

# Rough Fault with Viscoplasticity Benchmarks

TPV29 — Linear Elastic Case

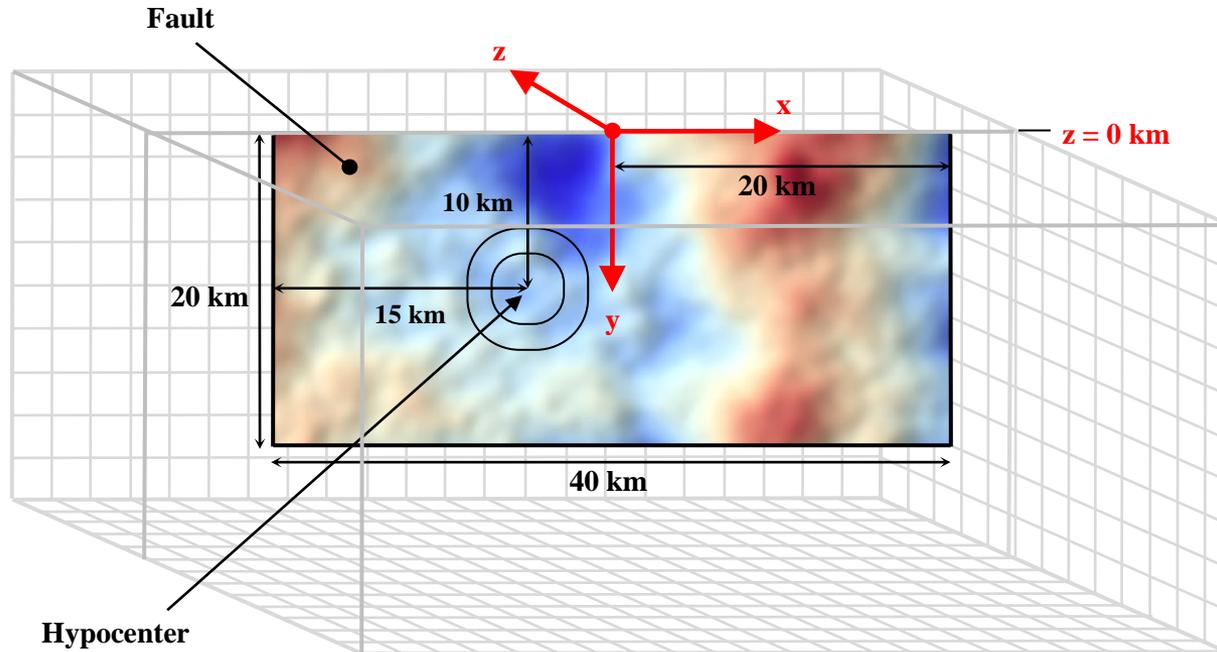
TPV30 — Viscoplastic Case

Michael Barall  
Invisible Software, Inc.

SCEC Rupture Dynamics Code Verification Workshop

March 23, 2015

# TPV29 (Elastic) and TPV30 (Viscoplastic) — Rough Fault with Viscoplasticity

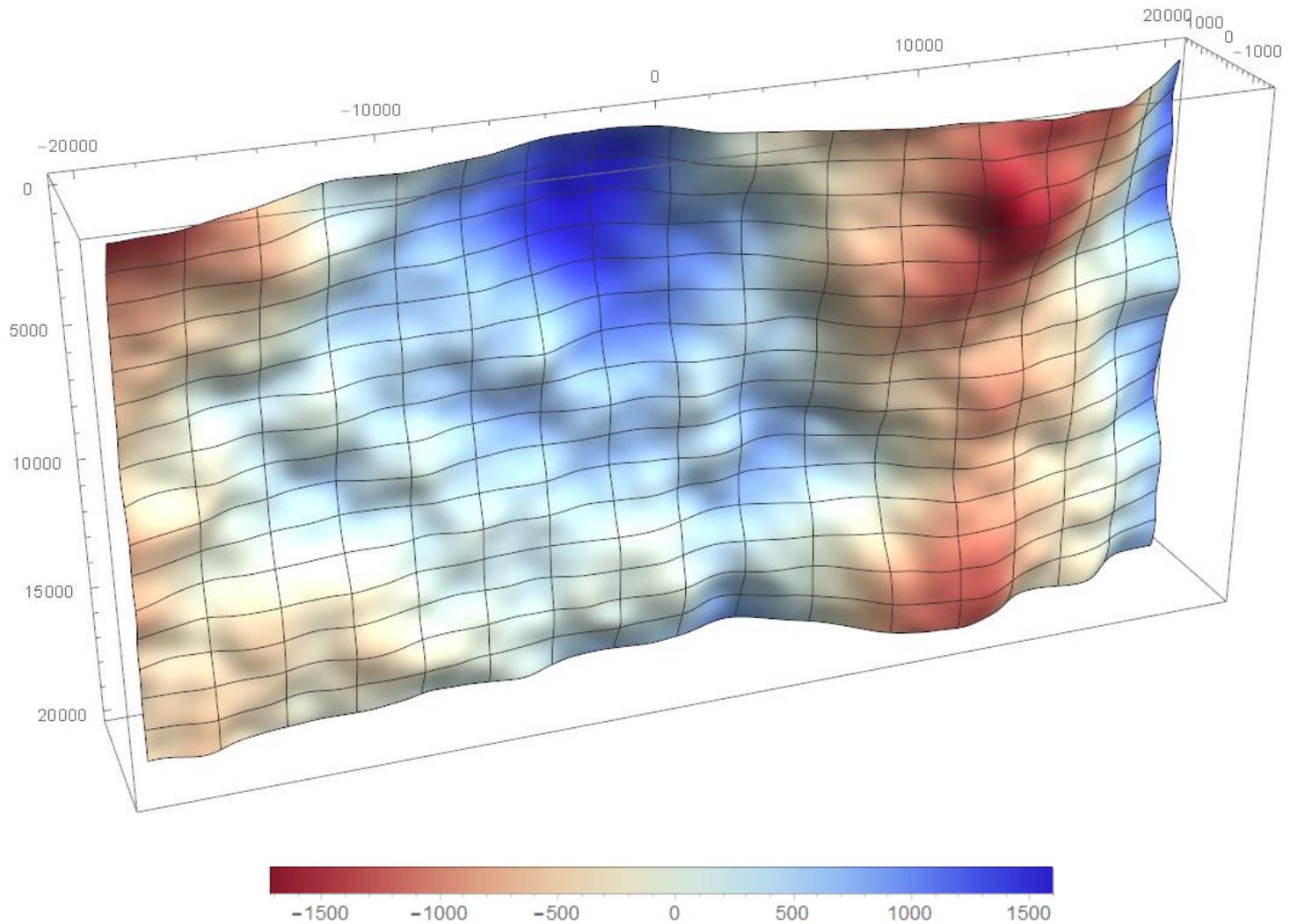


Right-lateral strike-slip fault in a uniform half-space.

TPV29 — Elastic material properties.

TPV30 — Viscoplastic material properties.

# TPV29-30 Fault Geometry



Note: Relief is exaggerated.

## TPV29-30 Fault Geometry Construction

The fault has fractal roughness, with Hurst exponent  $H = 1$ .

Roughness is bandwidth-limited to wavelengths between 1,000 m and 40,000 m.

It is constructed using a Fourier transform technique:

1. Begin with a square region, 400 km on a side, gridded at 25 m resolution.
2. Working in the frequency domain, each Fourier coefficient with wavelength between 1,000 m and 40,000 m is assigned a value proportional to  $k^{-(H+1)}$  and a random complex phase, where  $k$  is the wavenumber.
3. Perform a Fourier transform to get the fault roughness in the spatial domain.
4. Cut out an arbitrarily-chosen region the size of the fault surface.
5. Apply a moving-average filter, using a 2D filter kernel with a cosine shape and a half-width of 1,000 m.

You can think of the fault roughness as being a linear superposition of sine waves, each with random phase. The sine waves have wavelengths ranging from 1,000 m to 40,000 m, and amplitude proportional to the square of the wavelength.

# Viscoplasticity

## What is Viscoplasticity?

(Not mathematically rigorous)

A plastic or viscoplastic material has a *yield stress*  $Y$ .

- So long as the magnitude of the stress tensor  $\sigma$  remains less than  $Y$ , the material behaves like a linear elastic material.
- If the magnitude of the stress tensor  $\sigma$  exceeds  $Y$ , then the material *yields*.

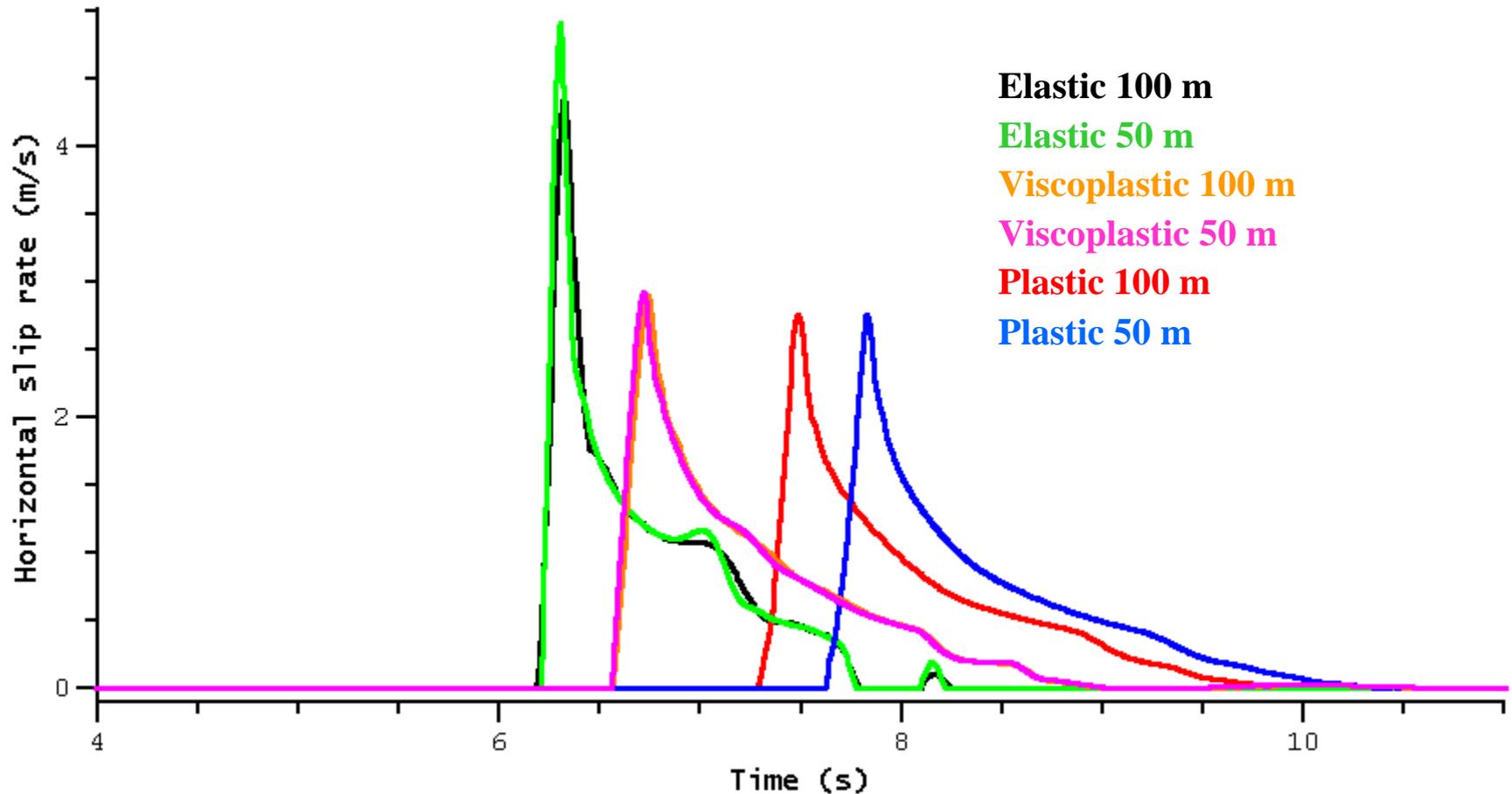
*Yielding* means that the stress tensor changes spontaneously (i.e., without application of strain), so that the magnitude of the stress tensor is reduced to the yield stress. Yielding is accompanied by the release of energy (heat).

Plasticity and viscoplasticity differ in how quickly the magnitude of the stress tensor is reduced to the yield stress.

- In plasticity, the magnitude of the stress tensor drops *instantaneously* to the yield stress.
- In viscoplasticity, the magnitude of the stress tensor decays gradually to the yield stress. The decay is exponential, with characteristic time  $T_v$ .

The time constant  $T_v$  is called the *relaxation time* of the viscoplastic material.

## Viscoplasticity is Needed for Numerical Convergence



The image shows modeling results for a vertical planar fault, at 17 km from the hypocenter.

Pure plasticity does not give good agreement between 100 m and 50 m resolutions.

# TPV29-30 Design

## Material Properties are the Only Difference Between TPV29 and TPV30

TPV29 uses linear elastic material:

$$\text{Density } \rho = 2670 \text{ kg/m}^3$$

$$\text{Shear-wave velocity } V_S = 3464 \text{ m/s}$$

$$\text{Pressure-wave velocity } V_P = 6000 \text{ m/s}$$

TPV30 uses viscoplastic material:

$$\text{Density } \rho = 2670 \text{ kg/m}^3$$

$$\text{Shear-wave velocity } V_S = 3464 \text{ m/s}$$

$$\text{Pressure-wave velocity } V_P = 6000 \text{ m/s}$$

$$\text{Cohesion } c = 1.18 \text{ MPa}$$

$$\text{Bulk friction } \nu = 0.1680$$

$$\text{Relaxation time } T_\nu = 0.05 \text{ s}$$

The viscoplastic material has a *yield stress*  $Y$ . When the magnitude of the stress tensor exceeds  $Y$ , the stress spontaneously decays so its magnitude approaches  $Y$ . The decay is exponential, with characteristic time  $T_\nu$ .

## Initial Stress Tensor

In a calculation with viscoplasticity:

- The initial stress tensor must be specified through the model volume.
- The shear and normal stress on the fault are obtained by resolving the stress tensor.

The initial stress tensor and fluid pressure are:

$$\text{Fluid pressure } P_f = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

Hydrostatic  
fluid pressure.

$$\text{Vertical stress } \sigma_{22} = -(2670 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

Lithostatic  
stress.

$$\text{Fault-parallel stress } \sigma_{11} = \Omega(\text{depth})(b_{11}(\sigma_{22} + P_f) - P_f) + (1 - \Omega(\text{depth}))\sigma_{22}$$

$$\text{Fault-normal stress } \sigma_{33} = \Omega(\text{depth})(b_{33}(\sigma_{22} + P_f) - P_f) + (1 - \Omega(\text{depth}))\sigma_{22}$$

$$\text{Horizontal shear stress } \sigma_{13} = \Omega(\text{depth})(b_{13}(\sigma_{22} + P_f))$$

Tapering coefficient  
(see next slide).

$$\text{Vertical shear stress } \sigma_{23} = 0$$

$$\text{Fault-parallel shear stress } \sigma_{12} = 0$$

Stress ratio coefficient  
(see next slide).

## Initial Stress Tensor

The tapering coefficient  $\Omega(\text{depth})$  causes the deviatoric component of stress to taper down to zero at depths between 17000 m and 22000 m.

$$\Omega(\text{depth}) = \begin{cases} 1, & \text{if depth} \leq 17000 \text{ m} \\ (22000 \text{ m} - \text{depth}) / (5000 \text{ m}), & \text{if } 17000 \text{ m} \leq \text{depth} \leq 22000 \text{ m} \\ 0, & \text{if depth} \geq 22000 \text{ m} \end{cases}$$

The coefficients  $b_{11}$ ,  $b_{33}$ , and  $b_{13}$  are the ratios of effective stress components.

Coefficient	Value for TPV29 and TPV30
$b_{11}$	1.025837
$b_{33}$	0.974162
$b_{13}$	-0.158649

$$\text{effective stress tensor} = \begin{pmatrix} \sigma_{11} + P_f & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} + P_f & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} + P_f \end{pmatrix}$$

Coefficients are chosen so that the mean fault plane is the optimum plane for fracture.

## Slip-Weakening Friction Parameters (With Time-Weakening Nucleation)

Static coefficient of friction  $\mu_s = 0.18$

Low friction coefficients due to high lithostatic stress.

Dynamic coefficient of friction  $\mu_d = 0.12$

Slip-weakening critical distance  $d_0 = 0.30$  m

Cohesion = 1.2 MPa at the earth's surface.

Cohesion  $C_0 = \begin{cases} 0.40 \text{ MPa} + (0.00020 \text{ MPa/m})(4000 \text{ m} - \text{depth}), & \text{if depth} \leq 4000 \text{ m} \\ 0.40 \text{ MPa}, & \text{if depth} \geq 4000 \text{ m} \end{cases}$

Radius of nucleation zone  $r_{\text{crit}} = 4000$  m

Time of forced rupture  $T = \begin{cases} \frac{r}{0.7 V_S} + \frac{0.081 r_{\text{crit}}}{0.7 V_S} \left( \frac{1}{1-(r/r_{\text{crit}})^2} - 1 \right), & \text{if } r < r_{\text{crit}} \\ 1.0\text{E}+9, & \text{if } r \geq r_{\text{crit}} \end{cases}$

Time-weakening decay time  $t_0 = 0.50$  s

( $r$  = distance from the hypocenter.)

Creates variable-speed expanding front of forced rupture in nucleation zone.

# Slip-Weakening Friction Law (With Time-Weakening Nucleation)

When the fault is sliding, the shear stress  $\tau$  is:

$$\tau = C_0 + \mu \max(0, \sigma_n - P_f)$$

The time-varying friction coefficient  $\mu$  is:

$$\mu = \mu_s + (\mu_d - \mu_s) \max(f_1, f_2)$$

$$f_1 = \begin{cases} D/d_0, & \text{if } D < d_0 \\ 1, & \text{if } D \geq d_0 \end{cases}$$

$$f_2 = \begin{cases} 0, & \text{if } t < T \\ (t - T)/t_0, & \text{if } T \leq t < T + t_0 \\ 1, & \text{if } t \geq T + t_0 \end{cases}$$

Where:

$\sigma_n$  = normal stress

$P_f$  = fluid pressure

$D$  = total distance the fault has slipped

$t$  = time since the start of the simulation

Linear slip-weakening.  
Starts weakening at slip  $D = 0$ .  
Stops weakening at slip  $D = d_0$ .

Linear time-weakening.  
Starts weakening at time  $t = T$ .  
Stops weakening at time  $t = T + t_0$ .

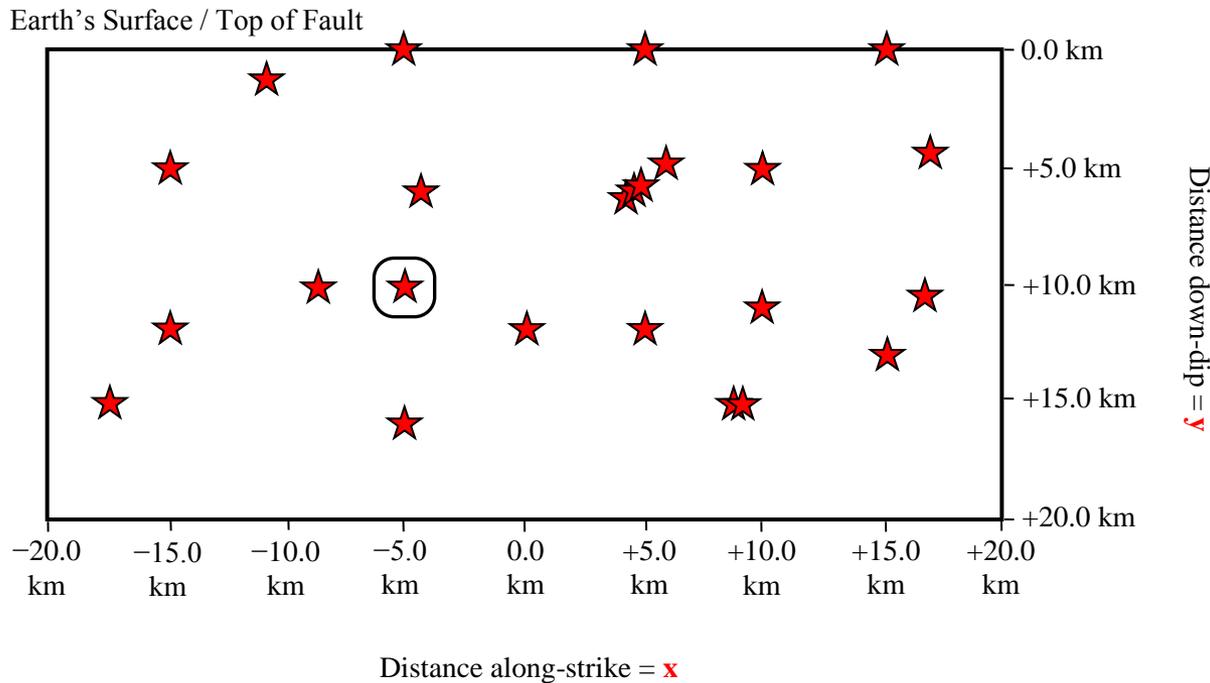
Outside nucleation zone,  $T = \infty$ ,  
so time-weakening does not occur.

## On-Fault Stations.

Modelers are asked to submit slip, slip rate, and stress as a function of time, for 24 stations on the fault.

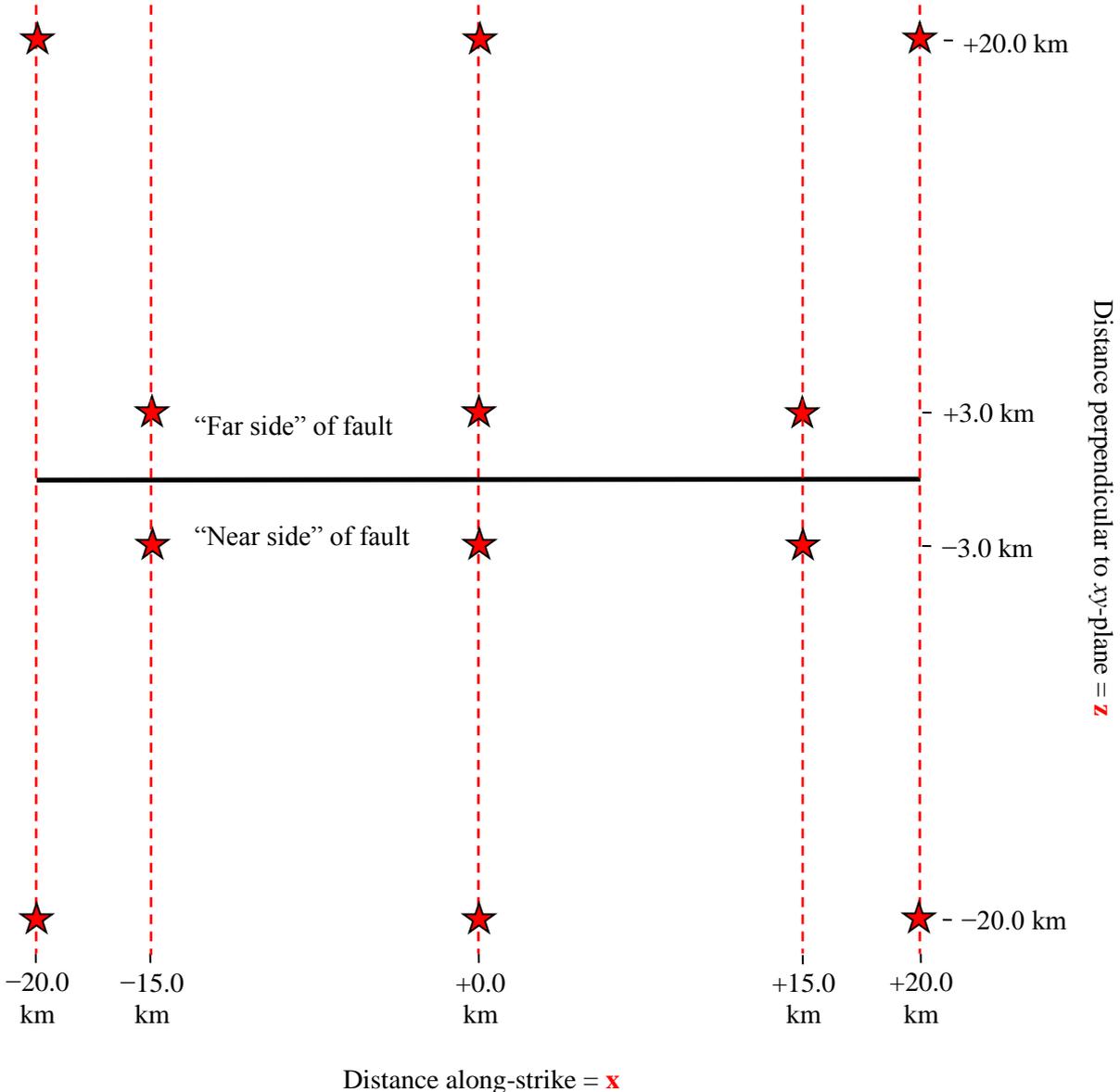
In addition, modelers are asked to submit the time at which each point on the fault begins to slip, from which we construct rupture contour plots.

We tried to select station locations that would be “interesting” given the shape of the fault.



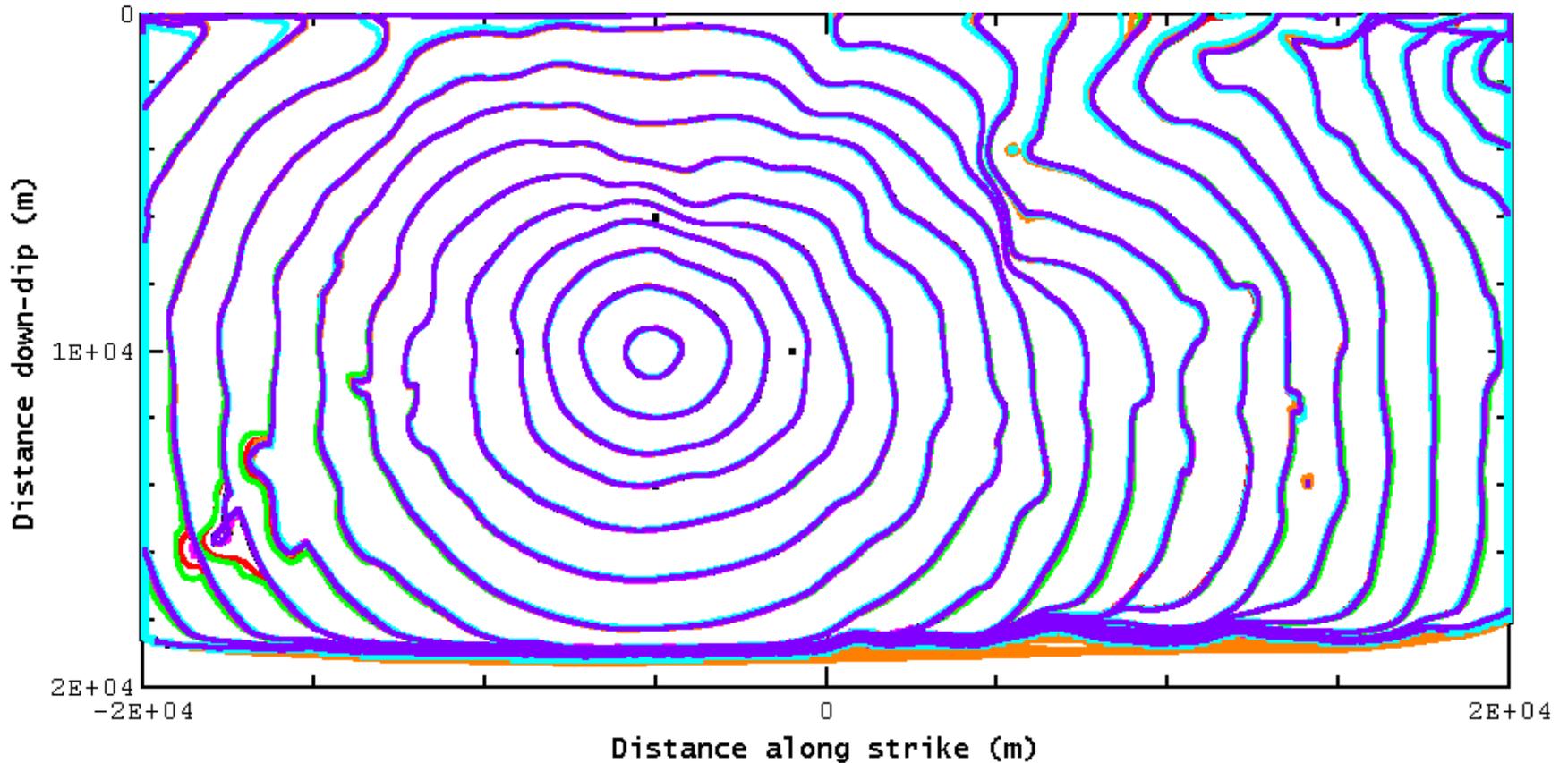
# Off-Fault Stations.

Modelers are asked to submit displacement and velocity as a function of time, for 12 stations on the earth's surface.



## TPV29-30 Rupture Contours

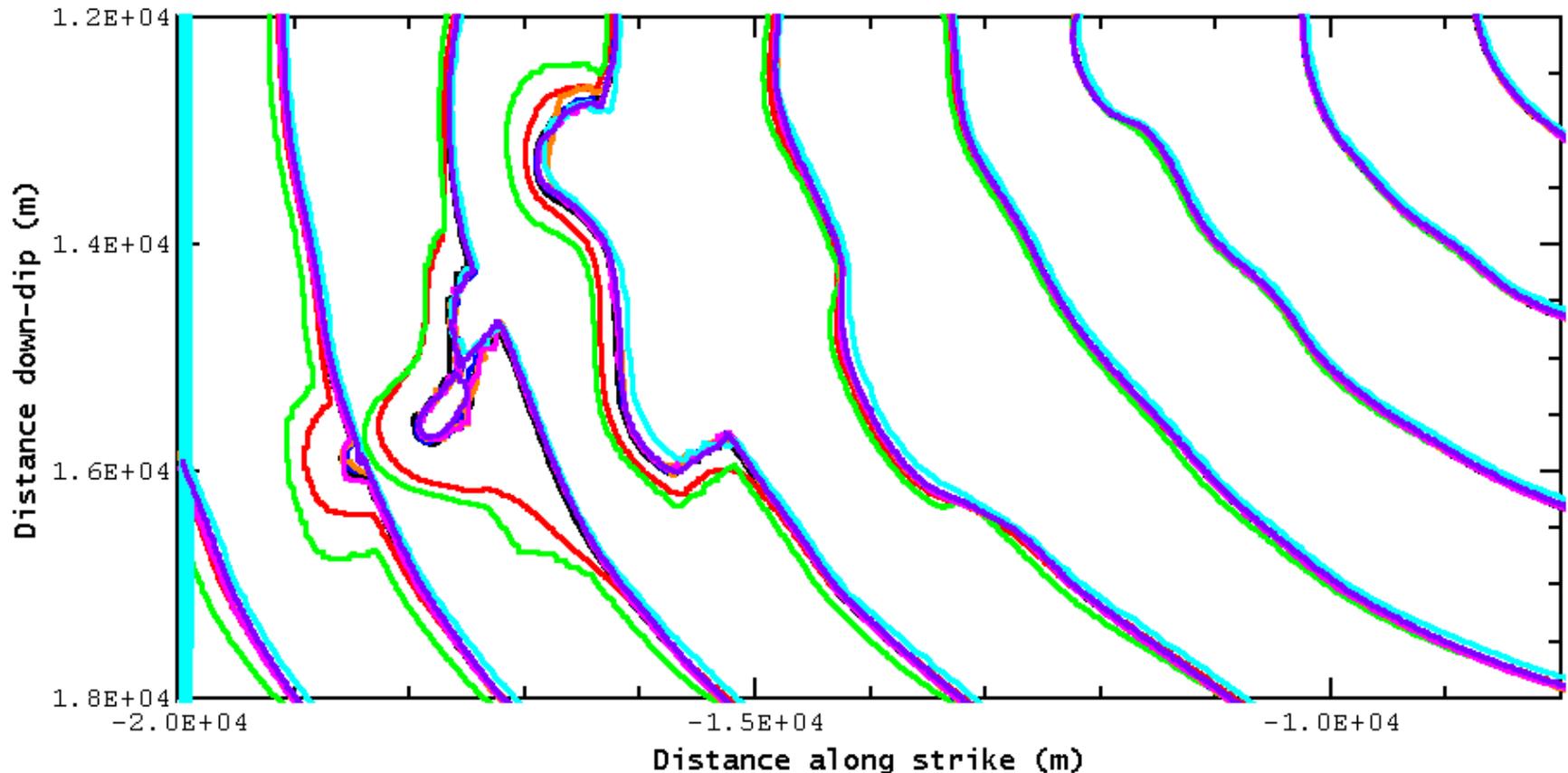
# TPV29 (Elastic) Rupture Contours — Highest Resolution from Each of 8 Modelers



- bai (Kangchen Bai - Spectral Element - SPECFEM3D 50m)
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- gabriel (Thomas Ulrich - Discontinuous Galerkin- Seissol 100m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)
- shi.2 (Zhejiang Shi - SORD-S - 25 m)

Contours agree very well, with some differences at the earth's surface (where one code deviates from the others) and at lower left.

## TPV29 (Elastic) Rupture Contours — Magnified Image of Lower Left Part of Fault



- bai (Kangchen Bai - Spectral Element - SPECFEM3D 50m)
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- gabriel (Thomas Ulrich - Discontinuous Galerkin- Seissol 100m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)
- shi.2 (Zheqiang Shi - SORD-S - 25 m)

Codes deviate from each other in this area.

As seen in TPV28, rupture contours tend to “heal” when they pass beyond an irregularity.

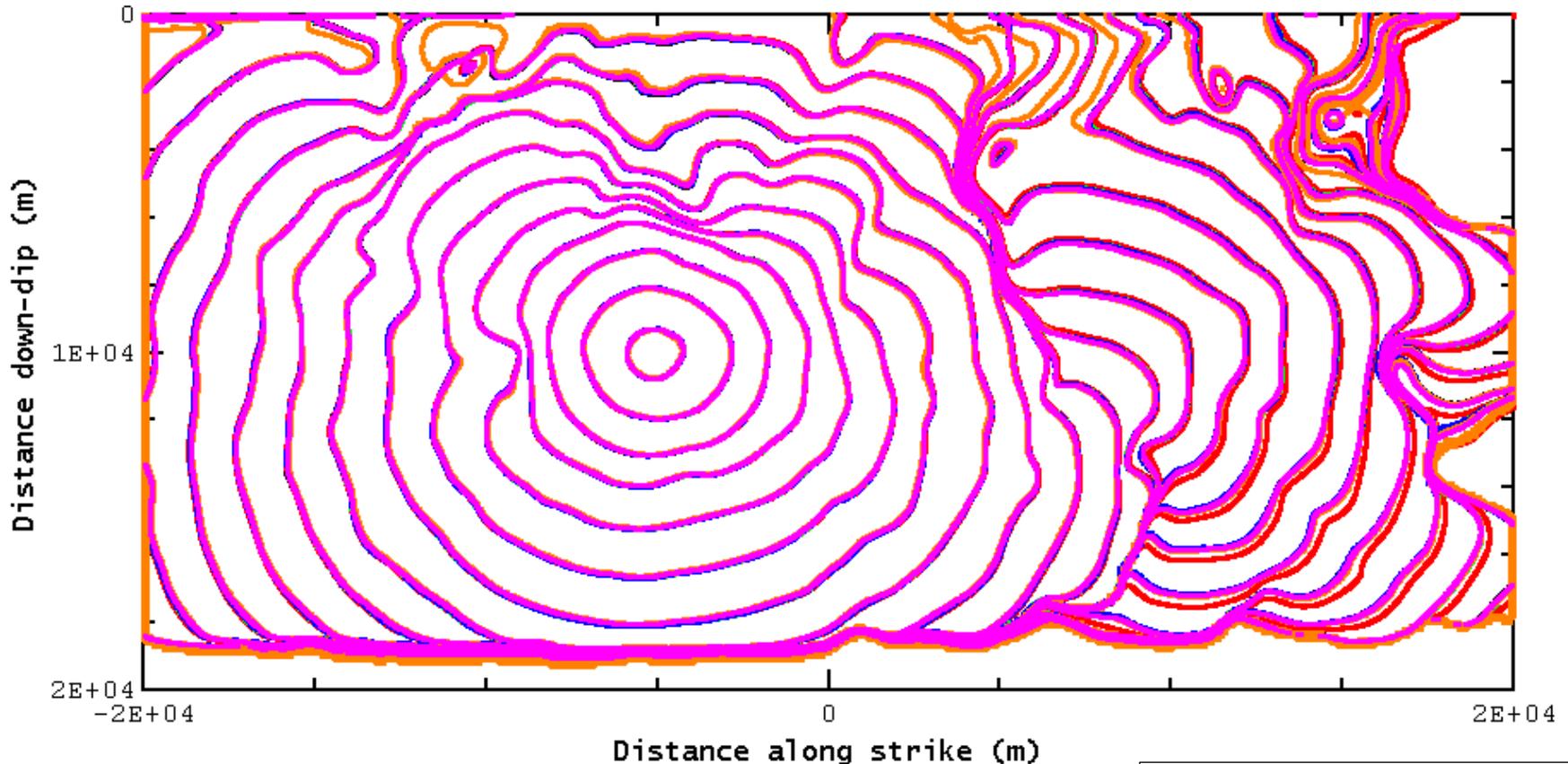
## TPV29 (Elastic) Rupture Contours — Metrics (RMS Difference in Rupture Time)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) bai		19.6	30.6	8.0	15.6	7.8	30.0	10.7
(2) barall.2	19.6		21.4	22.1	25.7	21.4	37.0	23.3
(3) dliu.2	30.6	21.4		32.1	36.7	31.2	48.8	33.5
(4) duru.2	8.0	22.1	32.1		15.4	5.5	30.4	7.6
(5) gabriel	15.6	25.7	36.7	15.4		15.3	22.3	15.7
(6) kozdon	7.8	21.4	31.2	5.5	15.3		30.0	9.3
(7) ma.2	30.0	37.0	48.8	30.4	22.3	30.0		28.6
(8) shi.2	10.7	23.3	33.5	7.6	15.7	9.3	28.6	

(1) bai	Kangchen Bai - Spectral Element - SPECFEM3D 50m
(2) barall.2	Michael Barall - FaultMod - 25 m
(3) dliu.2	Dunyu Liu - Finite Element - EQdyna-50m
(4) duru.2	Kenneth Duru - WaveQLab3D - 50 m
(5) gabriel	Thomas Ulrich - Discontinuous Galerkin- Seissol 100m
(6) kozdon	Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp
(7) ma.2	Shuo Ma - Finite Element - MAFE - 50m
(8) shi.2	Zheqiang Shi - SORD-S - 25 m

All values are less than 50 milliseconds, indicating good agreement. Codes bai, duru.2, kozdon, and shi.2 show excellent agreement among themselves, with metrics under 11 milliseconds.

## TPV30 (Viscoplastic) Rupture Contours — Highest Resolution from Each of 6 Modelers

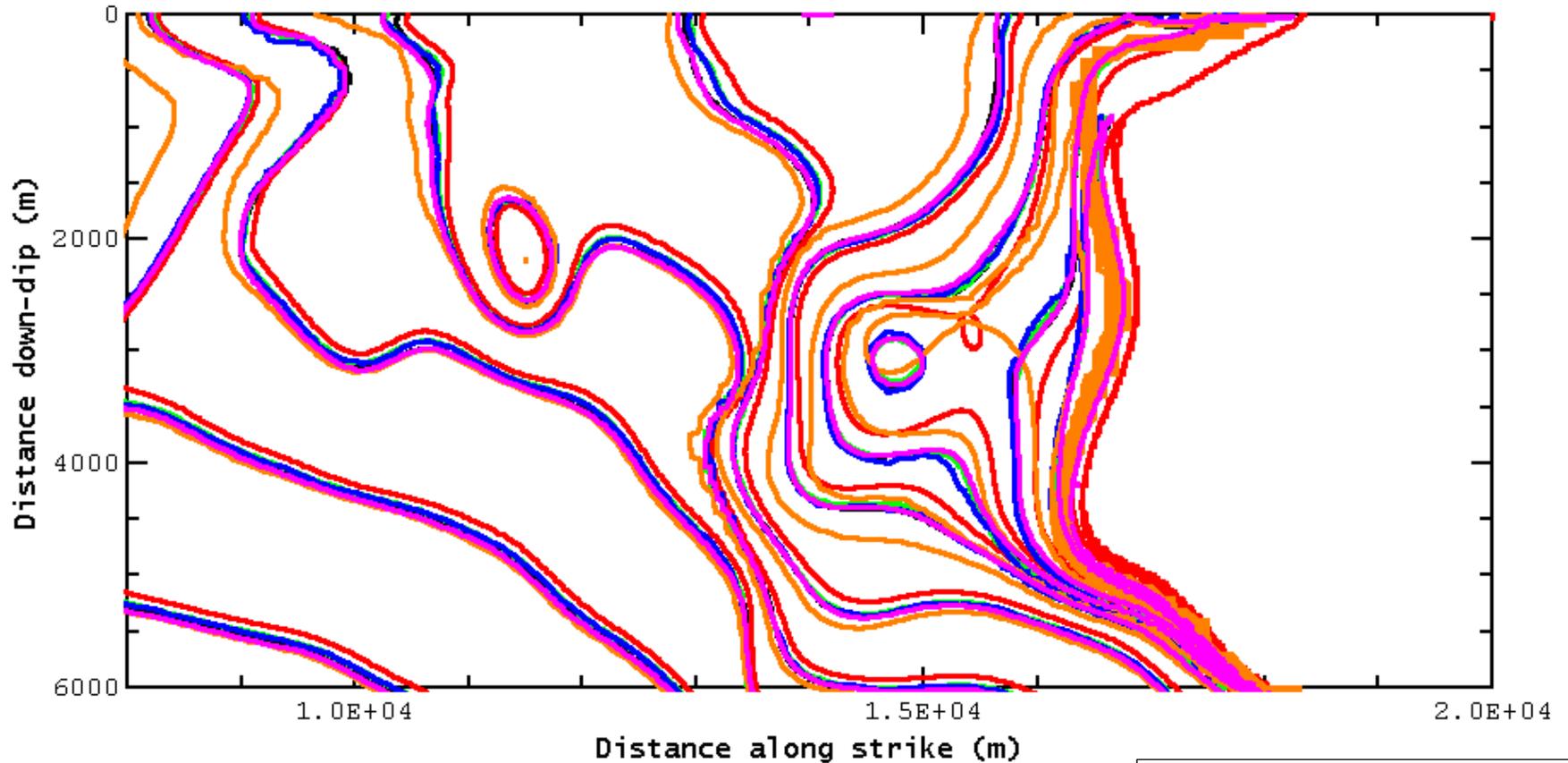


- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.3 (Dunyu Liu - Finite Element - EQdyna - 25m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)
- shi (Zheqiang Shi - Generalized Finite Difference - SORD)

Contours agree well, with some differences at top and lower right.

The irregularity at lower left, seen in TPV29, does not appear here.

# TPV30 (Viscoplastic) Rupture Contours — Magnified Image of Upper Right Part of Fault



- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.3 (Dunyu Liu - Finite Element - EQdyna - 25m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard ::  $p = 4$  :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)
- shi (Zheqiang Shi - Generalized Finite Difference - SORD)

Codes barall.2, duru.2, kozdon, and shi agree very closely with each other. The remaining two codes are a bit different.

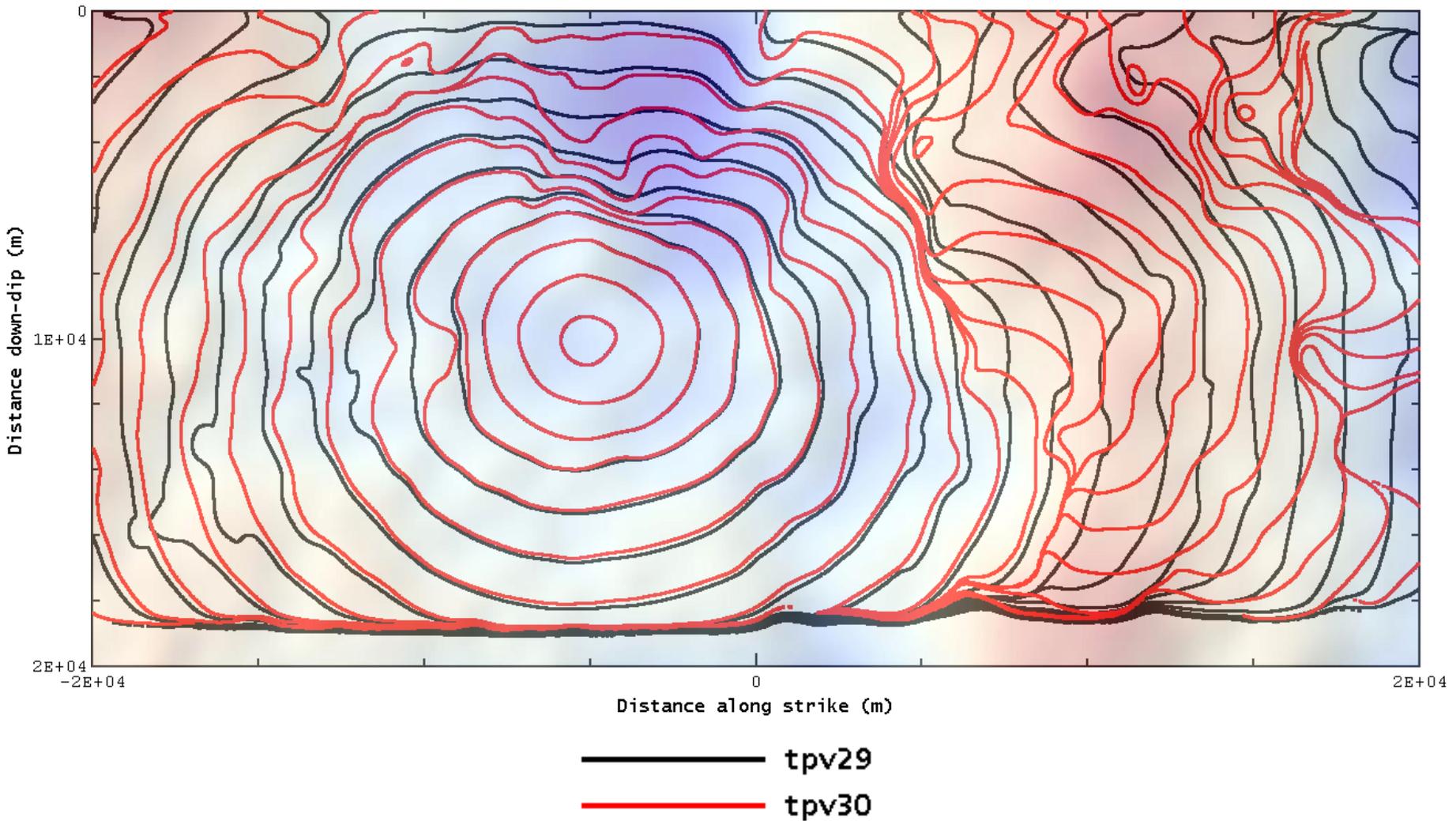
## TPV30 (Viscoplastic) Rupture Contours — Metrics (RMS Difference in Rupture Time)

	(1)	(2)	(3)	(4)	(5)	(6)
(1) barall.2		46.8	9.6	9.4	62.4	9.6
(2) dliu.3	46.8		40.2	45.0	82.3	45.4
(3) duru.2	9.6	40.2		7.2	62.9	11.8
(4) kozdon	9.4	45.0	7.2		61.5	13.3
(5) ma.2	62.4	82.3	62.9	61.5		59.0
(6) shi	9.6	45.4	11.8	13.3	59.0	

(1) barall.2	Michael Barall - FaultMod - 25 m
(2) dliu.3	Dunyu Liu - Finite Element - EQdyna - 25m
(3) duru.2	Kenneth Duru - WaveQLab3D - 50 m
(4) kozdon	Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp
(5) ma.2	Shuo Ma - Finite Element - MAFE - 50m
(6) shi	Zheqiang Shi - Generalized Finite Difference - SORD

Codes barall.2, duru.2, kozdon, and shi agree extremely well among themselves, with metric values under 14 milliseconds. Codes dliu.3 and ma.2 have higher metrics, but still show good agreement.

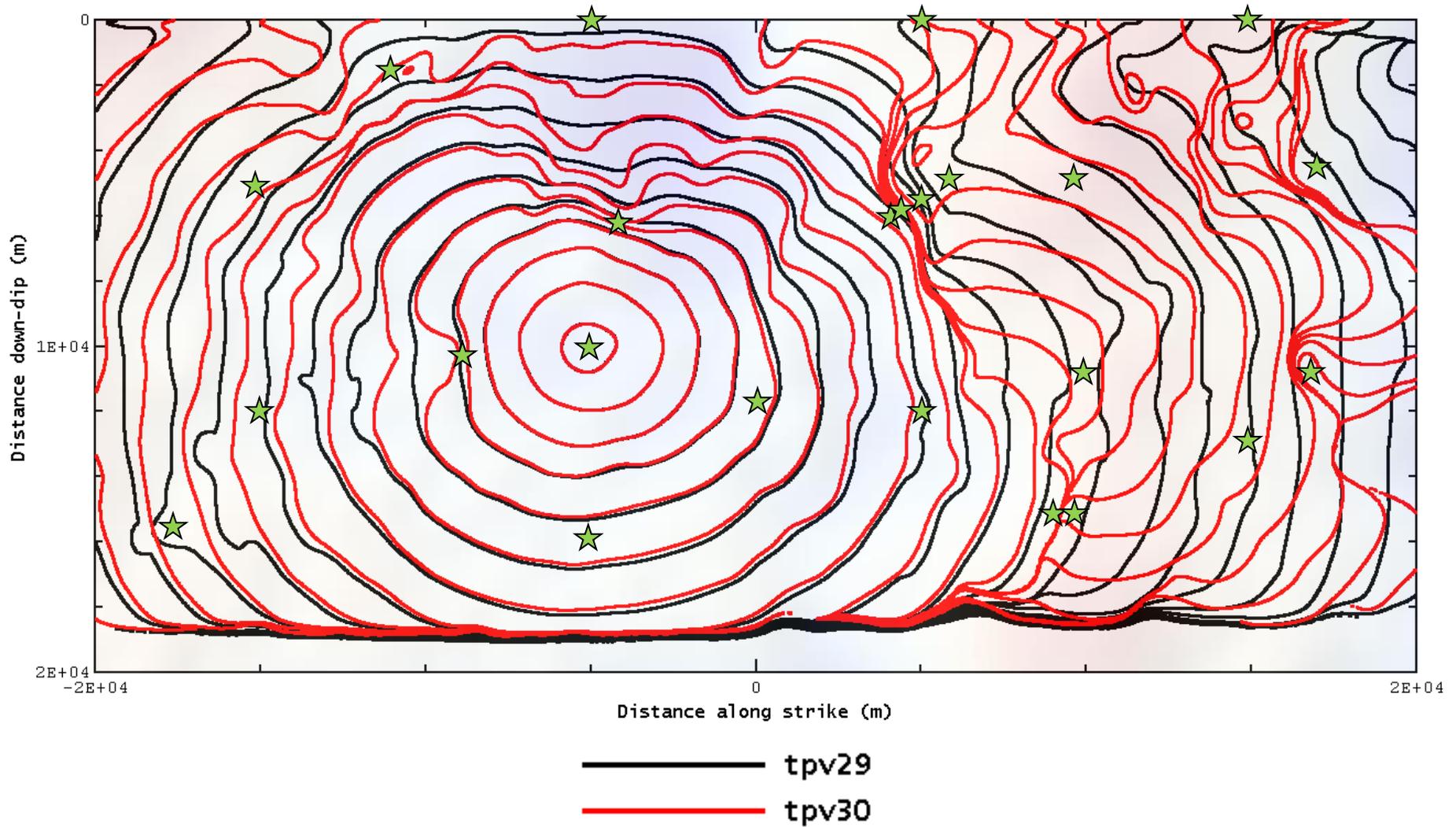
## Comparison of Elastic Case (TPV29), Viscoplastic Case (TPV30), and Geometry



The viscoplastic case has more difficulty propagating, and is more responsive to the fault roughness, than the elastic case. There is clearly some correlation between rupture contours and fault geometry, but it's hard to look at the geometry and predict what the rupture will do.

# Time Series: On-Fault Stations

## Locations of On-Fault Stations



We tried to select station locations where the rupture contours look “interesting.”

## TPV29 (Elastic) – Summary Metrics for All On-Fault Stations

	2d-stress	2d-rate	2d-slip	n-stress
<b>faultst-042dp061</b>	<u>15.9</u>	<u>3.7</u>	<u>2.4</u>	<u>21.8</u>
<b>faultst-050dp000</b>	<u>35.7</u>	<u>6.0</u>	<u>4.0</u>	<u>129.6</u>
<b>faultst-050dp100</b>	<u>28.5</u>	<u>3.3</u>	<u>1.6</u>	<u>48.8</u>
<b>faultst-050dp160</b>	<u>6.2</u>	<u>5.6</u>	<u>1.2</u>	<u>50.9</u>
<b>faultst-089dp101</b>	<u>10.9</u>	<u>3.8</u>	<u>2.2</u>	<u>8.0</u>
<b>faultst-110dp014</b>	<u>29.2</u>	<u>4.6</u>	<u>3.2</u>	<u>50.4</u>
<b>faultst-150dp050</b>	<u>9.3</u>	<u>4.1</u>	<u>1.7</u>	<u>20.6</u>
<b>faultst-150dp120</b>	<u>11.3</u>	<u>4.9</u>	<u>1.1</u>	<u>48.9</u>
<b>faultst-180dp156</b>	<u>9.1</u>	<u>12.1</u>	<u>1.1</u>	<u>95.3</u>
<b>faultst000dp120</b>	<u>7.0</u>	<u>2.6</u>	<u>1.1</u>	<u>29.4</u>
<b>faultst043dp062</b>	<u>32.8</u>	<u>5.8</u>	<u>3.1</u>	<u>68.9</u>
<b>faultst046dp060</b>	<u>10.3</u>	<u>5.4</u>	<u>2.8</u>	<u>40.7</u>

	2d-stress	2d-rate	2d-slip	n-stress
<b>faultst050dp000</b>	<u>26.9</u>	<u>7.5</u>	<u>5.3</u>	<u>53.9</u>
<b>faultst050dp120</b>	<u>14.8</u>	<u>3.5</u>	<u>1.6</u>	<u>103.3</u>
<b>faultst051dp057</b>	<u>9.7</u>	<u>4.7</u>	<u>3.8</u>	<u>14.7</u>
<b>faultst059dp047</b>	<u>14.3</u>	<u>3.0</u>	<u>2.0</u>	<u>72.4</u>
<b>faultst090dp153</b>	<u>11.6</u>	<u>10.4</u>	<u>2.1</u>	<u>51.2</u>
<b>faultst093dp153</b>	<u>9.3</u>	<u>9.2</u>	<u>2.3</u>	<u>11.5</u>
<b>faultst100dp050</b>	<u>30.1</u>	<u>2.9</u>	<u>1.9</u>	<u>121.8</u>
<b>faultst100dp110</b>	<u>12.5</u>	<u>4.5</u>	<u>1.9</u>	<u>41.7</u>
<b>faultst150dp000</b>	<u>36.8</u>	<u>9.1</u>	<u>6.5</u>	<u>77.5</u>
<b>faultst150dp130</b>	<u>12.4</u>	<u>6.2</u>	<u>0.7</u>	<u>96.4</u>
<b>faultst167dp105</b>	<u>20.6</u>	<u>8.6</u>	<u>3.3</u>	<u>13.0</u>
<b>faultst170dp045</b>	<u>20.9</u>	<u>5.1</u>	<u>2.9</u>	<u>85.2</u>

These are the waveform comparison metrics, averaged over all pairs of codes, using the best version from each of 7 codes.

Low numbers in the “2d-rate” and “2d-slip” columns indicate good agreement on slip history at all stations. But high numbers in “n-stress” and “2d-stress” indicate poor agreement on stresses.

## TPV30 (Viscoplastic) – Summary Metrics for All On-Fault Stations

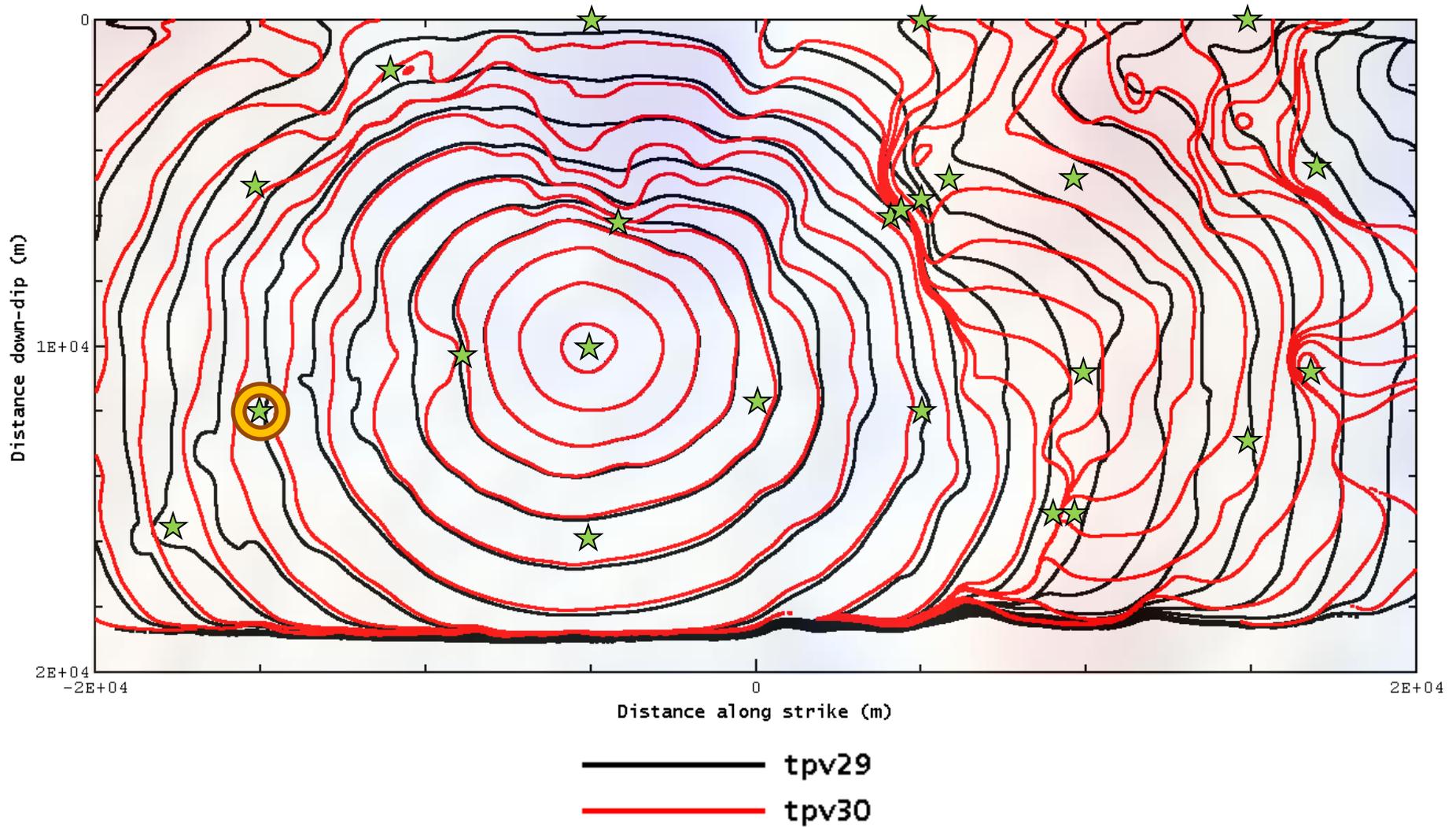
	2d-stress	2d-rate	2d-slip	n-stress
<b>faultst-042dp061</b>	<u>7.5</u>	<u>7.1</u>	<u>4.7</u>	<u>7.6</u>
<b>faultst-050dp000</b>	<u>25.2</u>	<u>8.8</u>	<u>6.0</u>	<u>116.5</u>
<b>faultst-050dp100</b>	<u>11.0</u>	<u>4.7</u>	<u>2.5</u>	<u>18.8</u>
<b>faultst-050dp160</b>	<u>2.7</u>	<u>2.6</u>	<u>2.1</u>	<u>34.4</u>
<b>faultst-089dp101</b>	<u>7.9</u>	<u>5.7</u>	<u>4.1</u>	<u>6.1</u>
<b>faultst-110dp014</b>	<u>22.1</u>	<u>7.8</u>	<u>5.8</u>	<u>4.6</u>
<b>faultst-150dp050</b>	<u>7.2</u>	<u>7.5</u>	<u>6.1</u>	<u>16.9</u>
<b>faultst-150dp120</b>	<u>7.6</u>	<u>4.4</u>	<u>4.2</u>	<u>45.8</u>
<b>faultst-180dp156</b>	<u>3.8</u>	<u>3.7</u>	<u>4.0</u>	<u>25.4</u>
<b>faultst000dp120</b>	<u>5.4</u>	<u>3.3</u>	<u>1.9</u>	<u>26.5</u>
<b>faultst043dp062</b>	<u>7.8</u>	<u>8.3</u>	<u>5.1</u>	<u>19.0</u>
<b>faultst046dp060</b>	<u>9.8</u>	<u>8.7</u>	<u>7.2</u>	<u>43.2</u>

	2d-stress	2d-rate	2d-slip	n-stress
<b>faultst050dp000</b>	<u>26.4</u>	<u>10.0</u>	<u>7.7</u>	<u>36.6</u>
<b>faultst050dp120</b>	<u>5.6</u>	<u>3.5</u>	<u>2.1</u>	<u>78.7</u>
<b>faultst051dp057</b>	<u>7.5</u>	<u>7.4</u>	<u>6.6</u>	<u>4.6</u>
<b>faultst059dp047</b>	<u>4.8</u>	<u>5.5</u>	<u>4.0</u>	<u>42.7</u>
<b>faultst090dp153</b>	<u>15.3</u>	<u>16.3</u>	<u>6.3</u>	<u>16.5</u>
<b>faultst093dp153</b>	<u>12.8</u>	<u>5.5</u>	<u>3.9</u>	<u>10.4</u>
<b>faultst100dp050</b>	<u>7.0</u>	<u>4.4</u>	<u>3.2</u>	<u>73.9</u>
<b>faultst100dp110</b>	<u>6.7</u>	<u>5.6</u>	<u>3.2</u>	<u>35.7</u>
<b>faultst150dp000</b>	<u>23.0</u>	<u>20.7</u>	<u>12.2</u>	<u>42.7</u>
<b>faultst150dp130</b>	<u>4.2</u>	<u>5.0</u>	<u>2.5</u>	<u>56.7</u>
<b>faultst167dp105</b>	<u>16.0</u>	<u>11.3</u>	<u>12.3</u>	<u>18.6</u>
<b>faultst170dp045</b>	<u>10.4</u>			<u>25.6</u>

These are the waveform comparison metrics, averaged over all pairs of codes, using the best version from each of 5 codes.

Low numbers in the “2d-rate” and “2d-slip” columns indicate good agreement on slip history at all stations. But high numbers in “n-stress” and “2d-stress” indicate poor agreement on stresses.

## Example of Disagreement in Stress

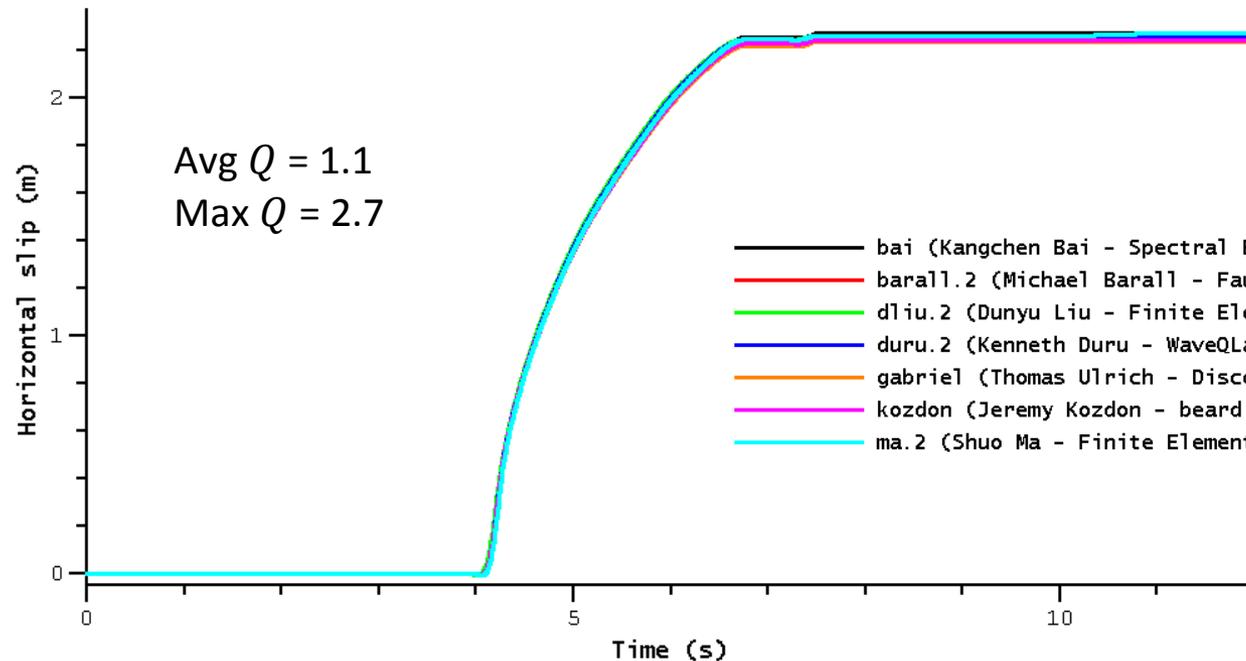
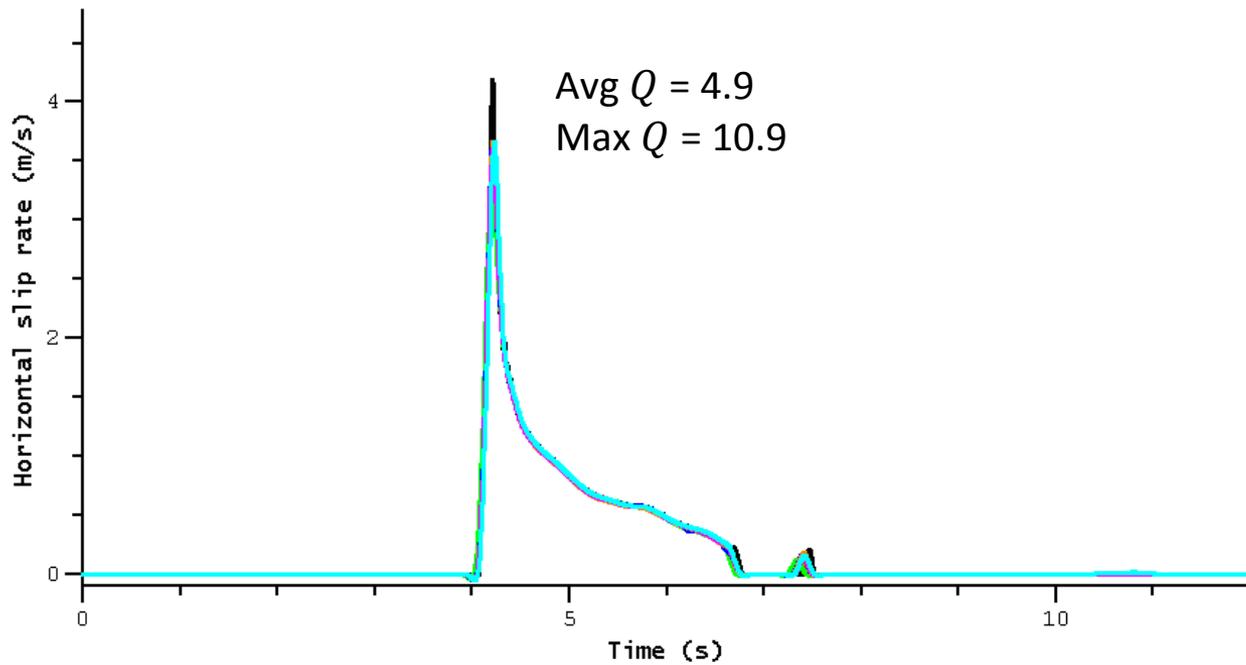


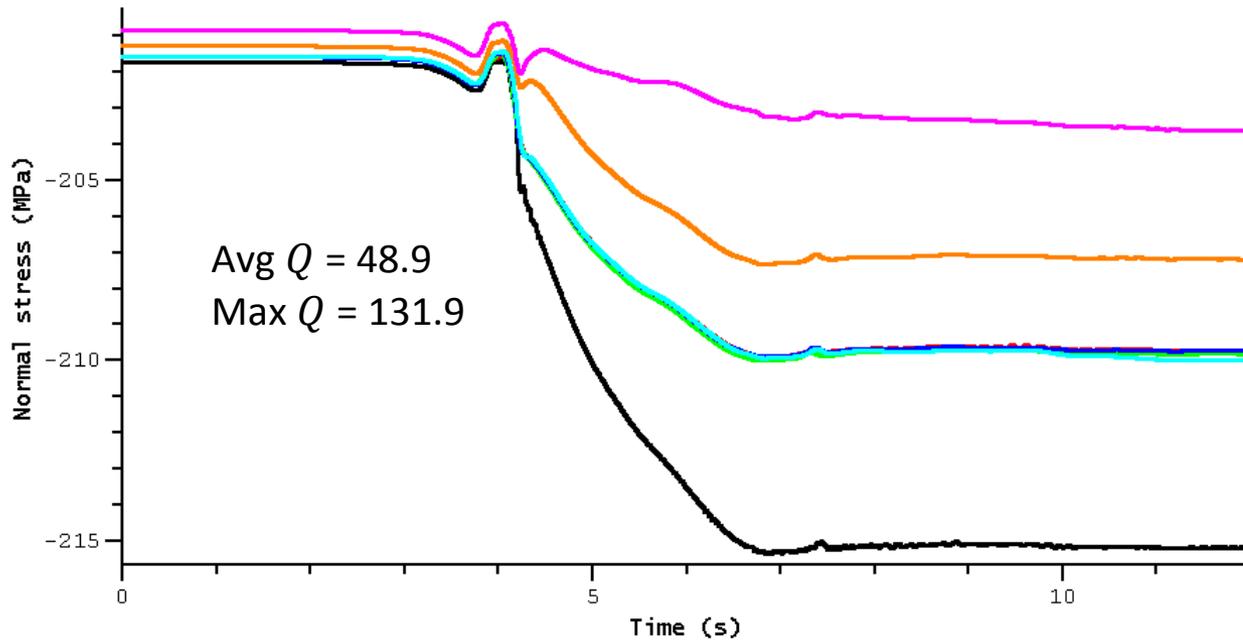
To explore the high metric values for stress, we look at slip and stress at the marked station.

**TPV29 (Elastic) –  
faultst-150dp120 –  
Example of Disagreement  
in Stress**

There is excellent agreement  
in slip history and final slip  
among the 7 codes.

But ... (see next slide)

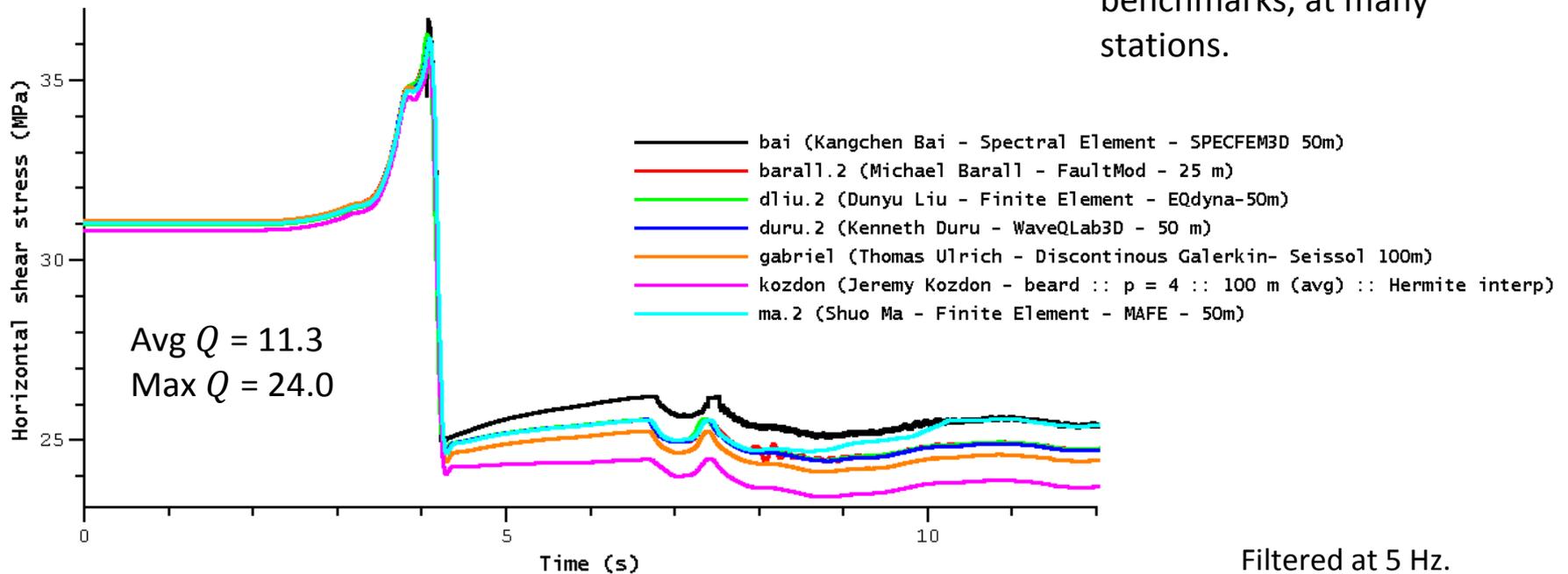




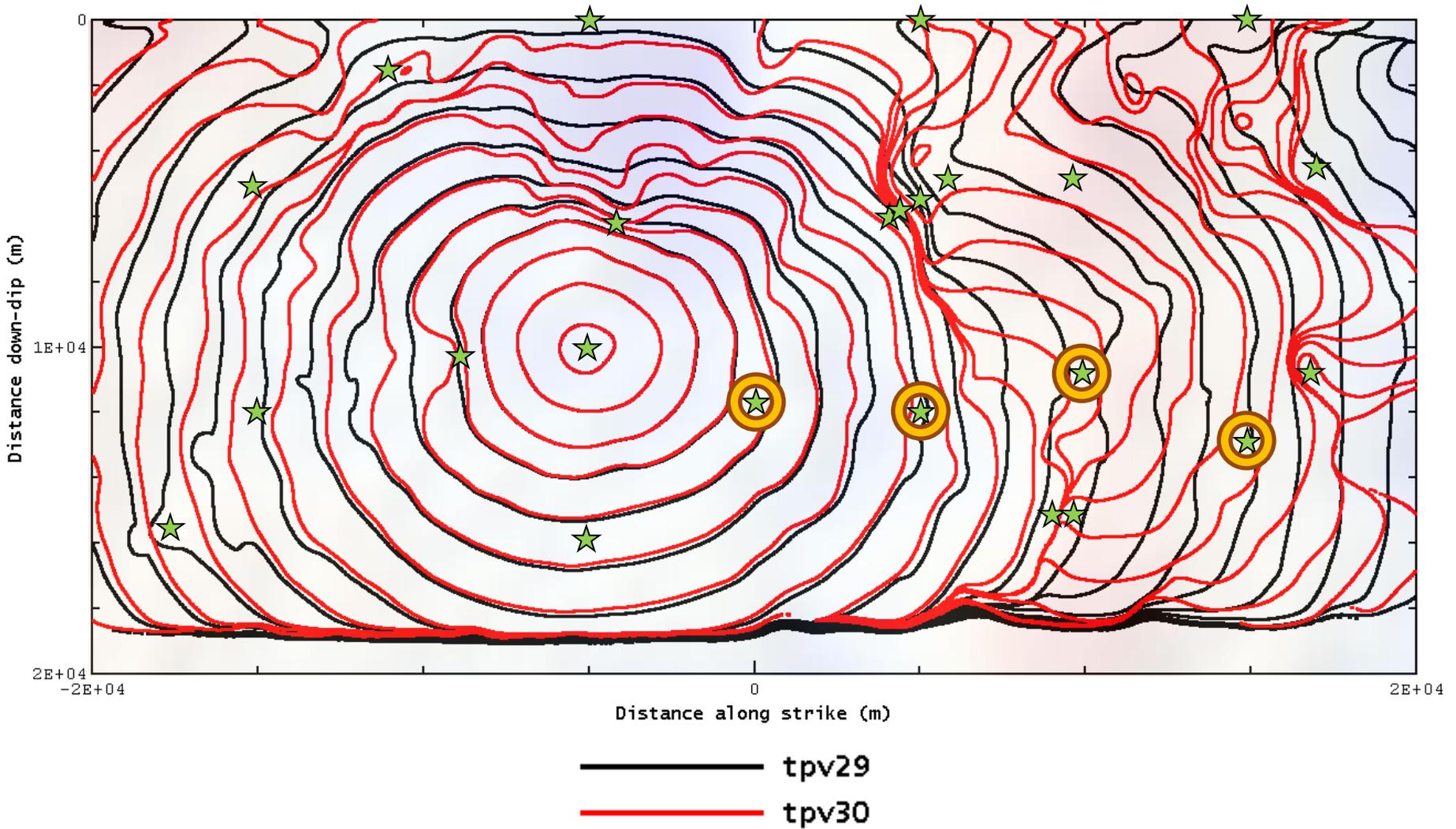
## TPV29 (Elastic) – faultst-150dp120 – Example of Disagreement in Stress

There is great disagreement  
in normal stress, and  
corresponding disagreement  
in shear stress.

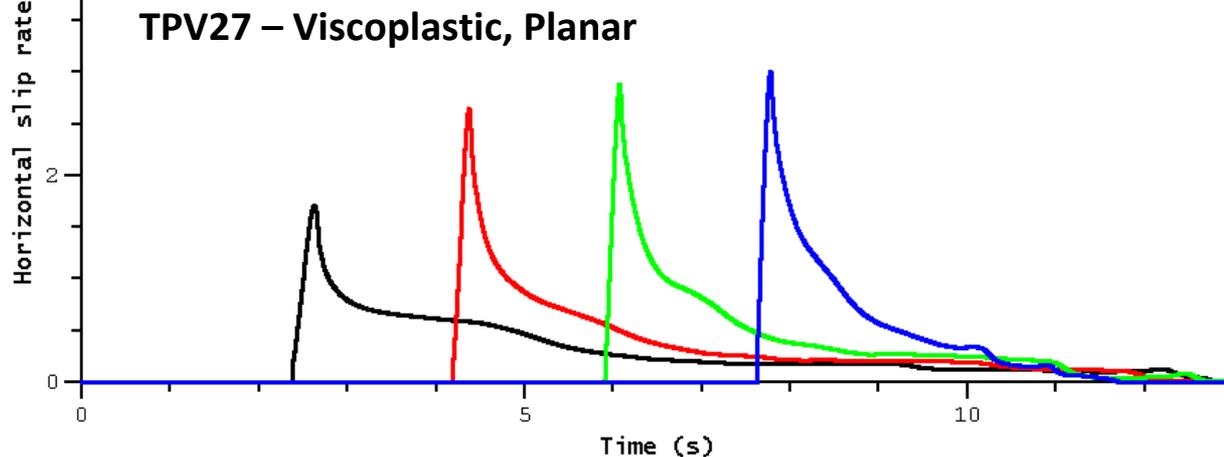
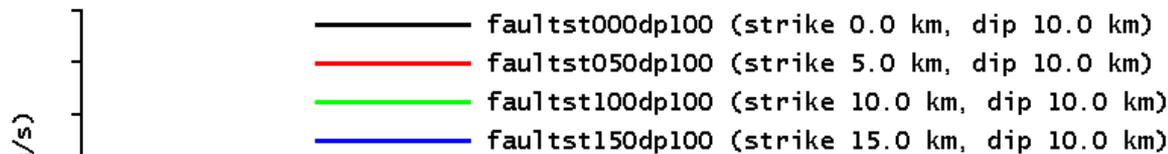
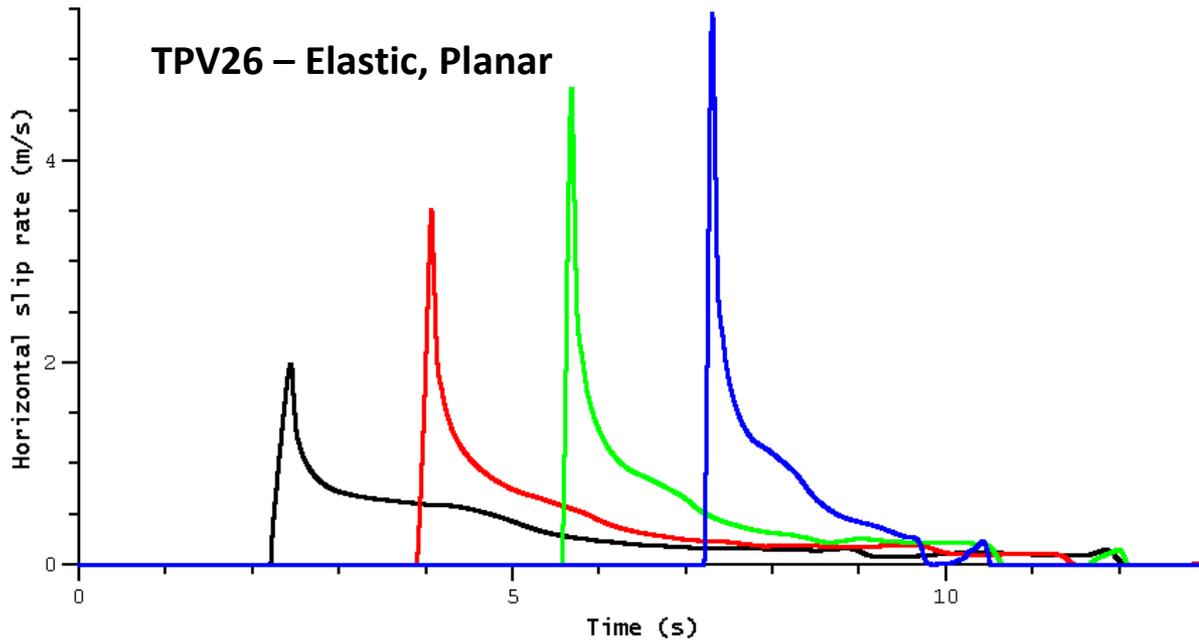
This is repeated in both  
benchmarks, at many  
stations.



# Behavior of Rupture as it Propagates



Marked stations are at intervals of 5 km along-strike, along a path of rupture propagation.

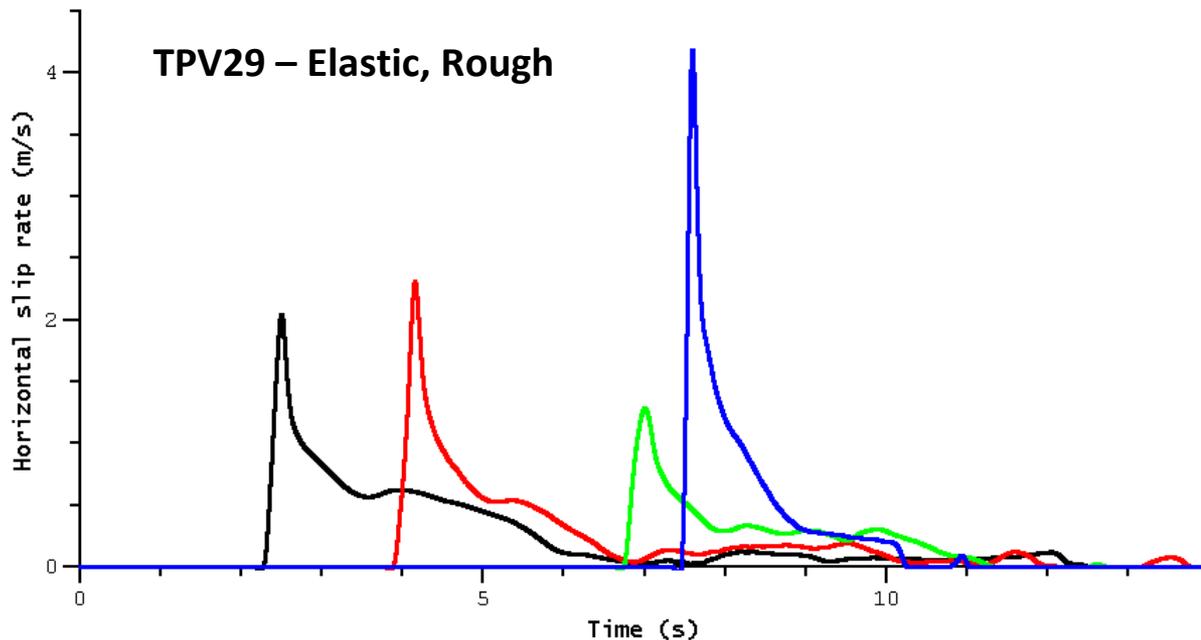


**Reminder – TPV26 and TPV27**

TPV26 and TPV27 have a planar fault. The plots show slip rate at stations spaced 5 km apart along-strike.

In the elastic case (TPV26), the peak slip rate increases approximately linearly with distance, and the peak gets progressively narrower.

In the viscoplastic case (TPV27), the peaks are much lower, and they tend to level off with increasing distance from the hypocenter.



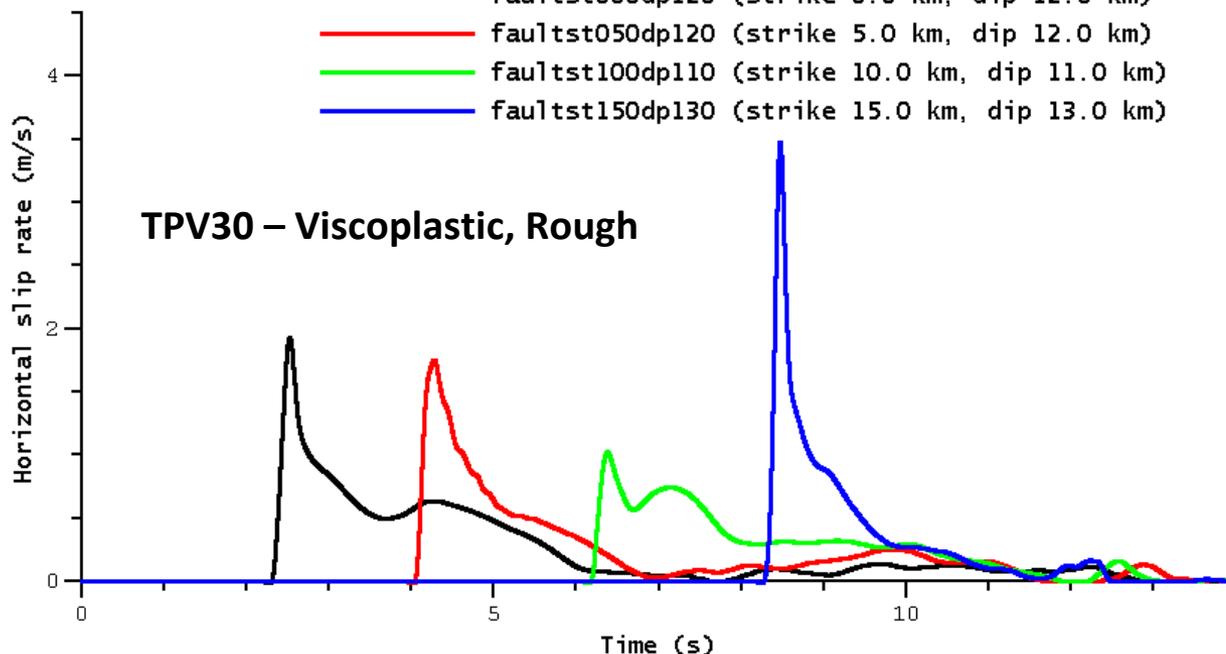
## Propagation on a Rough Fault

The pattern seen in TPV26 and TPV27 is not repeated here.

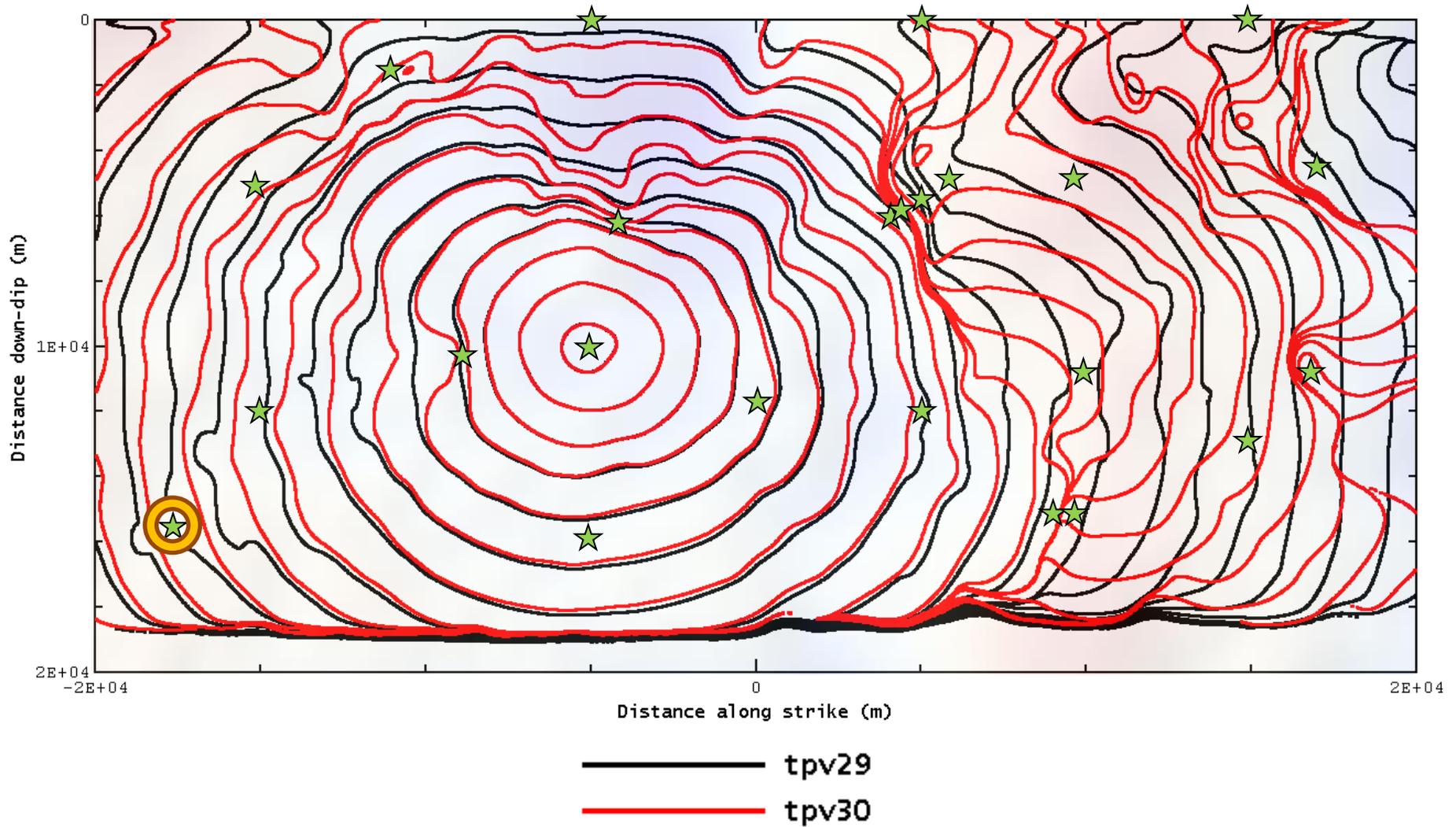
The viscoplastic peaks are almost as high as the corresponding elastic peaks.

The peak slip rate does not increase steadily with increasing from the hypocenter.

Instead, the peaks seem controlled more by local roughness and stress, than by propagation distance.

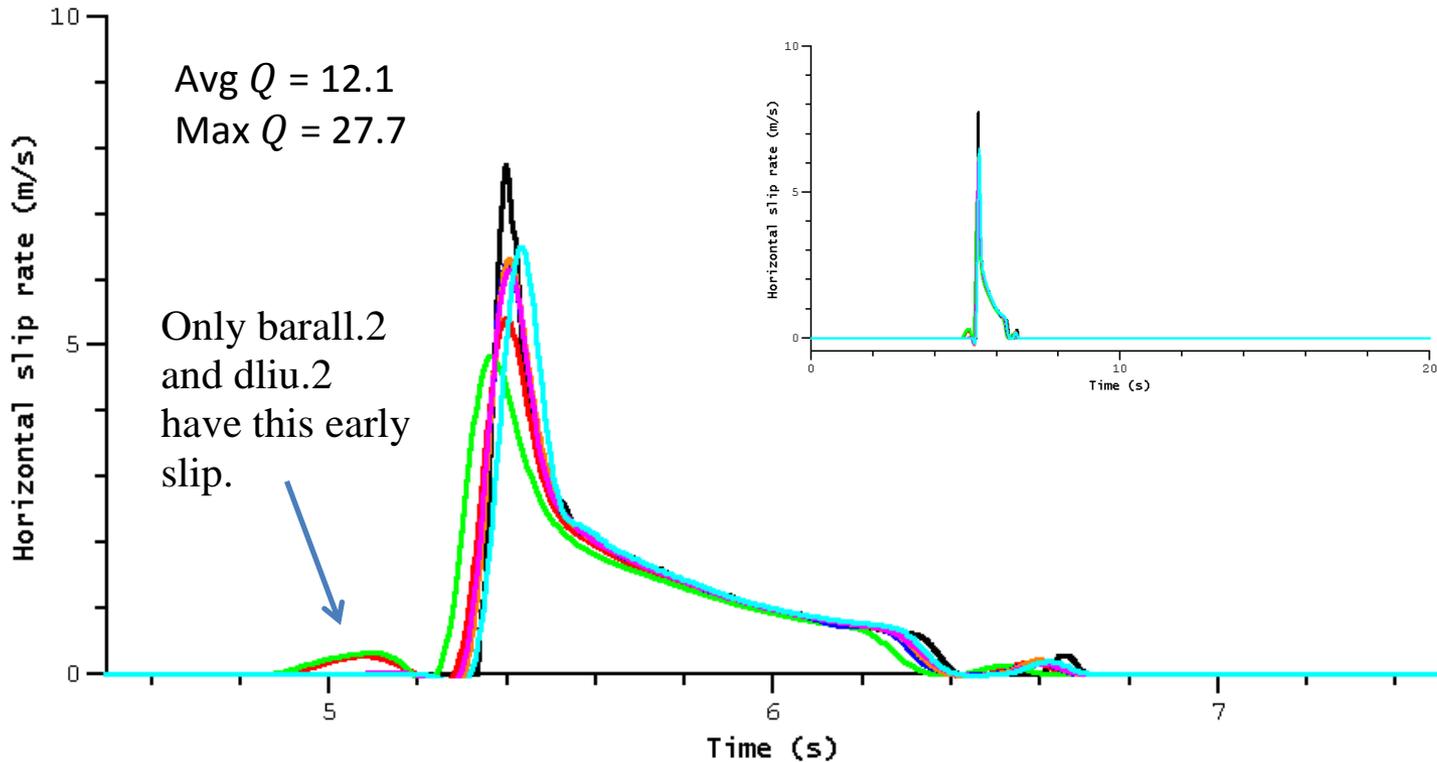


## Differences Between Codes in TPV29 (Elastic) Rupture Contours



The marked station is where codes had different rupture contours for TPV29.

# TPV29 (Elastic) – faultst-180dp156 – Horizontal Slip Rate



This station has the highest average  $Q$  of all on-fault stations in TPV29.

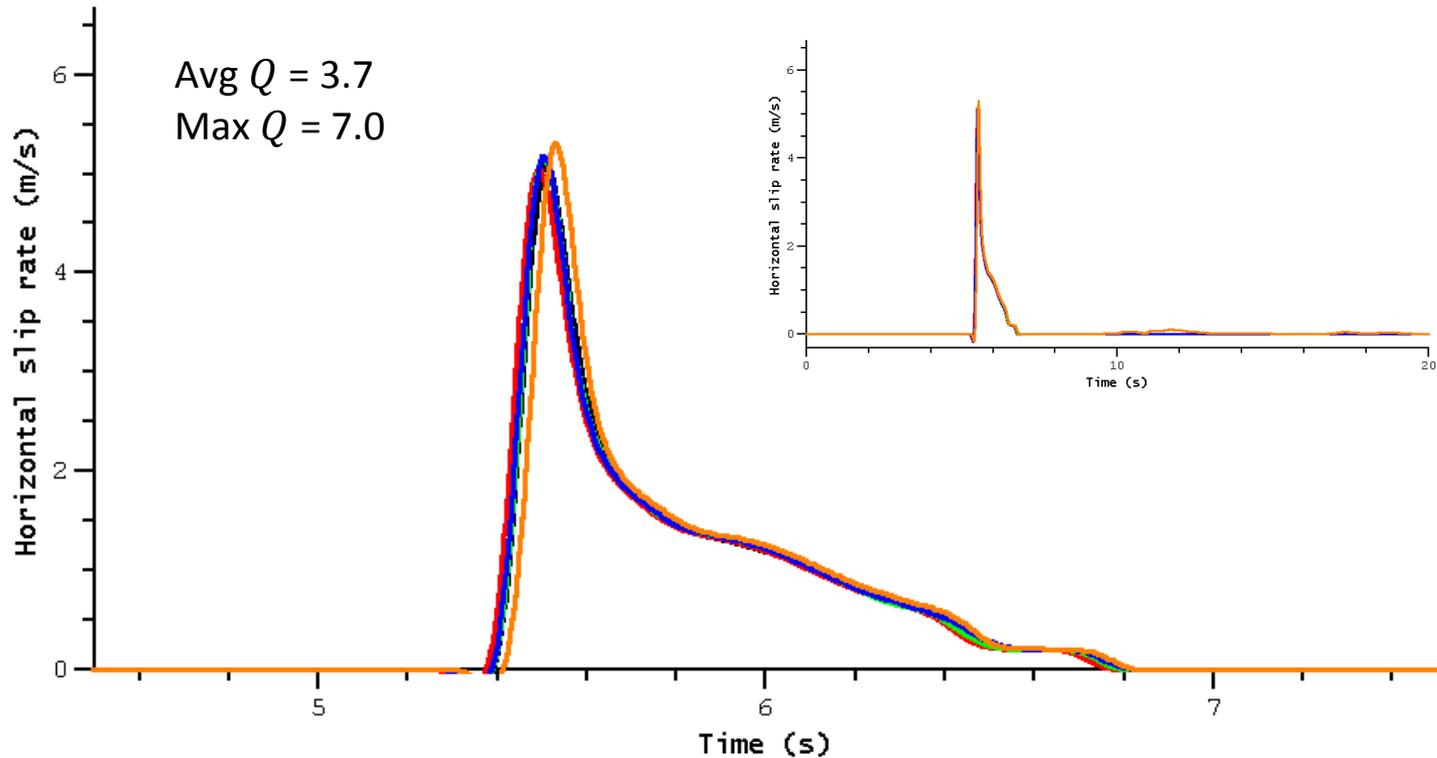
Difference in rupture contours is due to early small slip in 2 codes.

The main part of the waveform agrees reasonably well, not perfectly.

- bai (Kangchen Bai - Spectral Element - SPECFEM3D 50m)
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- gabriel (Thomas Ulrich - Discontinuous Galerkin- SeisSol 100m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)

Filtered at 5 Hz.

# TPV30 (Viscoplastic) – faultst-180dp156 – Horizontal Slip Rate

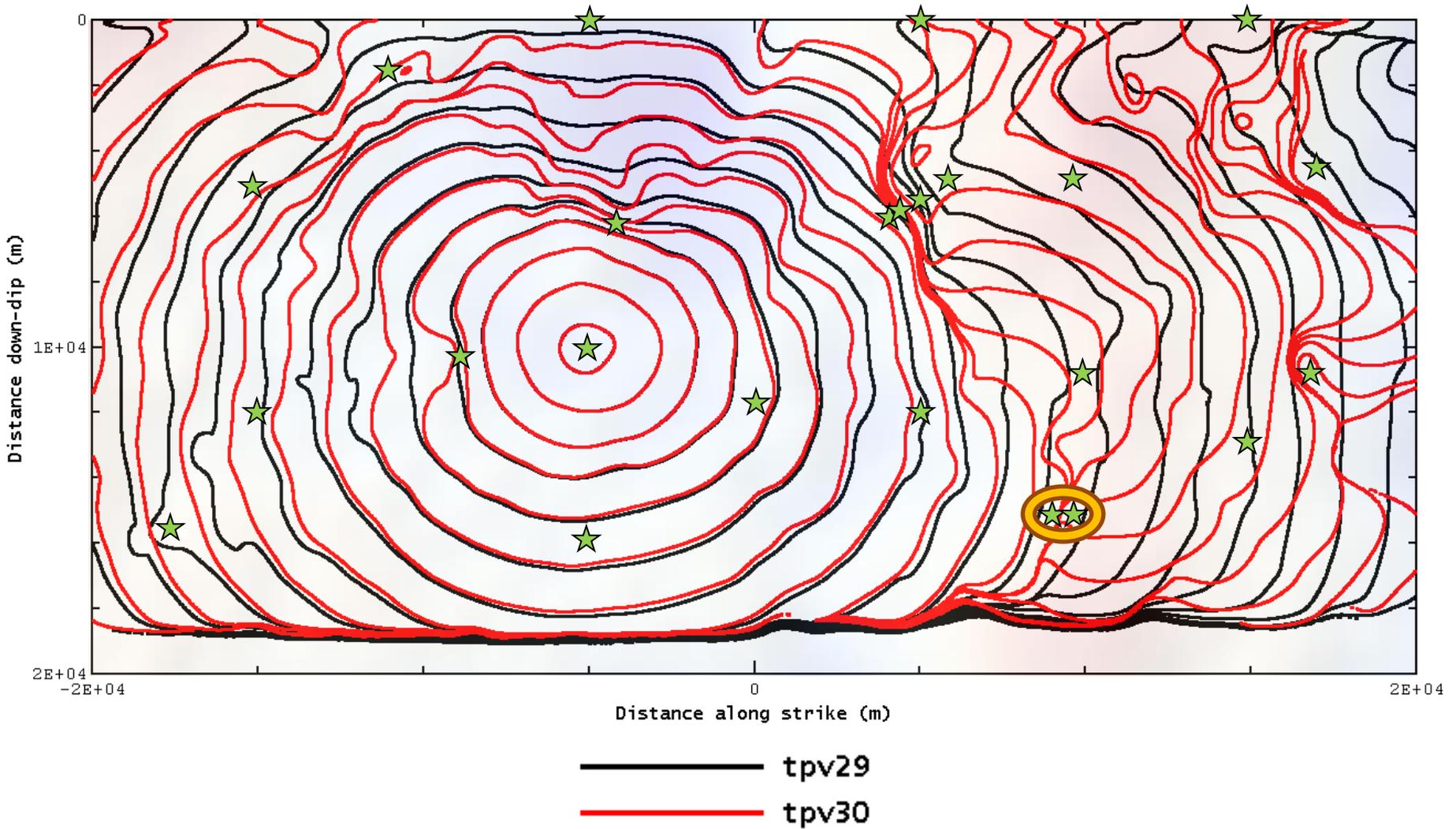


Unlike TPV29, there is no early slip, so the rupture contours agree.

Low metric  $Q$  values indicate an excellent waveform match.

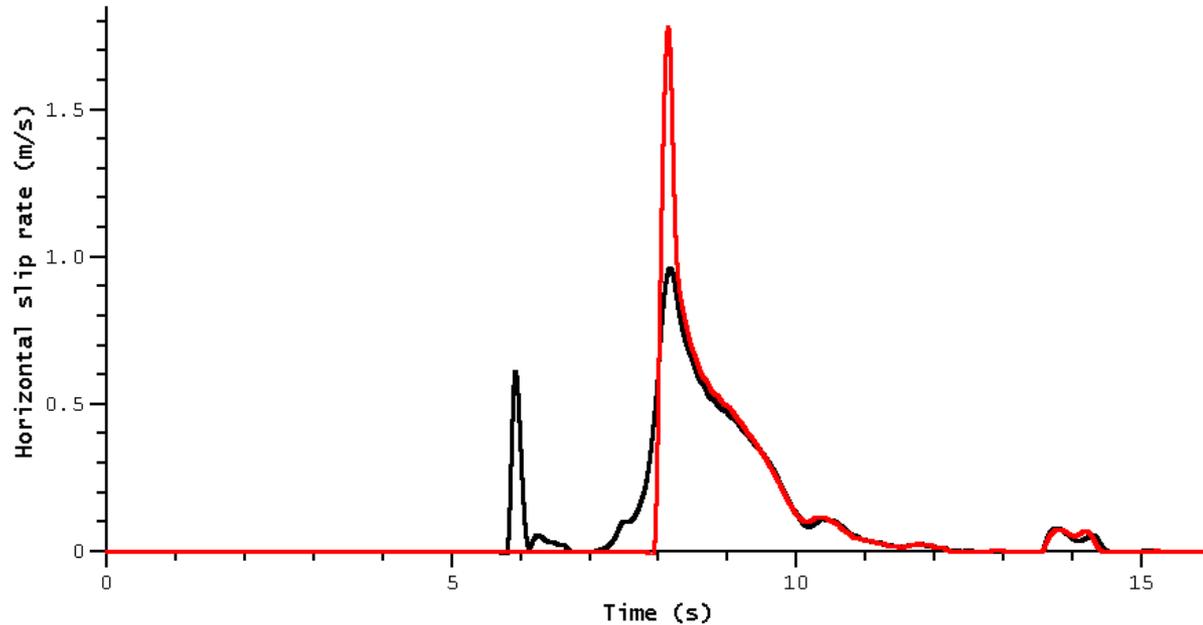
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.3 (Dunyu Liu - Finite Element - EQdyna - 25m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)

## 2-Second Stall in Viscoplastic Case (TPV30)



The marked stations are 300 m apart, yet it takes the rupture 2 seconds to get from one to the other, for TPV30.

## 2-Second Stall in TPV30 (Viscoplastic Case)

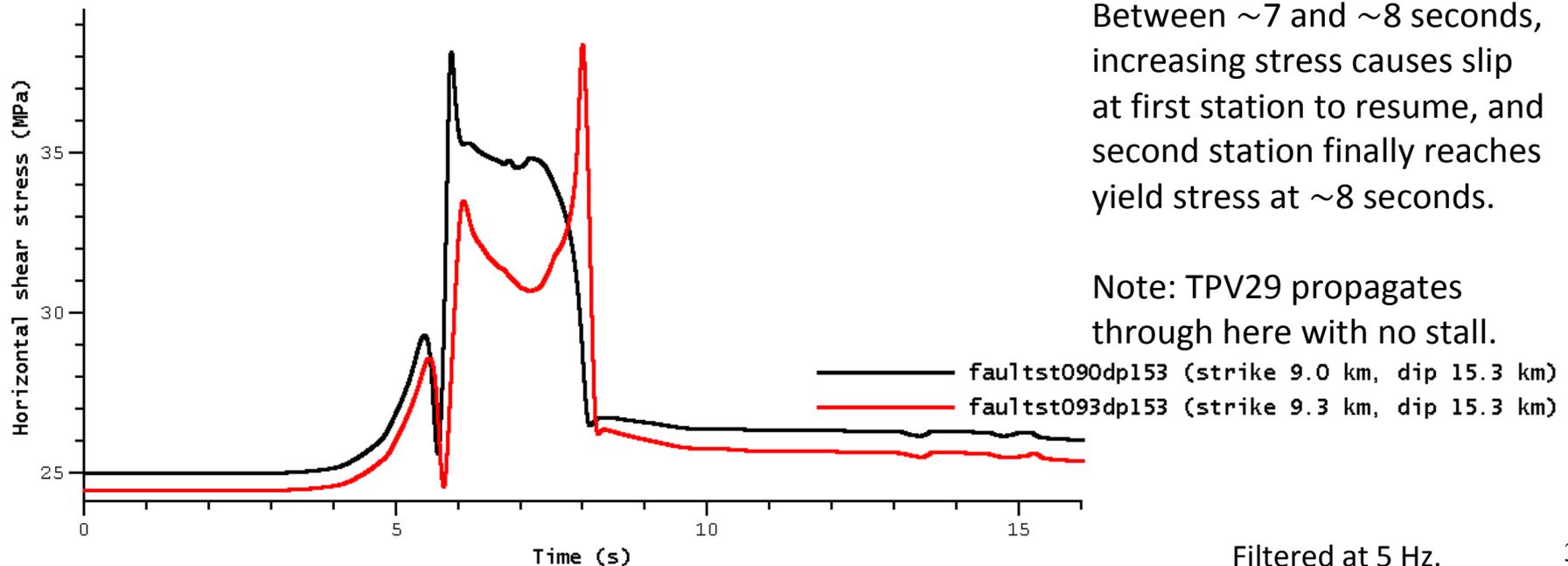


At ~6 seconds, rupture reaches first station (**black**) and slip begins, but stress at second station (**red**) never reaches yield stress.

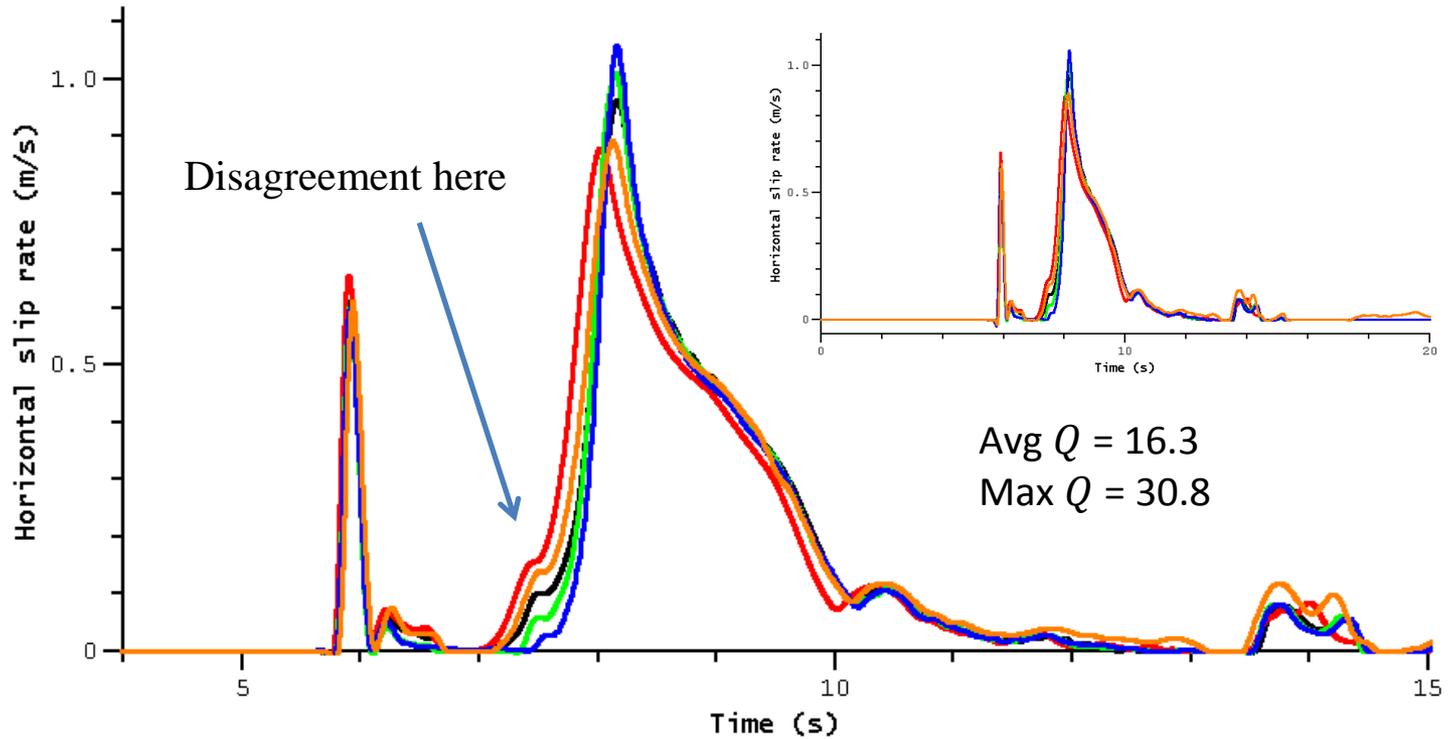
At ~7 seconds, slip at first station stops, without reaching critical slip distance.

Between ~7 and ~8 seconds, increasing stress causes slip at first station to resume, and second station finally reaches yield stress at ~8 seconds.

Note: TPV29 propagates through here with no stall.



# TPV30 (Viscoplastic) – faultst090dp153 (First Station) – Horizontal Slip Rate

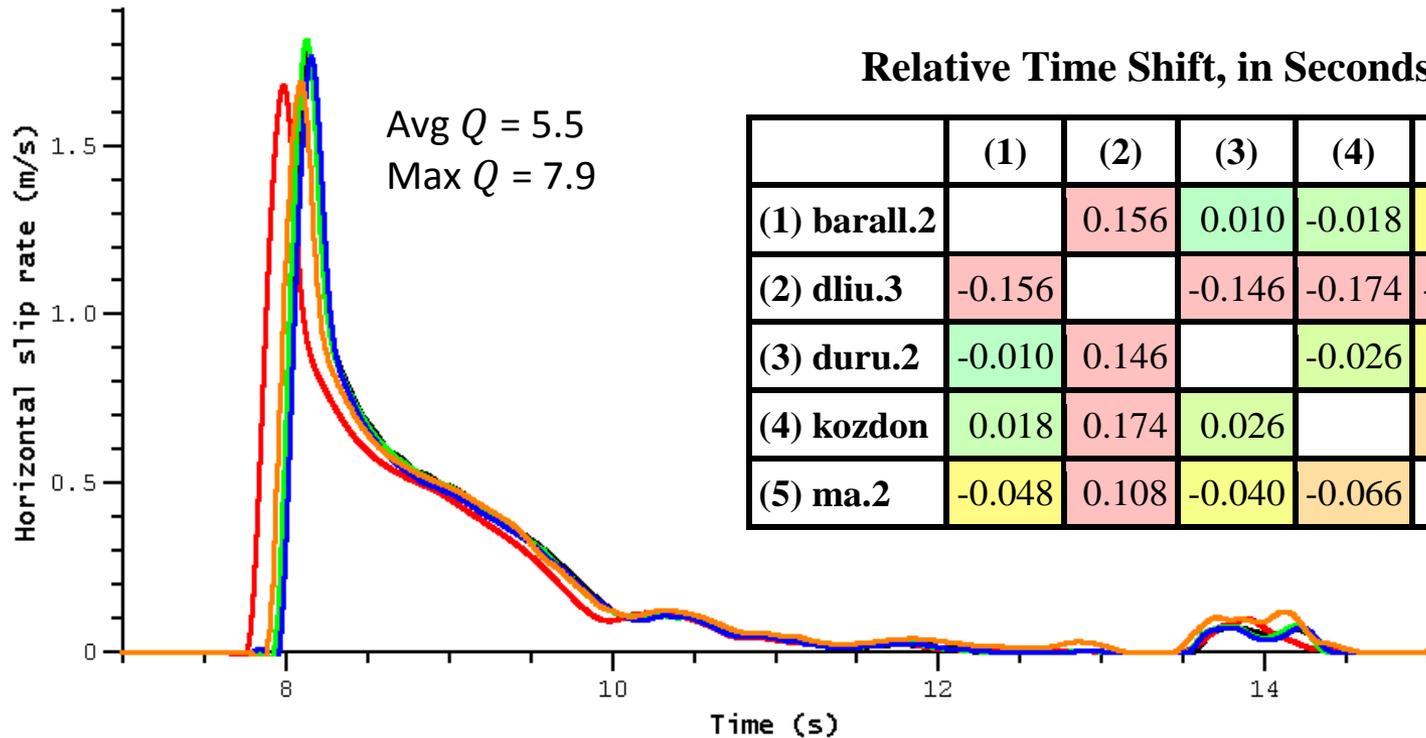


Codes disagree between ~7 seconds and ~8 seconds during the time that slip is resuming.

This station has the second-highest average  $Q$  value of all on-fault stations in TPV30.

- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.3 (Dunyu Liu - Finite Element - EQdyna - 25m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)

# TPV30 (Viscoplastic) – faultst093dp153 (Second Station) – Horizontal Slip Rate

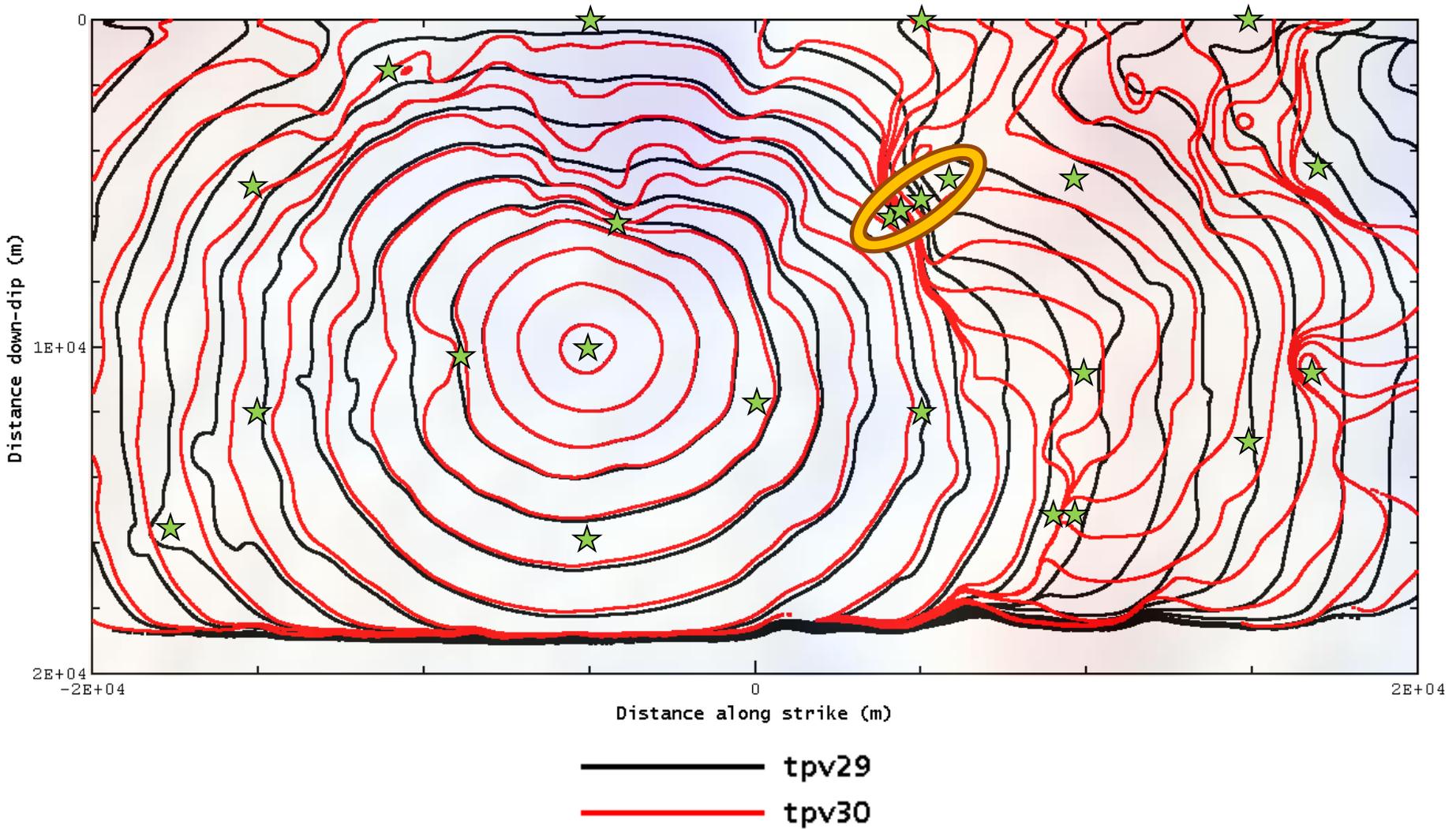


Agreement is much better than at the first station, and the metric  $Q$  values are much lower.

There is a significant relative time shift between the codes. (Recall that  $Q$  removes time shifts.)

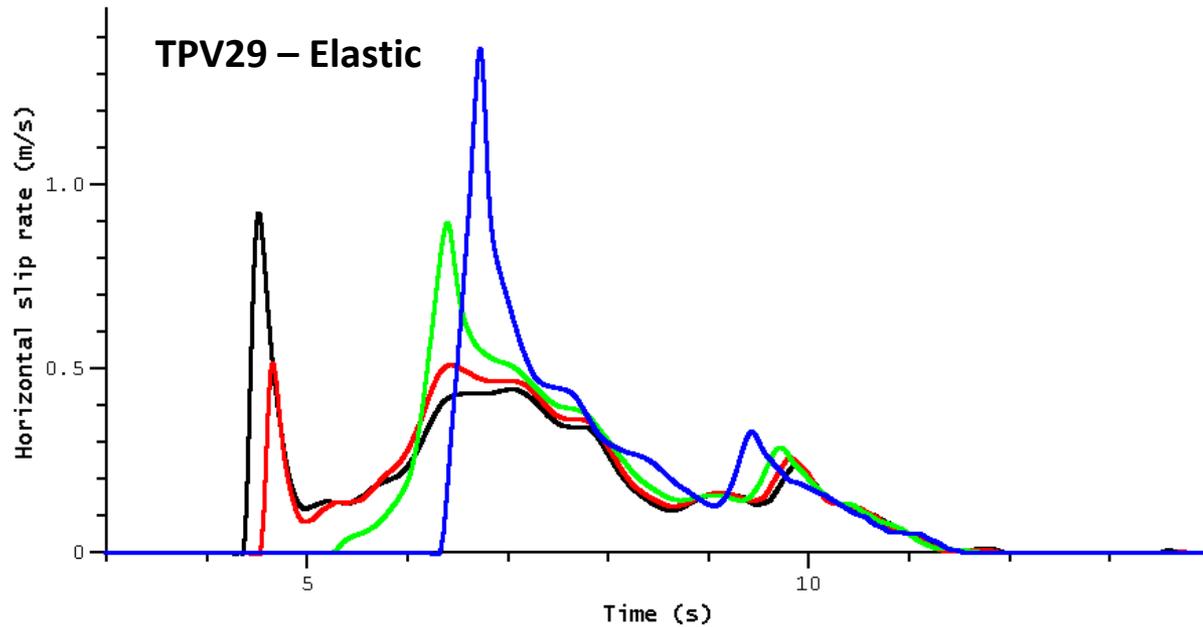
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.3 (Dunyu Liu - Finite Element - EQdyna - 25m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)
- ma.2 (Shuo Ma - Finite Element - MAFE - 50m)

## Rupture Slowdown



The rupture slows while traversing the four marked stations, for both benchmarks. The viscoplastic benchmark (TPV30) slows more, and earlier.

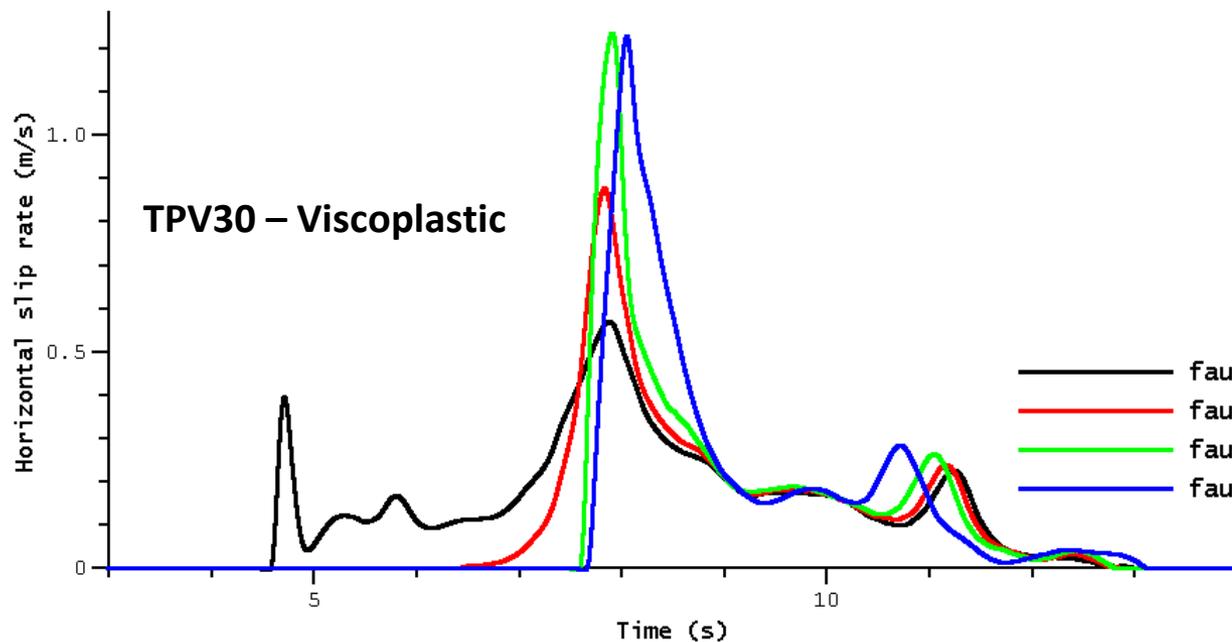
## Rupture Slowdown



The two benchmarks both slow down, but in different ways.

TPV29 has about a 2-second gap between the second (**red**) and fourth (**blue**) stations.

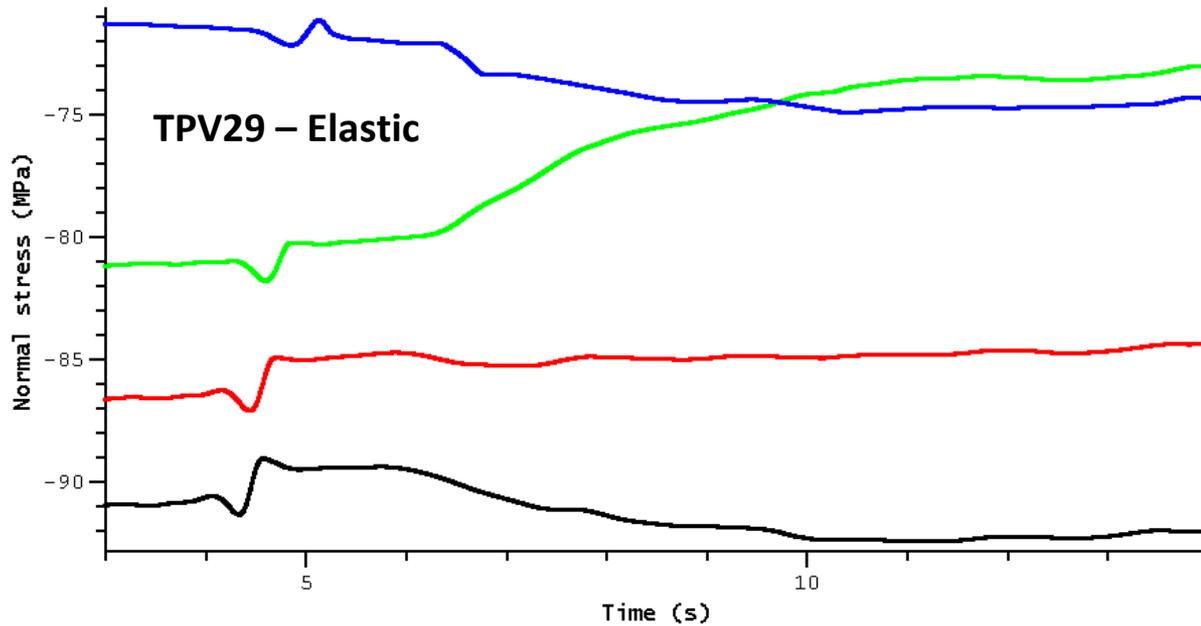
TPV30 has about a 3-second gap between the first (**black**) and third (**green**) stations.



Slip on the first station slows down but does not stop.

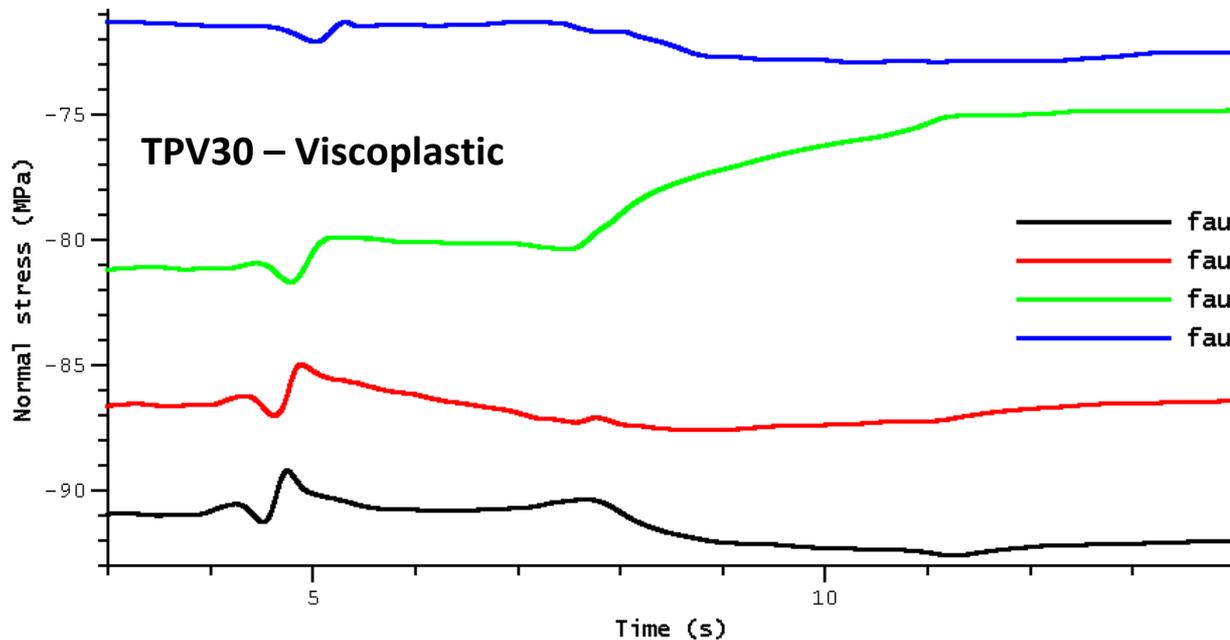
The four stations span a distance of about 2.2 km.

## Rupture Slowdown – Normal Stress



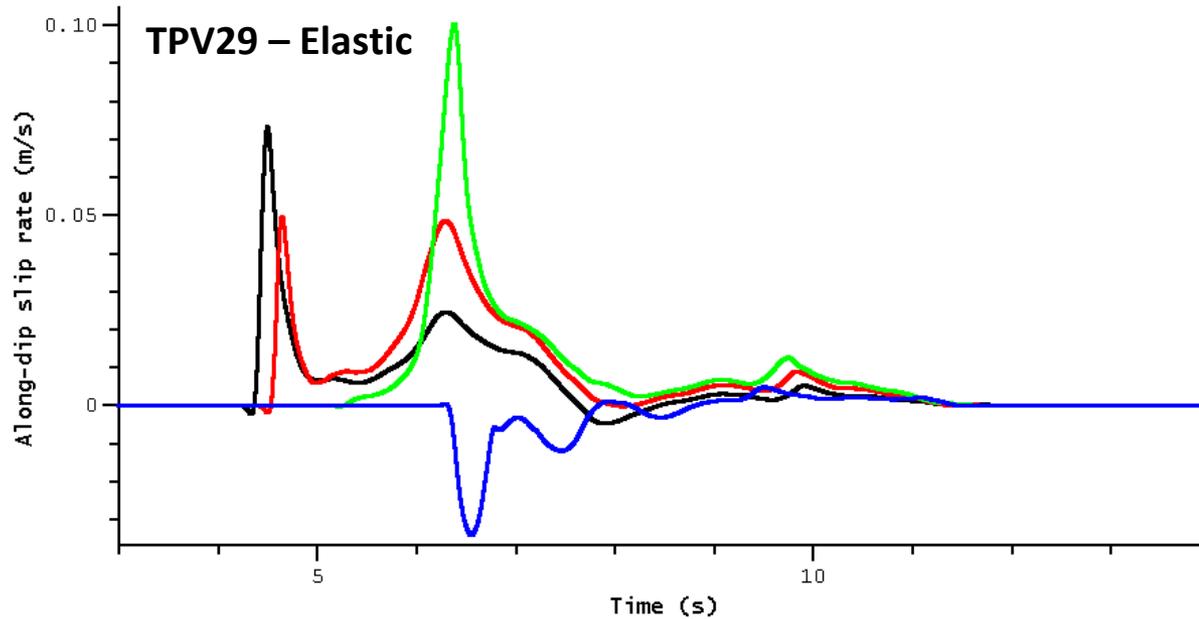
The rough fault geometry goes from clamping (**black**) to unclamping (**green**) to clamping (**blue**) in the space of 2.2 km.

The viscoplastic case has less clamping/unclamping than the elastic case. This may be due partly to the fact that it has about 15% less slip, and partly to plastic yielding.



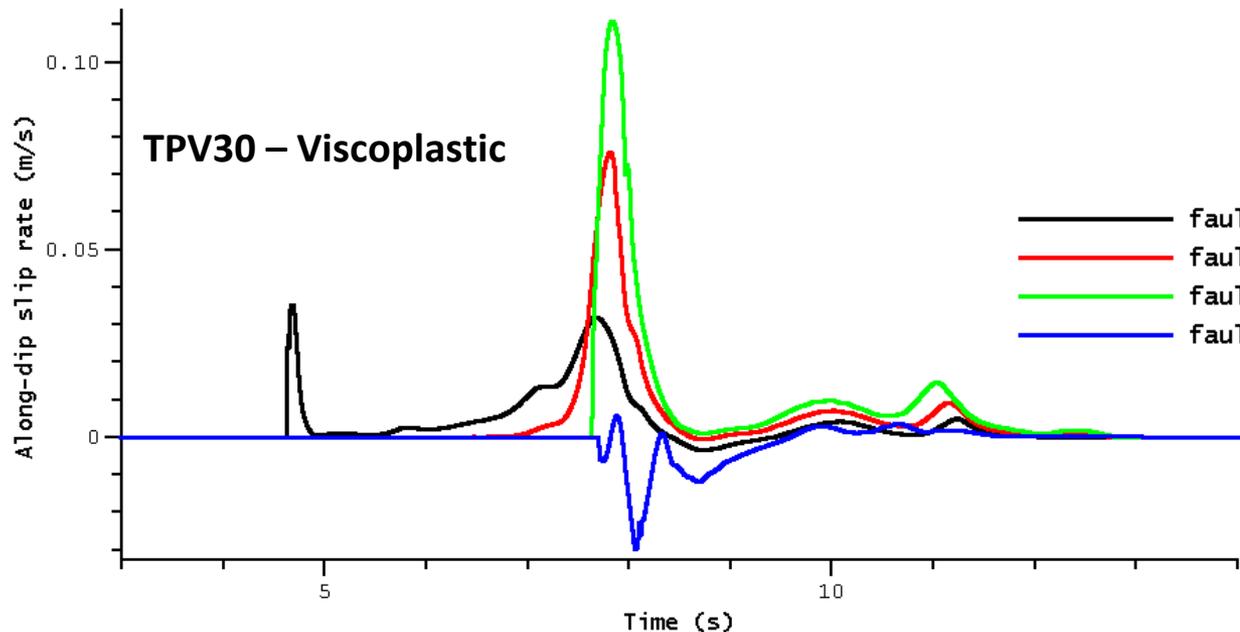
- faultst043dp062 (strike 4.3 km, dip 6.2 km)
- faultst046dp060 (strike 4.6 km, dip 6.0 km)
- faultst051dp057 (strike 5.1 km, dip 5.7 km)
- faultst059dp047 (strike 5.9 km, dip 4.7 km)

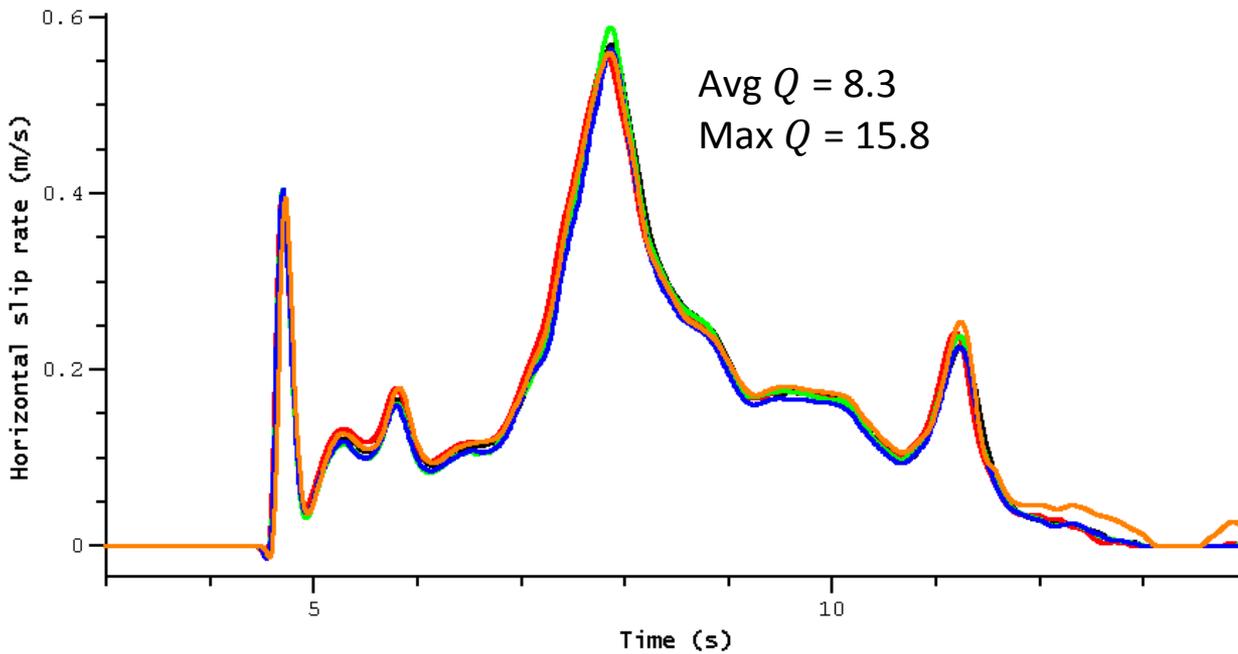
## Rupture Slowdown – Vertical Slip Rate



The rough fault geometry causes the vertical slip to reverse direction between the third (**green**) and fourth (**blue**) stations, which are about 1.3 km apart.

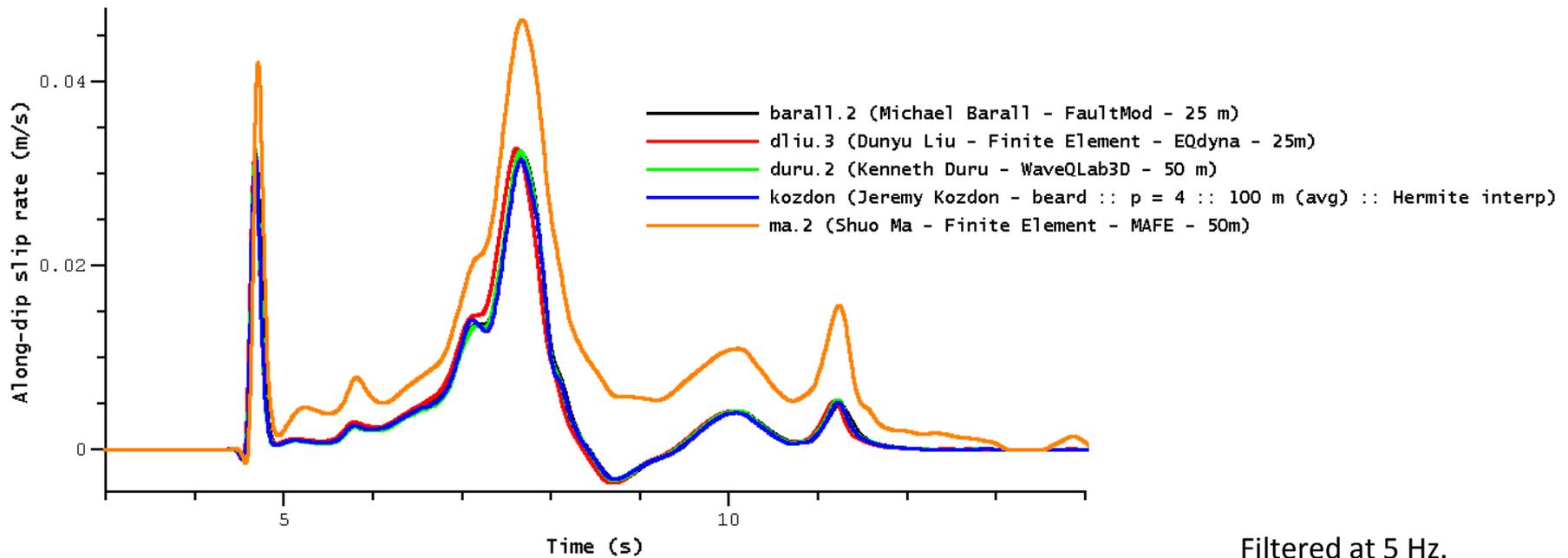
So the fault roughness creates a variable rake angle.

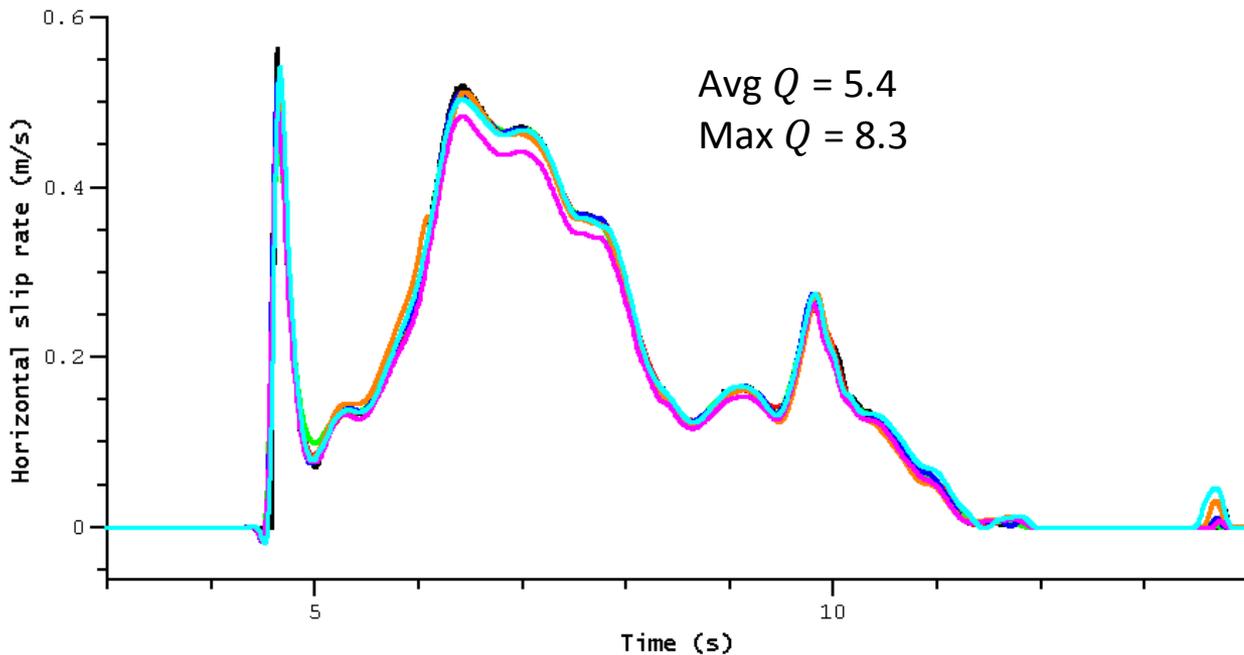




**TPV30 (Viscoplastic) –  
faultst043dp062  
(First Station) Slip Rate**

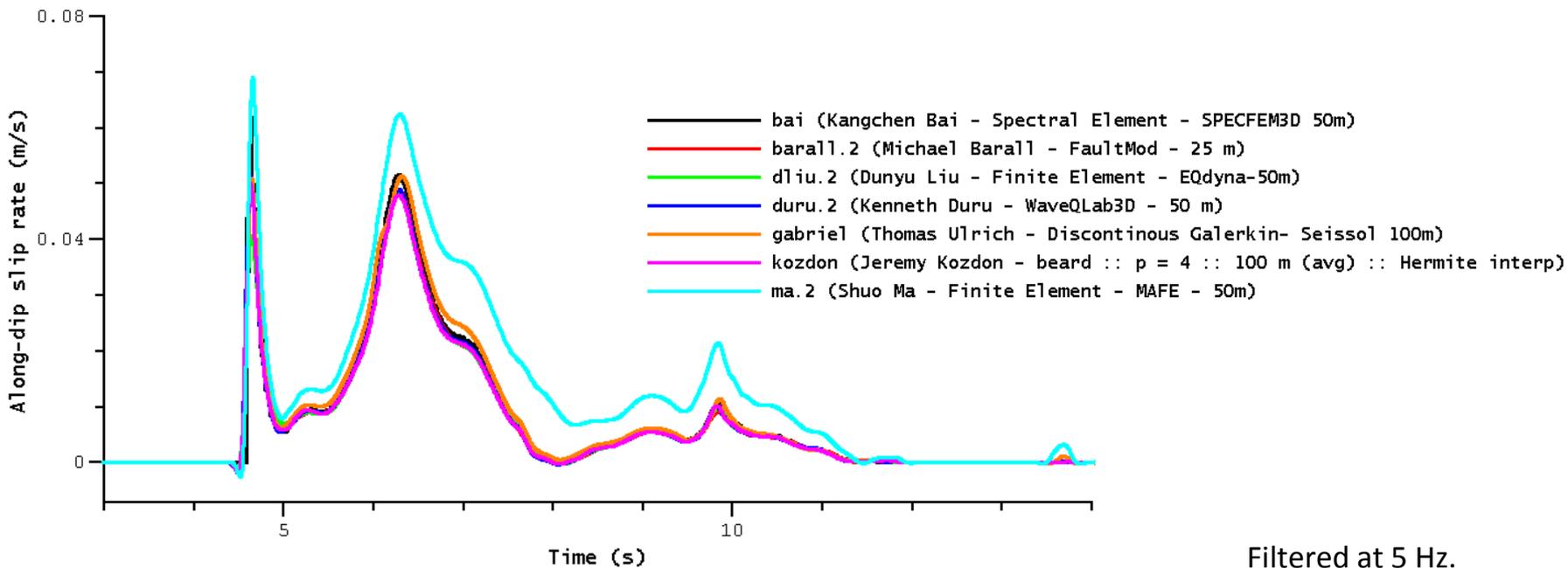
Good agreement among all codes in horizontal component, one code differs from the others in vertical component.

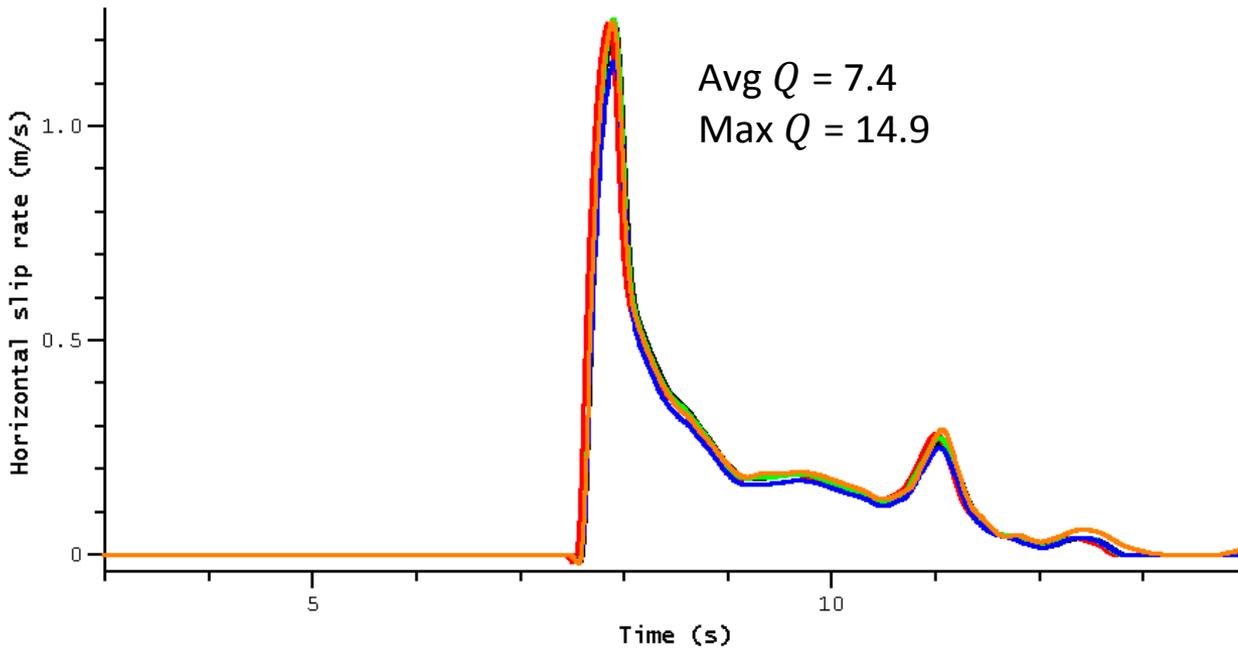




## TPV29 (Elastic) – faultst046dp060 (Second Station) Slip Rate

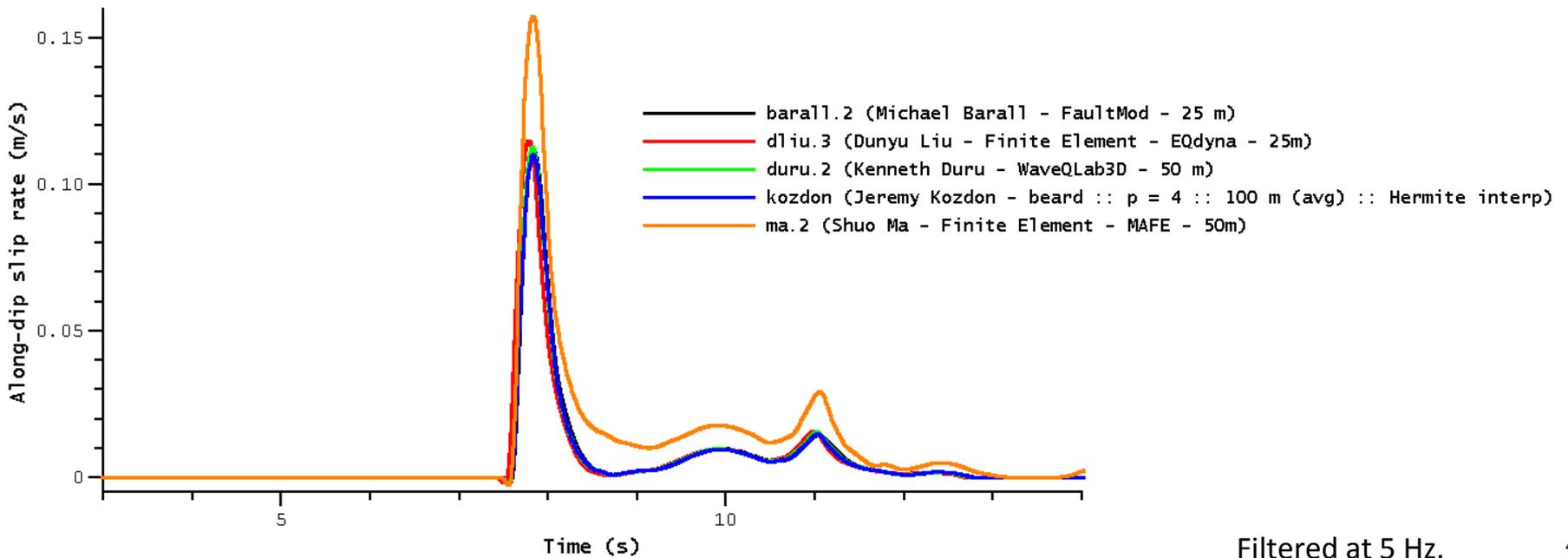
Metrics indicate very good agreement, although each component has one code that looks a bit different from the others.

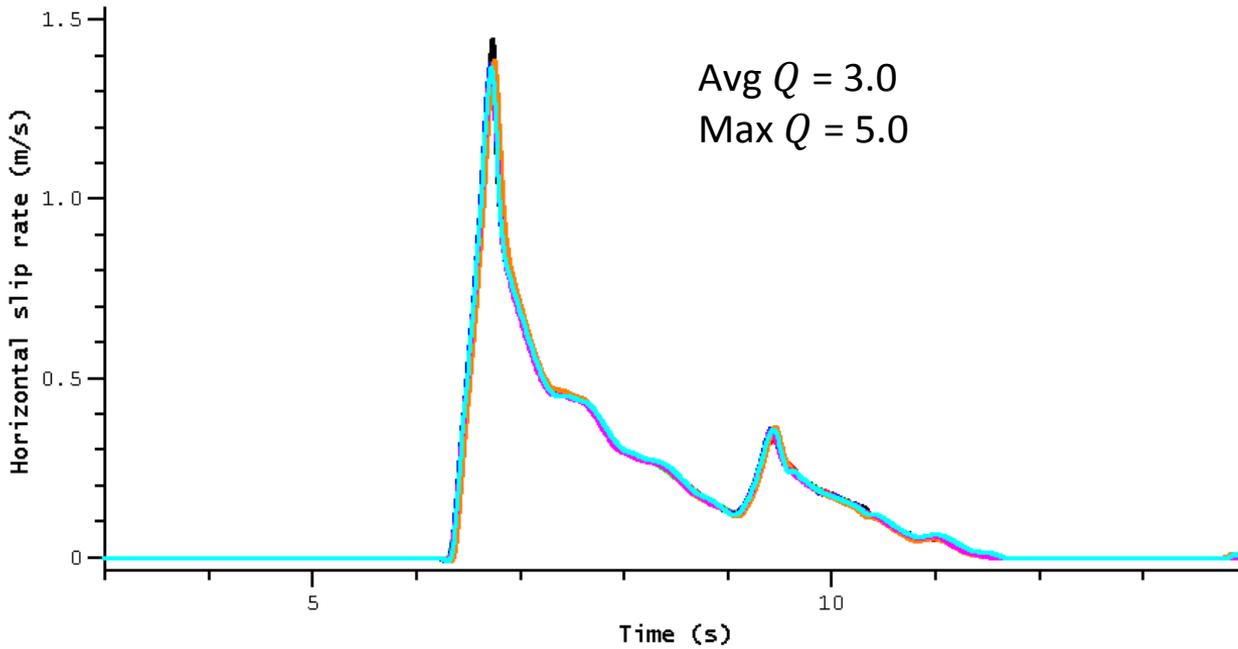




**TPV30 (Viscoplastic) –  
faultst051dp057  
(Third Station) Slip Rate**

Good agreement among all codes in horizontal component, one code differs from the others in vertical component.

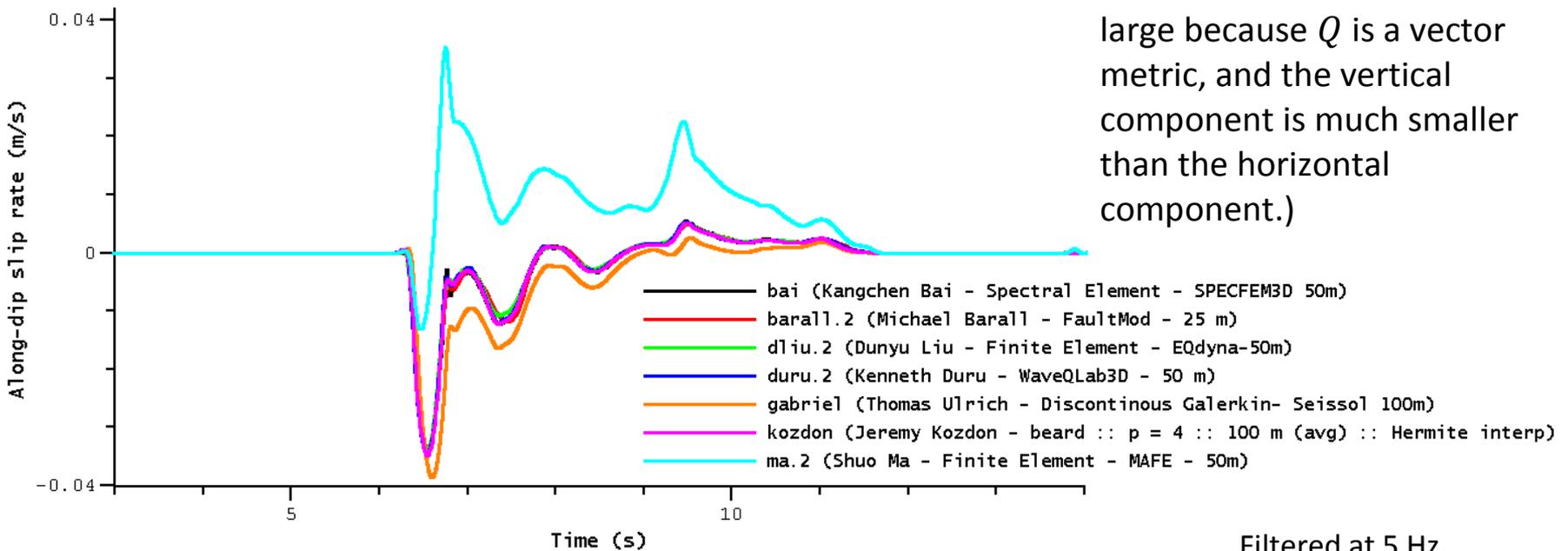




## TPV29 (Elastic) – faultst059dp047 (Fourth Station) Slip Rate

Excellent agreement among codes, with extremely low  $Q$  metric value.

One code has mostly reversed sign in vertical component. (This does not cause the metric value to be large because  $Q$  is a vector metric, and the vertical component is much smaller than the horizontal component.)

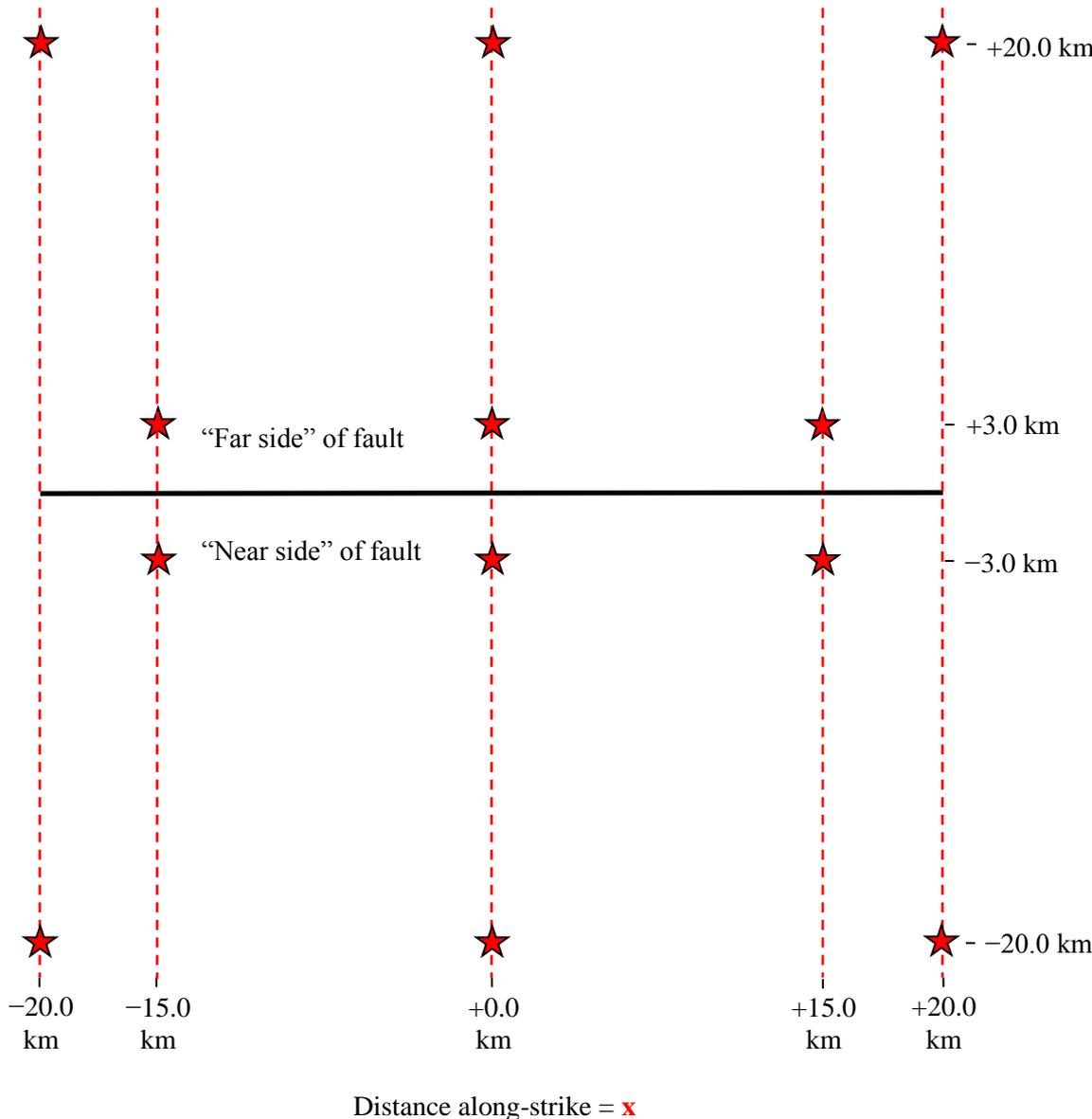


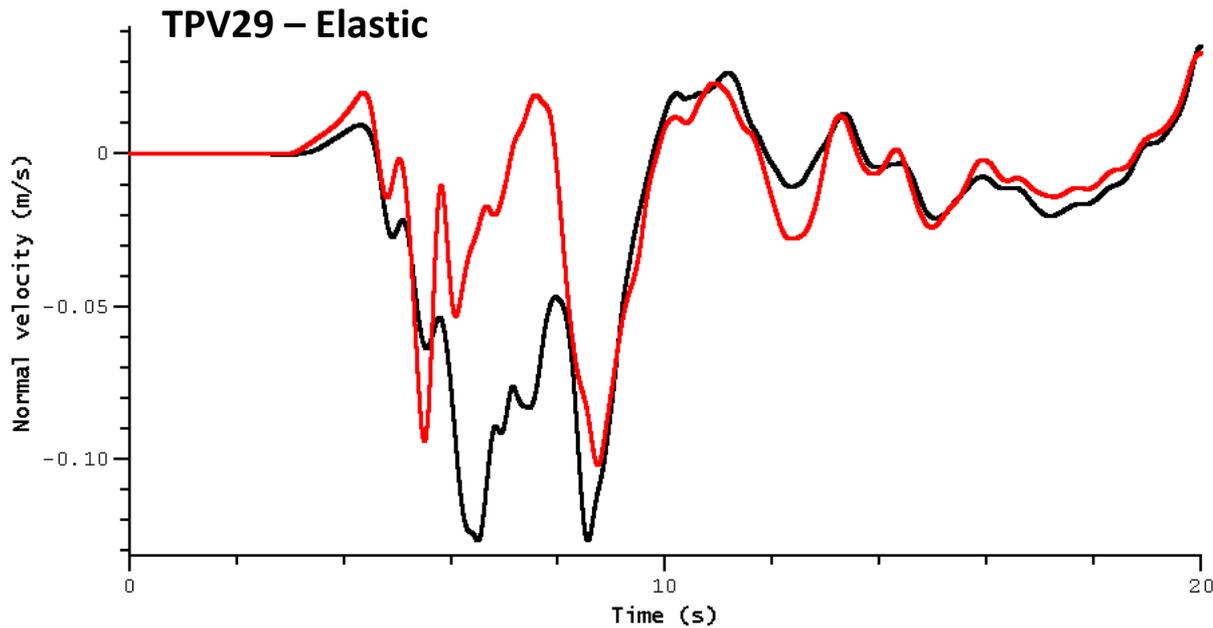
# Time Series: Off-Fault Stations

# Off-Fault Stations.

We have nearby stations located 3 km from the fault, and distant stations located 20 km from the fault.

Note that each station has a counterpart located on the opposite side of the fault. If the fault were planar, then each station and its counterpart would have the same normal velocity, and opposite-signed horizontal and vertical velocity.

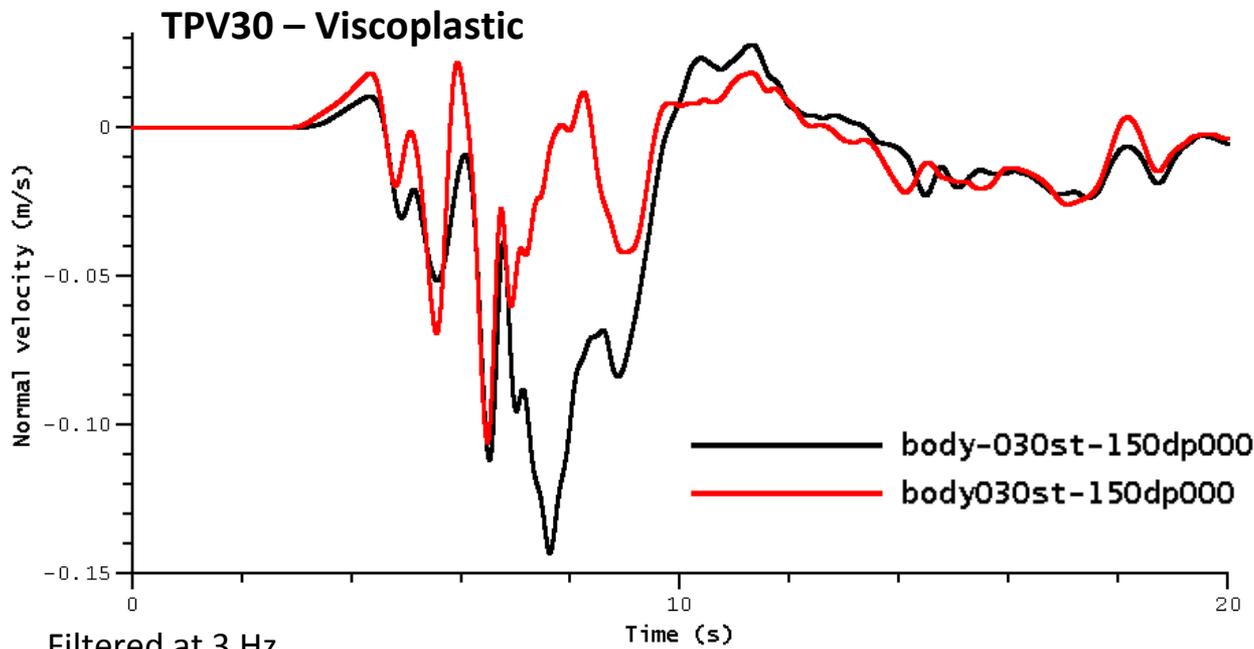




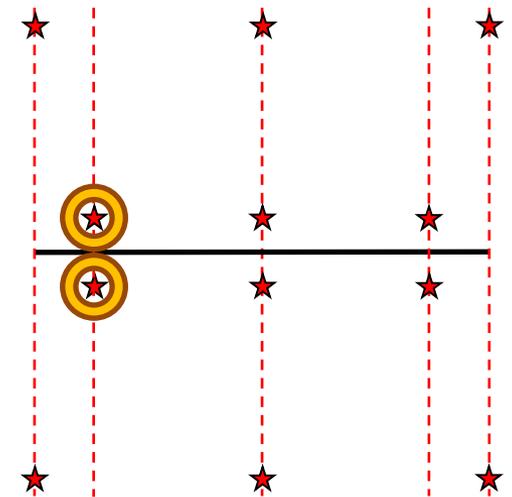
## Broken Symmetry – Nearby Stations

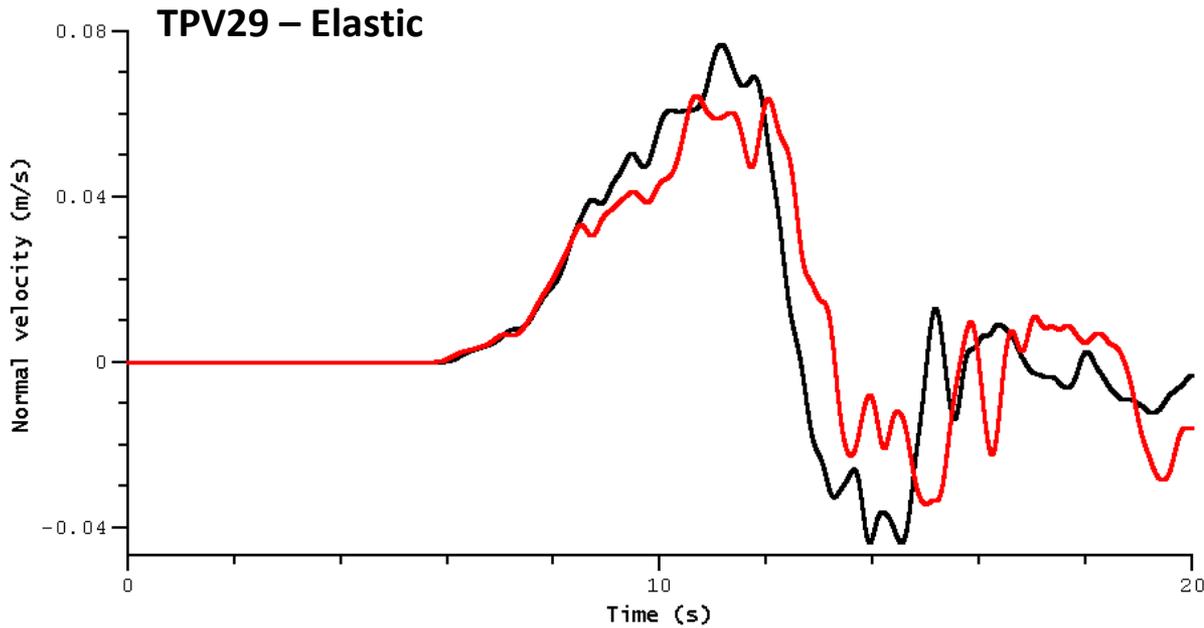
The graphs show normal velocity at a pair of stations 3 km from the fault.

If the fault were planar, the red and black curves would coincide.



Filtered at 3 Hz.

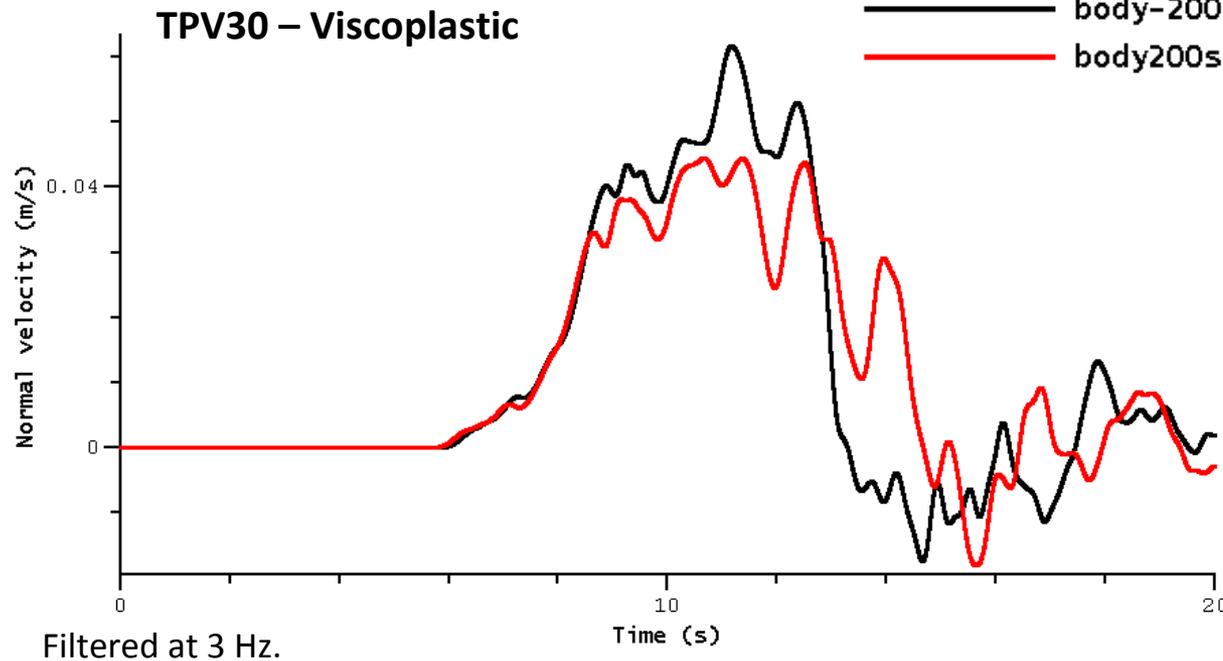




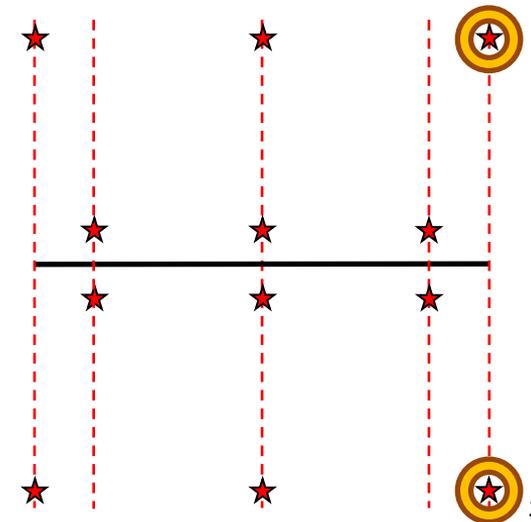
## Broken Symmetry – Distant Stations

The graphs show normal velocity at a pair of stations 20 km from the fault.

If the fault were planar, the red and black curves would coincide.



Filtered at 3 Hz.



## TPV29 (Elastic) – Summary Metrics for All Off-Fault Stations

	3d-vel	5 codes
body-030st-150dp000	<u>5.3</u>	<u>3.9</u>
body-030st000dp000	<u>4.7</u>	<u>3.2</u>
body-030st150dp000	<u>10.2</u>	<u>6.3</u>
body030st-150dp000	<u>6.8</u>	<u>5.0</u>
body030st000dp000	<u>6.1</u>	<u>4.2</u>
body030st150dp000	<u>11.5</u>	<u>8.1</u>

	3d-vel	5 codes
body-200st-200dp000	<u>14.4</u>	<u>9.7</u>
body-200st000dp000	<u>13.9</u>	<u>9.0</u>
body-200st200dp000	<u>23.0</u>	<u>10.5</u>
body200st-200dp000	<u>15.4</u>	<u>11.2</u>
body200st000dp000	<u>13.4</u>	<u>7.8</u>
body200st200dp000	<u>27.3</u>	<u>13.5</u>

These are the waveform comparison metrics, averaged over all pairs of codes, using the best version from each of 6 codes. The “3d-vel” column is the average 3D velocity metric for all 6 codes. The “5 codes” column is the average metric for only the 5 best-matching codes.

The stations 20 km from the fault (right table) have larger metric  $Q$  values, and therefore more disagreement, than the stations 3 km from the fault (left table).

## TPV30 (Viscoplastic) – Summary Metrics for All Off-Fault Stations

	3d-vel
body-030st-150dp000	<u>6.4</u>
body-030st000dp000	<u>6.5</u>
body-030st150dp000	<u>10.9</u>
body030st-150dp000	<u>8.4</u>
body030st000dp000	<u>7.4</u>
body030st150dp000	<u>11.8</u>

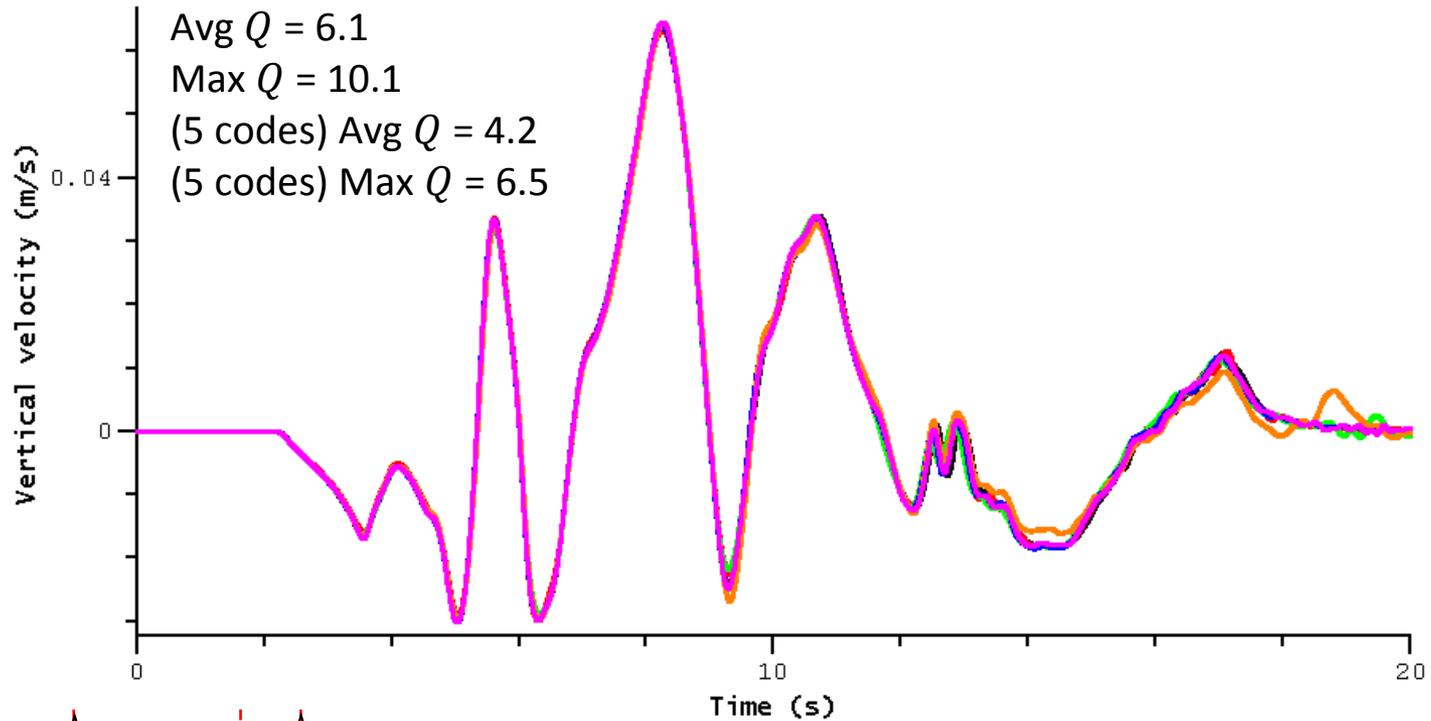
	3d-vel
body-200st-200dp000	<u>18.6</u>
body-200st000dp000	<u>12.9</u>
body-200st200dp000	<u>18.6</u>
body200st-200dp000	<u>18.8</u>
body200st000dp000	<u>13.1</u>
body200st200dp000	<u>20.1</u>

These are the waveform comparison metrics, averaged over all pairs of codes, using the best version from each of 4 codes. The “3d-vel” column is the average 3D velocity metric for all 4 codes.

The stations 20 km from the fault (right table) have larger metric  $Q$  values, and therefore more disagreement, than the stations 3 km from the fault (left table).

These metric values are considerably higher than the “5 codes” metrics for TPV29.

# TPV29 (Elastic) – body030st000dp000 – Vertical Velocity

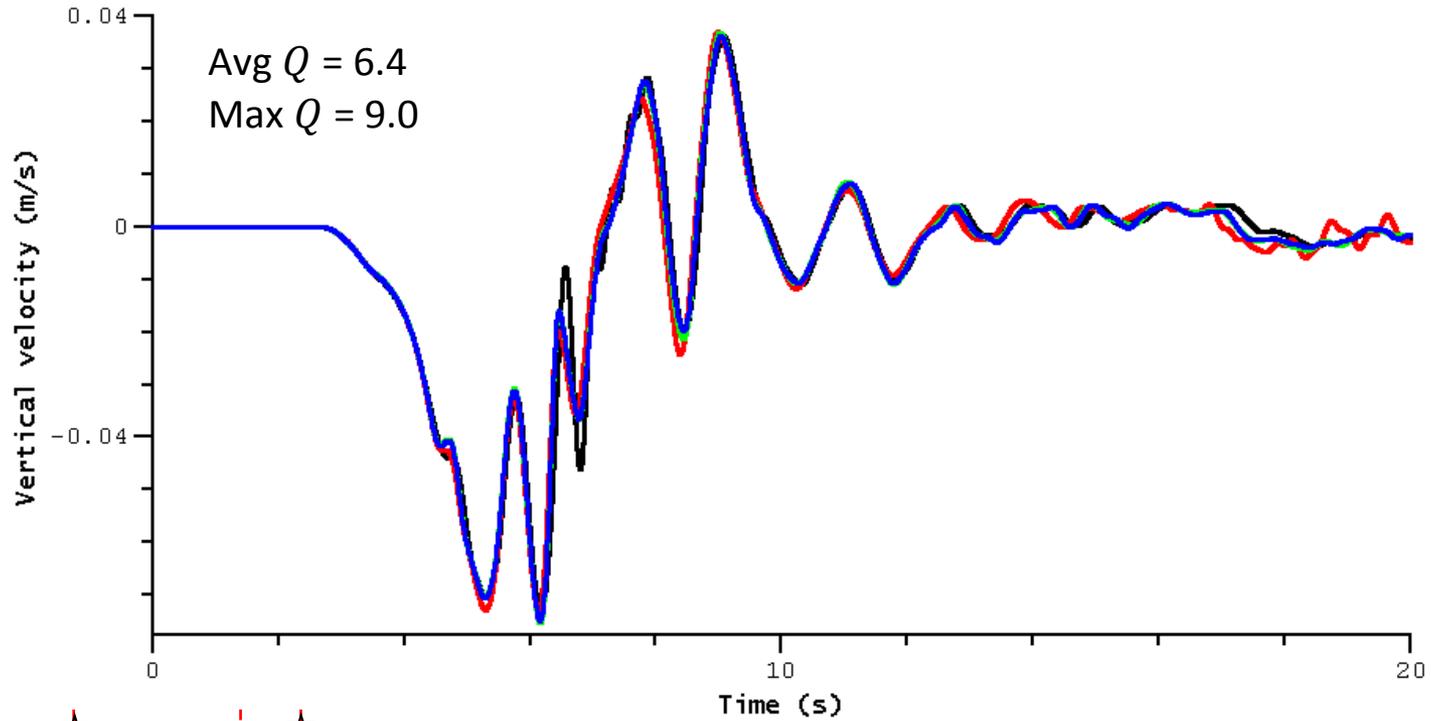


Codes show excellent agreement close to the fault, with small differences appearing late in the simulation. Note the complicated waveform.

- bai (Kangchen Bai - Spectral Element - SPECFEM3D 50m)
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- gabriel (Thomas Ulrich - Discontinuous Galerkin- SeisSol 100m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)

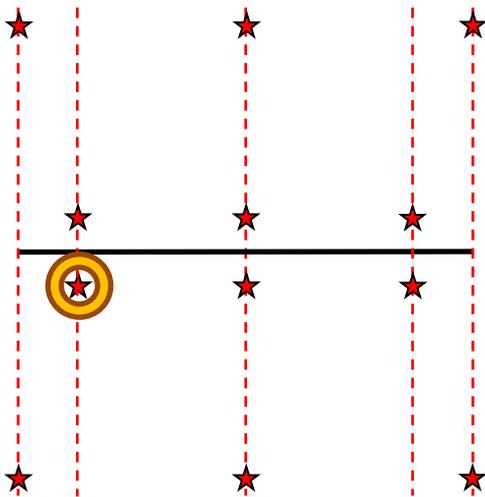
Filtered at 3 Hz.

# TPV30 (Viscoplastic) – body-030st-150dp000 – Vertical Velocity

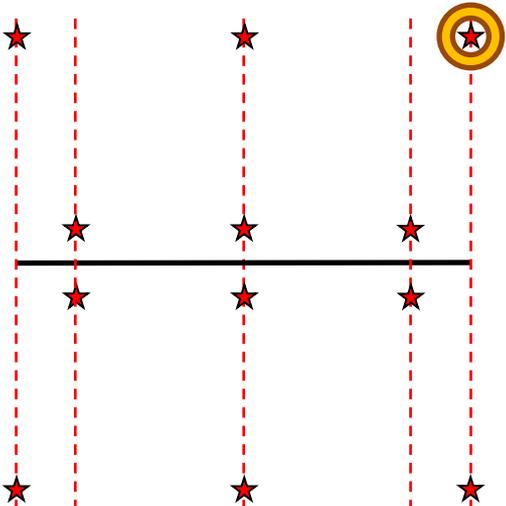
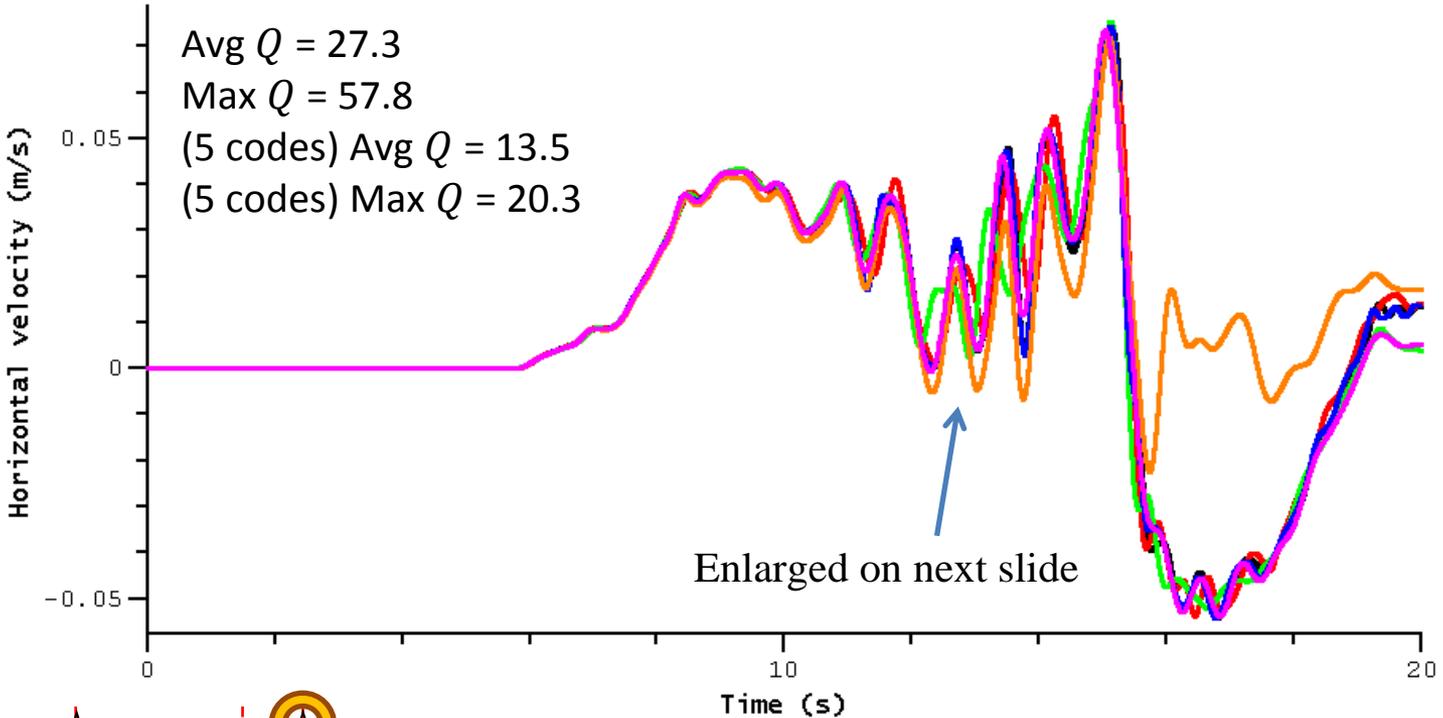


Codes show excellent agreement close to the fault in the viscoplastic case, although the match is not quite as good as for the best-matching codes in the elastic case.

- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard ::  $p = 4$  :: 100 m (avg) :: Hermite interp)



# TPV29 (Elastic) – body200st200dp000 – Horizontal Velocity

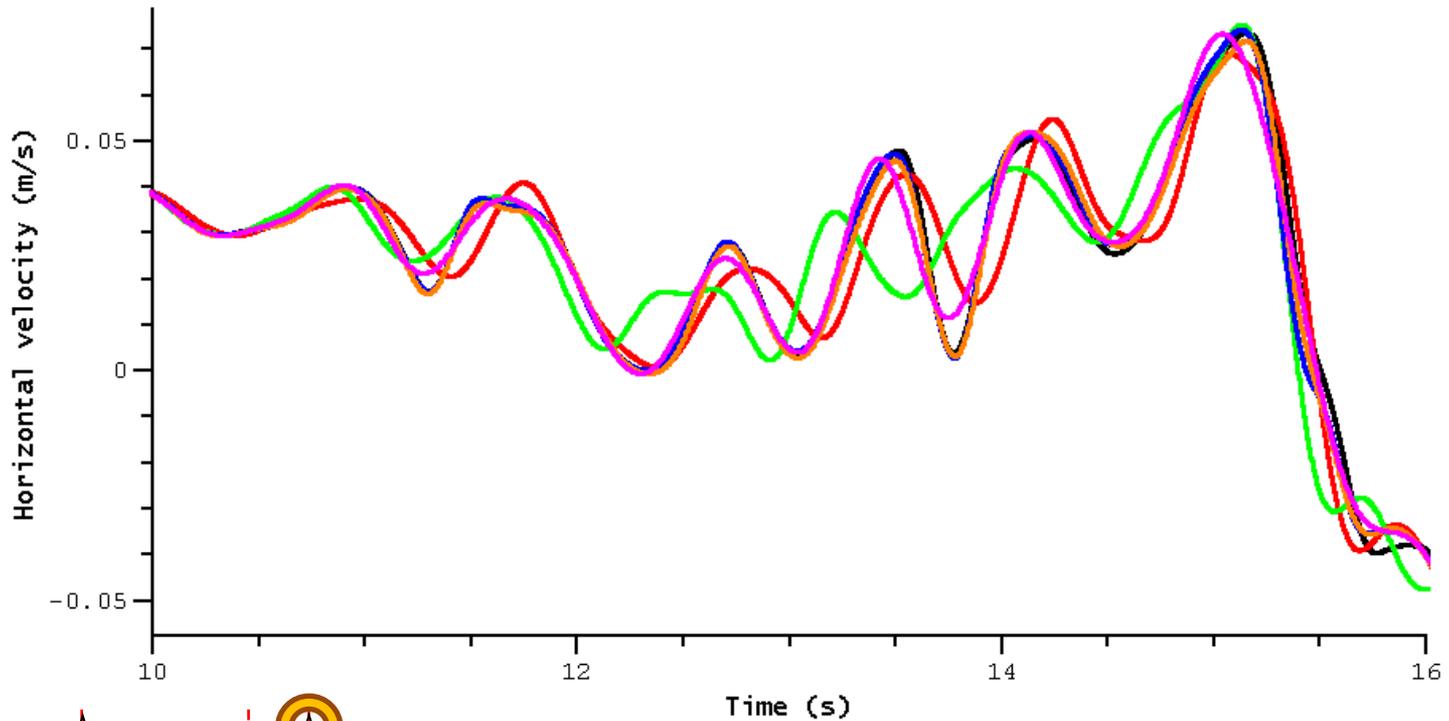


Far from the fault, agreement is noticeably worse than close to the fault. Codes show overall agreement, except one code that differs from the other five codes late in the simulation.

- bai (Kangchen Bai - Spectral Element - SPECFEM3D 50m)
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- gabriel (Thomas Ulrich - Discontinuous Galerkin- SeisSol 100m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)

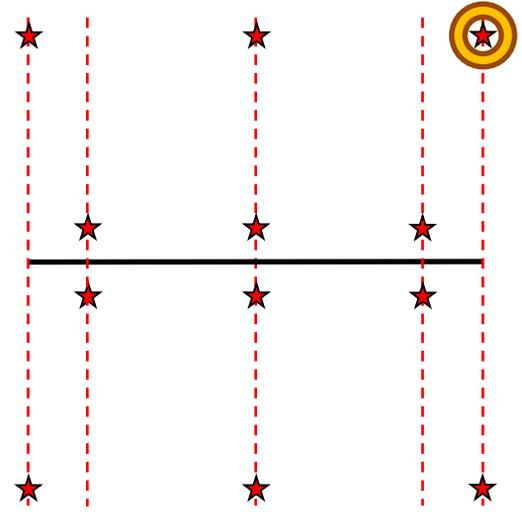
Filtered at 3 Hz.

# TPV29 (Elastic) – body200st200dp000 – Horizontal Velocity – Enlarged

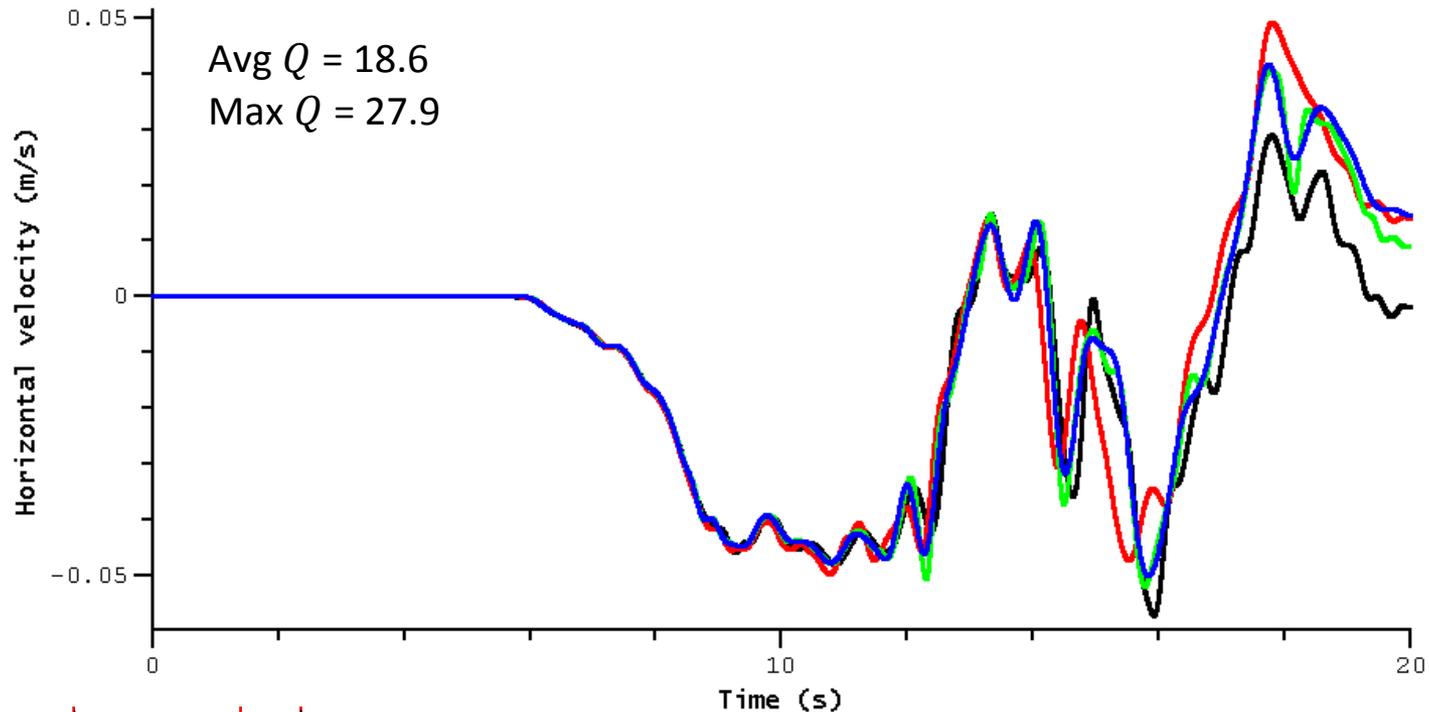


Four of the codes are in fairly good agreement on the amplitude and phase of these oscillations, which are about 1.4 Hz. The other two codes show some difference in phase.

- bai (Kangchen Bai - Spectral Element - SPECFEM3D 50m)
- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- gabriel (Thomas Ulrich - Discontinuous Galerkin- SeisSol 100m)
- kozdon (Jeremy Kozdon - beard :: p = 4 :: 100 m (avg) :: Hermite interp)

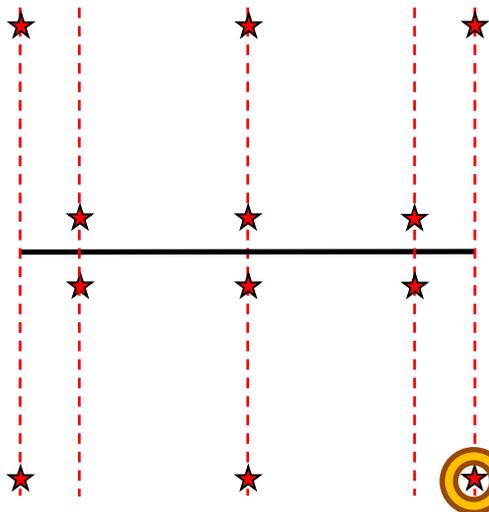


# TPV30 (Viscoplastic) – body-200st200dp000 – Horizontal Velocity

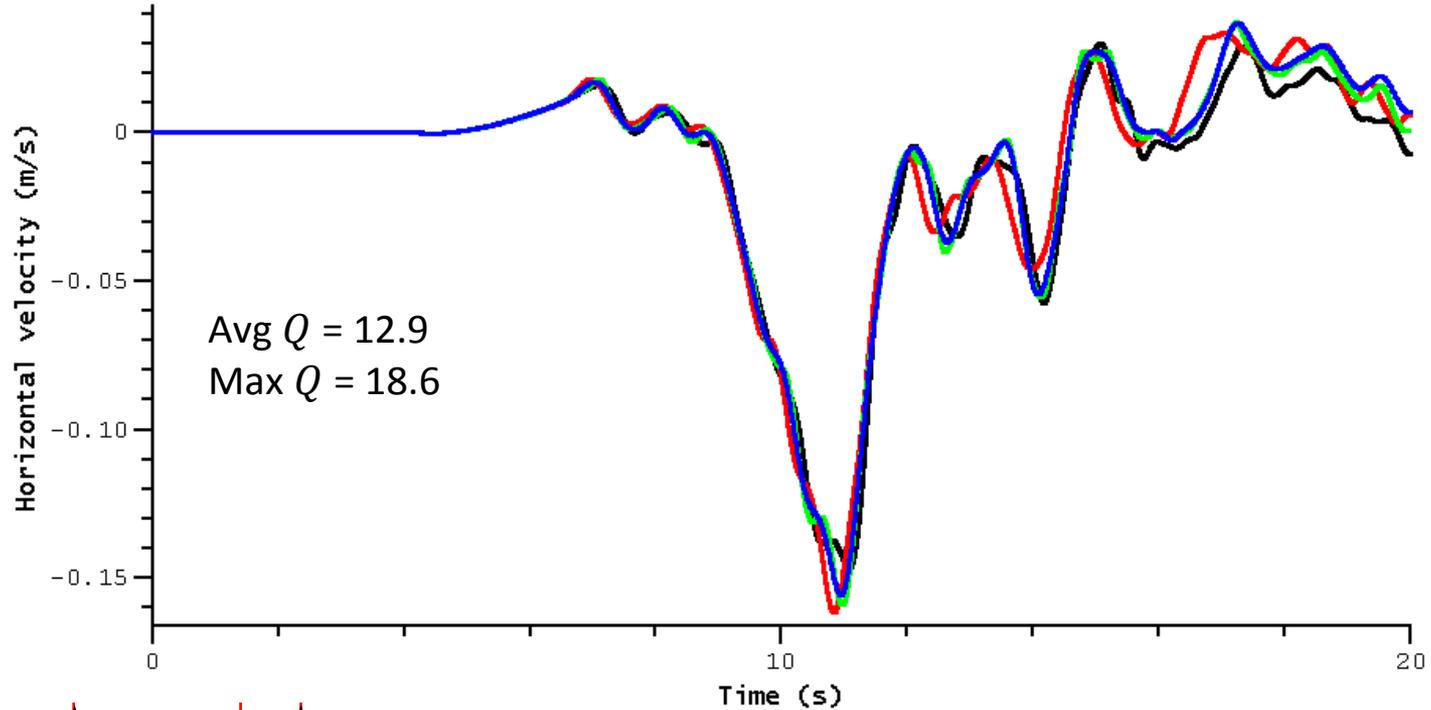


In the viscoplastic case far from the fault, agreement has worsened compared to the elastic case and to the stations close to the fault. But the codes still produce the same general waveform shape.

- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard ::  $p = 4$  :: 100 m (avg) :: Hermite interp)

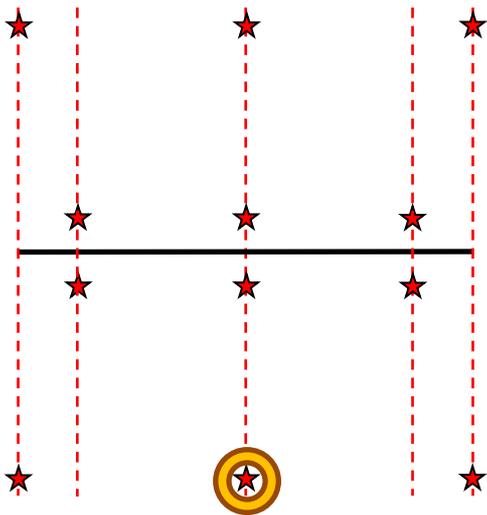


# TPV30 (Viscoplastic) – body-200st000dp000 – Horizontal Velocity



The peak velocity is three times as much as the station on the previous slide. The metric values are lower, indicating better agreement, but still not as good as close to the fault.

- barall.2 (Michael Barall - FaultMod - 25 m)
- dliu.2 (Dunyu Liu - Finite Element - EQdyna-50m)
- duru.2 (Kenneth Duru - WaveQLab3D - 50 m)
- kozdon (Jeremy Kozdon - beard ::  $p = 4$  :: 100 m (avg) :: Hermite interp)



# Conclusions

## Conclusions

1. We did two benchmarks using a fault with stochastic roughness:
  - TPV29 – Rough fault in a linear elastic medium.
  - TPV30 – Rough fault in a Drucker-Prager viscoplastic medium.
2. Rupture front contours are distorted by the rough fault geometry.
  - Distortion is greater in the viscoplastic case than in the elastic case.
  - Codes agree well in both cases, although better in the elastic case.
  - There are places where the rupture front slows or pauses for 2 or 3 seconds.
3. At on-fault stations, codes are generally in excellent agreement on slip rates. Most differences appear to be associated with rupture front slowdowns.
4. At on-fault stations, there are large disagreements between codes on shear and normal stresses.
5. These are our first benchmarks with distant off-fault stations.
  - Agreement is distinctly worse at distant stations than at stations close to the fault.
  - But even as agreement worsens, codes continue to produce the same general waveform shape and many of the same “wiggles.”