Heterogeneous Initial Stress Conditions

for Spontaneous Rupture Modeling

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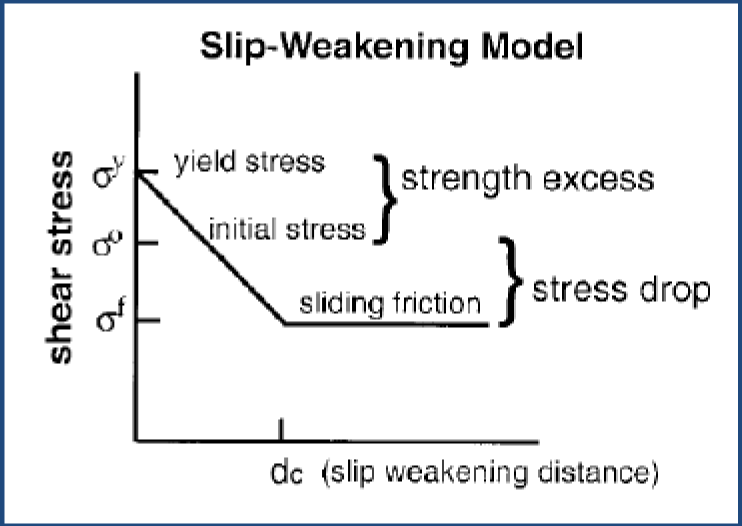
Outline

- Spatial heterogeneity with 1-point and 2-point statistics
  - Do we pay enough attention to 1-point statistics?
  - How to constrain them from data?

- Two-step approach
  - Quasi-dynamic multi-cycle simulation with RS friction law
  - Full-dynamic single-event simulation with SW friction law

Spontaneous Rupture Modeling
with Slip-weakening Friction Law



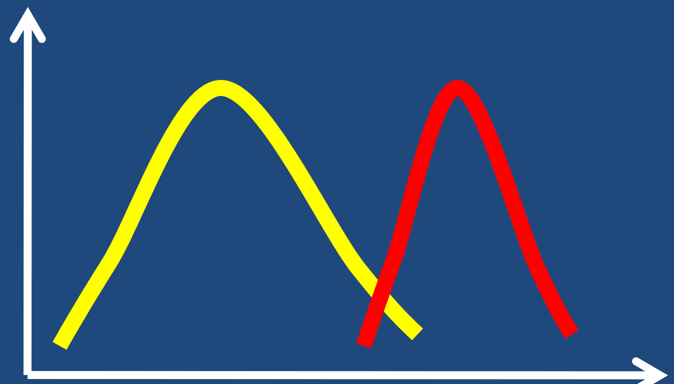
(Ida, 1972; Andrews, 1976)

- Stress drop: from given slip models, or assumed stochastic model (e.g., $k^{-\nu}$)
- Fracture energy: somewhat arbitrary, i.e., S parameter, constant yield stress, strength excess, d_c, etc.

Earthquake Source Statistics

1-Point Statistics

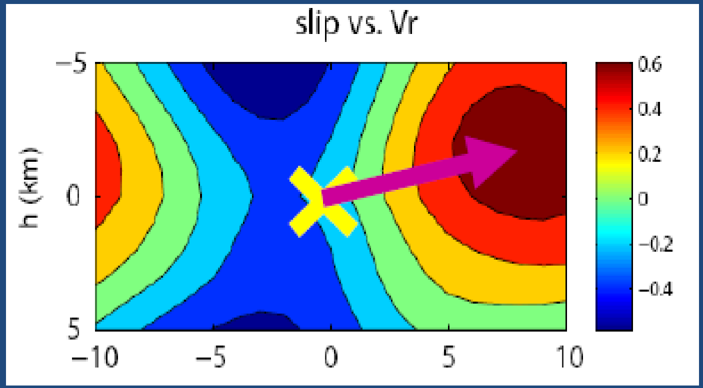
- Scaling of mean slip and sigma with earthquake size
- Supershear and subshear
- Crack-like and pulse-like rupture
- Stick-slip and creeping



(slip, V_r, V_{max}, rT)

2-Point statistics

- *Auto-coherence*: define heterogeneity of source parameters
- *Cross-coherence*: control coupling between different parameters



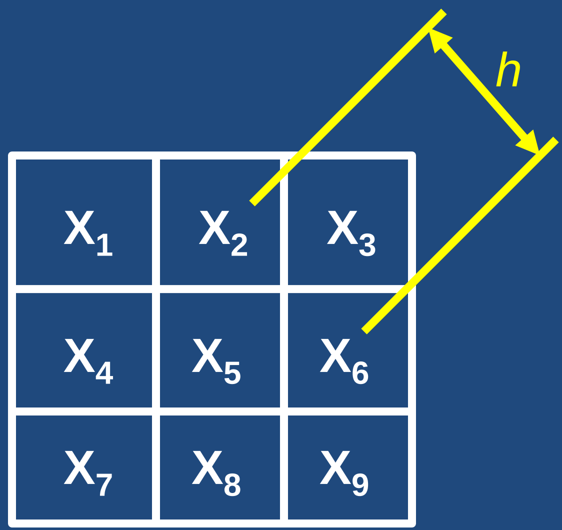
Spatial Random Field Model

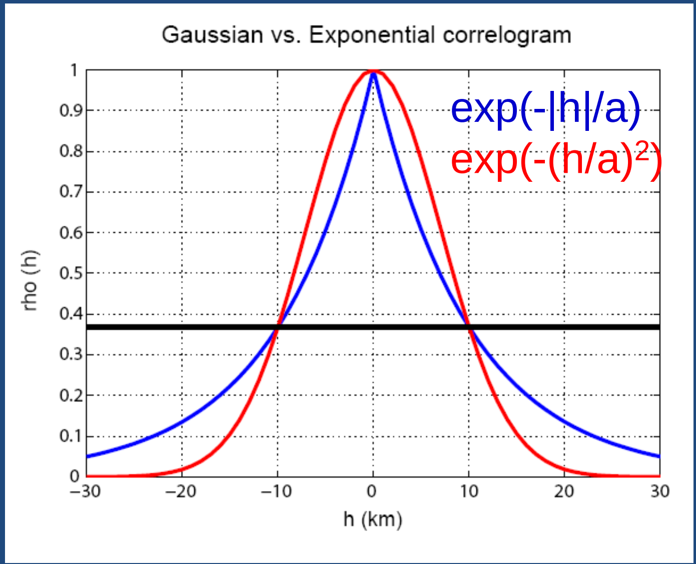
$$\mathbf{X} \sim \mathcal{N}(\mu, \Sigma)$$

$$\mu = (\mu_1, \mu_2, \mu_3, \ldots, \mu_9)^T$$
$$= (\mu, \mu, \mu, \ldots, \mu)^T$$

$$\Sigma = \begin{bmatrix} \sigma_1\sigma_1\rho_{11}, \sigma_1\sigma_2\rho_{12}, \ldots, \sigma_1\sigma_9\rho_{19} \\ \sigma_2\sigma_1\rho_{21}, \sigma_2\sigma_2\rho_{22}, \ldots, \sigma_2\sigma_9\rho_{29} \\ \ldots \\ \sigma_9\sigma_1\rho_{91}, \sigma_9\sigma_2\rho_{92}, \ldots, \sigma_9\sigma_9\rho_{99} \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 1, \rho_{12}, \ldots, \rho_{19} \\ \rho_{21}, 1, \ldots, \rho_{29} \\ \ldots \\ \rho_{91}, \rho_{92}, \ldots, 1 \end{bmatrix}$$



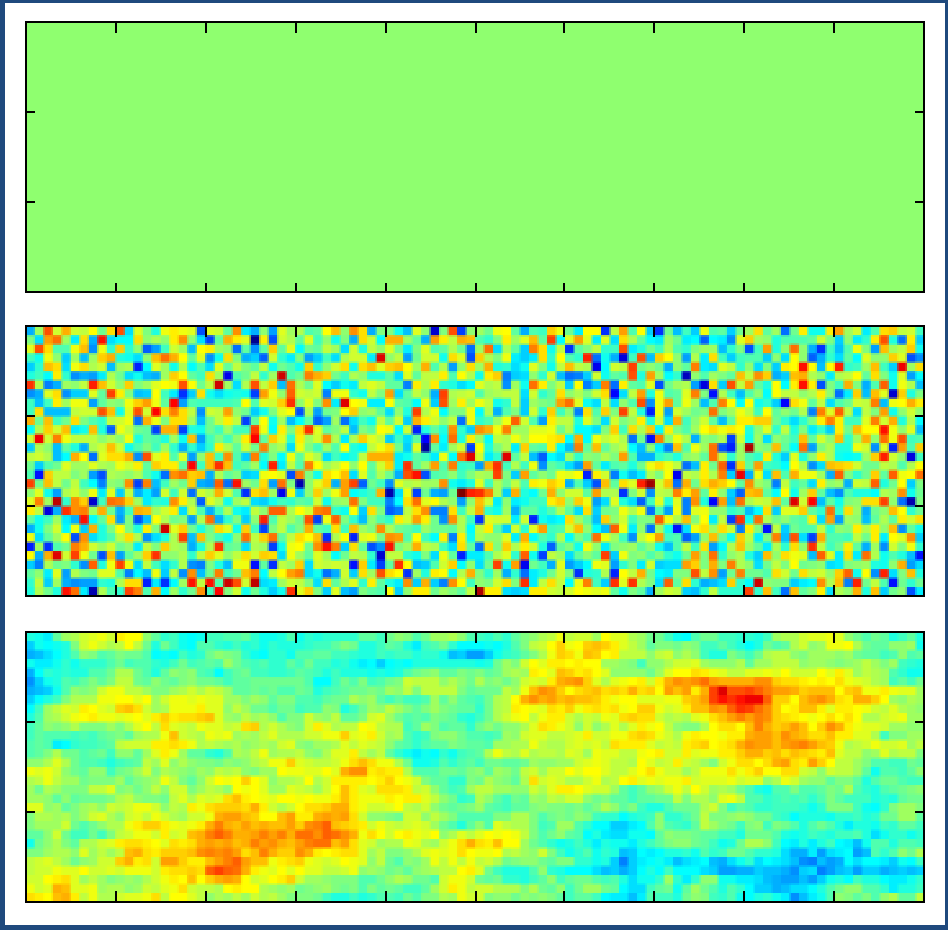


Reproduction of
1-point and 2-point statistics

1) $X \sim \mathcal{N}(\mu, 0)$

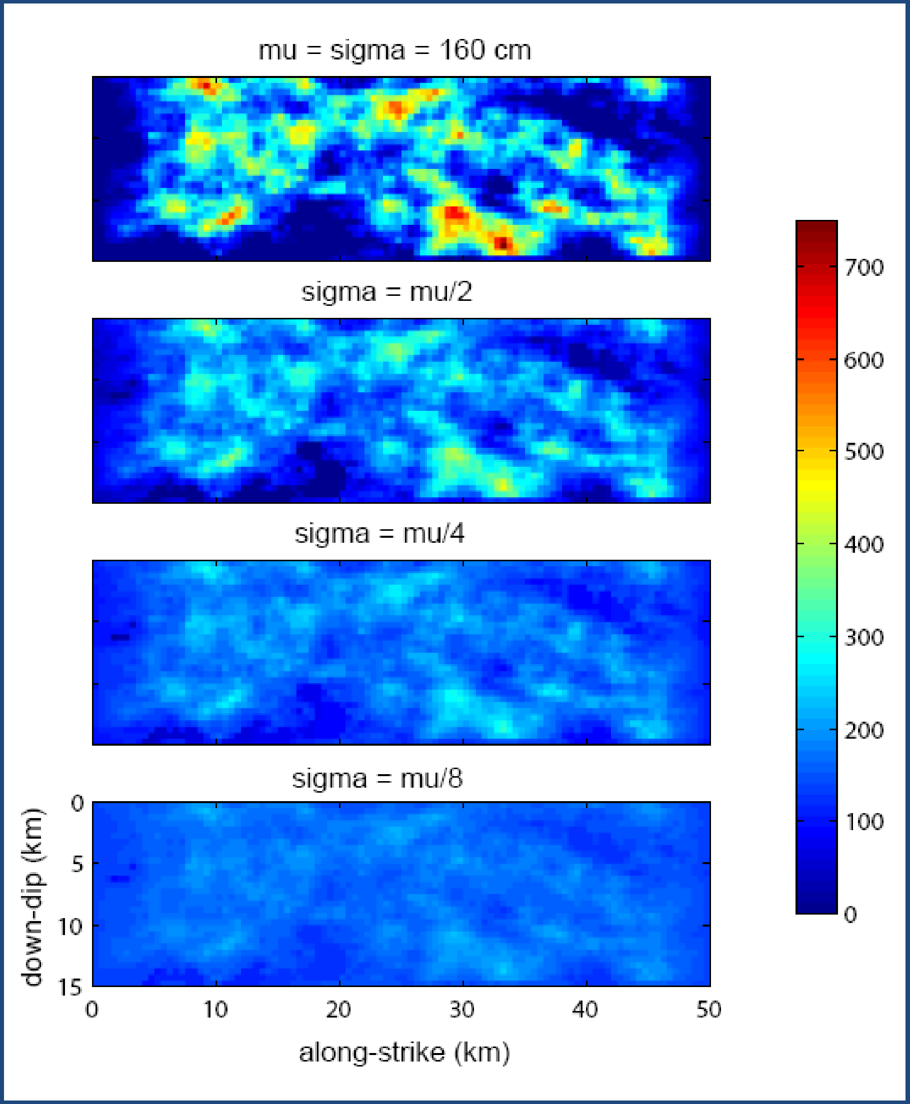
2) $X \sim \mathcal{N}(\mu, \sigma^2 I)$

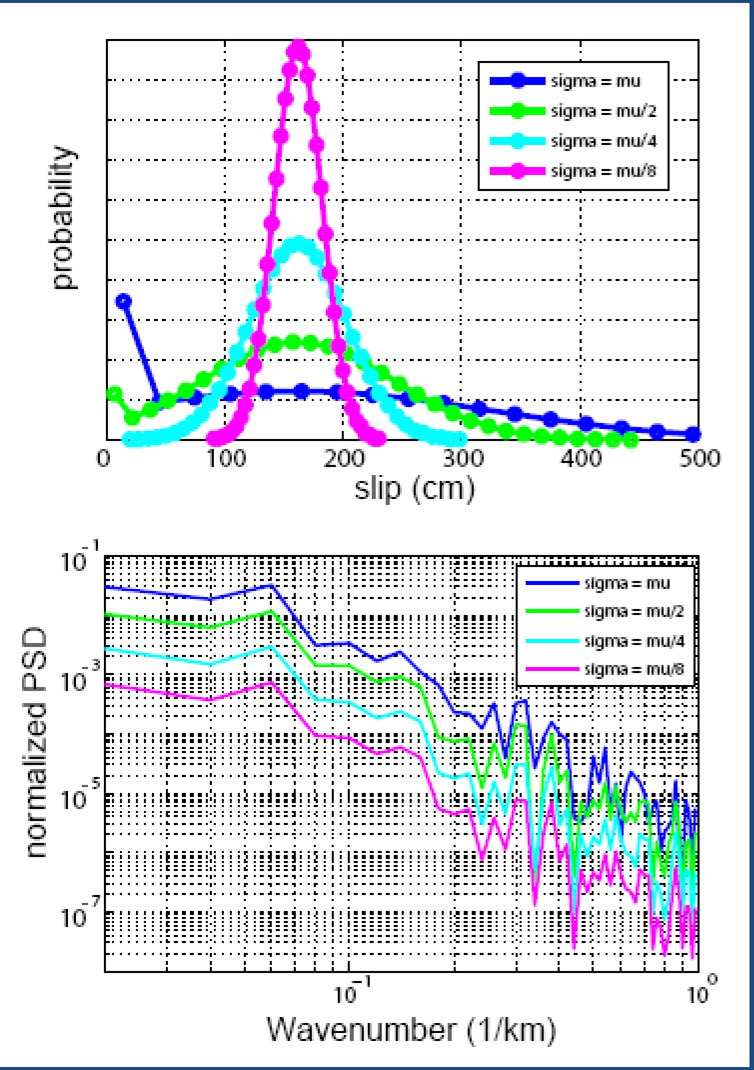
3) $X \sim \mathcal{N}(\mu, \Sigma)$



Slip Realizations

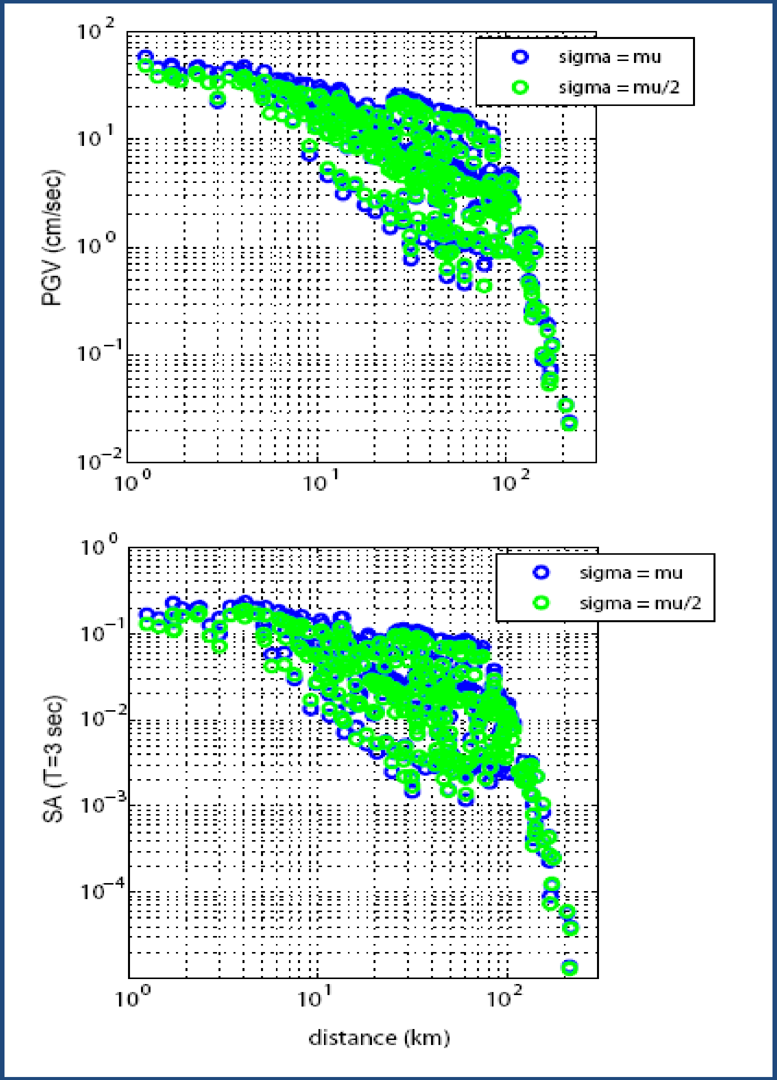
with the Same Spectral Decay (2-point stats)

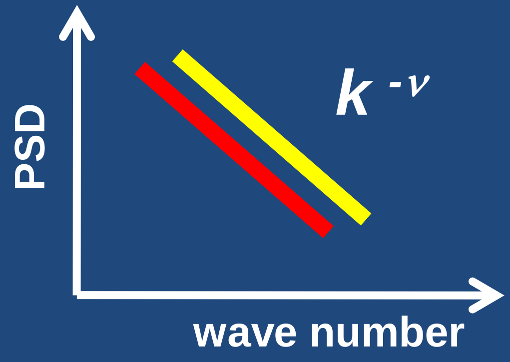


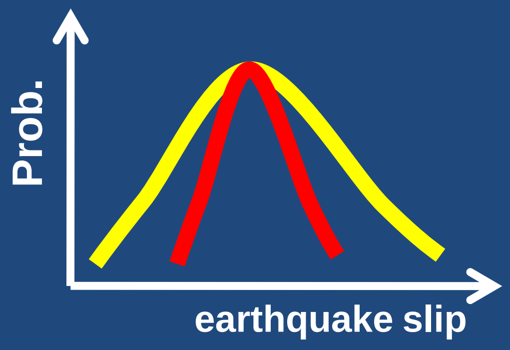


Variability in Ground Motion

with different 1-point statistics







Scaling law for sigma?

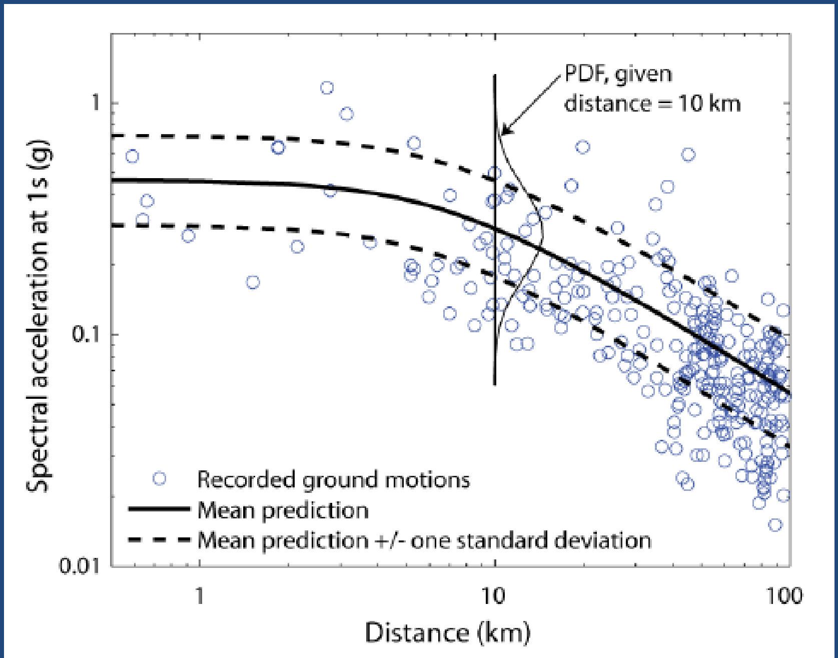
Log (mu) = a M + b [Somerville et al., 1999]

Log (sigma) = a M + b **missing!!**

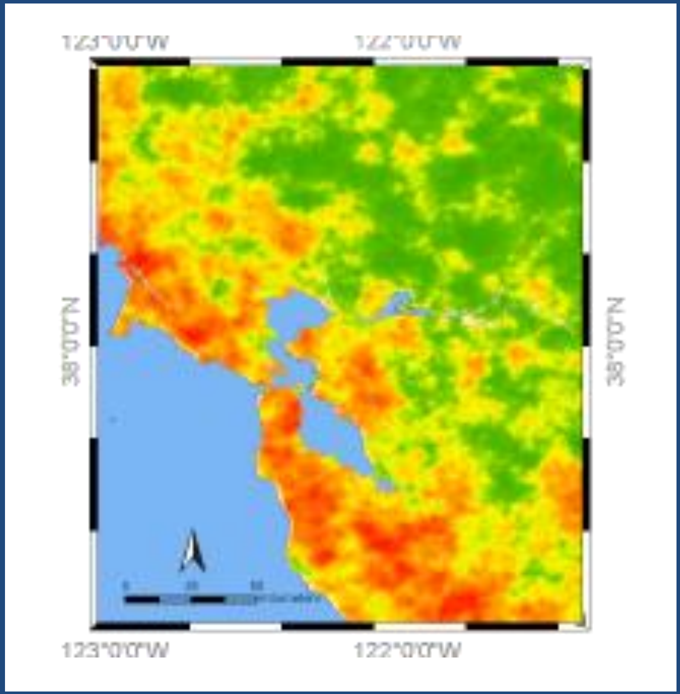
Log (a_x) = a M + b [Mai and Beroza, 2002]

1-point and 2-point statistics
in Ground Motion Prediction

1-point statistics in GMP



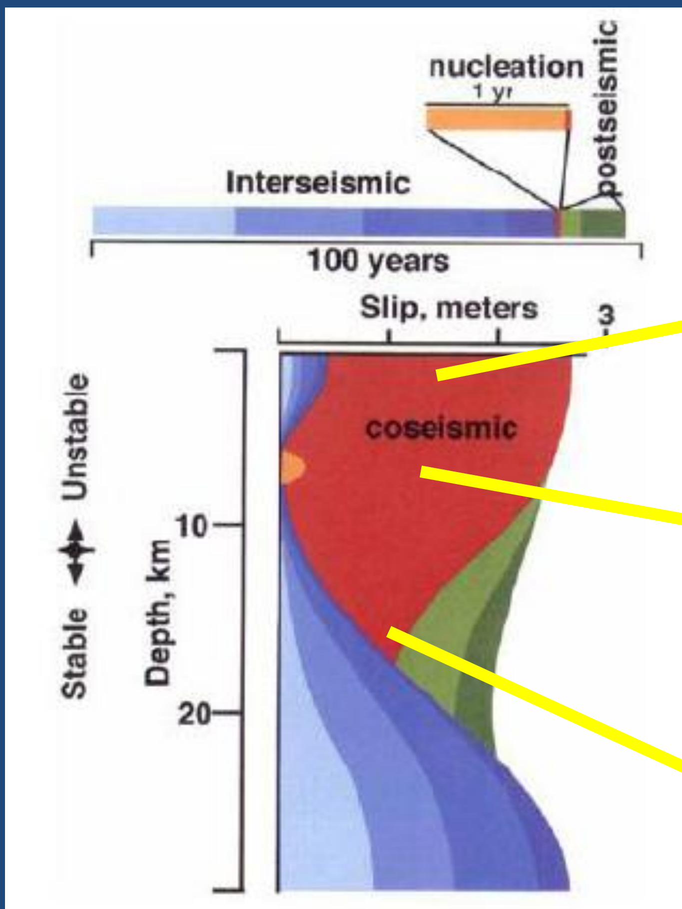
2-point statistics in GMP



(Image source: J. Baker's website at Stanford Univ.)

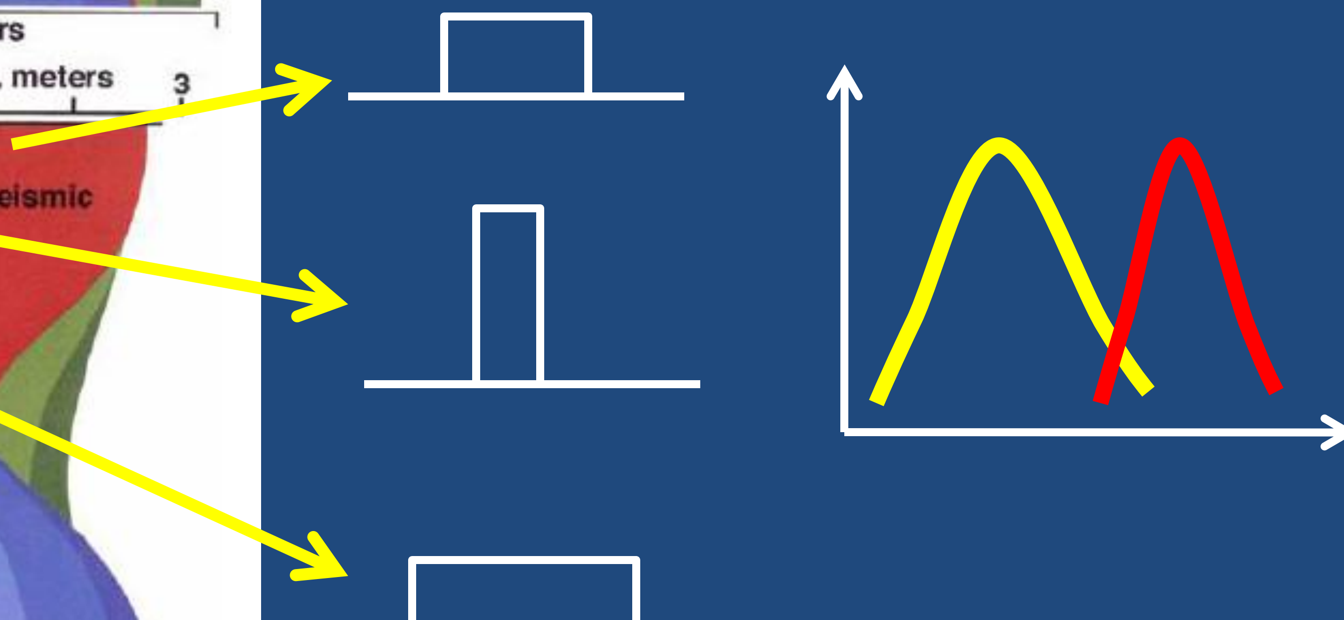
Depth-dependency (Non-stationarity)

of earthquake source statistics

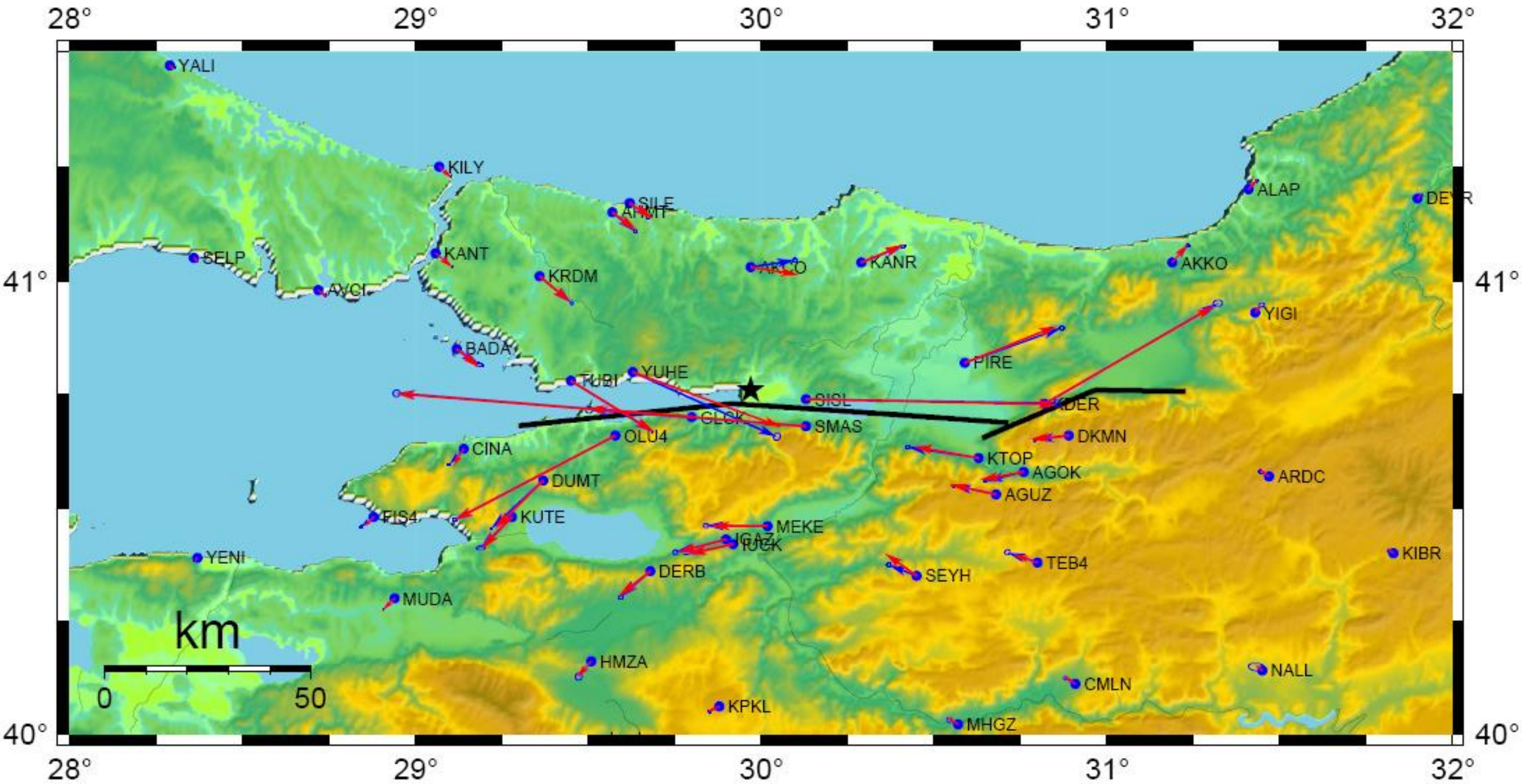


[Scholz, 2002]

Slip Velocity Function

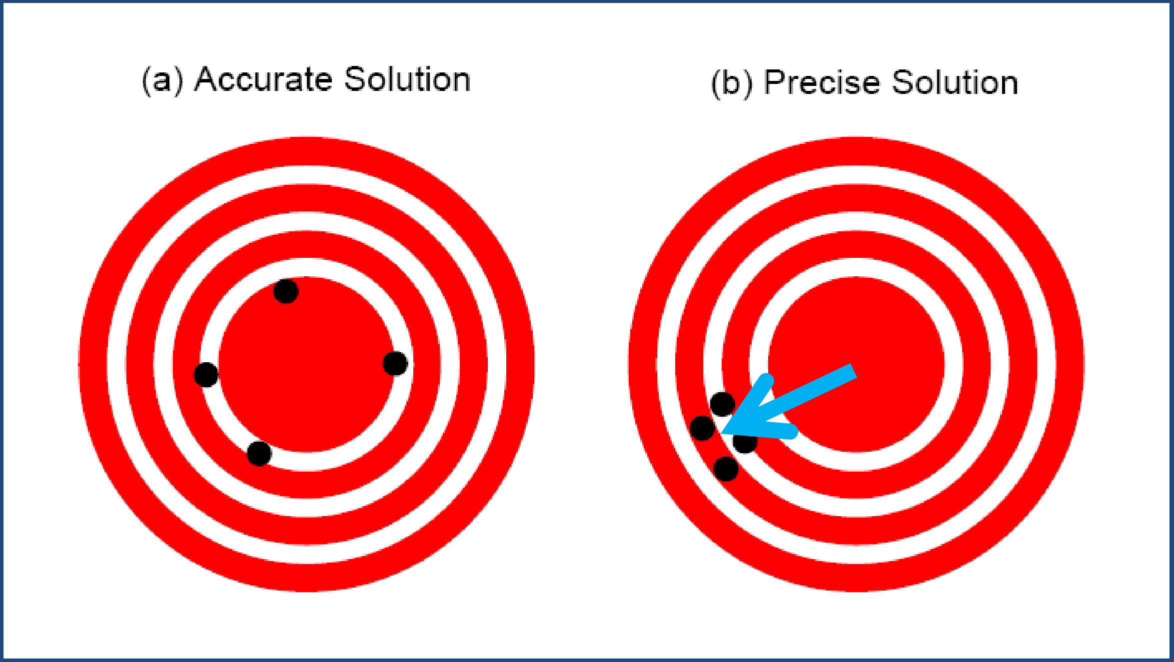


Constraining 1-point and 2-point statistics
with Bayesian inversion



(1999 Izmit, Turkey, event)

Accurate vs. Precise Solutions



$$\min \|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2 + \alpha^2 \|\mathbf{L}\mathbf{m}\|_2^2 \,,\ \hat{\mathbf{m}} = (\mathbf{G}^T\mathbf{G} + \alpha^2 \mathbf{L}^T\mathbf{L})^{-1}\mathbf{G}^T\mathbf{d}$$

$E(\hat{\boldsymbol{m}}) \neq \boldsymbol{m}^{true}$ => *biased estimator!!*

Mean squared error:

$$MSE(\hat{m}) = E((\hat{m} - m^{true})^2) = Var(\hat{m}) + (Bias(\hat{m}))^2$$

Tikhonov Regularization

- Advantage: improve the stability of inversion, otherwise very ill-posed inverse problems

$$\min \|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2 + \alpha^2\|\mathbf{L}\mathbf{m}\|_2^2\,, \hat{\mathbf{m}} = (\mathbf{G}^T\mathbf{G} + \alpha^2\mathbf{L}^T\mathbf{L})^{-1}\mathbf{G}^T\mathbf{d}$$

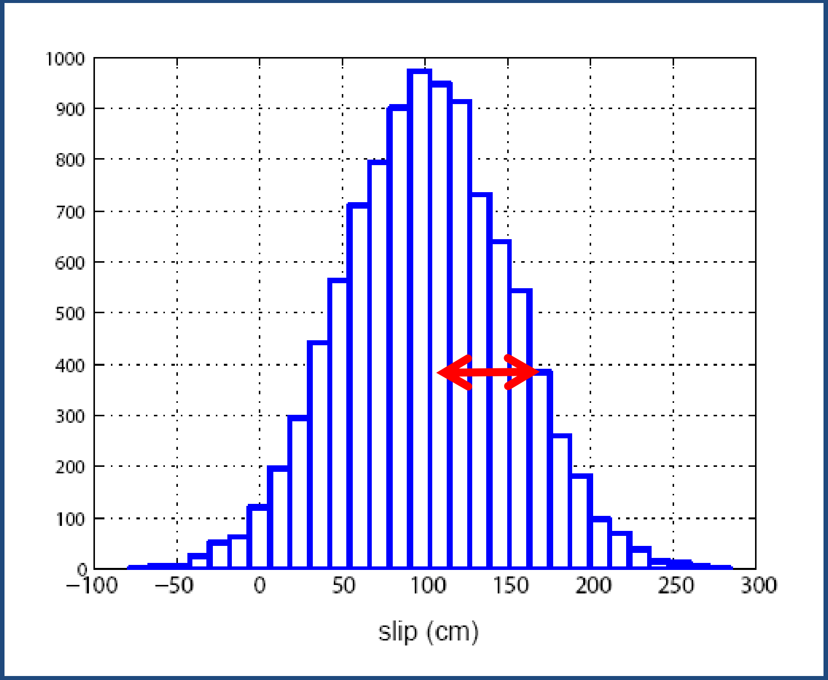
- By-products:
  - Lower resolution => inaccurate estimation of solutions
  - Biased => inaccurate estimation of uncertainty
  - Contaminates 1-point and 2-point stats of earthquake slip (and stress drop)

Prior model distribution

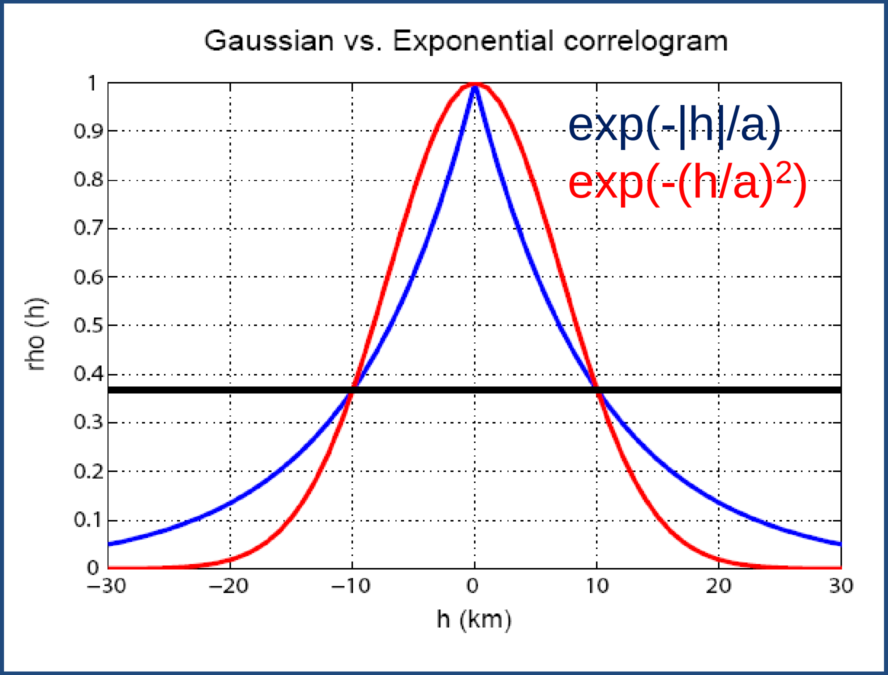
$$\sigma_M(\boldsymbol{m}) = k\rho_M(\boldsymbol{m}) \cdot \rho_D(\boldsymbol{g}(\boldsymbol{m}))$$

$$\rho_M(\boldsymbol{m}) = N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

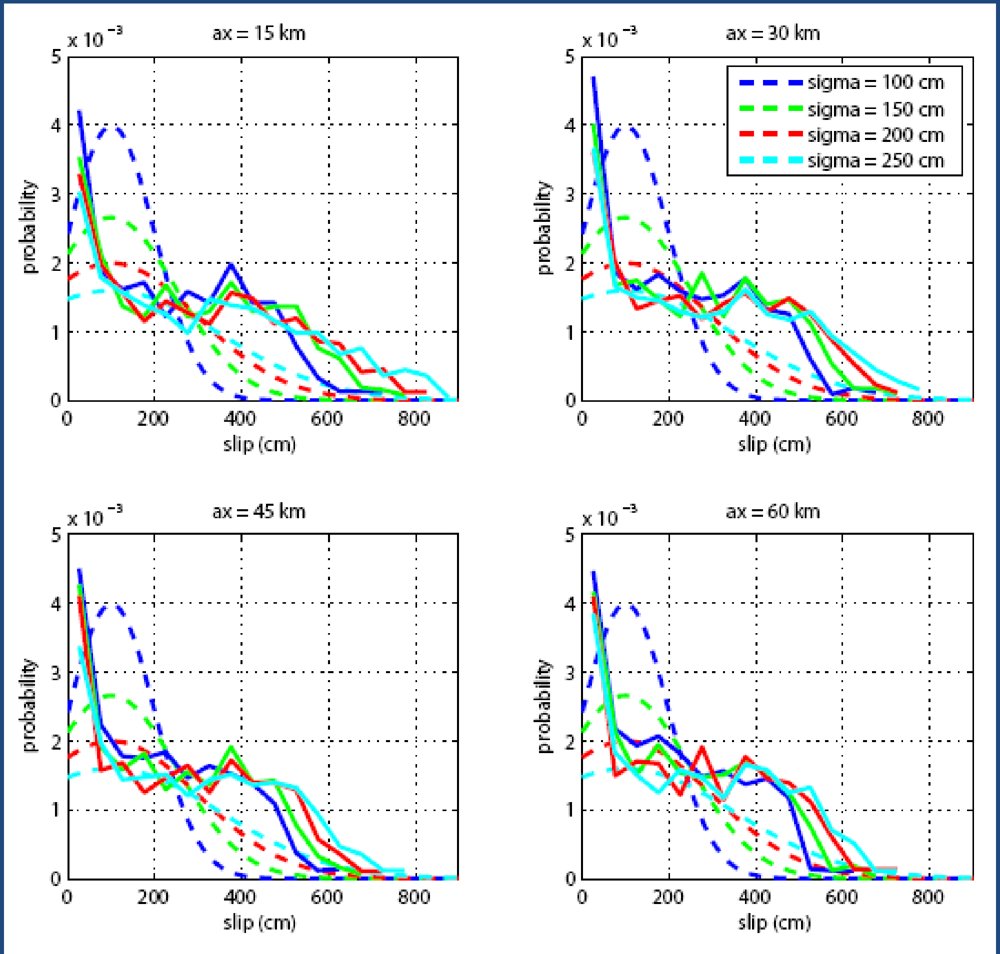
1-point statistics



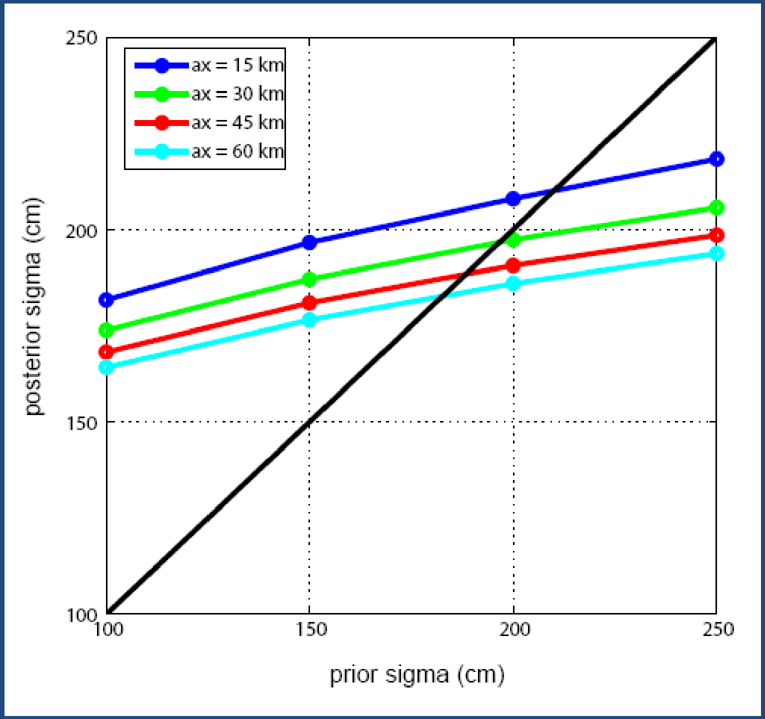
2-point statistics



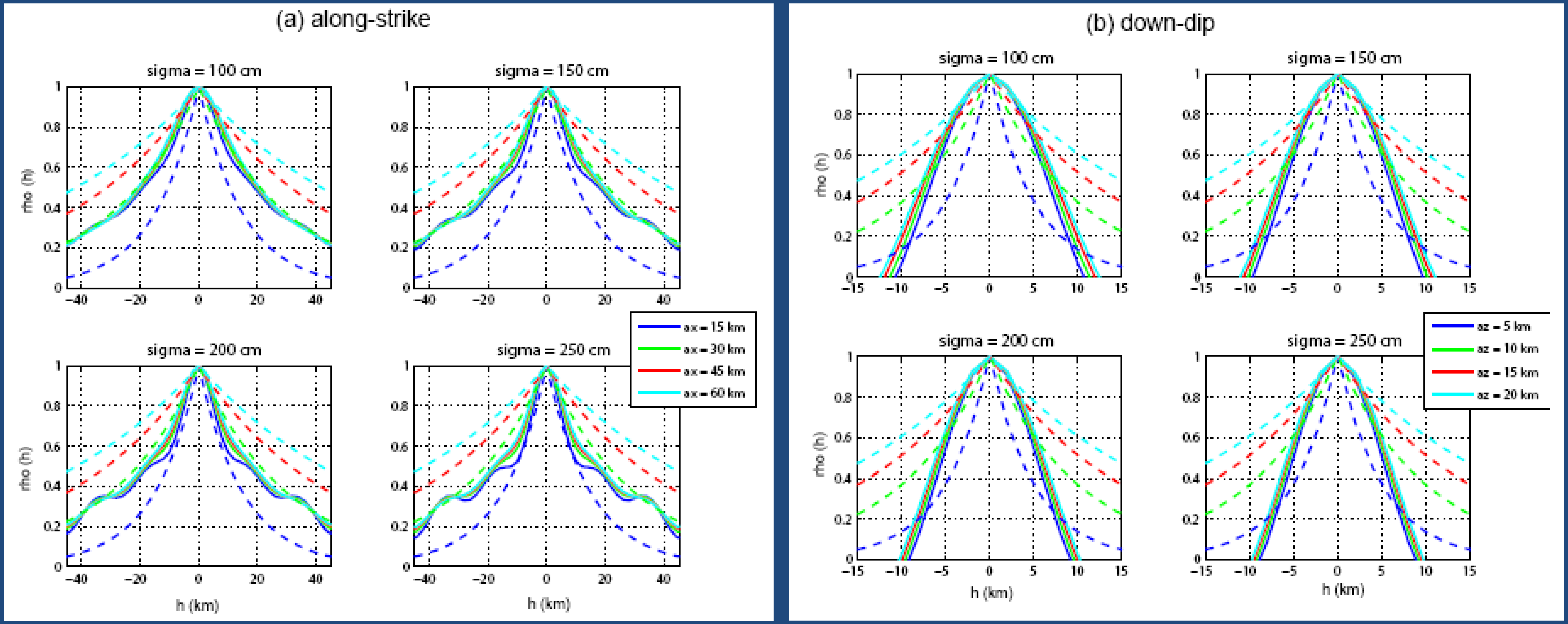
1-Point Statistics



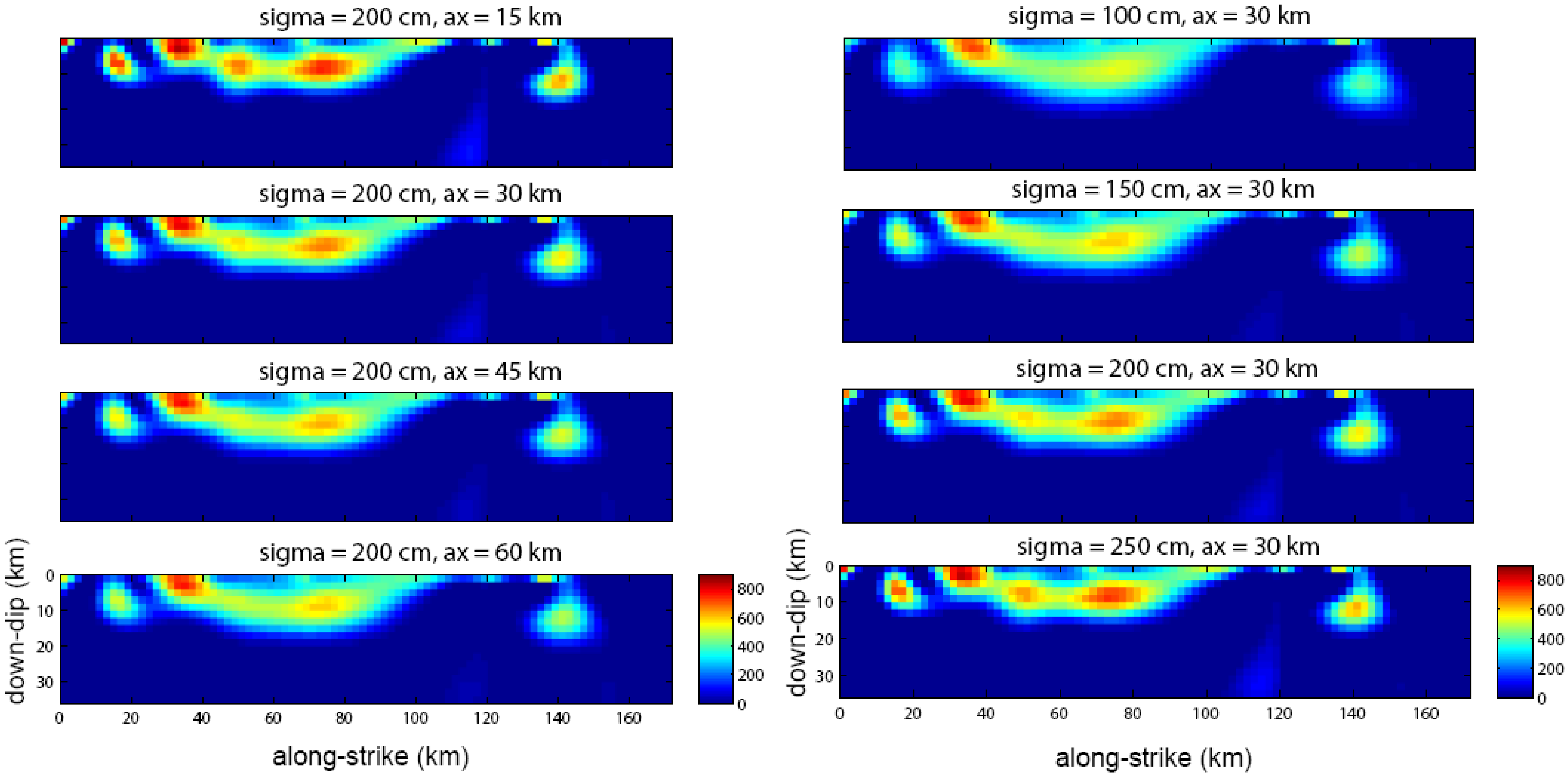
Dashed: prior
Solid: posterior



Spatial Coherence (2-point stats)
(dashed: prior, solid: posterior)



Estimated Slip Distributions



Inferring dynamic parameters
from kinematic rupture models

$\Sigma =$

slip vs. slip	slip vs. V_r	slip vs. V_{max}	slip vs. rT
V_r vs. slip	V_r vs. V_r	V_r vs. V_{max}	V_r vs. rT
V_{max} vs. slip	V_{max} vs. V_r	V_{max} vs. V_{max}	V_{max} vs. rT
rT vs. slip	rT vs. V_r	rT vs. V_{max}	rT vs. rT

(Song and Somerville, 2010)

$\mathbf{u}(\underline{\mathbf{x}}, t) \Rightarrow \mathbf{T}(\underline{\mathbf{x}}, t) \Rightarrow$ sd, SE, d_c

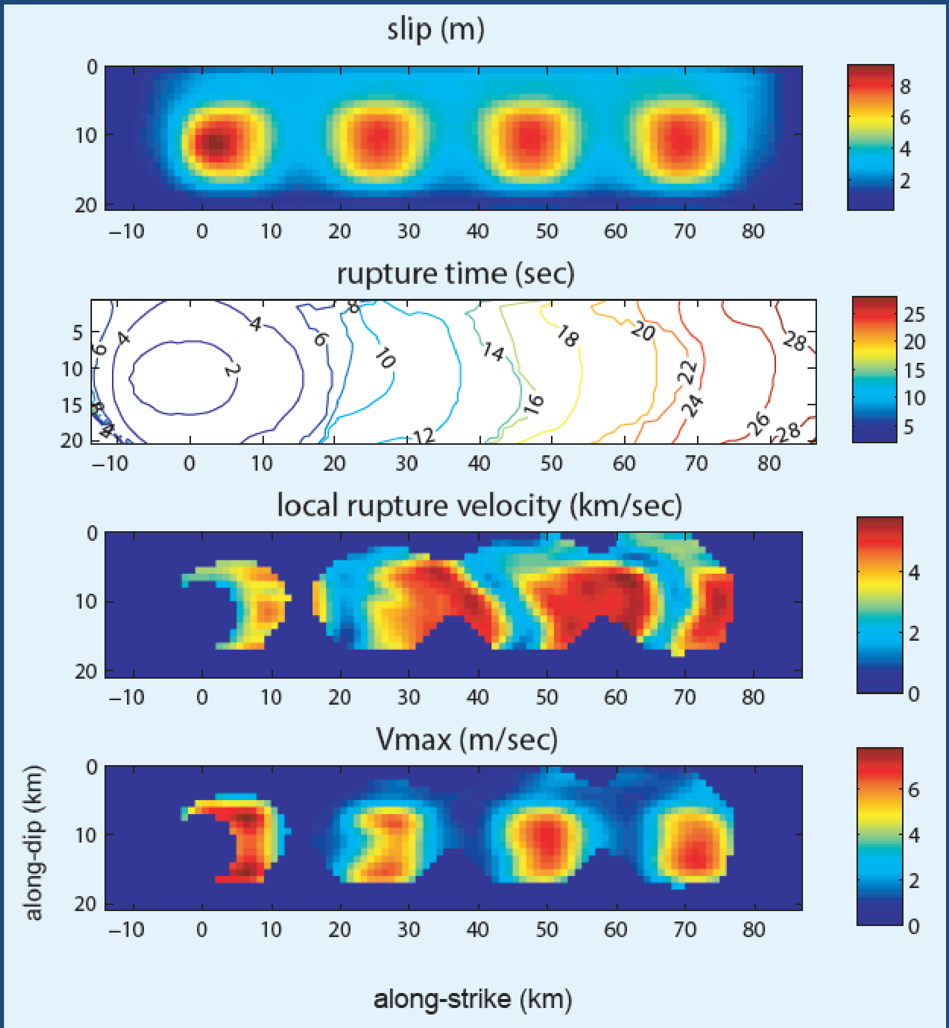
(Ide and Takeo, 1997; Tinti et al., 2005)

$\Sigma =$

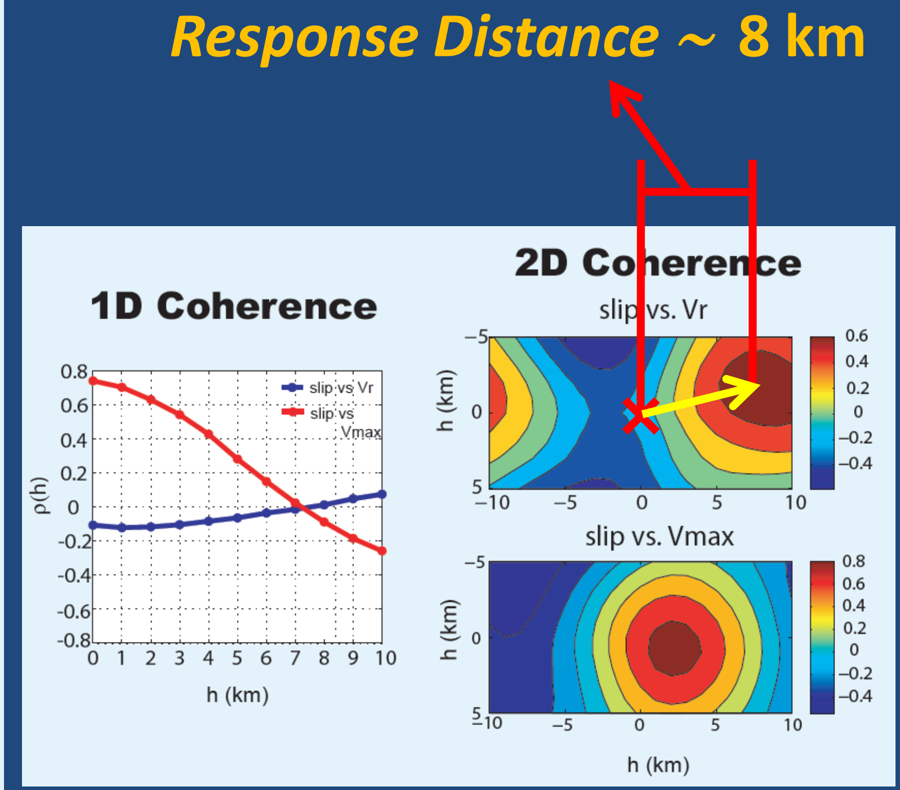
sd vs. sd	**sd vs. G_c**
G_c vs. sd	**G_c vs. G_c**

Spatial coherence

from dynamic rupture models



(Dalguer et al., 2008)



(Song and Somerville, 2010)

Summary (Part I)

- We should pay more attention to 1-point statistics and its non-stationarity.
- We may better constrain 1-point and 2-point statistics of source parameters by regularizing the model space with the same form of 1-point and 2-point statistics with Bayesian inversion.

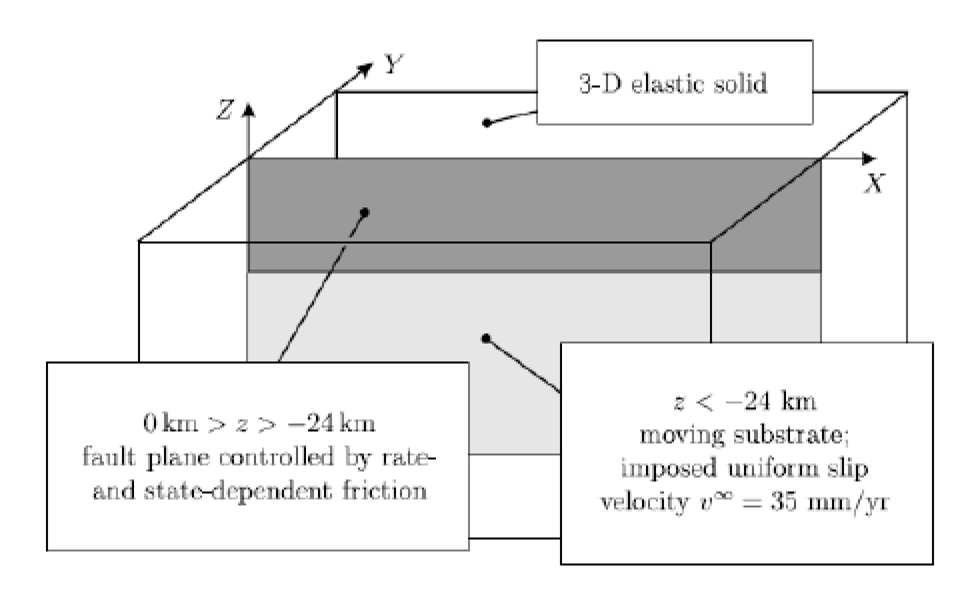
Outline

- **Spatial heterogeneity with 1-point and 2-point statistics**
  - **Do we pay enough attention to 1-point statistics?**
  - **How to constrain them from data?**
- **Two-step approach**
  - **Quasi-dynamic multi-cycle simulation with RS friction law**
  - **Full-dynamic single-event simulation with SW friction law**
  - Contributors: G. Hillers, A. Pitarka, P.M. Mai, L.A. Dalguer, P. Somerville
  - Supported by Japan Nuclear Energy Safety (JNES) through Geo Research Institute (GRI)

Quasi-dynamic multi-cycle simulation

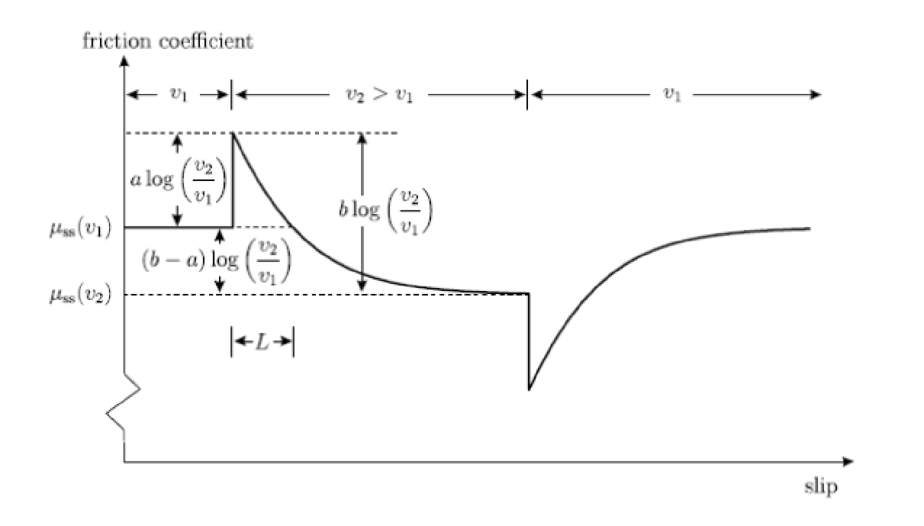
?

a, b, L => sd, SE, d_c => slip, V_r, V_{max} => PGV, PGA, SA



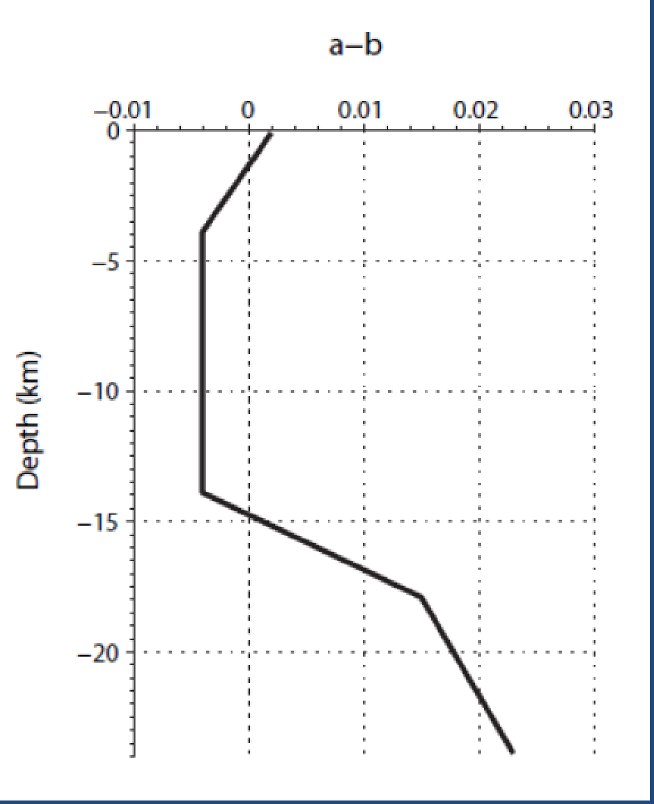
(model setup from Hillers et al., 2006)

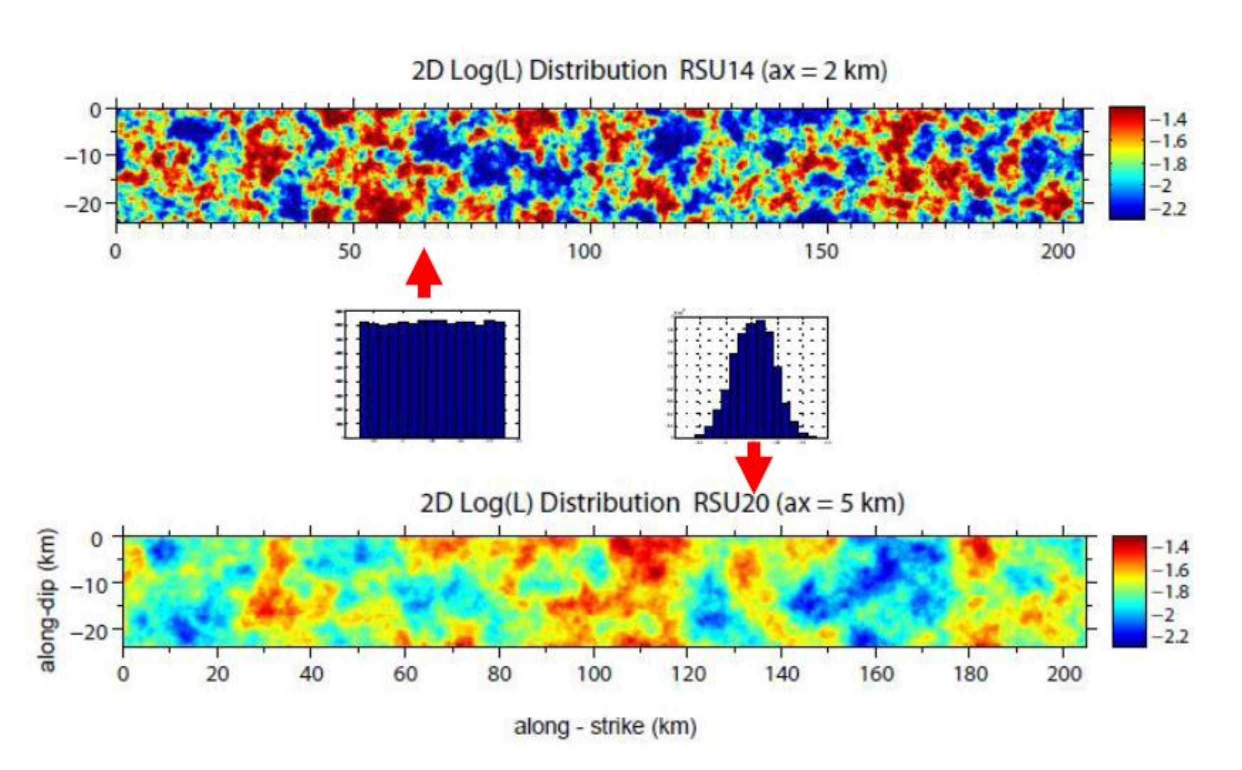
Rate and state-dependent friction law



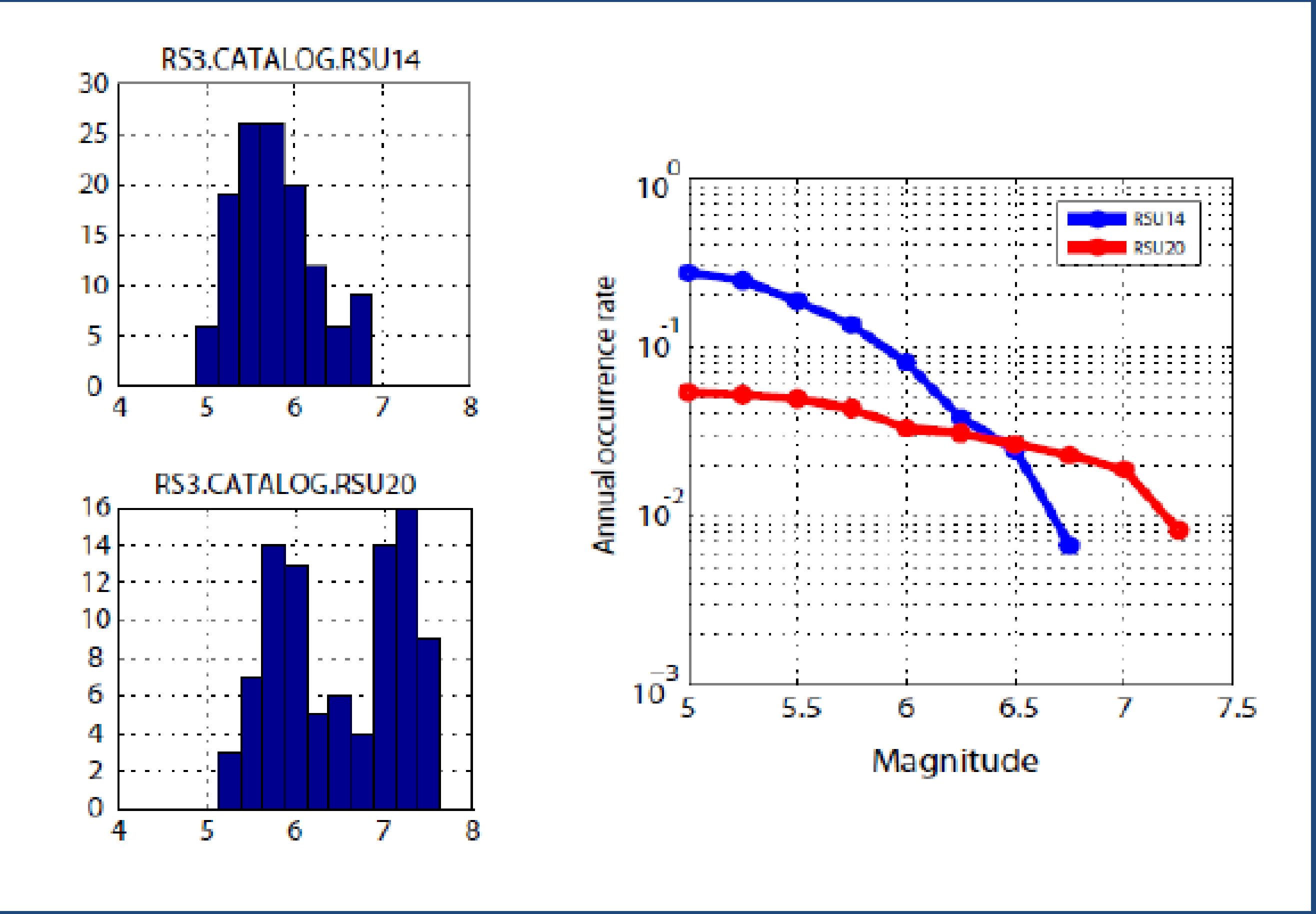
(Dieterich, 1979; Ruina, 1983)

Input parameters (*a, b, L*)
in the multi-cycle simulation

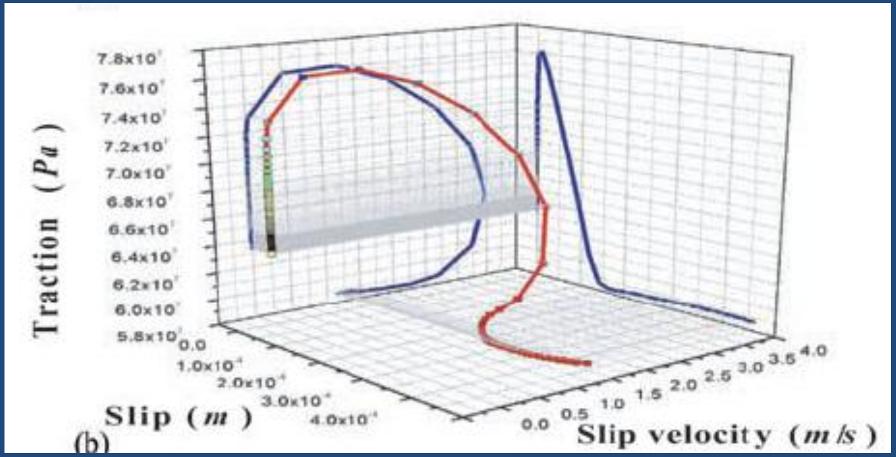


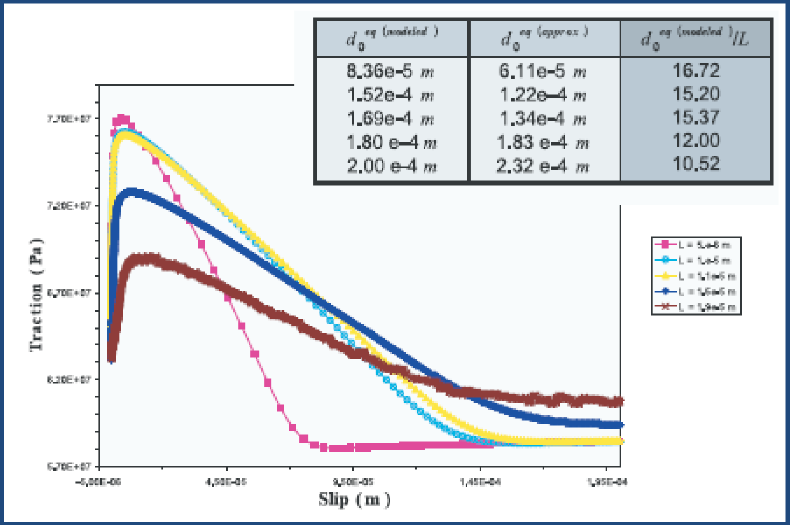


Gutenberg-Richter vs. Characteristics?



L to d_c conversion





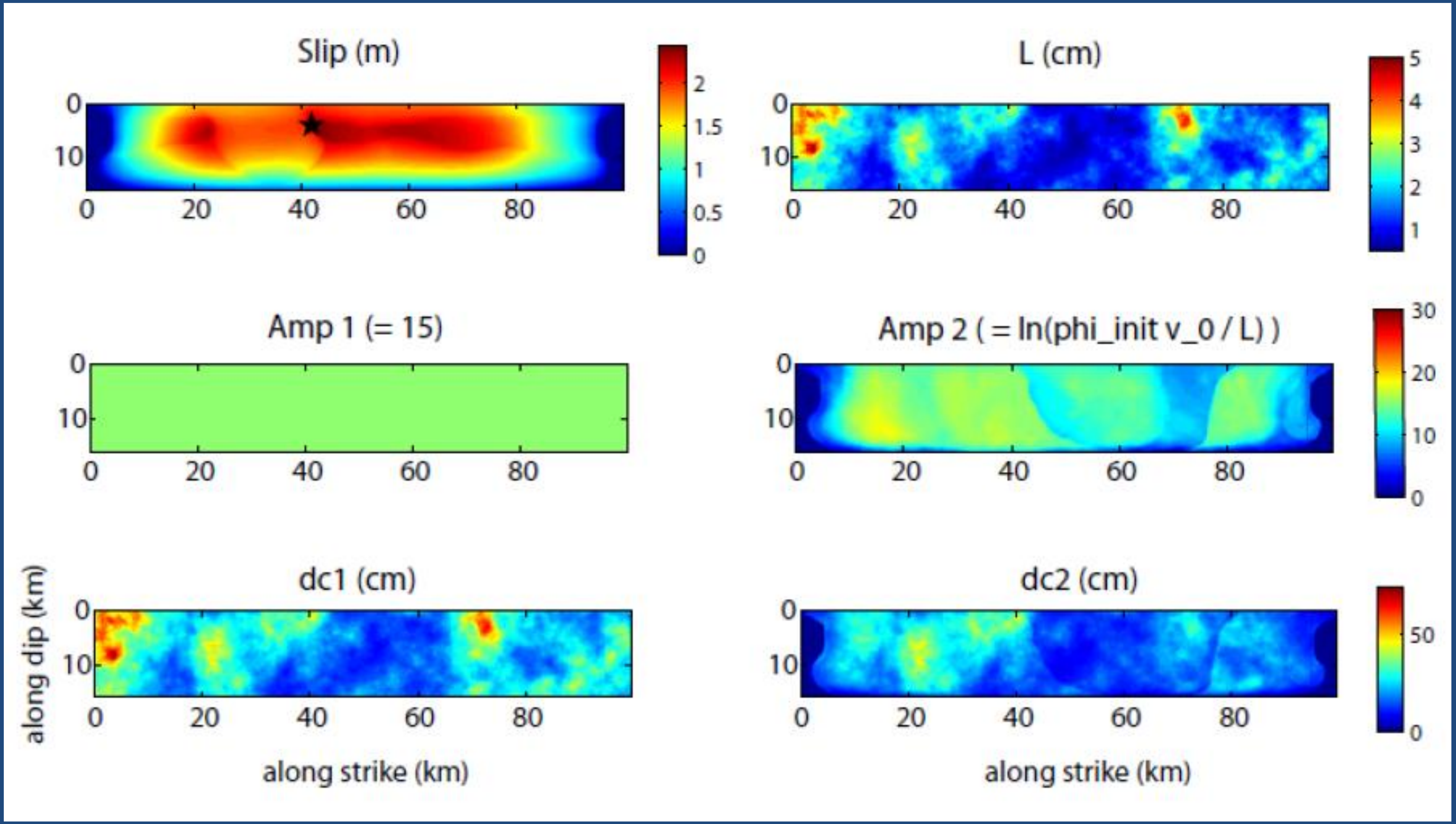
$d_0^{eq\ (modeled)}$	$d_0^{eq\ (approx)}$	$d_0^{eq\ (modeled)}/L$
8.36e-5 m	6.11e-5 m	16.72
1.52e-4 m	1.22e-4 m	15.20
1.69e-4 m	1.34e-4 m	15.37
1.80 e-4 m	1.83 e-4 m	12.00
2.00 e-4 m	2.32 e-4 m	10.52

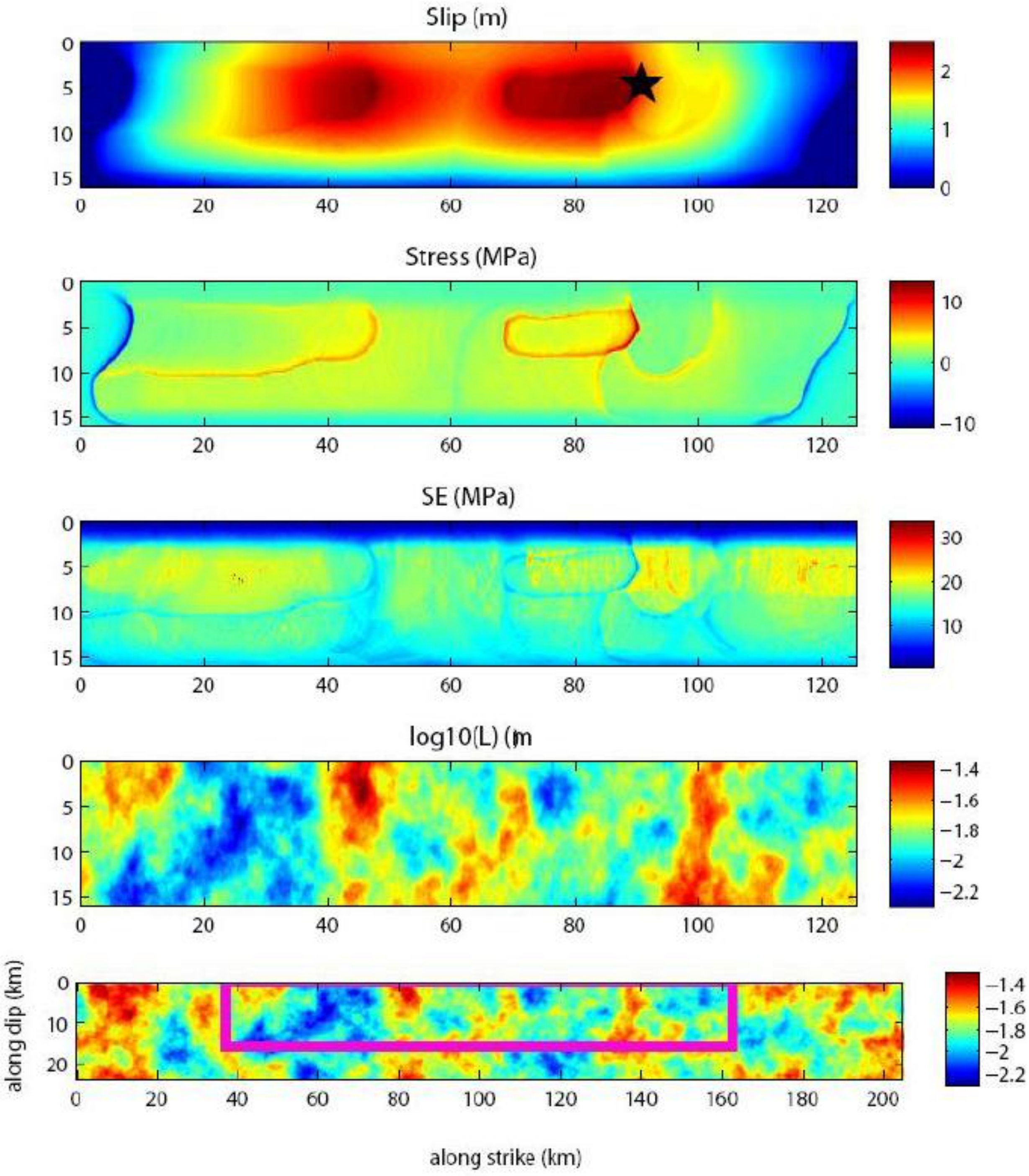
$$d_{c1} \cong L * 15$$

$$d_{c2} \cong L * \ln\left(\frac{\theta_{init}\ v_0}{L}\right)$$

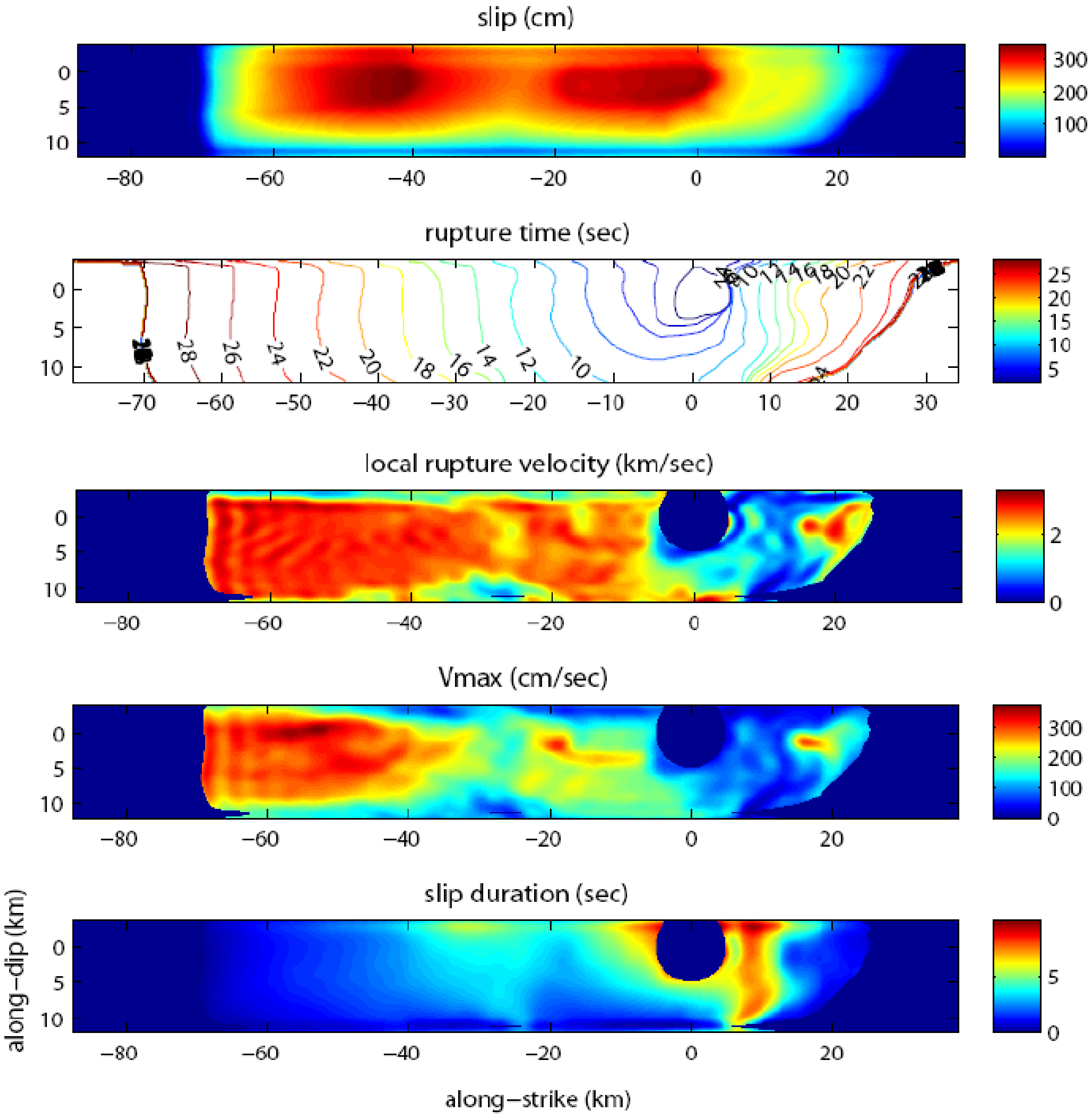
(Cocco and Bizzarri, 2002; Bizzarri and Cocco, 2003)

L to d_c conversion



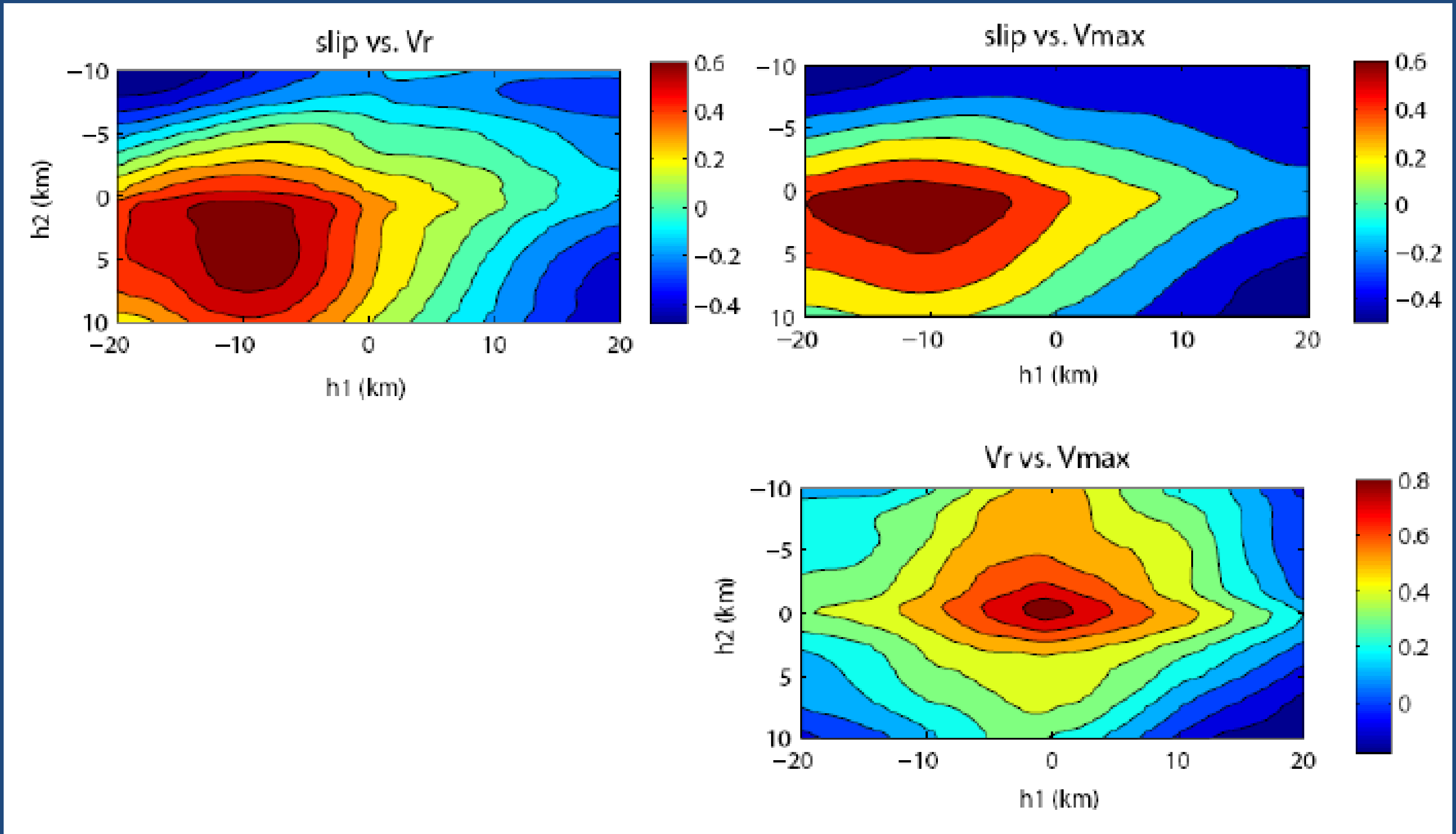


Slip (m)
Stress (MPa)
SE (MPa)
log10(L) (m
along dip (km)
along strike (km)



slip (cm)
rupture time (sec)
local rupture velocity (km/sec)
Vmax (cm/sec)
slip duration (sec)
along−dip (km)
along−strike (km)

Spatial cross-coherence



Summary (Part II)

- Physically self-consistent dynamic input parameters inferred from multi-cycle simulation
- Generate a series of events occurring on a single fault system through a cycle of the fault evolution
- **Applicability to "100-runs"**
  - Currently applied to strike-slip
  - Magnitude range