
Heterogeneous Stress
on Faults
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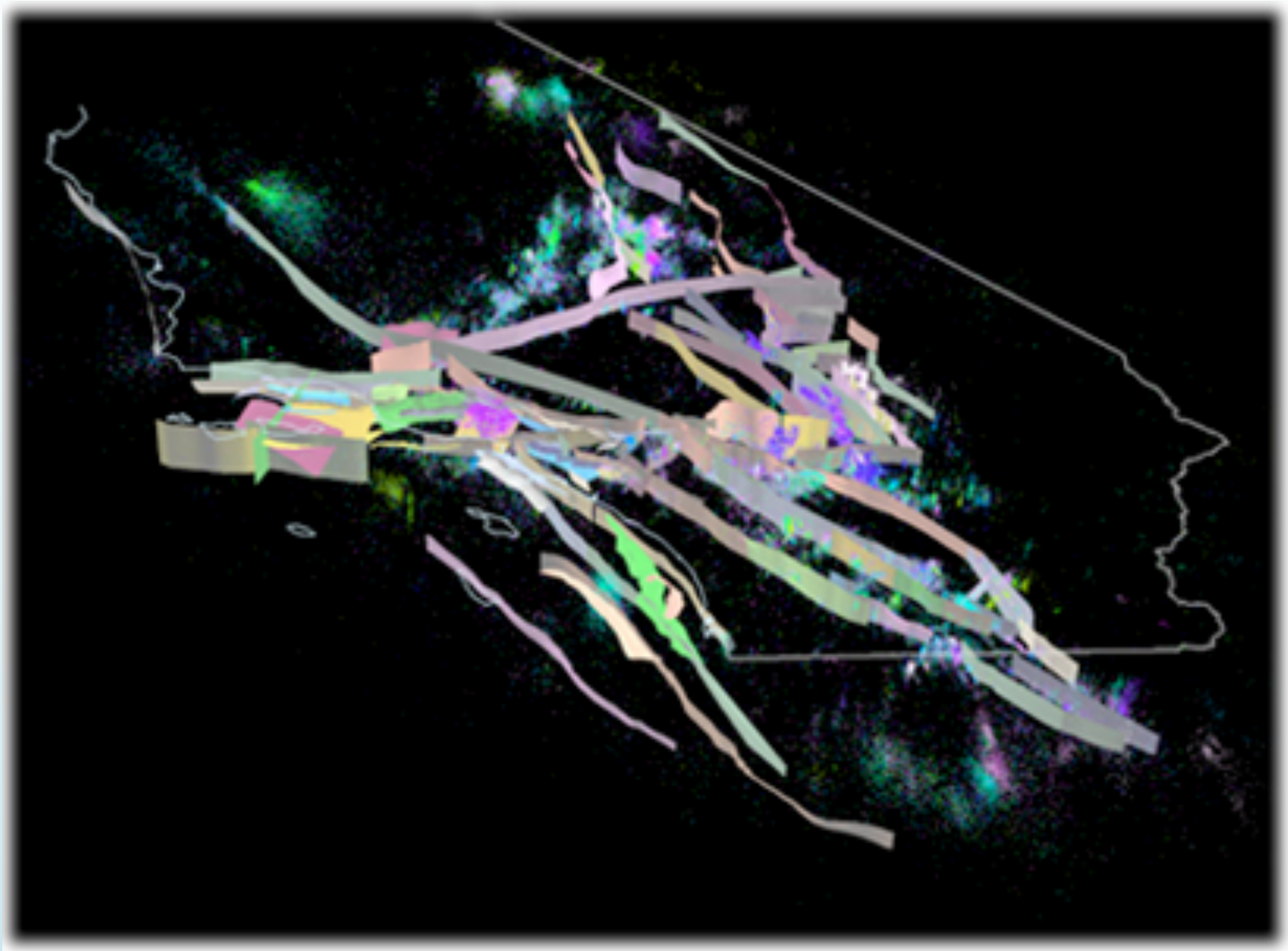
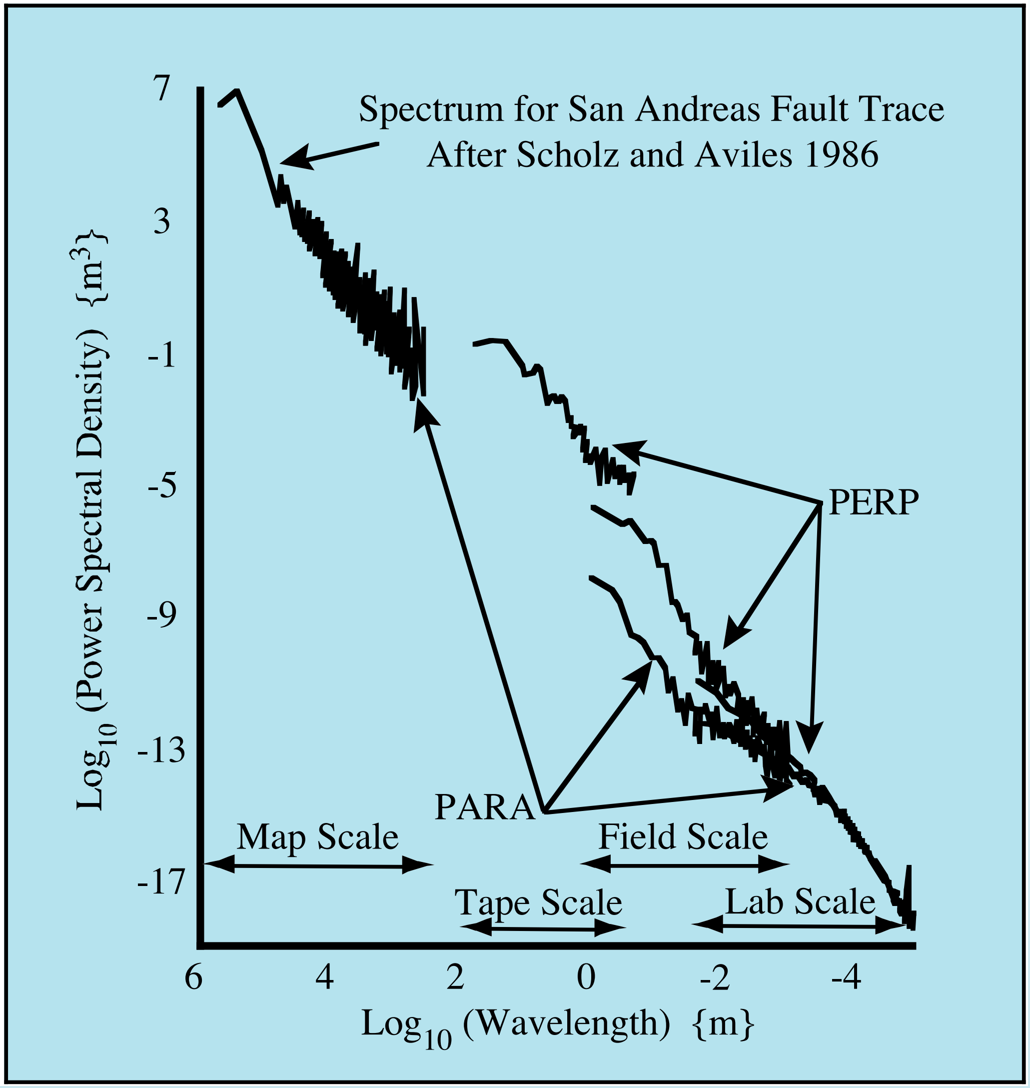
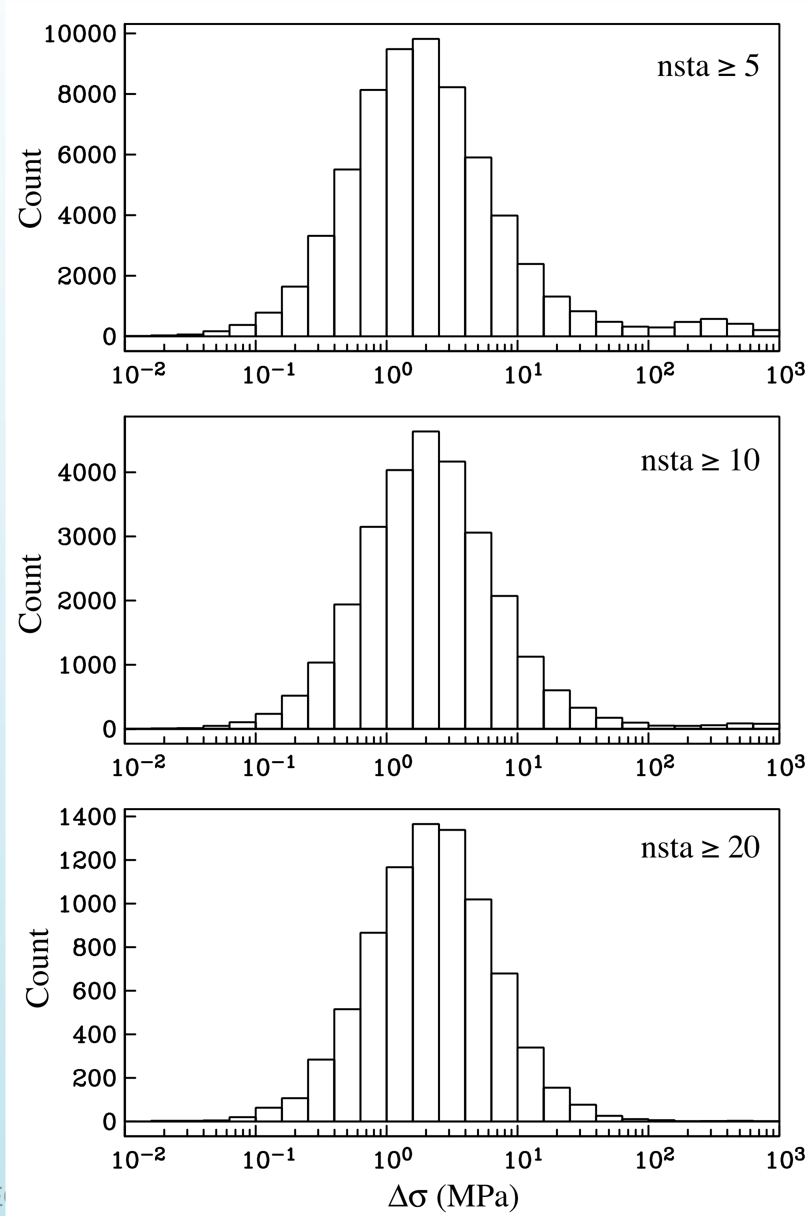
SCEC CFM

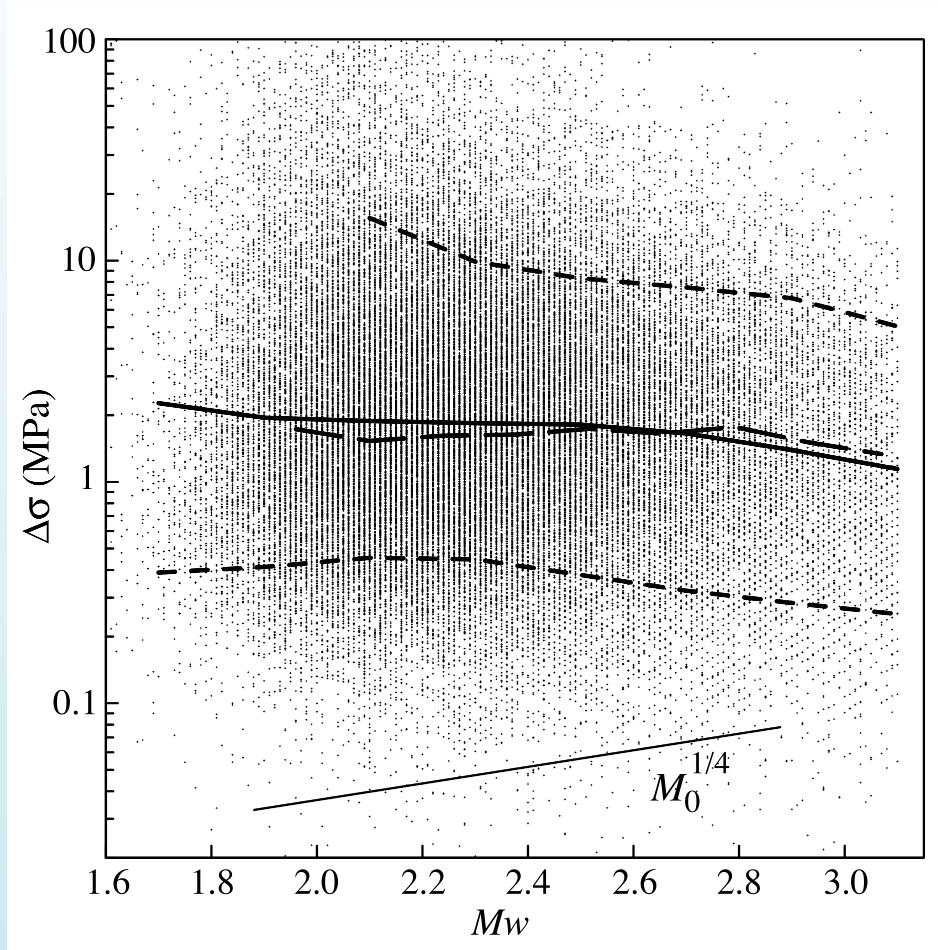


Roughness at All Scales



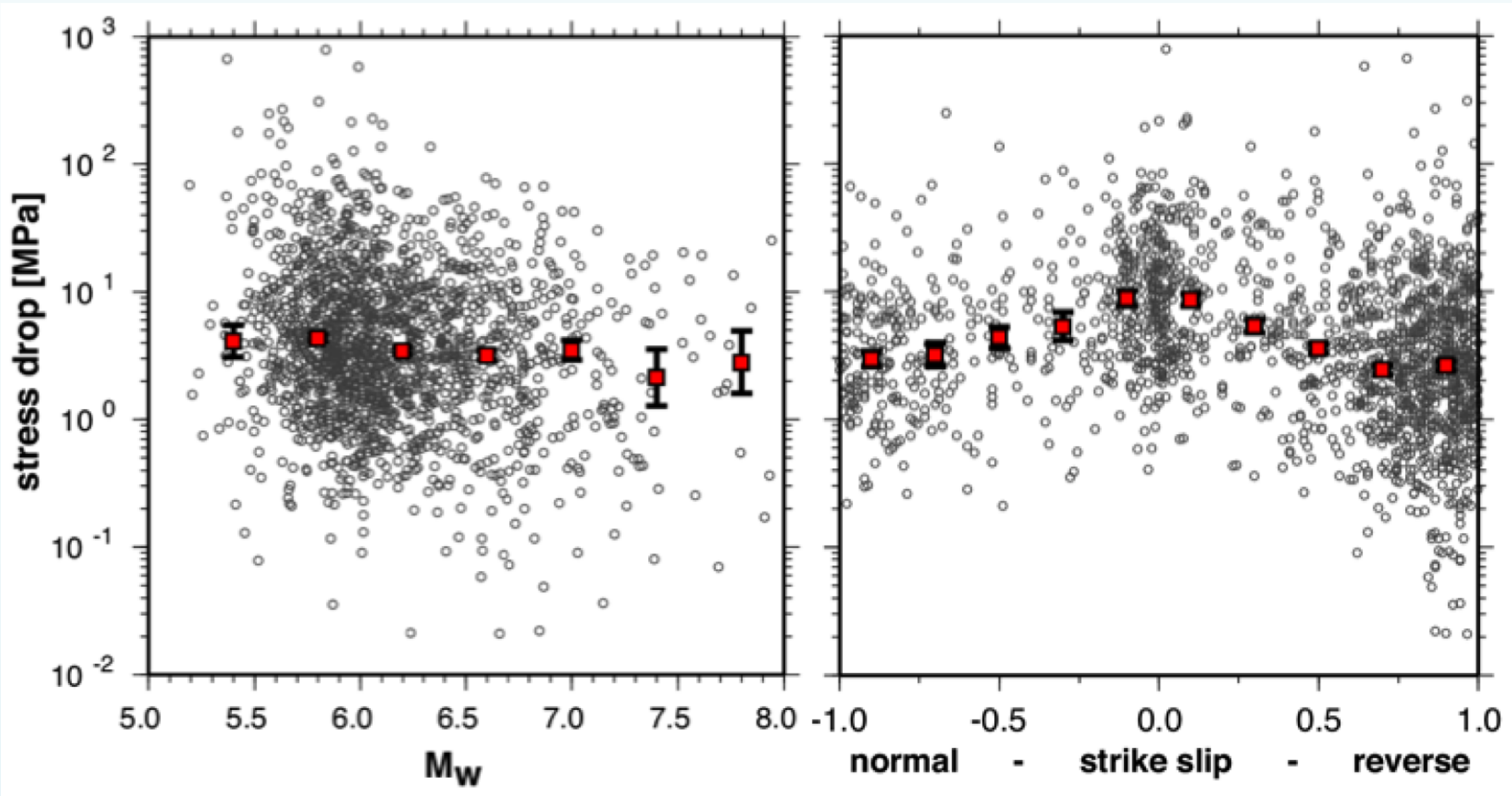
Southern California Stress Drops





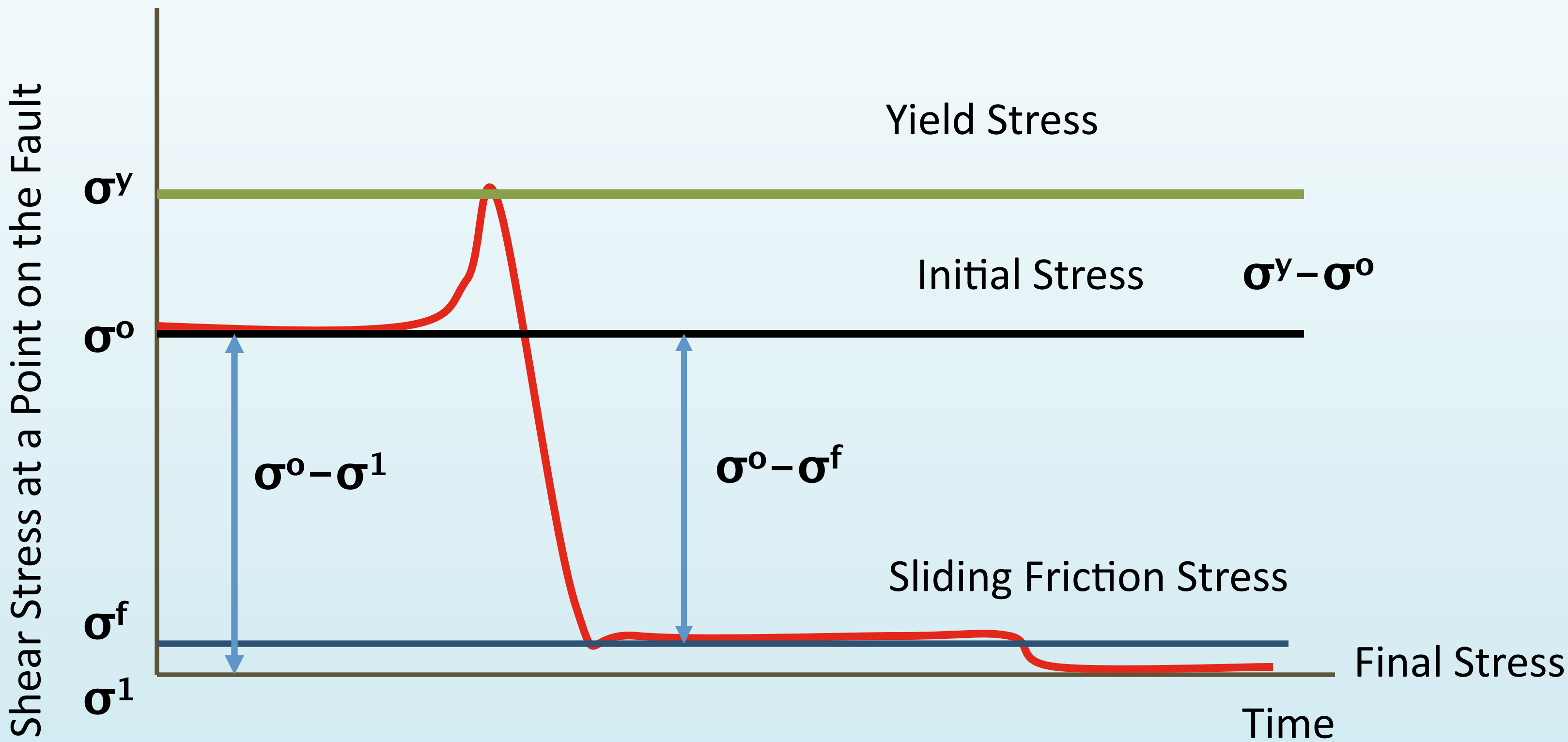
Shearer, Prieto, Hauksson, 2006

Global Stress Drops



Allmann and Shearer, 2009, JGR

Stress at a Point on the Fault



S=$\sigma^y - \sigma^o$ / $\sigma^o - \sigma^f$ =Strength Excess/Stress Drop

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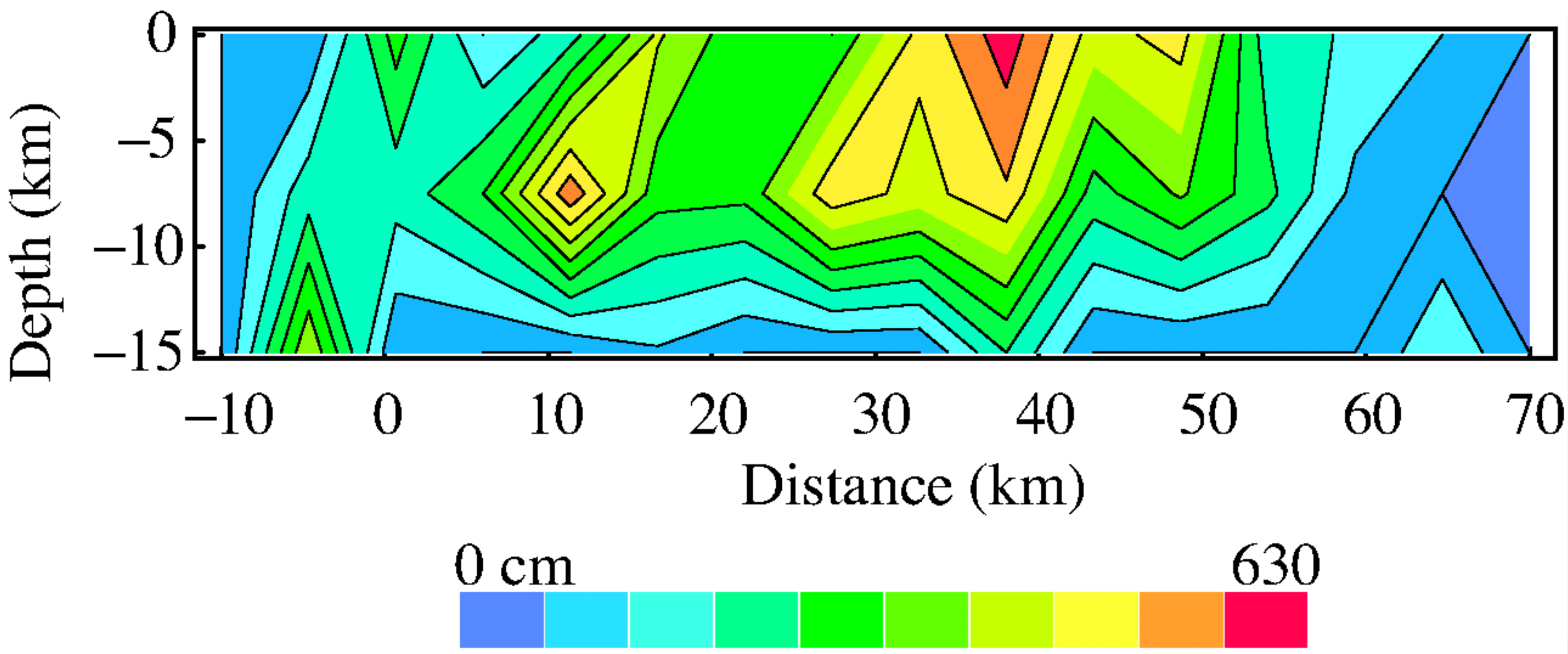
Random model of slip heterogeneity

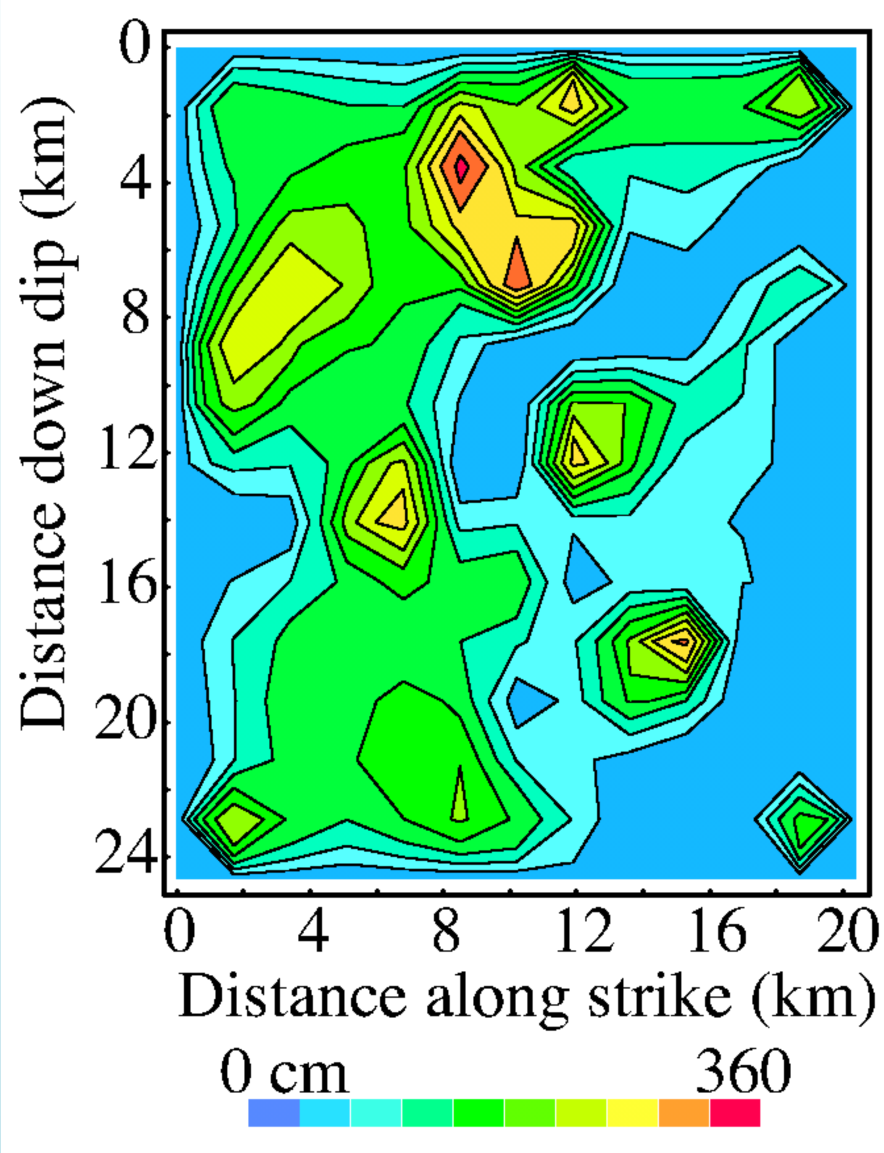
- One point-statistics: probability law governing the distribution of the random variables.
- Two-point statistics: correlation function or power spectrum.
- Three-point statistics and so on.

Gusev, Tectonophysics, 1992

Landers: Slip

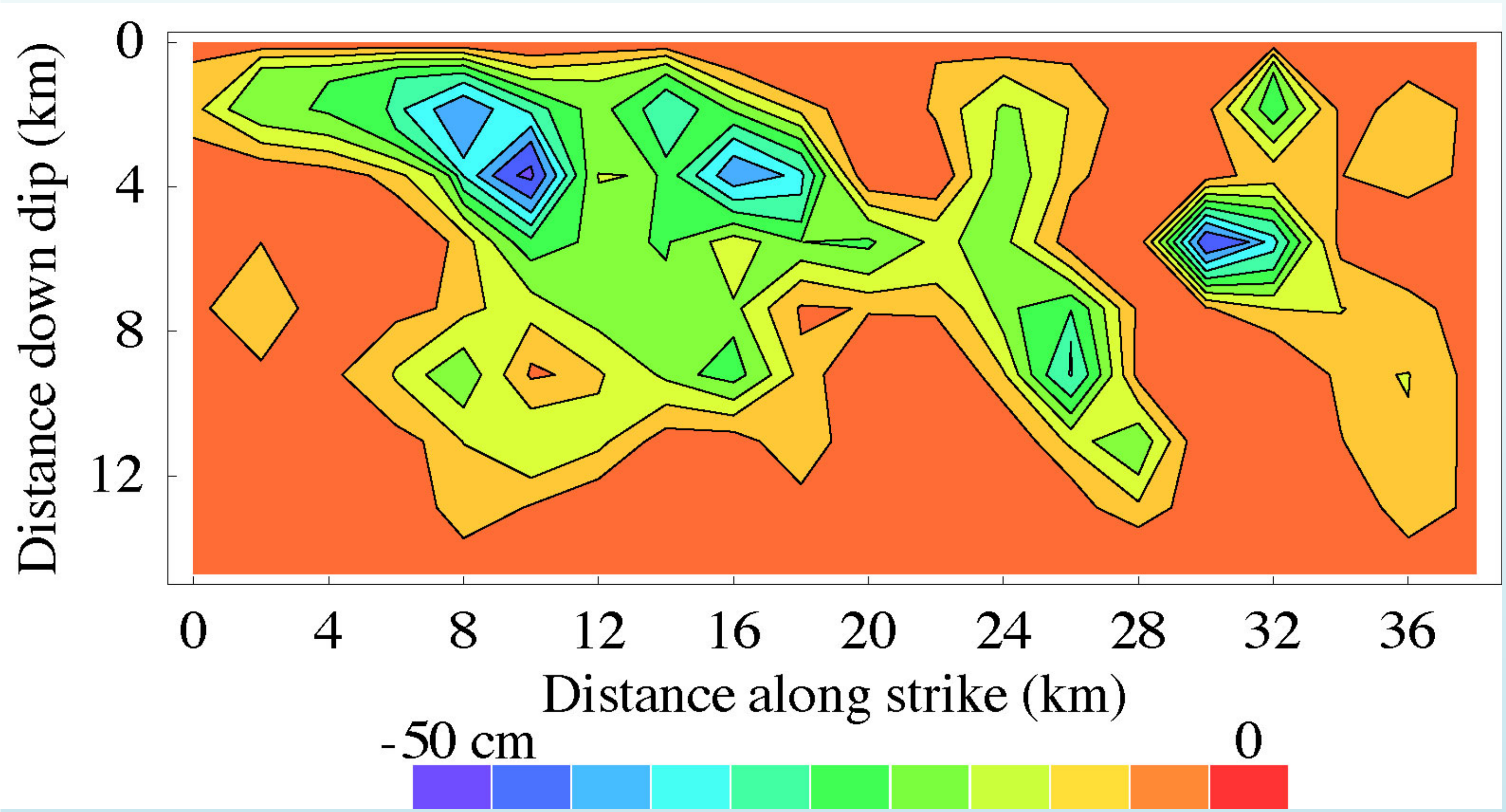


Northridge: Slip

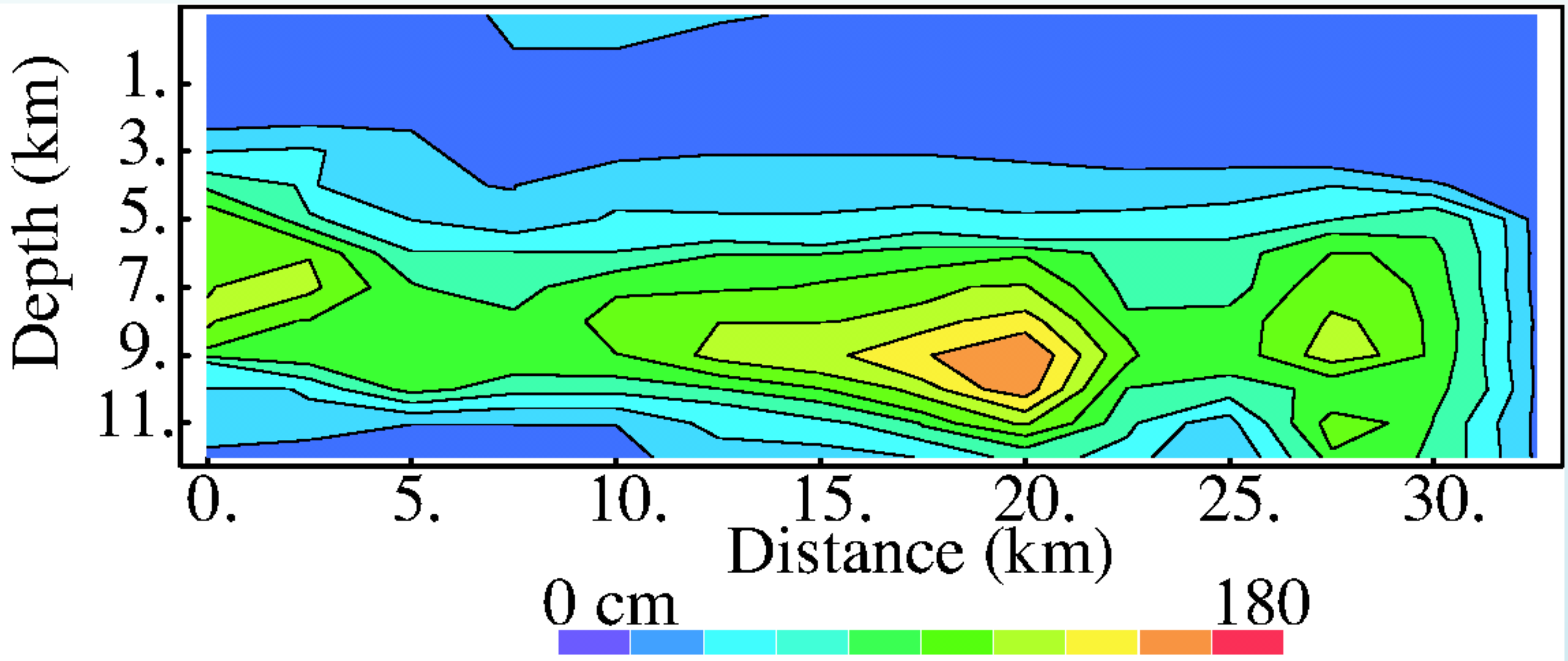


Liu and Archuleta, 2000

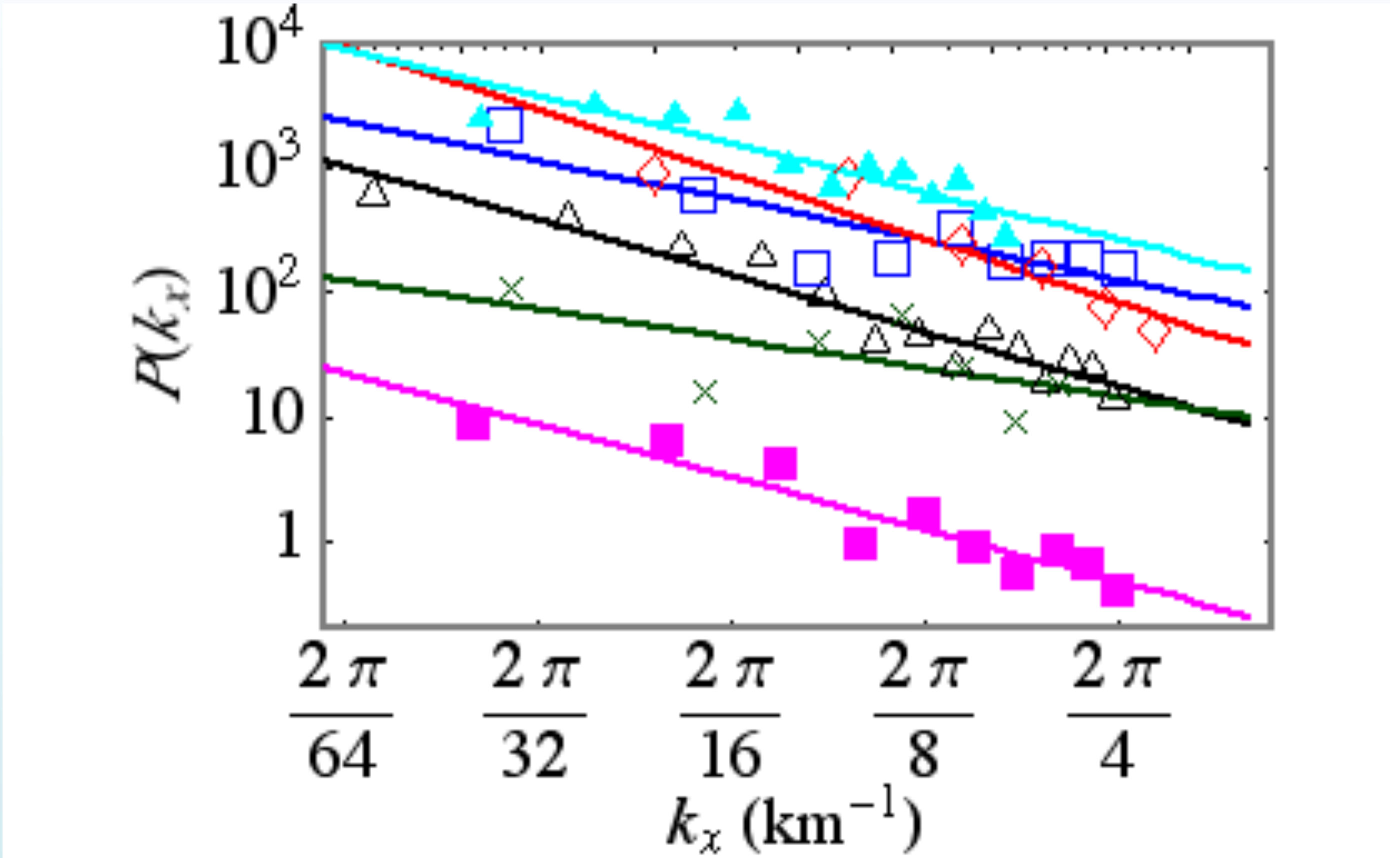
2004 Parkfield Earthquake: Strike slip



Custodio et al., Seism. Res. Lett., 2005

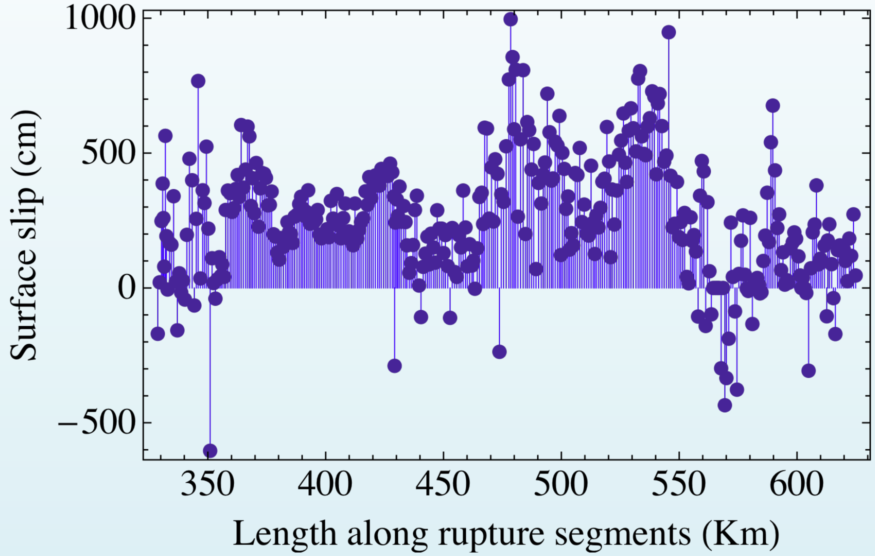

Depth (km)
1.
3.
5.
7.
9.
11.
0.
5.
10.
15.
20.
25.
30.
Distance (km)
0 cm
180

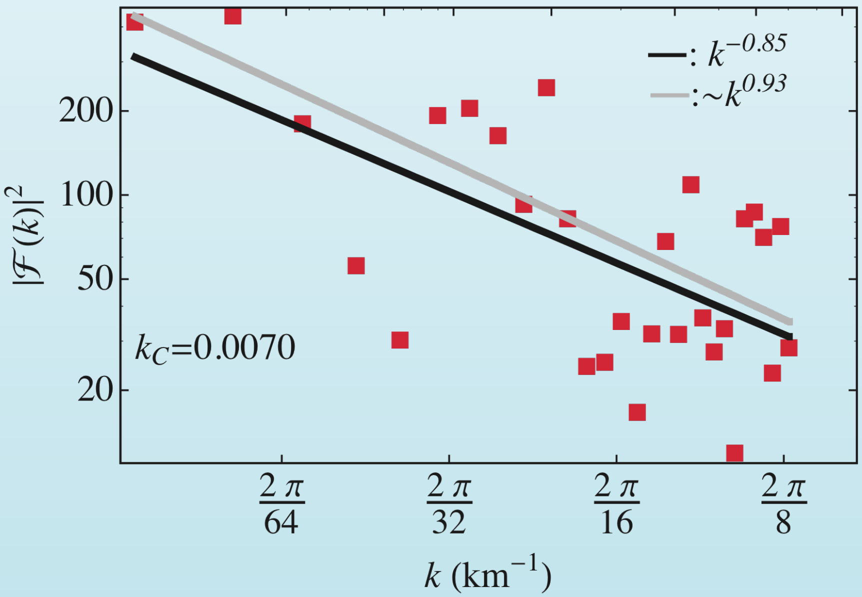
Spectrum: Strike slip component



1995 Hyogo-ken Nanbu earthquake (△), 1979 Imperial Valley earthquake (X), 1989 Loma Prieta earthquake (□), 1994 Northridge earthquake (◊), 1999 Chi Chi earthquake (▲), and the 2004 Parkfield earthquake (■).

1D Spectrum: Slip profile recorded at the surface: 2001 Kokoxili earthquake





Klinger *et al.*, . *Earth and Planetary Science Letters*, 2006

Gray line: von Karman function
Black line: Power law

Interpolation

Somerville et al., Seism. Res. Lett., 1999, and Mai and Beroza, JGR, 2002, obtained values of scaling exponent for the power spectrum that range between 3 and 4. In both studies, the slip data have been interpolated.

$$P_D(k) \propto k^{-\eta_1} k^{-\eta_2} \propto k^{-\eta_1 - \eta_2}$$

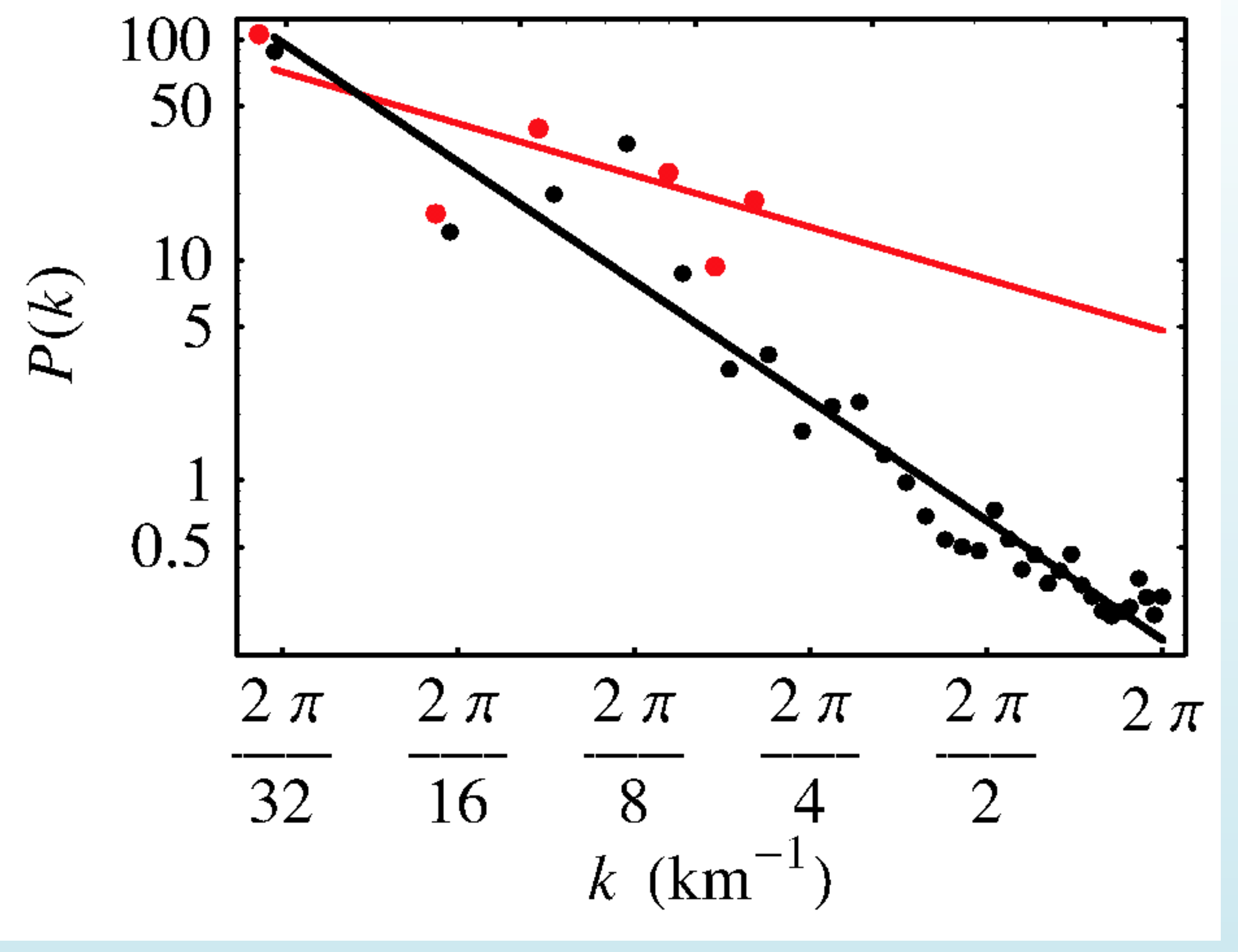
P_D: 2D power spectrum density.
η_1: Intrinsic scaling exponent.
η_2: Scaling exponent induced by interpolating the data.

$$\eta_1 + \eta_2 \sim 4$$

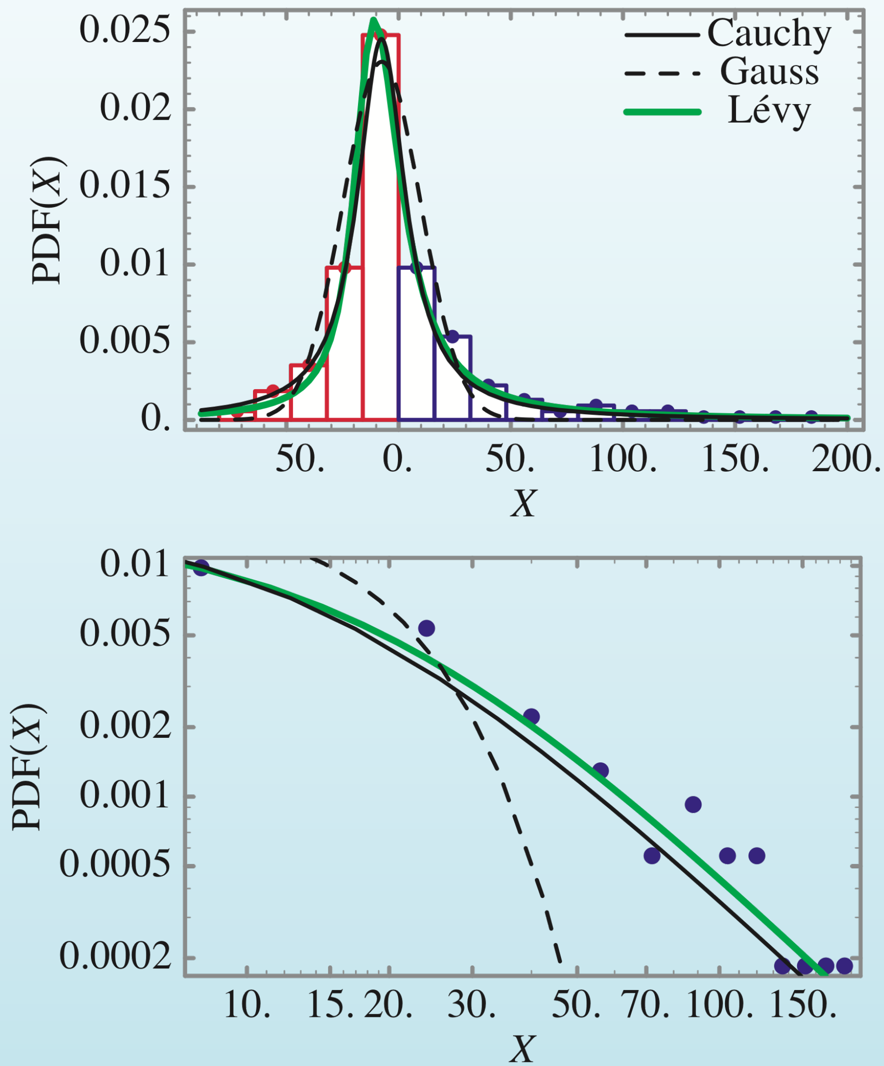
$$\eta_1 \sim 4 - \eta_2 < 4$$

Lavallée et al., 2006, GJI

Imperial Valley: Spectrum



Probability law for the strike slip: 1999 Chi Chi earthquake



- *X*: Independent random variable (white noise)
- Slip is proportional to a linear combination of *X*
- *X* (and thus Slip) are distributed according to a non Gaussian law
- Independent confirmation of this result for the same earthquake by Raghu Kanth and Ivengar, *JGR*, 2008.
- Similar conclusions is reached for other earthquakes (Lavallée *et al*., *GJI*, 2006.

Question: Is there any evidence that shows that a Gaussian is the appropriate PDF?

Consequences of formulating the PDF for slip being distributed according to a Lévy law:

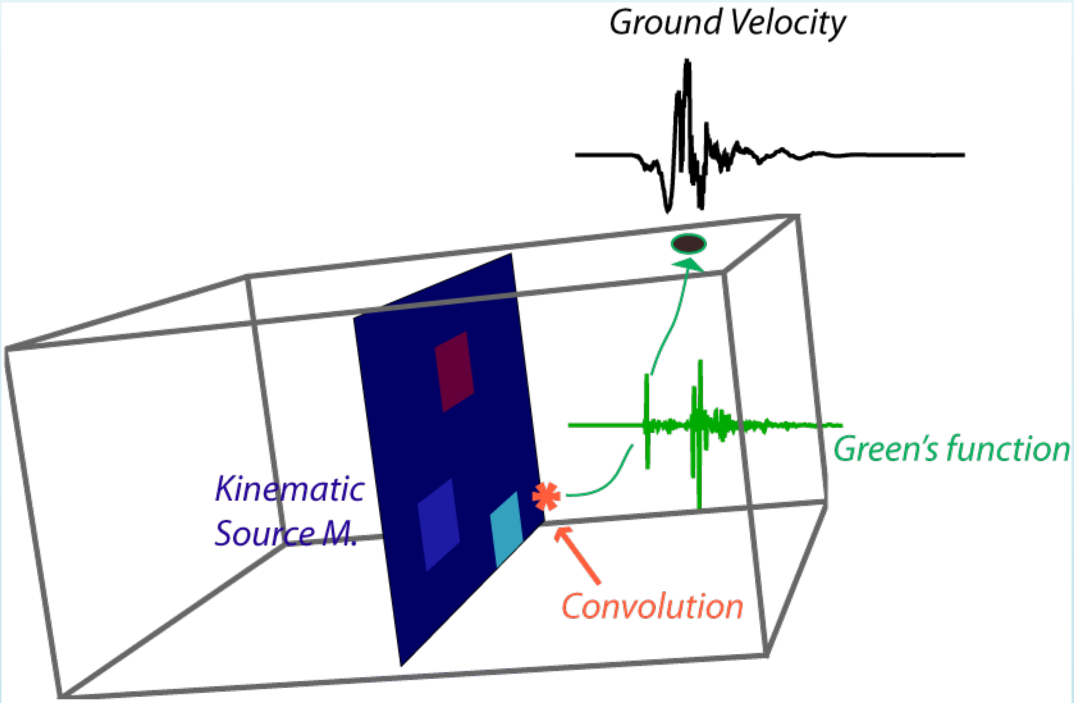
1- Representation theorem

2- Lévy law independent of the length scales at which the slip is observed or generated. (Parameters of the Lévy law may or may not be length scale dependent.)

Representation Theorem

$$u_n(\boldsymbol{x},t) = \int_{-\infty}^{\infty} d\tau \iint_{\Sigma} [u_i(\boldsymbol{\xi},\tau]\, c_{ijpq} \nu_j \partial G_{np}(\boldsymbol{x},t-\tau;\boldsymbol{\xi},0) / \partial \xi_q \; d\Sigma$$

Convolve slip rate function with Green's functions of elastic medium and sum over whole fault to get ground motion at free surface.



Aki and Richards, 1980

Representation Theorem

$$\boldsymbol{u}(t) = \sum_{i=1}^{N} \mu_i a_i D_i \dot{s}(t - t_{ri}, \tau_i) * [\cos(\lambda_i)\boldsymbol{g}_i^1(t) + \sin(\lambda_i)\boldsymbol{g}_i^2(t)]$$

N is the number of point sources; μ_i, a_i, λ_i are the rigidity, area, and rake angle for *i*th point source, respectively; the asterisk * indicates convolution; $\dot{s}$ is the slip rate time function; $\boldsymbol{g}_i^{1,2}(t)$ are the Green's functions for unit slip in the strike direction and down-dip direction, respectively; D_i, t_{ri}, τ_i and are the slip amplitude, rupture time, and rise time, respectively.

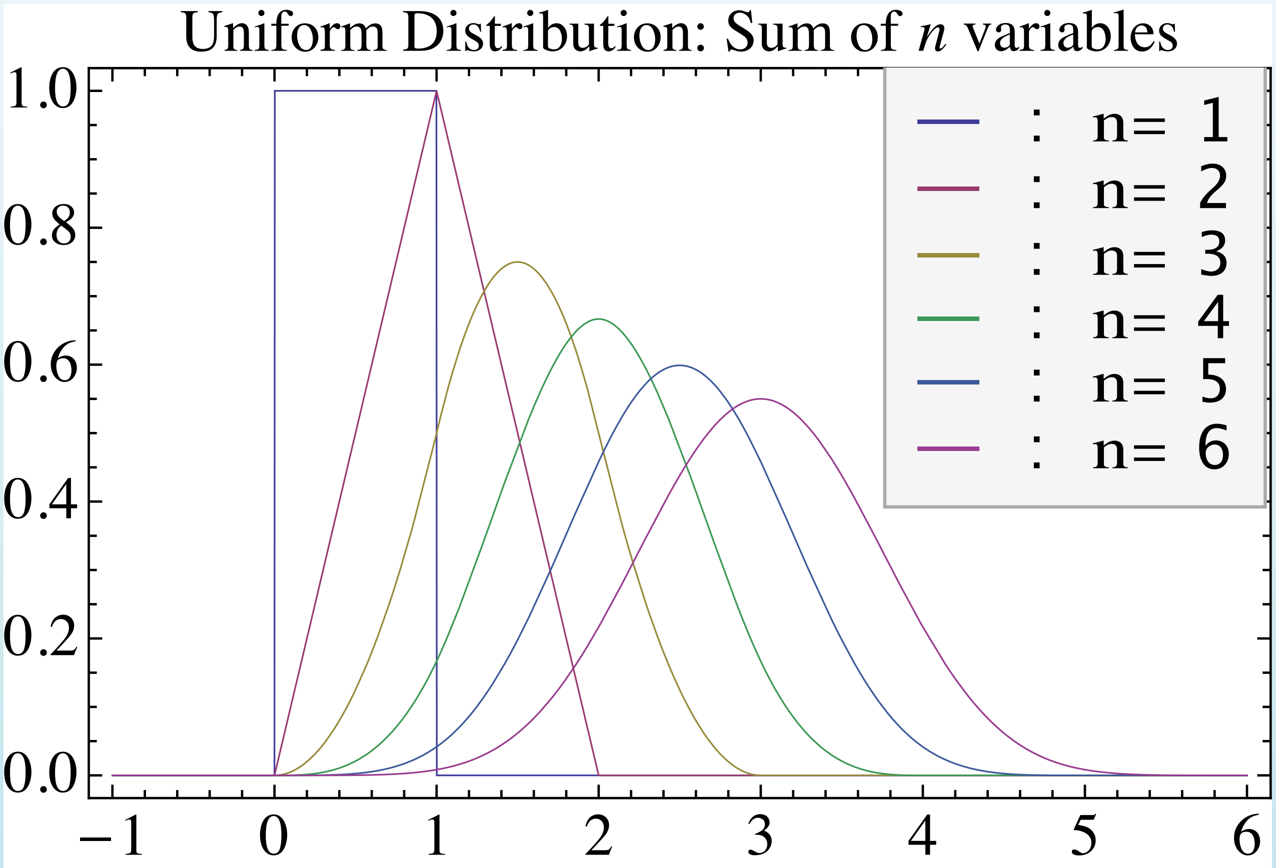
The amplitude of slip is taken to be a random variable, specified by a PDF.

The critical point is that the Representation Theorem represents a sum of independent random variables.

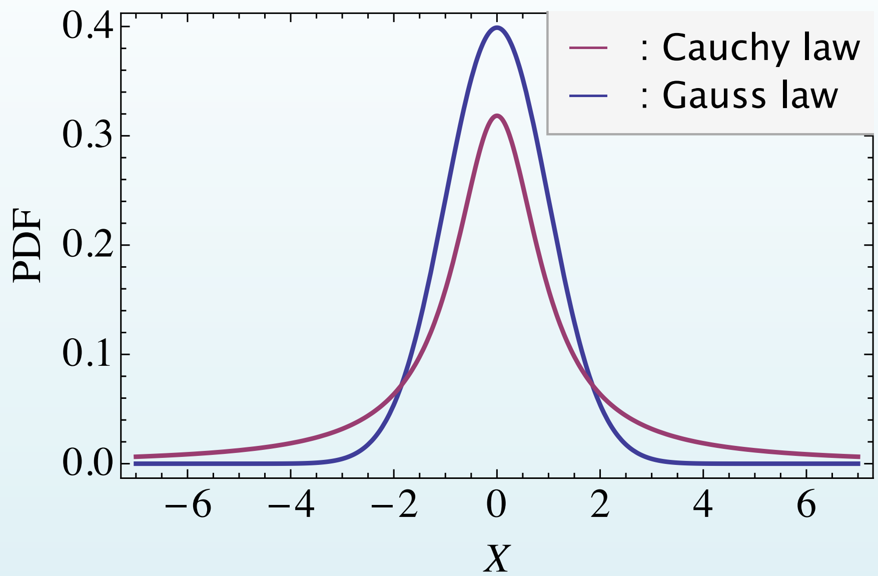
Aki and Richards, 1980

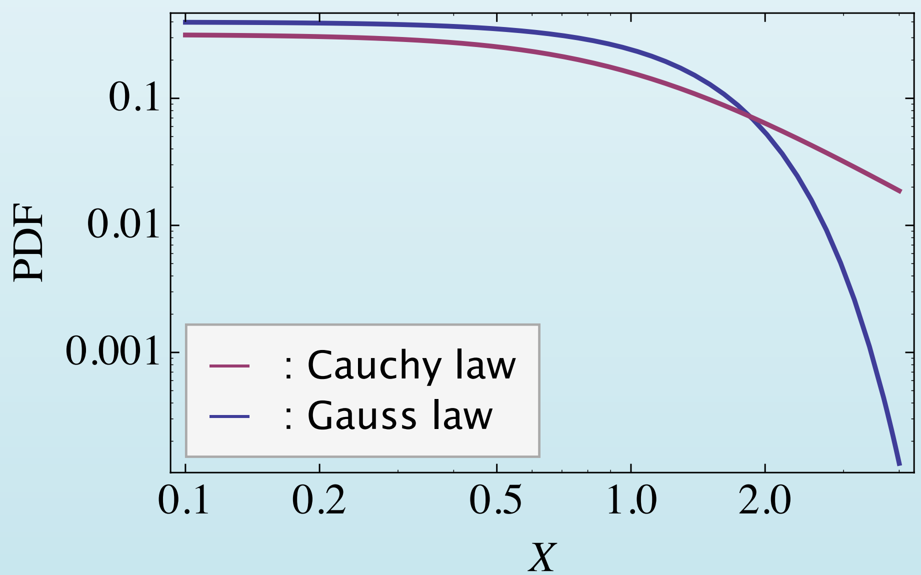
Sum of n Random Variables: All with a Uniform Distribution

Additional tests considering uniformly distributed random stress values indicated no significant differences in the model response compared to the case of Gaussian distributions.
--Ripperberger et al., JGR, 2007.



Cauchy and Gauss PDF





- α with $0< \alpha \leq 2$:
- The Lévy exponent or the characteristic exponent or the stable exponent.
- Controls the rate of fall off of the probability density function.
- Larger is the value of α, the less likely it is to find a random variable far away from the central location.
- Gauss law: $\alpha=2$ and $\beta=0$
- Cauchy law: $\alpha=1$ and $\beta=0$
- Half-normal law: $\alpha=1/2$ and $\beta=1$

Summary of Parameters for Different Laws:

Summary for the Dip-slip component

	Scaling Exponent	Gauss law		Cauchy law		Lévy law			
	ν	μ	σ	γ	μ	**α**	**β**	γ	μ
1989 Loma Prieta	**0.94**	-8.5	22.	16.4	-10.2	**1.31**	**0.84**	34.5	14.3
1994 Northridge	**1.18**	-1.5	13.9	9.7	-1.13	**1.34**	**-0.05**	21.3	-2.2
1995 Hyogo-ken Nanbu	**1.47**	0.63	9.91	7.3	1.2	**1.50**	**-0.2**	18.2	0.2
1999 Chi Chi	**1.11**	9.2	25.9	19.1	9.8	**.95**	**-0.3**	14.6	79.
2004 Parkfield	**0.92**	10.	171.	101.	4.	**1.26**	**0.0**	387.	14.

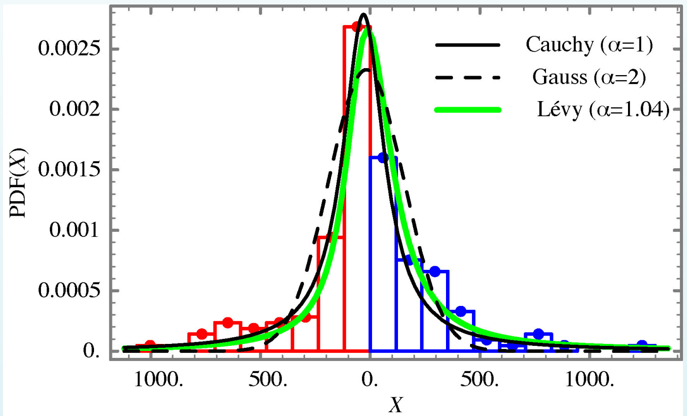
Summary for the strike slip component

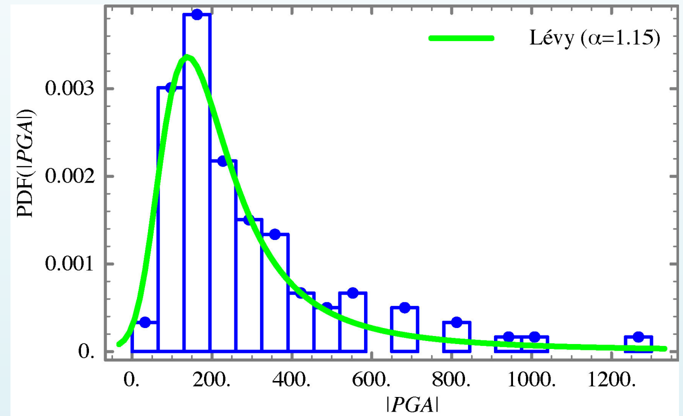
	Scaling Exponent	Gauss law		Cauchy law		Lévy law			
	ν	μ	σ	γ	μ	**α**	**β**	γ	μ
1979 Imperial Valley	**0.78**	-0.7	4.55	3.3	-0.47	**1.14**	**-0.04**	3.75	-1.0
1989 Loma Prieta	**1.07**	-5.5	19.	13.5.	-8.3	**1.07**	**0.6**	15.2	63.7
1994 Northridge	**1.71**	-0.1	6.3	4.29	-0.56	**1.17**	**0.07**	6.41	1.1
1995 Hyogo-ken Nanbu	**1.48**	-2.2	6.4	4.8	-2.6	**1.56**	**0.85**	9.8	0.05
1999 Chi Chi strike slip	**1.27**	-7.5	17.3	13.	-7.7	**1.0**	**0.3**	12.3	9.7
2004 Parkfield	**1.40**	25.	86.	64.	23.	**1.11**	**0.0**	93.	20.

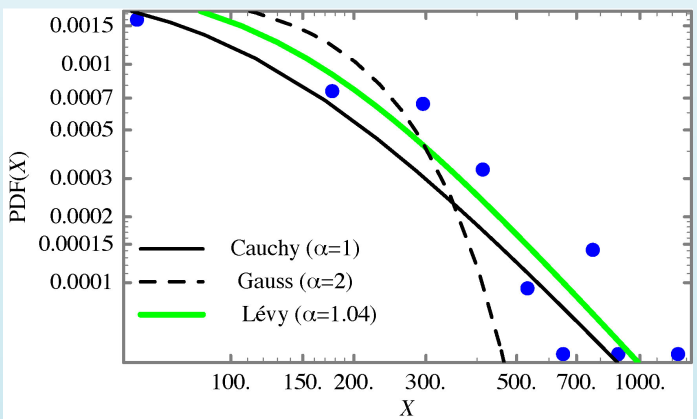
2004 Parkfield Earthquake

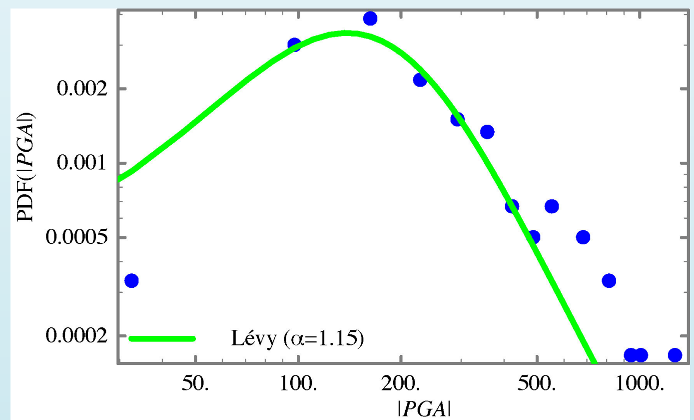
Random model for the filtered slip *X*

Random model for the ground motion |*PGA*|









Both PDF tails decrease with a power law given by α ≈ 1.

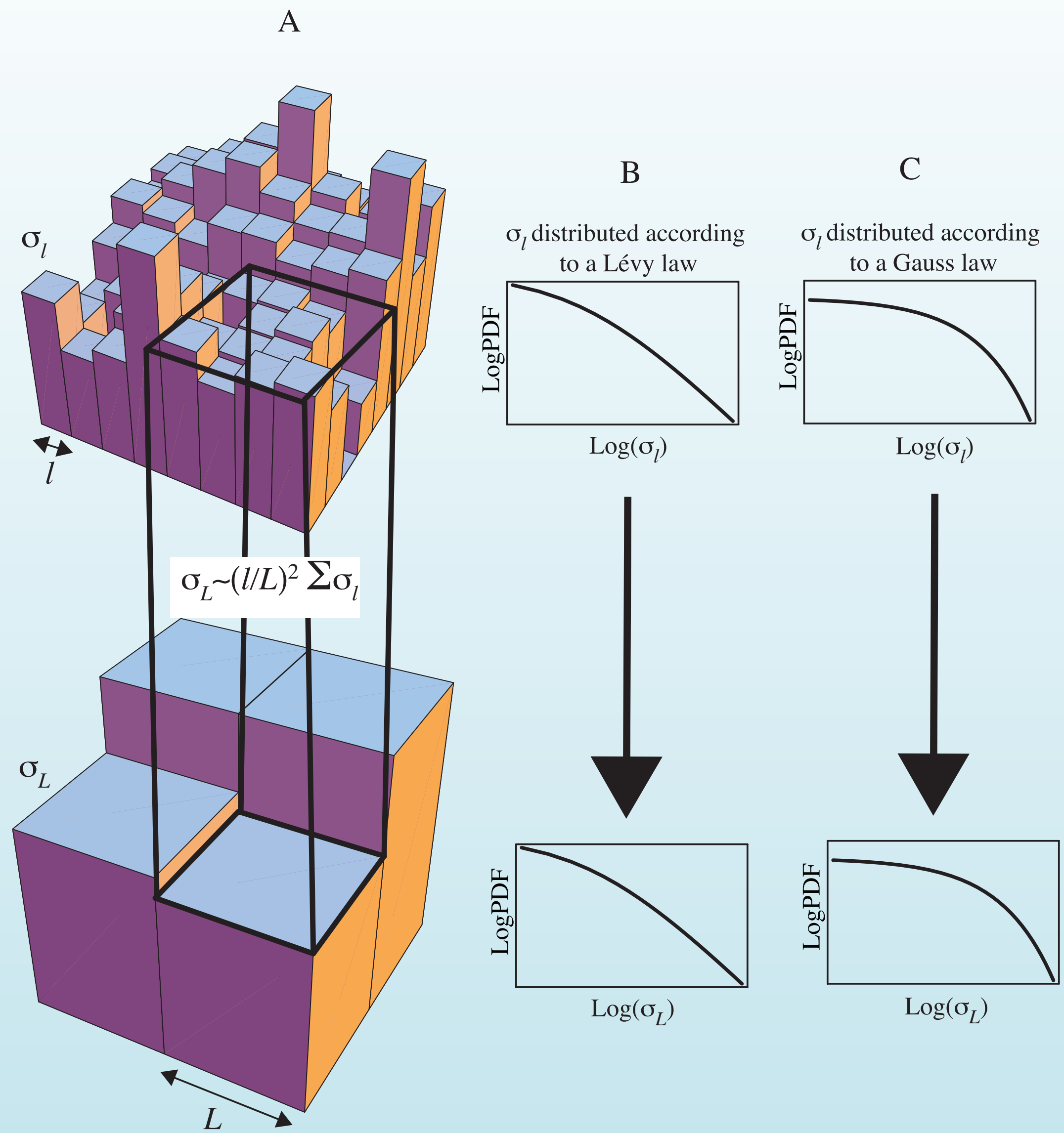
$$A_1 X_1(\alpha,\beta,\gamma,\mu) + A_2 X_2(\alpha,\beta,\gamma,\mu) + \ldots \overset{d}{=} X(\alpha,\beta',\gamma',\mu')$$

with A_1 and A_2 real constants.

Assumptions

- Scaling (or power law) behavior of the slip delimited by the characteristic length scale of the brittle seismogenic region (~10^4 m) and the grain size of the medium (~10^{-2} m) —Andrews, *JGR*, 1980. (10^{-4} m^{-1} < k < 10^2 m^{-1}).
- Scaling law can only be observed and computed in the average.
- The Lévy PDF are truncated, indicating that random variables are bounded between minimum and maximum values.

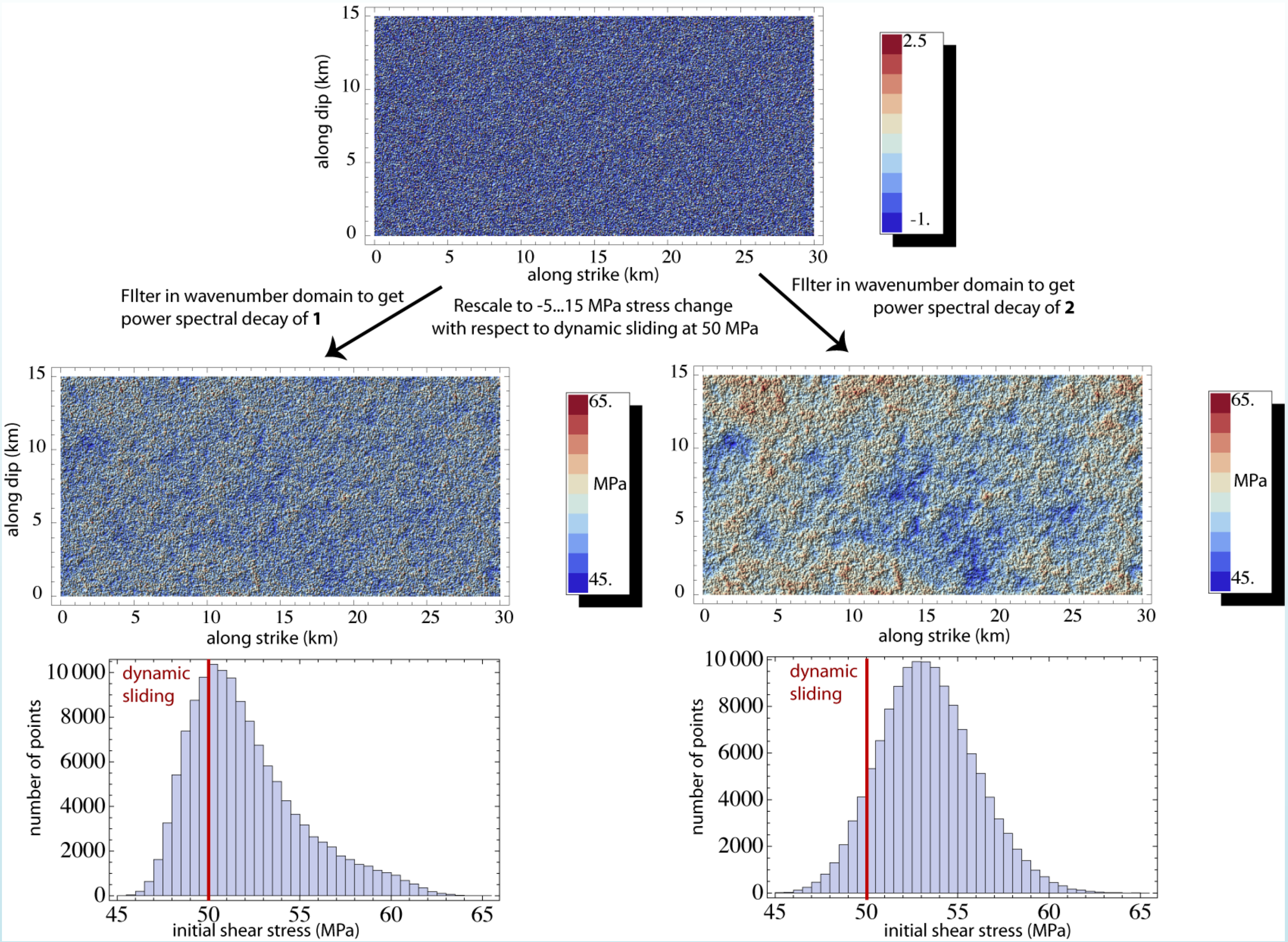
Average: Central Limit Theorem



Recipe for the random model (1D)

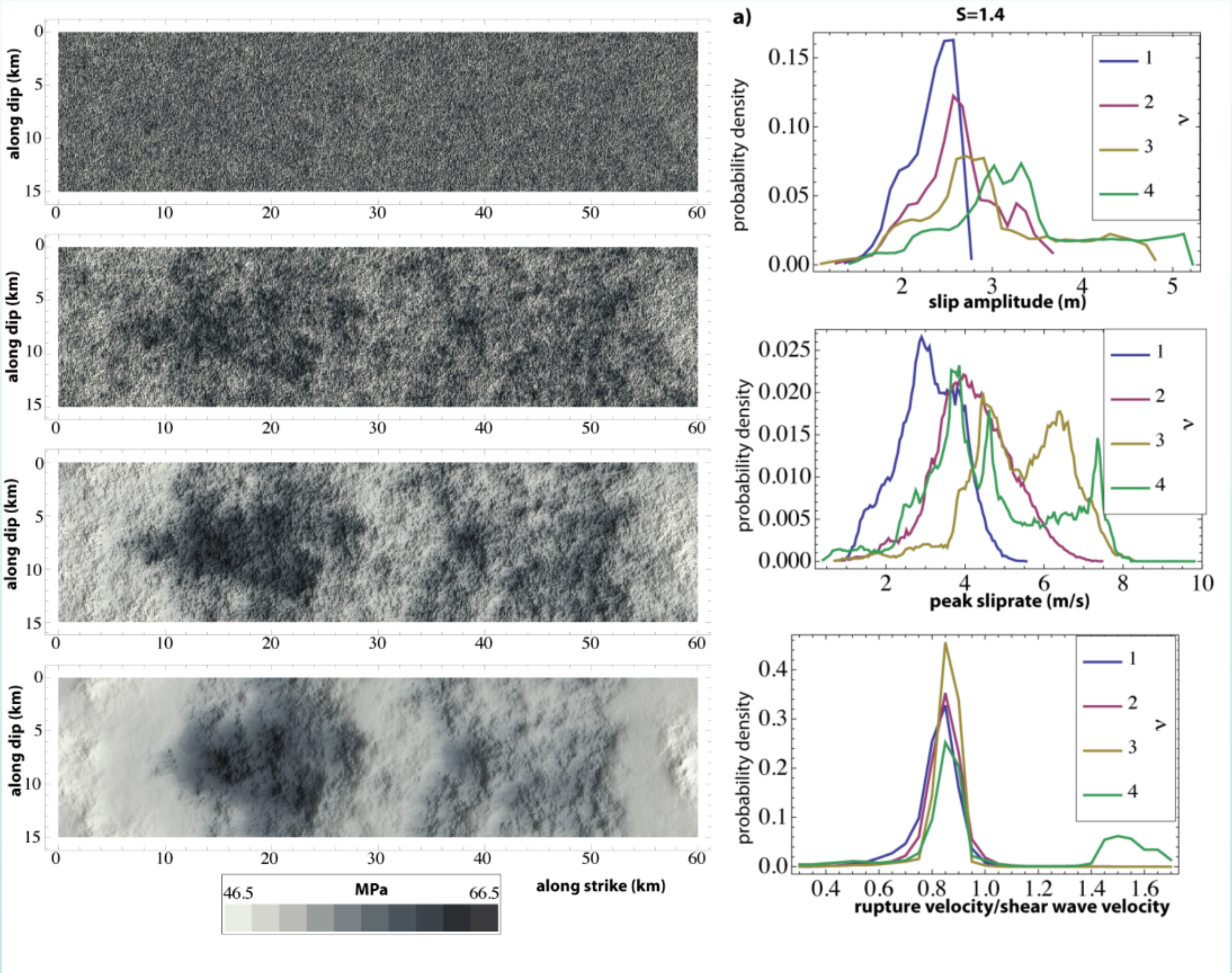
1. Generate white noise
2. Compute the Fourier transform of the white noise.
3. In the Fourier space, multiply by $k_x^{-\nu/2}$ (to get a power spectrum $P(k_x) \approx k_x^{-\nu}$).
4. Compute the inverse Fourier.

Heterogeneous Stress



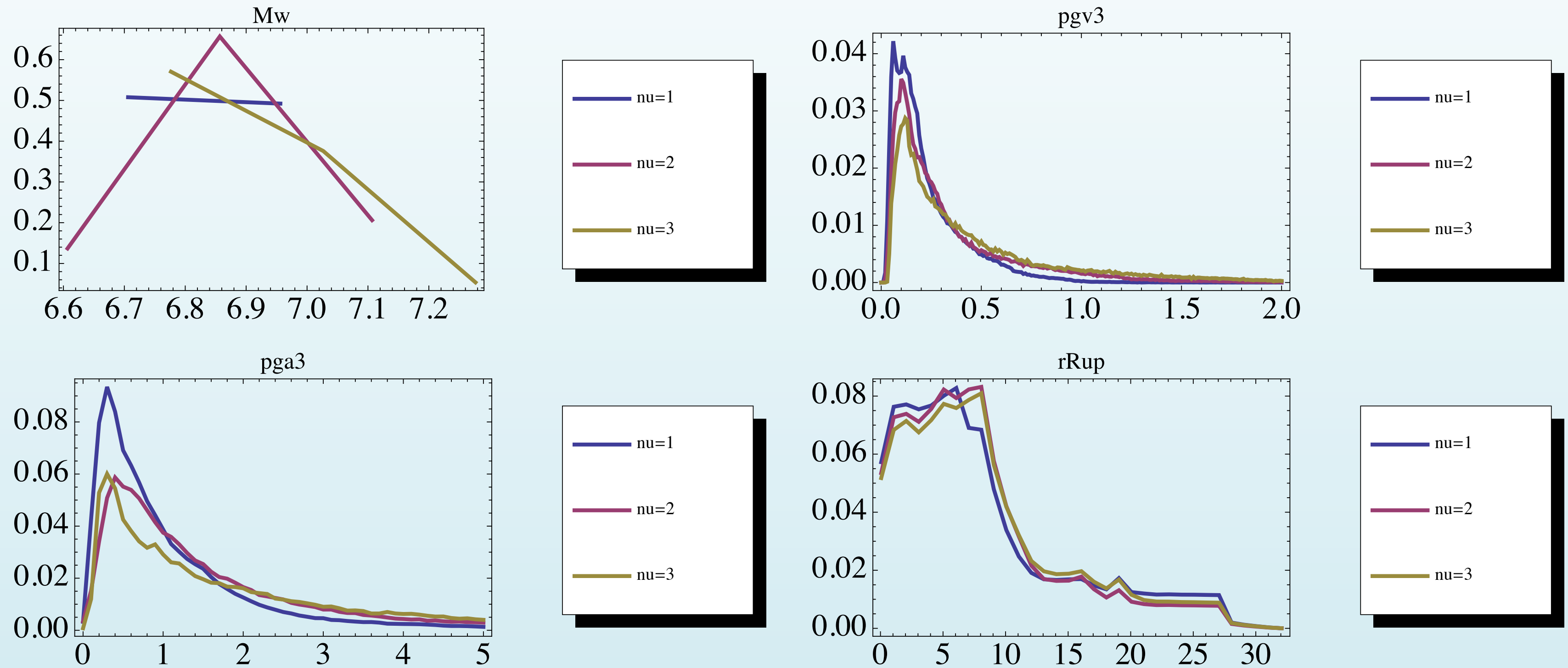
Schmedes et al., JGR, 2010

Stress with Different Autocorrelation



Schmedes et al., Tectonophysics, 2010

PDF's from 114 Dynamic Simulations in a Halfspace (5Hz)



Summary

The basic question is what will be used for the one-point statistic and the two-point statistic.

"However, the Levy distributions are characterized by four parameters instead of the two needed for a Gaussian distribution, thus constituting a higher degree of complexity." --Ripperger *et al*., *JGR*, 2007.

In fact, the "higher degree of complexity" in the probability law does not exist for a Cauchy distribution that requires only two parameters, same as Gauss.

The issue is not the degree of complexity, rather the degree that is most consistent with the data. The data clearly indicate that a Gaussian is deficient for accounting for the wide variation of the stresses; the more extreme values of stress are those that are likely to lead to the higher ground motions.

The two-point statistic, i.e., the autocorrelation of the stress has a power that is more like 2 in 2D and not 4. This indicates that the stresses on the fault are less correlated. In dynamic simulations of strike-slip faults by Schmedes et al., (2010), a higher correlation is associated with faster ruptures and larger ground motion.

Summary

Examples of this approach:

Kinematic modeling: Slip distributed according to a (truncated) Cauchy law, spectrum given by von Karman function: Liu et al., *Bull. Seism. Soc.*, 2006.
Dynamic modeling of rupture: Slip distributed according to a (truncated) Cauchy law. Schmedes et al., *JGR*, 2010.
Dynamic modeling of rupture: Slip distributed according to a (truncated) Cauchy law, spectrum given by power law function: Schmedes et a*l*., to be published in *Tectonophysics*, *JGR*, 2010; Liu *et al*., , *Seismol. Res. Lett.*, 2010.

Question

What do the dynamic models predict that can be validated by comparing with data?