Cigma: CIG Model Analyzer

Quantifying Error in Finite-Element Models

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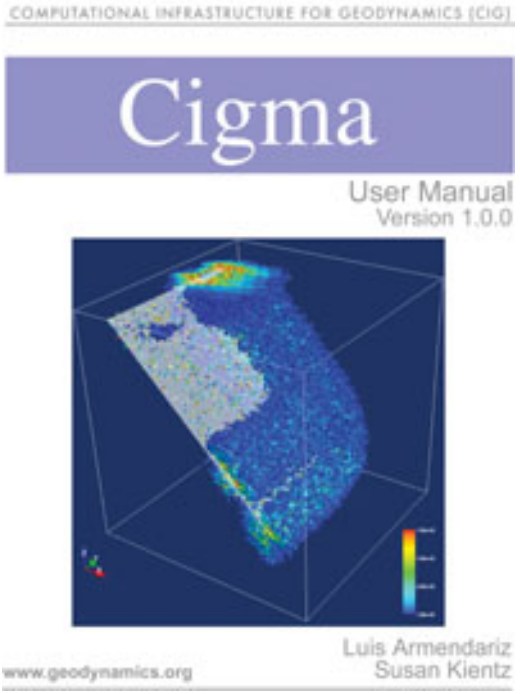
Cigma: CIG Model Analyzer

Developed by Luis Armendariz (CIG)

Designed to quantify errors in finite-element solutions for various cell types and discretizations

- Compare solutions w/different discretizations
  - Meshes (vertices need not coincide)
  - Cell types (hex vs. tet)
  - Basis functions (linear versus quadratic)
- Open-source w/binaries
- Command line driven tool



Computational Infrastructure for Geodynamics

Quantifying the Error

Use integrated L2 norm

$$E = \frac{1}{V}\sqrt{\int_V \|u(\vec{x}) - v(\vec{x})\|^2 dV}$$

Evaluate integral using finite-element formulation

$$E \approx \frac{\sqrt{\sum_{cells}\sum_{quadpts} \|u(\vec{x}_q) - v(\vec{x}_q)\|^2 \, |J(\vec{x}_q)| \, w_q}}{\sum_{cells} V^e}$$

Local error Integrate error within each cell

Global error Integrate error over entire domain

Comparing Solutions Between Meshes

Mesh geometry need not be the same

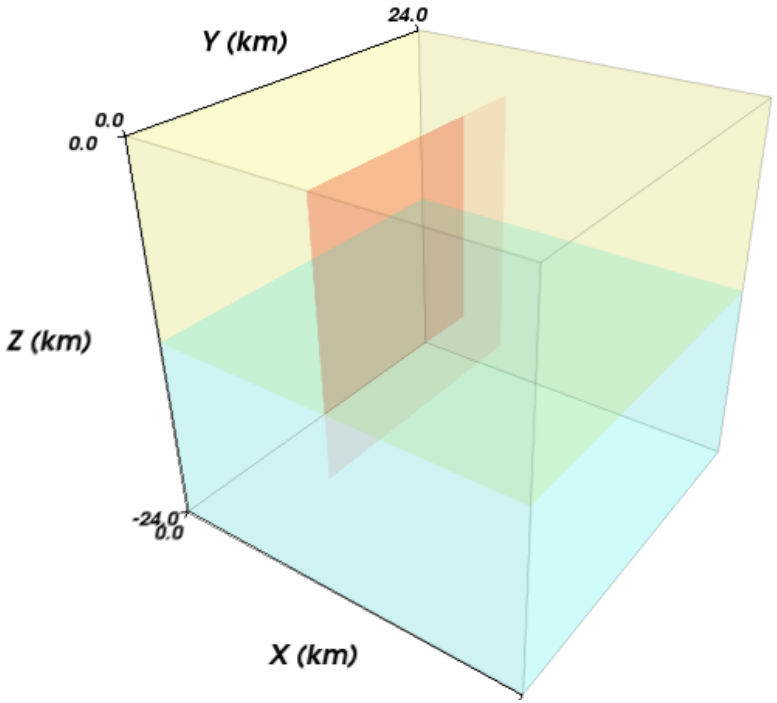
- Use finite-element basis (interpolation) functions to project solution into common geometry
  - Geometry of model A or model B
  - Independent model geometry
    - Subset of volume
    - Fault surface
    - Ground surface

- Integrate difference (error) using common geometry

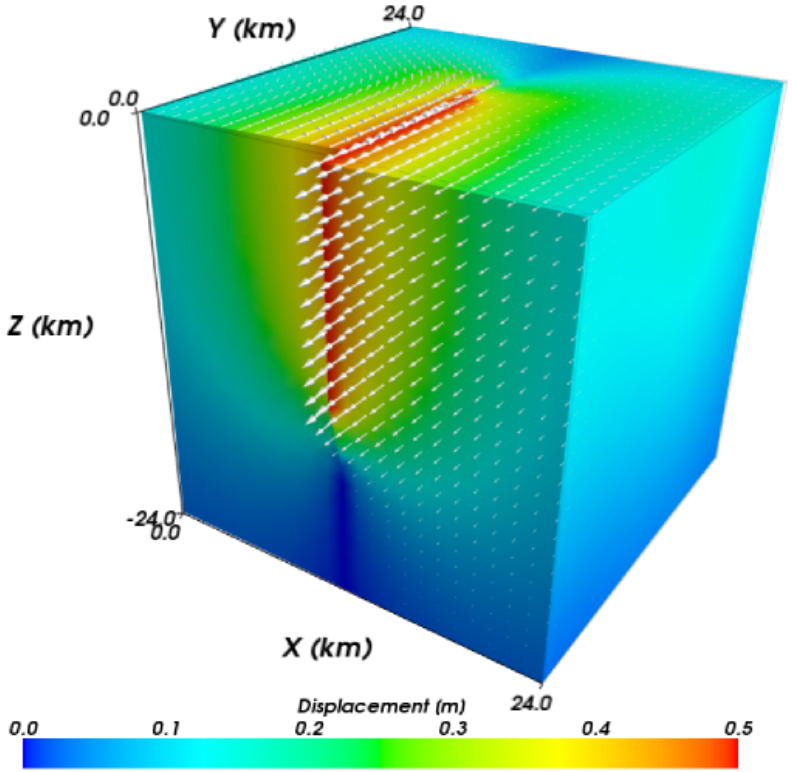
Example: Crustal Deformation Benchmark

Viscoelastic relaxation for strike-slip faulting

Geometry



Solution



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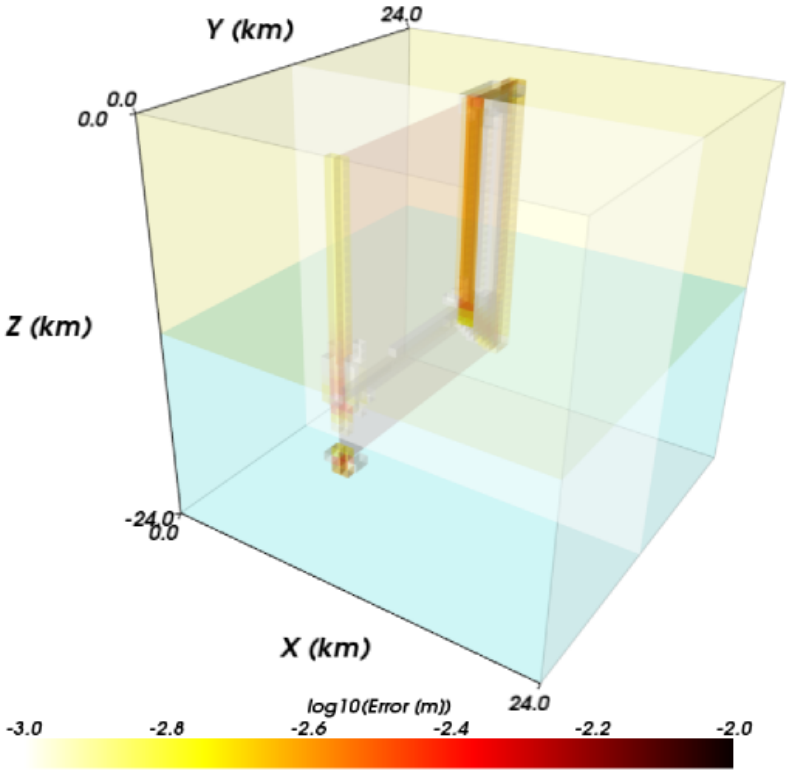
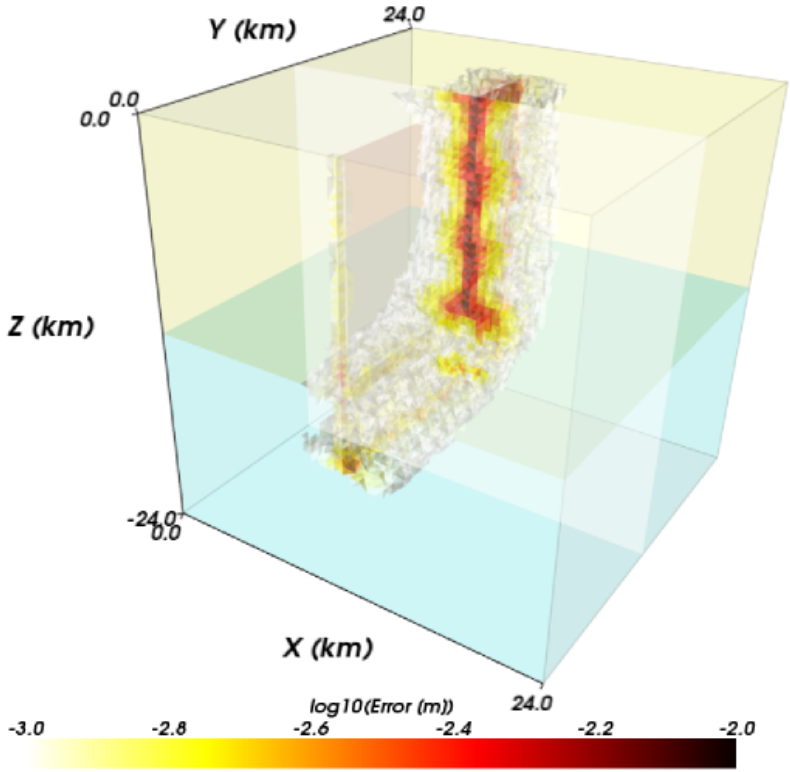
Example: Crustal Deformation Benchmark

Local error in elastic solution (500m resolution) wrt Okada

Hex8 Cells



Tet4 Cells



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Application to Spontaneous Rupture Benchmarks

Facilitate quantifying errors and differences among codes

- Quantify local and global errors/differences over desired domain
  - Fault surface
  - Ground surface
  - Volume
- Requires extending Cigma
  - Finite-difference discretizations
  - Define suitable error metric(s)