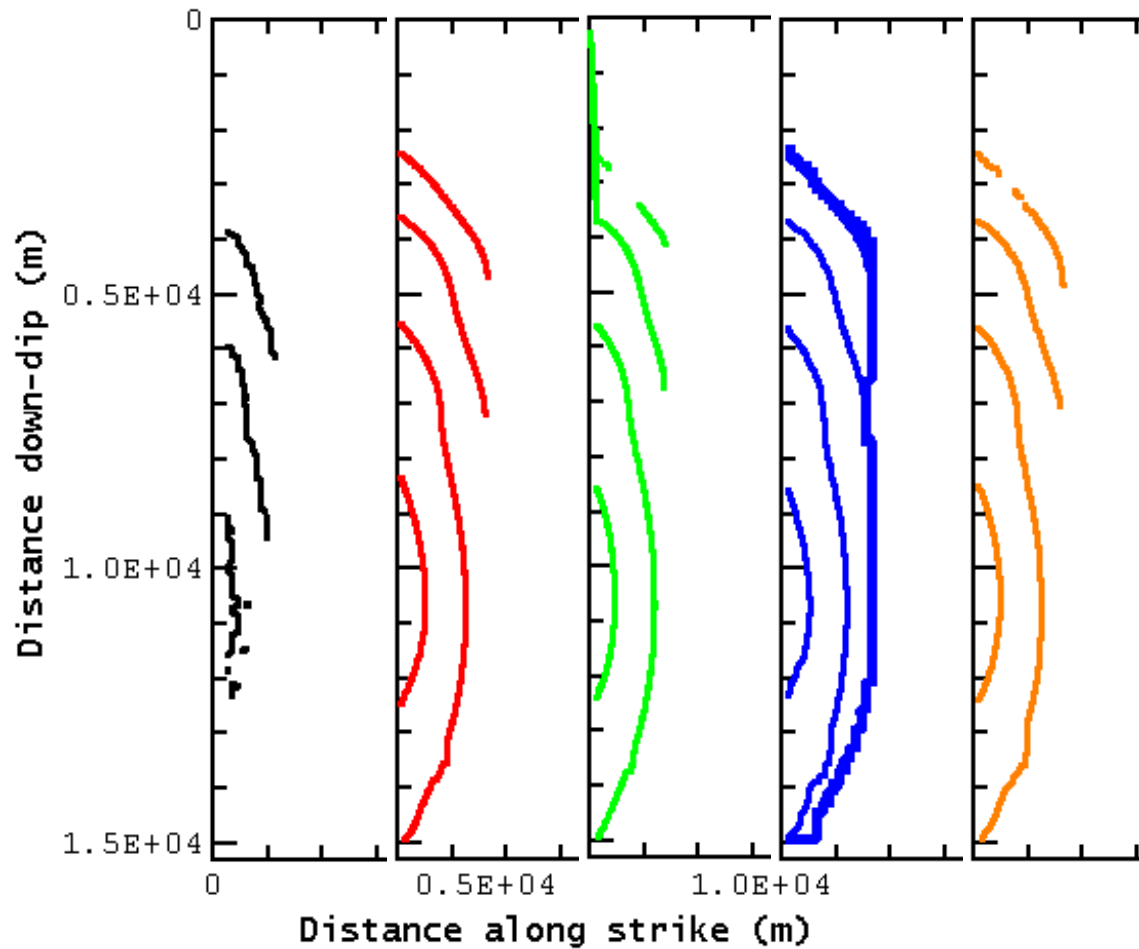
- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))
- somala (Surendra Somala - Spectral Element - SESAME (100m))

Branch Fault



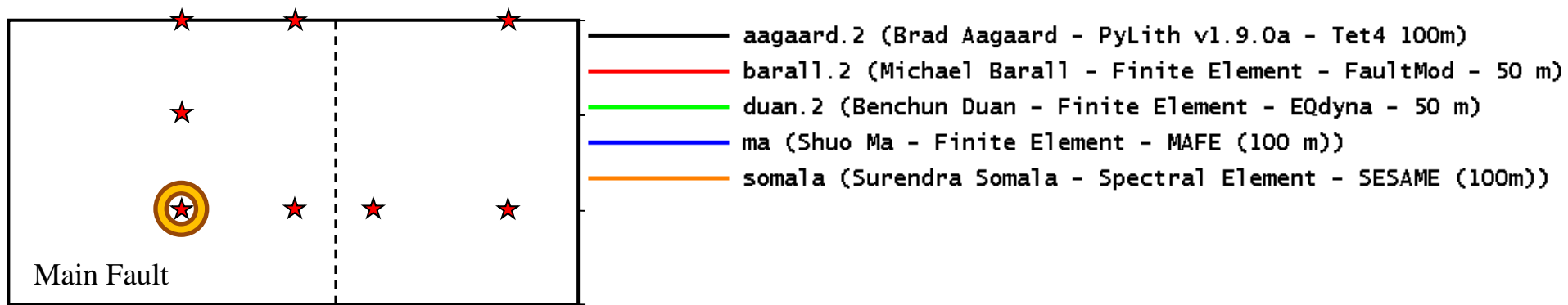
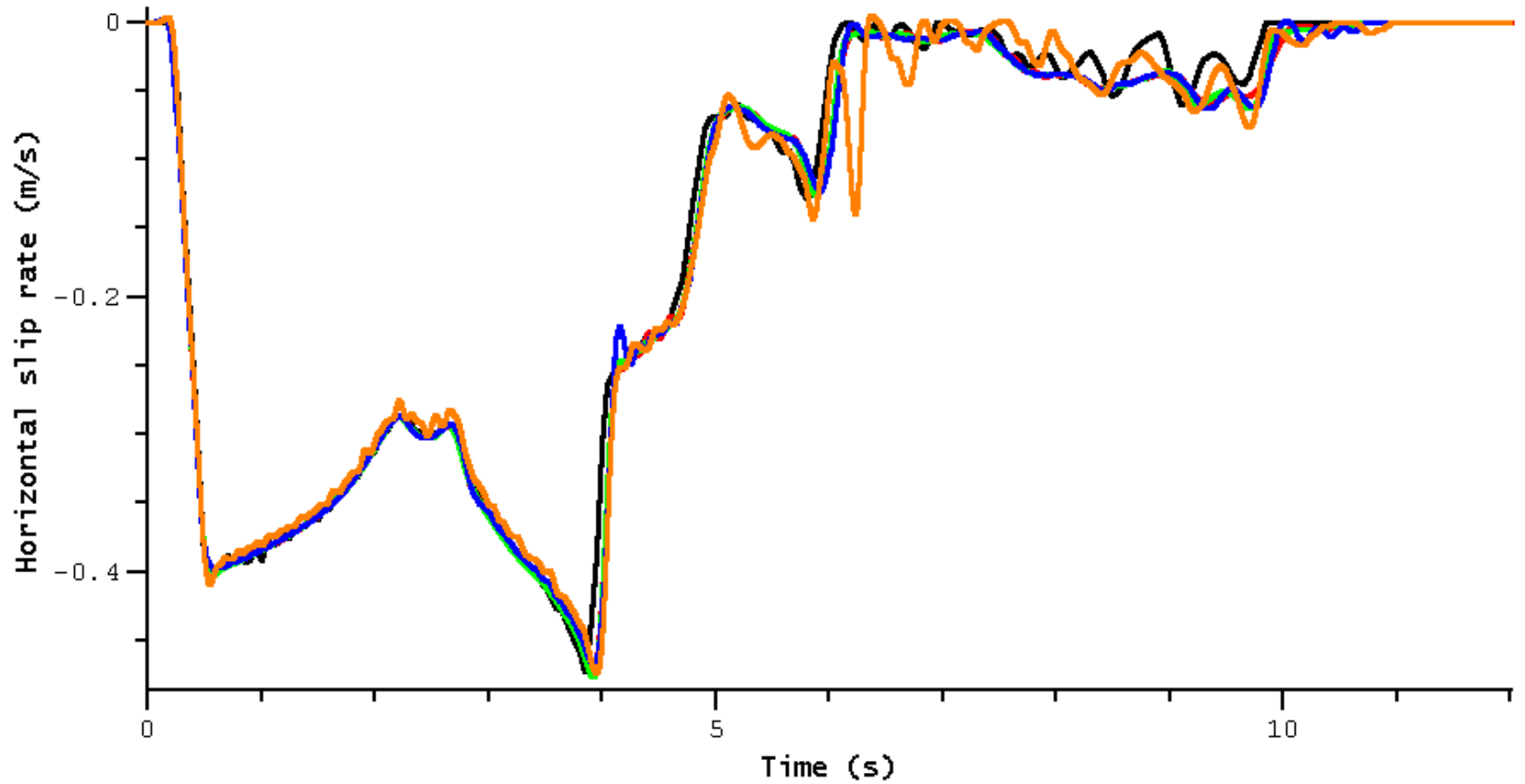
- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))
- somala (Surendra Somala - Spectral Element - SESAME (100m))

Branch Fault



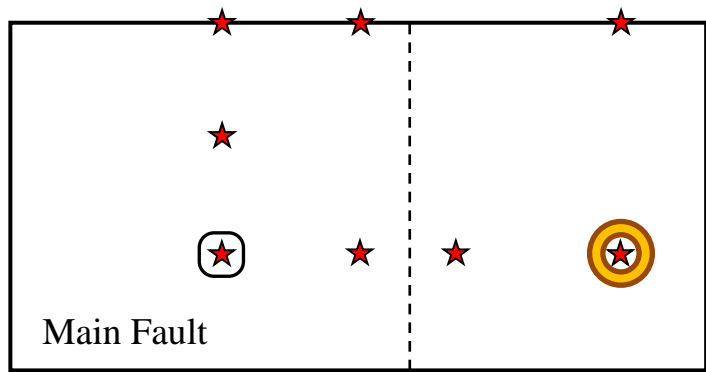
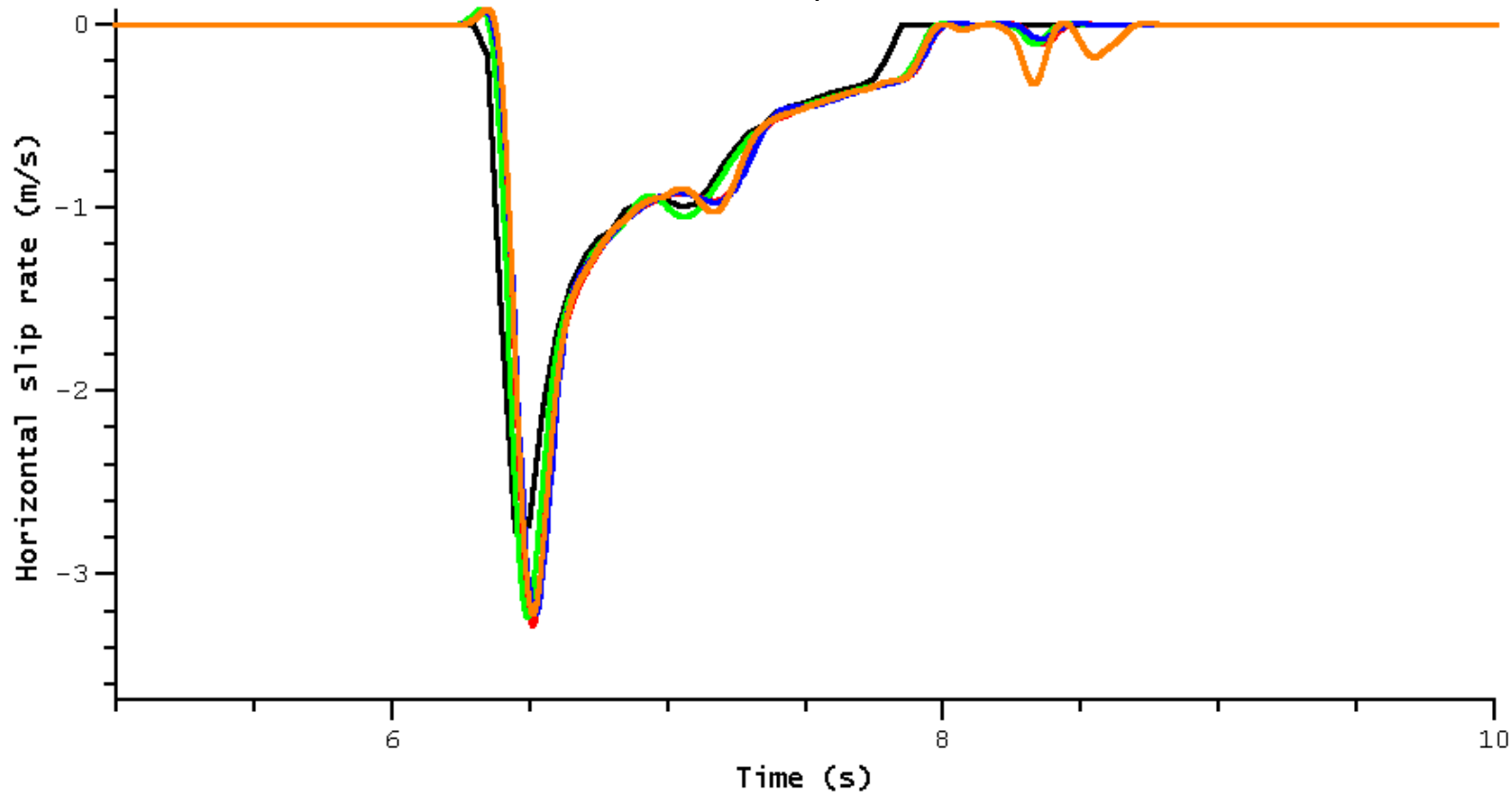
- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))
- somala (Surendra Somala - Spectral Element - SESAME (100m))

faultst-080dp100 (hypocenter)



5 Hz low-pass filter applied to all time series.

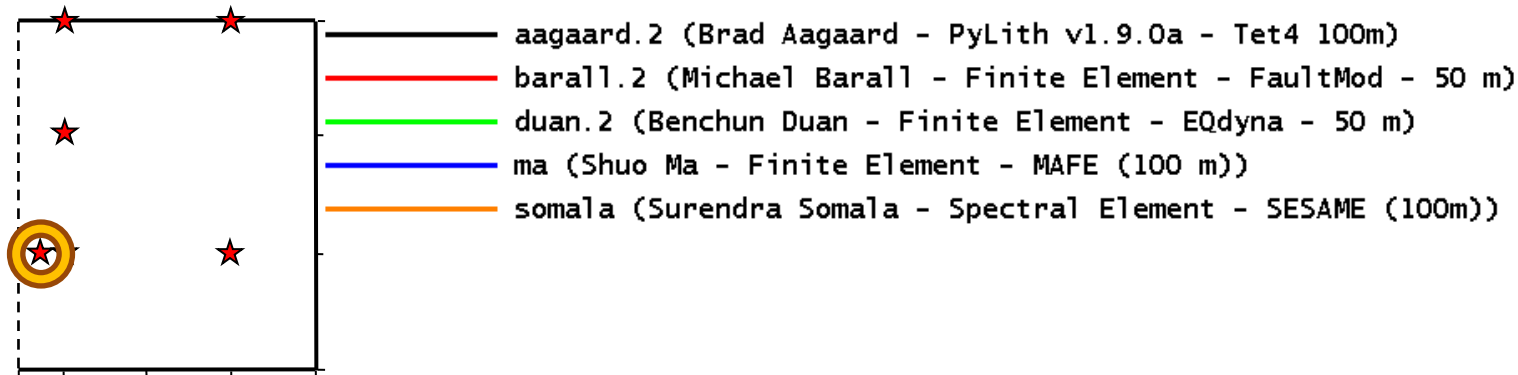
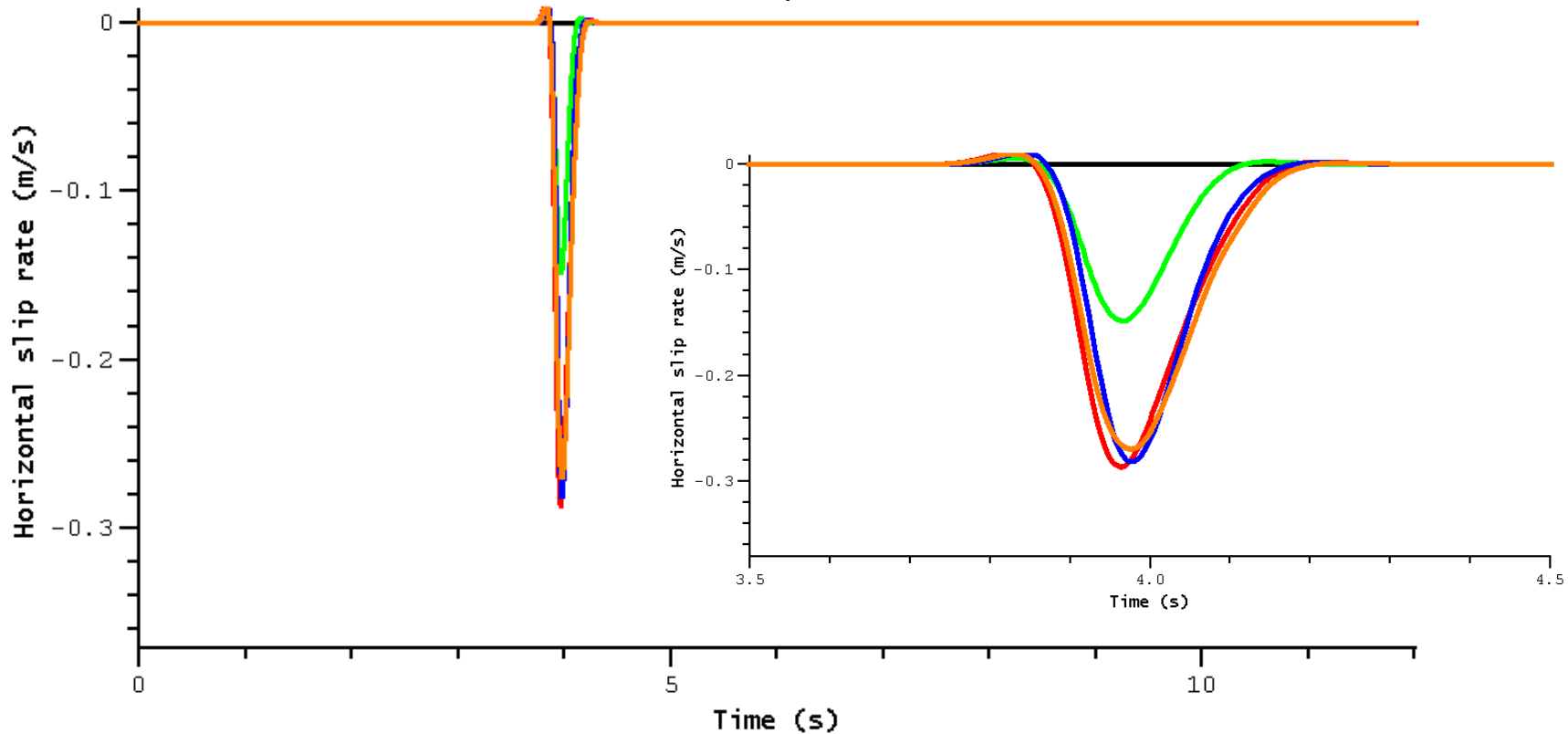
faultst090dp100



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))
- somala (Surendra Somala - Spectral Element - SESAME (100m))

5 Hz low-pass filter applied to all time series.

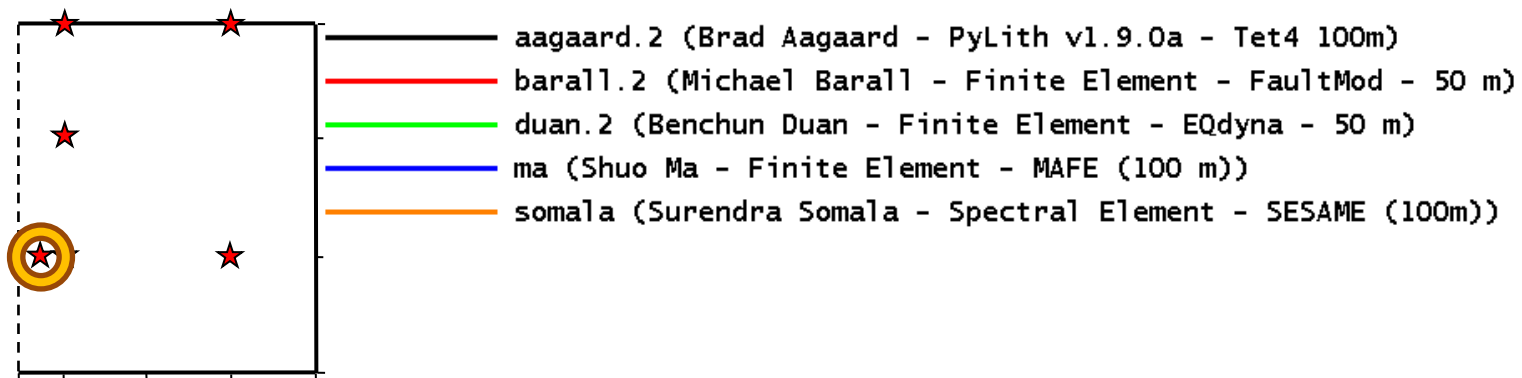
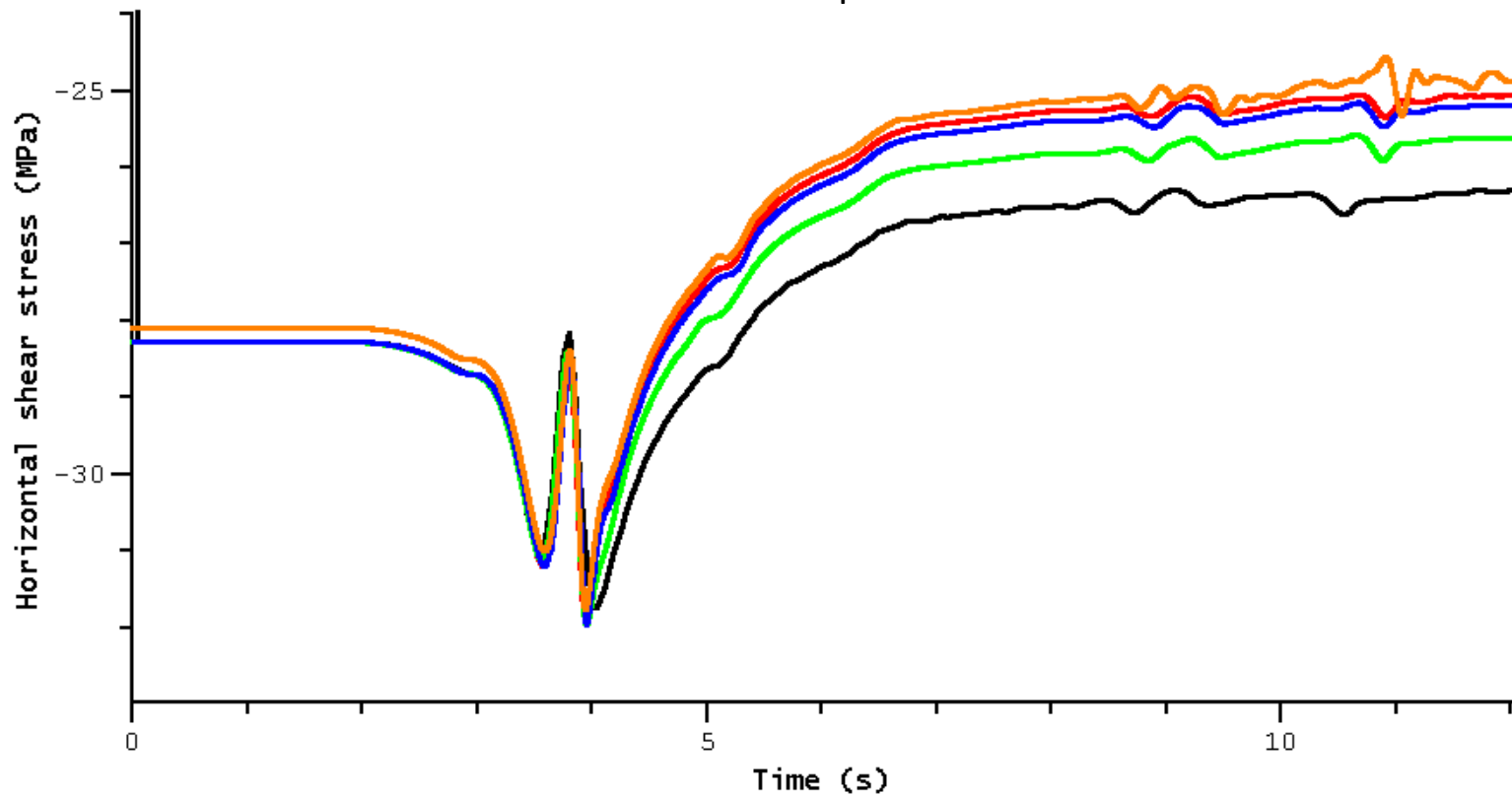
branchst010dp100



Branch Fault

5 Hz low-pass filter applied to all time series.

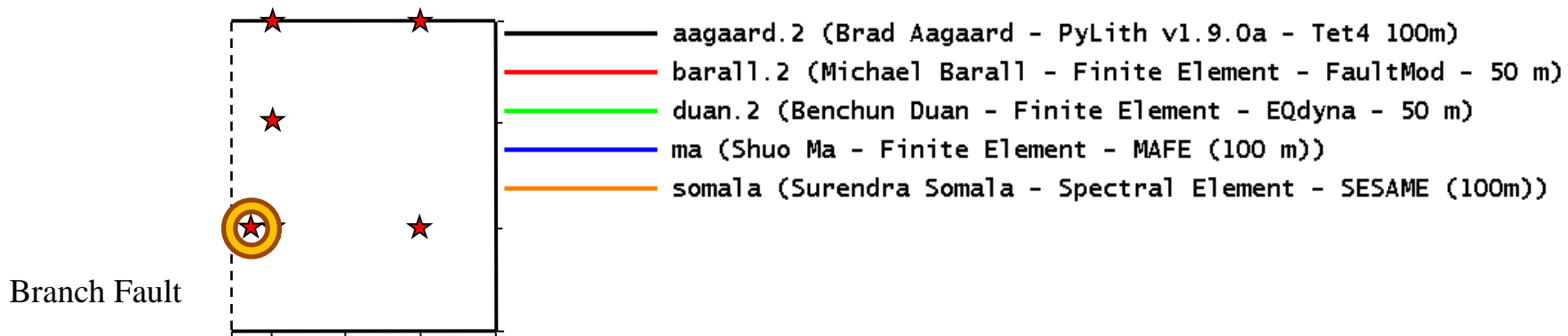
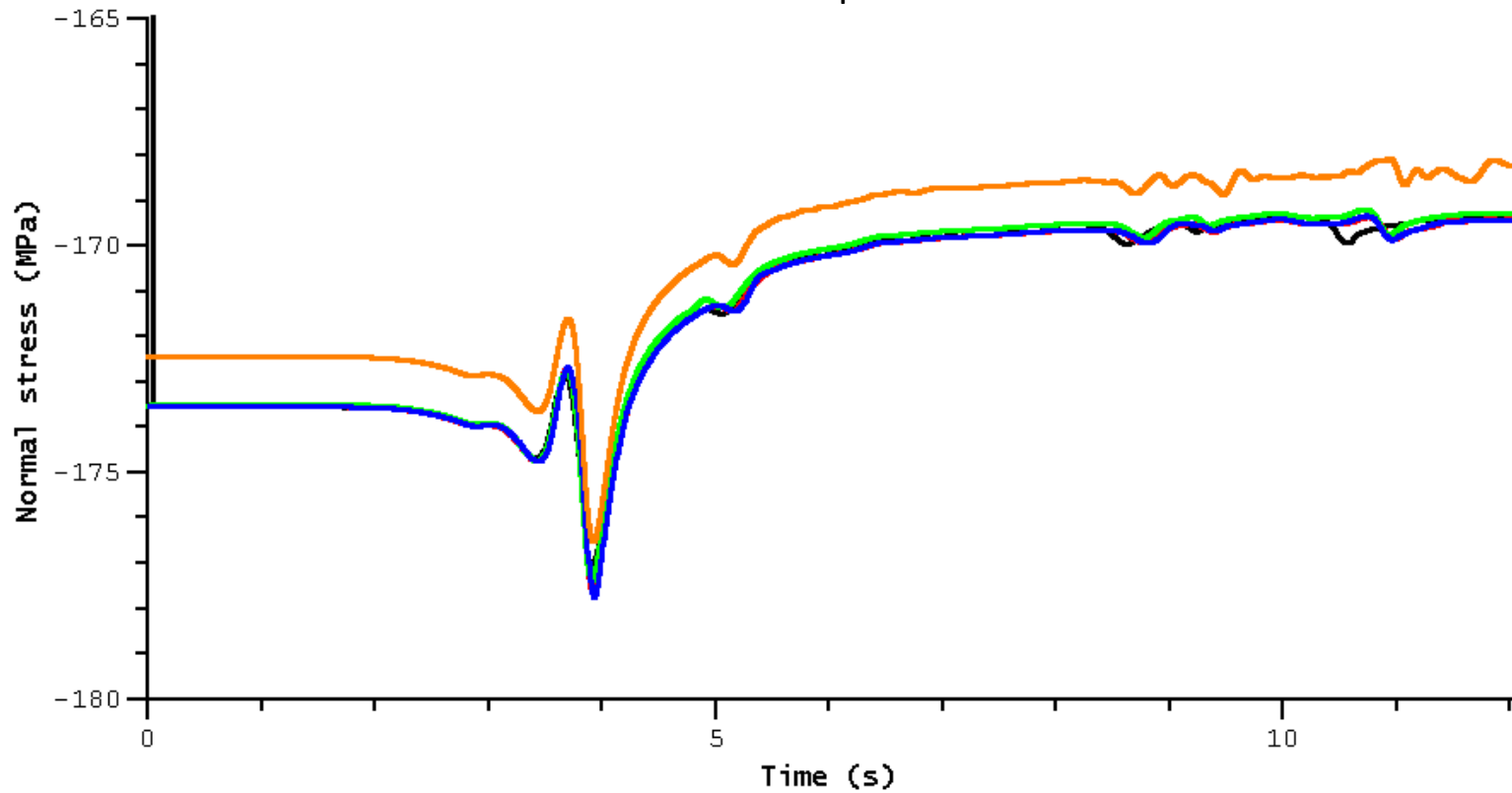
branchst010dp100



Branch Fault

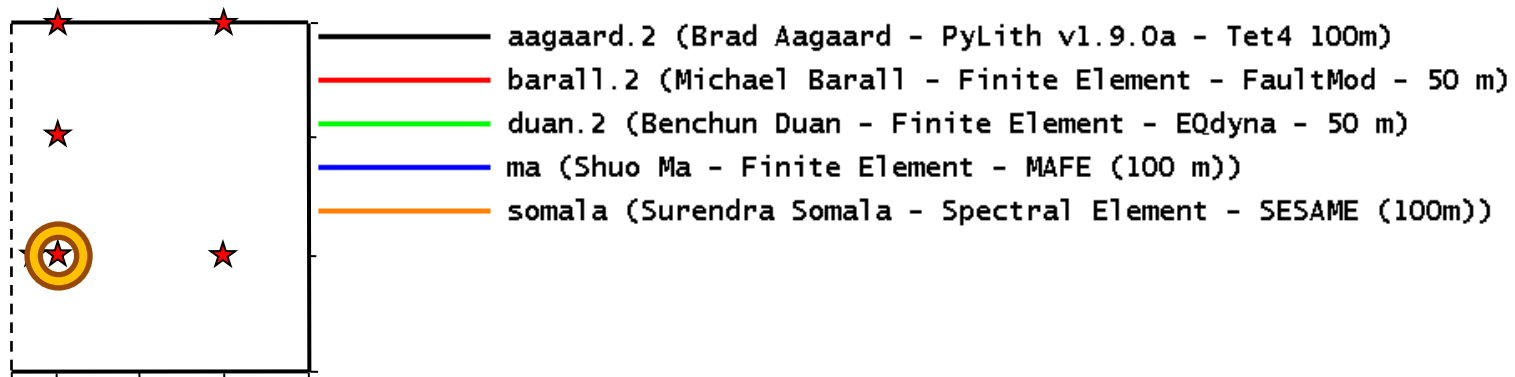
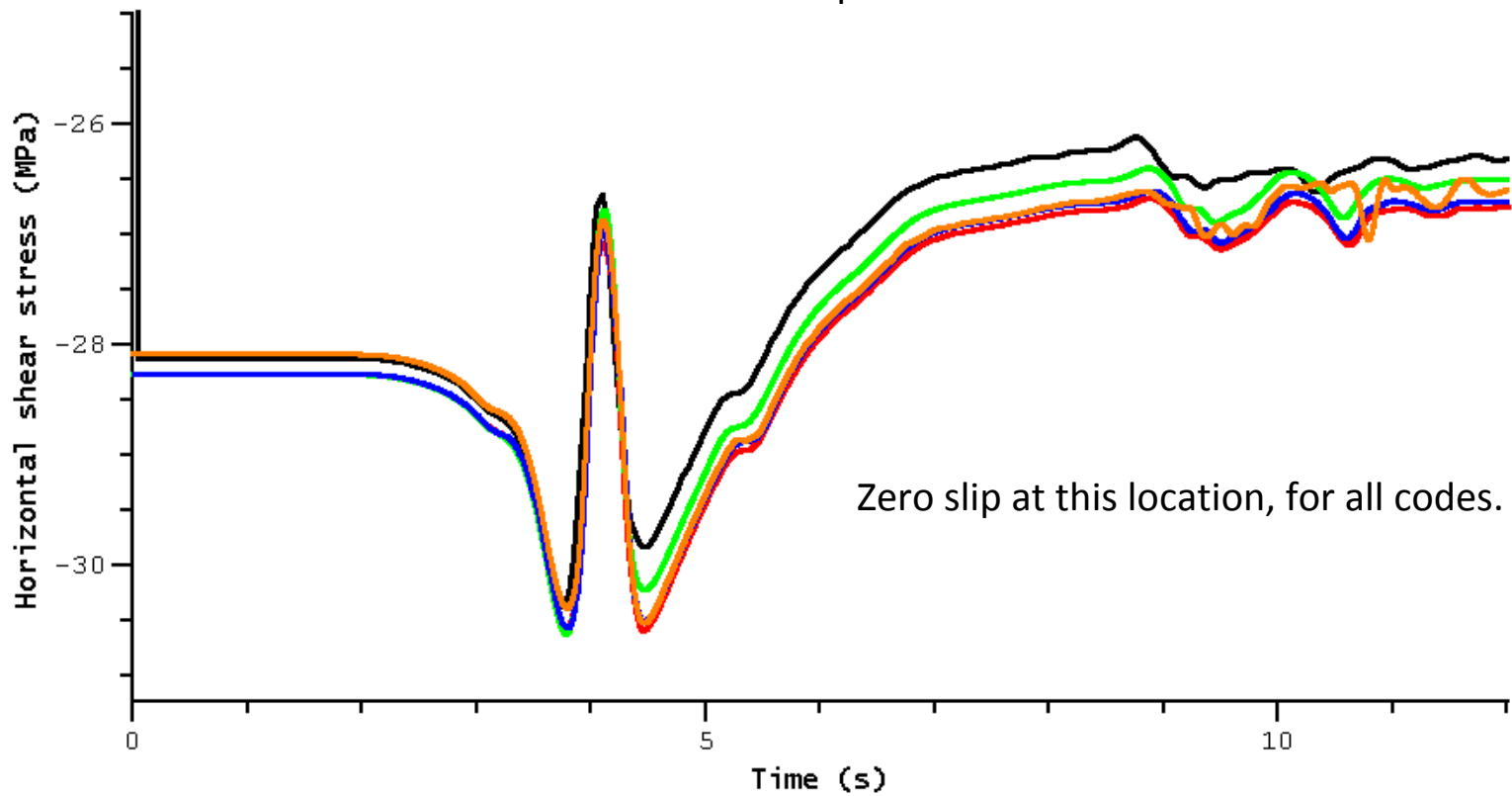
5 Hz low-pass filter applied to all time series.

branchst010dp100



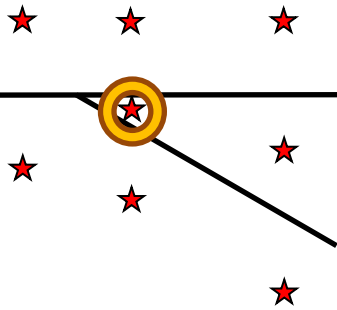
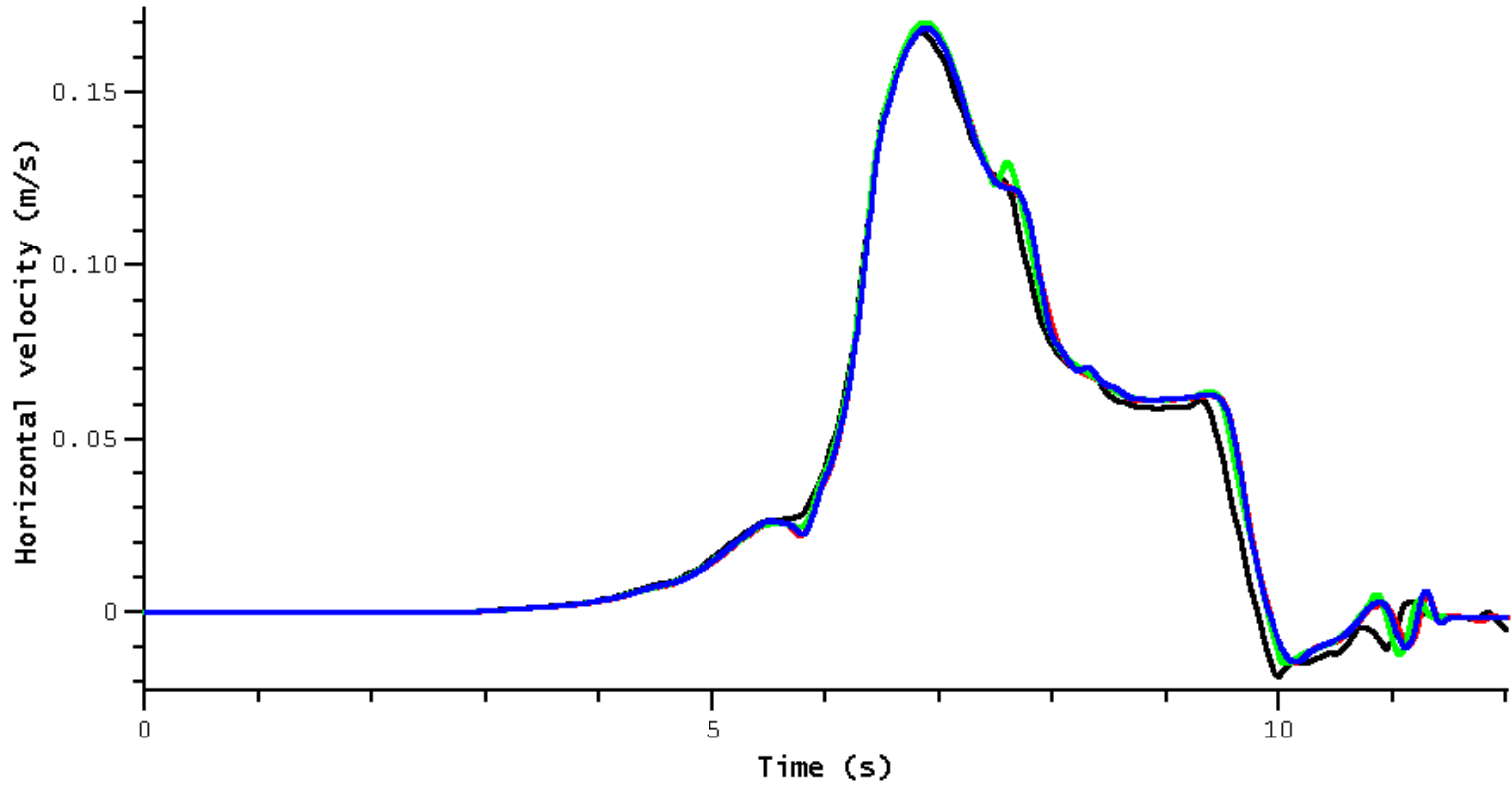
5 Hz low-pass filter applied to all time series.

branchst020dp100



5 Hz low-pass filter applied to all time series.

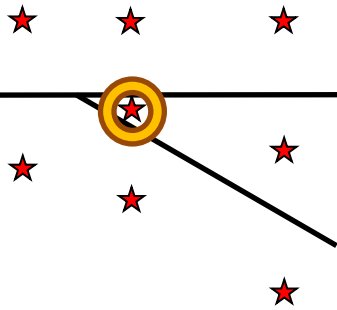
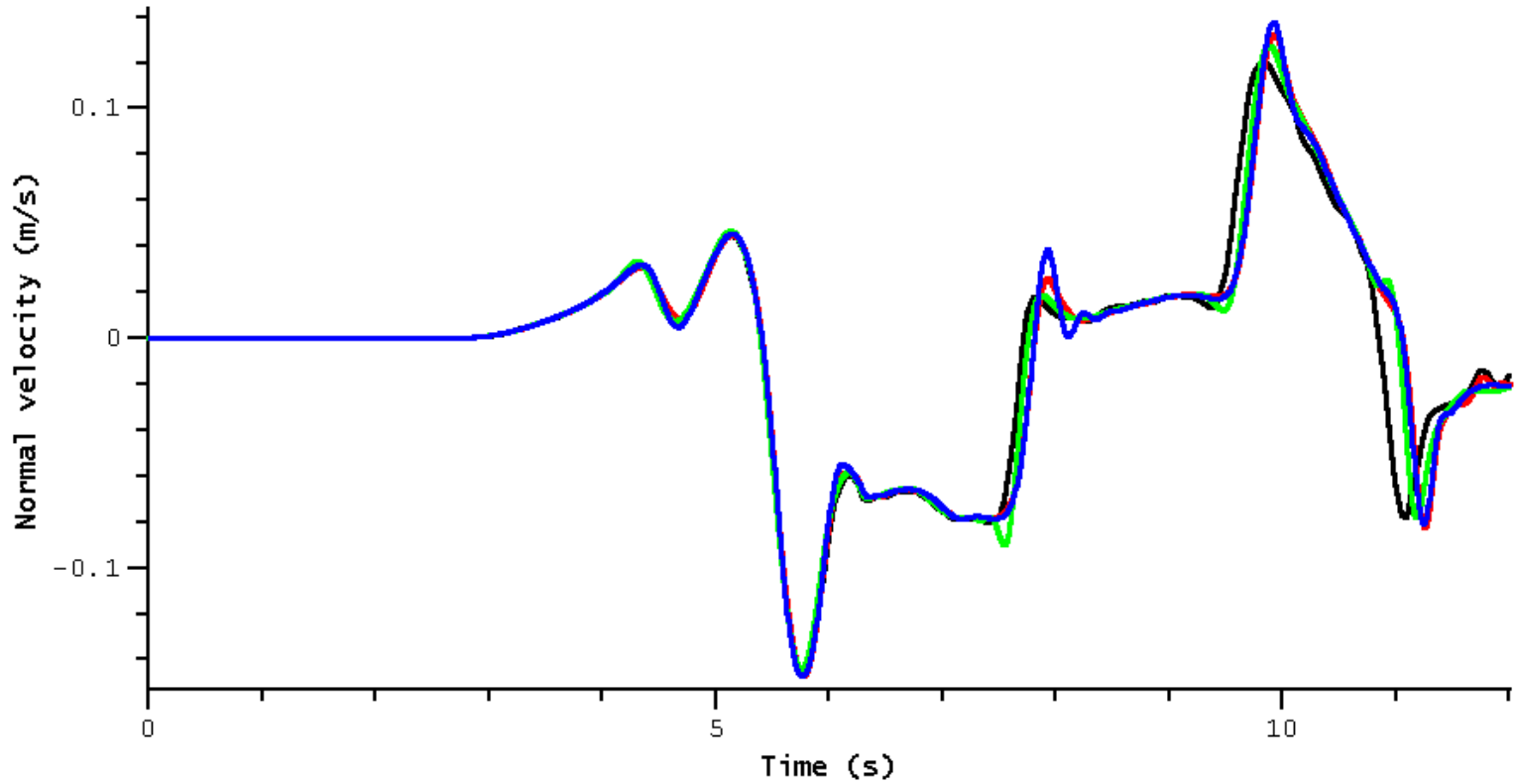
body-006st020dp000



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))

5 Hz low-pass filter applied to all time series.

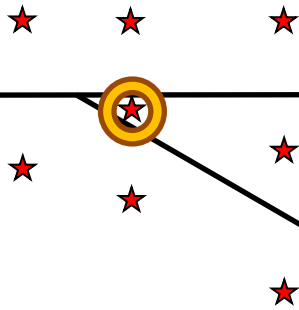
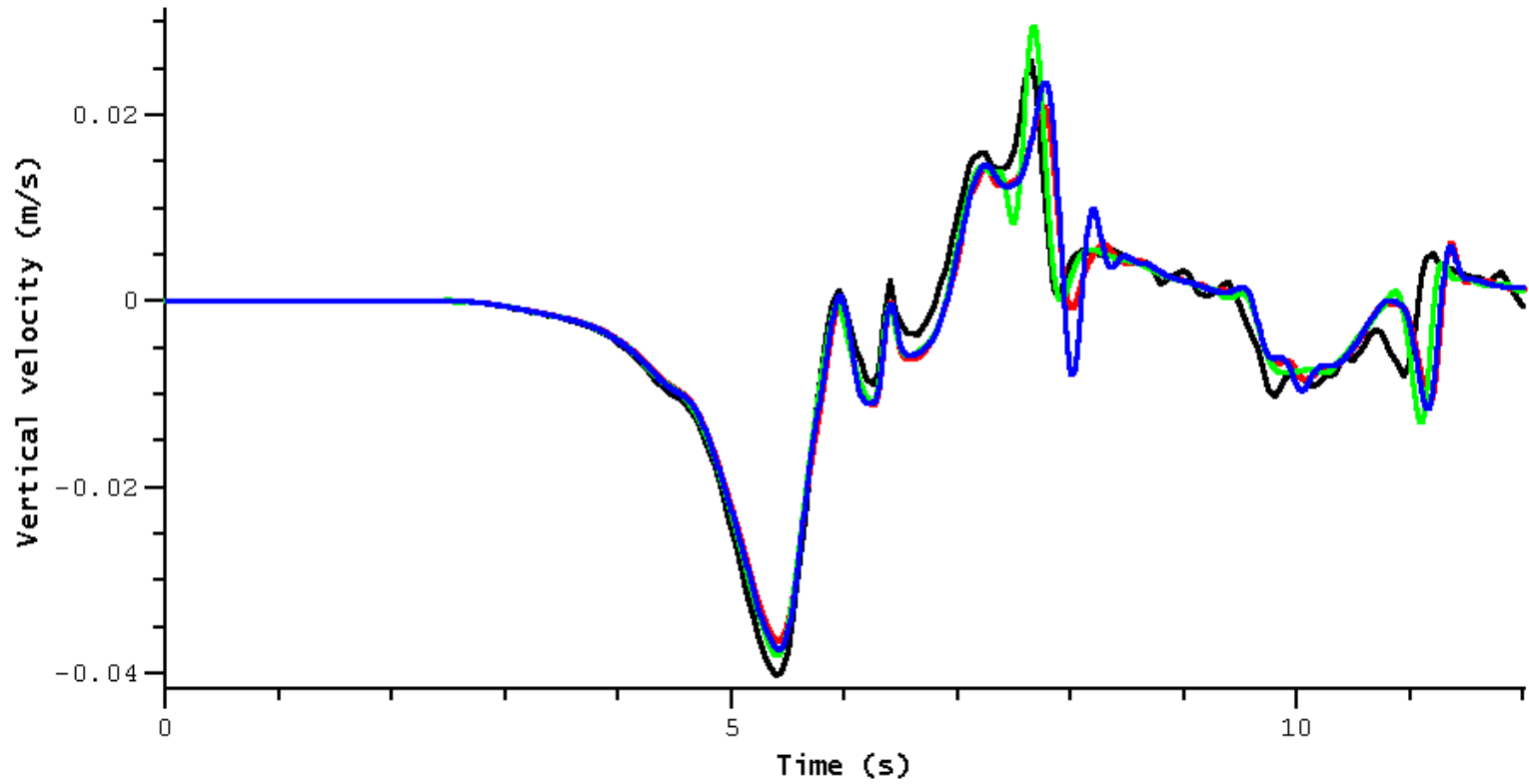
body-006st020dp000



- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))

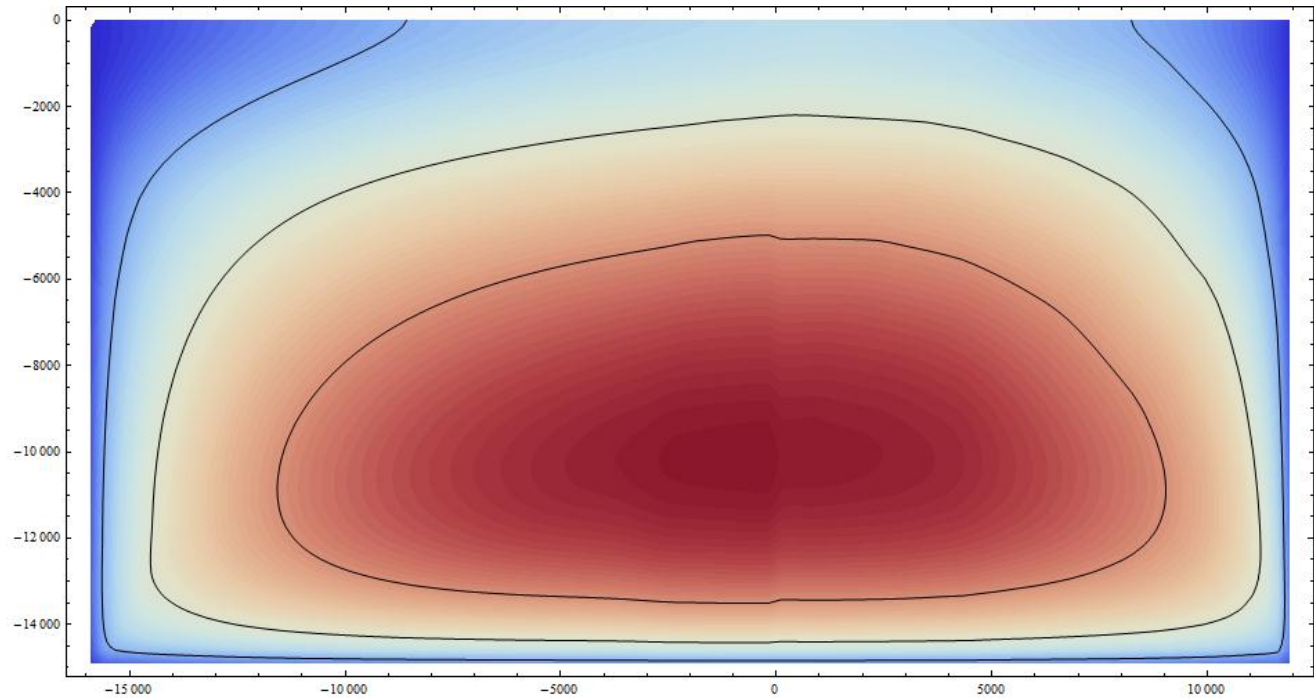
5 Hz low-pass filter applied to all time series.

body-006st020dp000

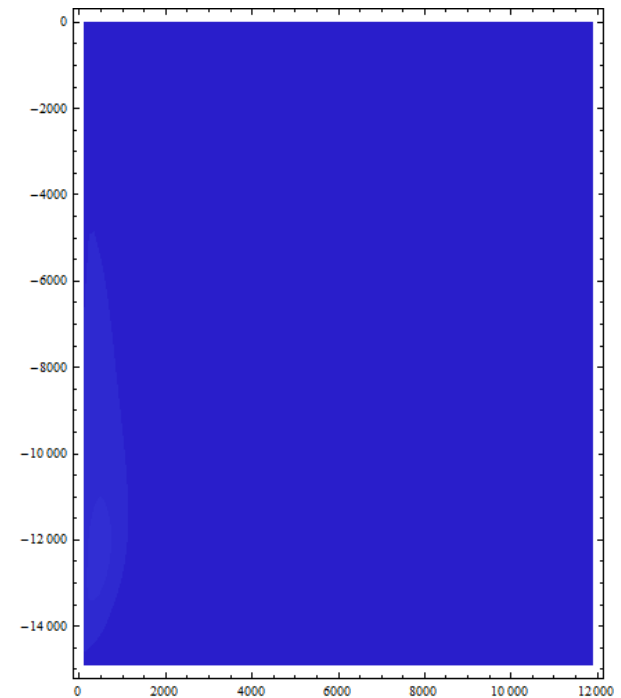
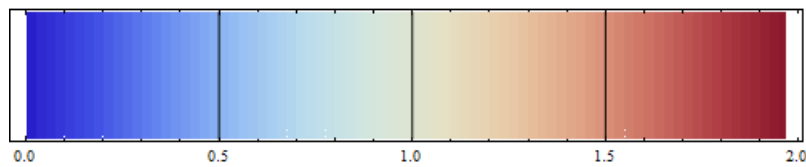


- aagaard.2 (Brad Aagaard - PyLith v1.9.0a - Tet4 100m)
- barall.2 (Michael Barall - Finite Element - FaultMod - 50 m)
- duan.2 (Benchun Duan - Finite Element - EQdyna - 50 m)
- ma (Shuo Ma - Finite Element - MAFE (100 m))

5 Hz low-pass filter applied to all time series.



Final slip on main and branch faults for TPV25



Conclusions

Our branched-fault benchmarks are:

TPV24 = Right-lateral, releasing branch.

TPV25 = Left-lateral, restraining branch.

The benchmarks are linear elastic but are designed like plastic benchmarks, with gravitational loading, fluid pressure, and an initial stress tensor specified throughout the medium.

These multi-fault benchmarks are designed to avoid loss of numerical precision, which may occur if there is a significant part of the branch fault where shear stress is near the minimum required to sustain a rupture.

We nucleate by gradually reducing the friction within the nucleation zone, to create a forced rupture with variable speed.

With selected parameters, the releasing case ruptures the entire branch fault, while the restraining case ruptures only a small part of the branch fault.

Comparison of 50 m and 100 m results shows good agreement, indicating the benchmarks are well resolved at the requested resolutions.

Comparison between different codes shows good agreement.