Modelling frictional faults as plastic shear bands in nonlinear media

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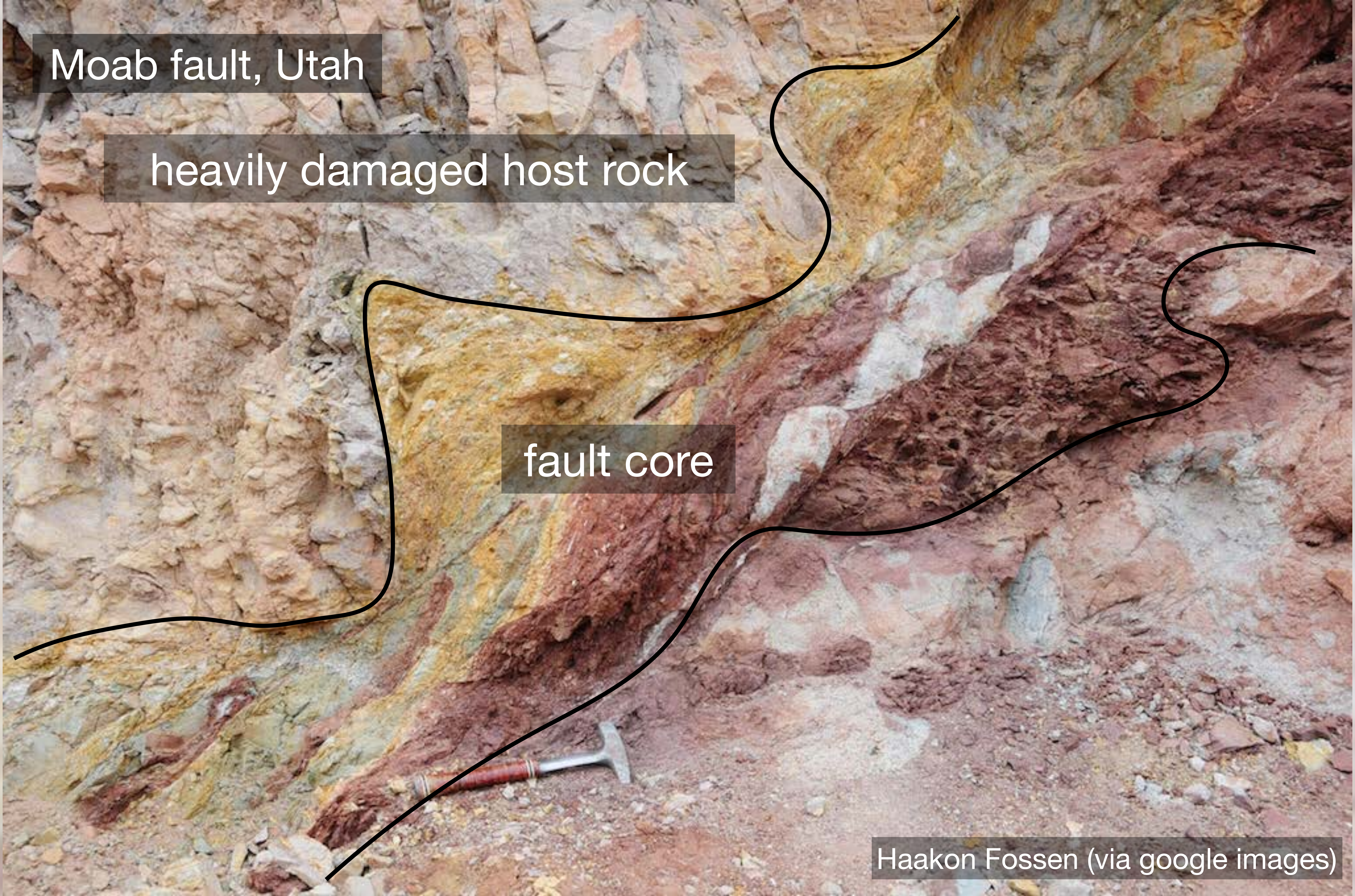
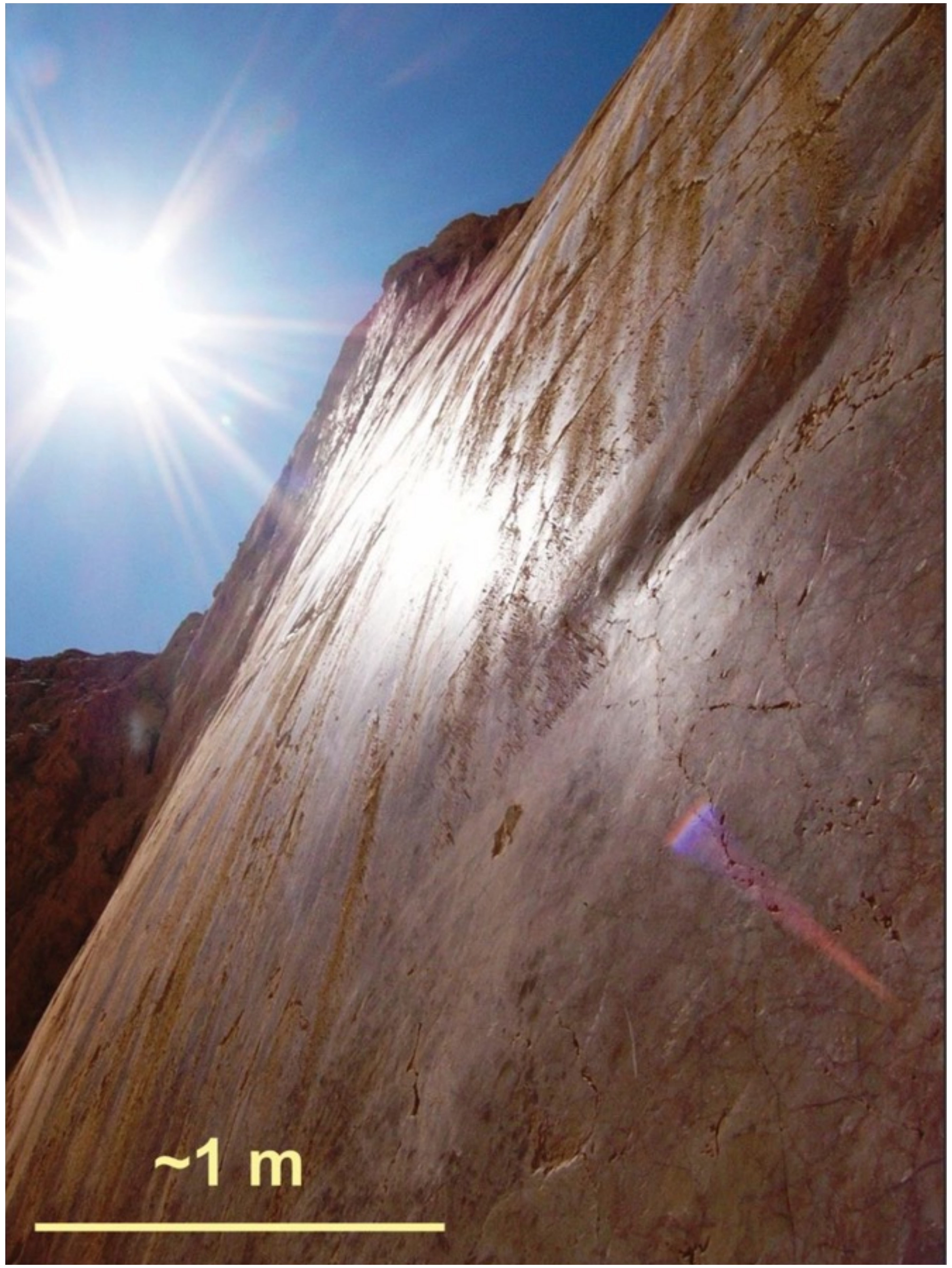
Moab fault, Utah

Haakon Fossen (via google images)


Moab fault, Utah
heavily damaged host rock
fault core
Haakon Fossen (via google images)

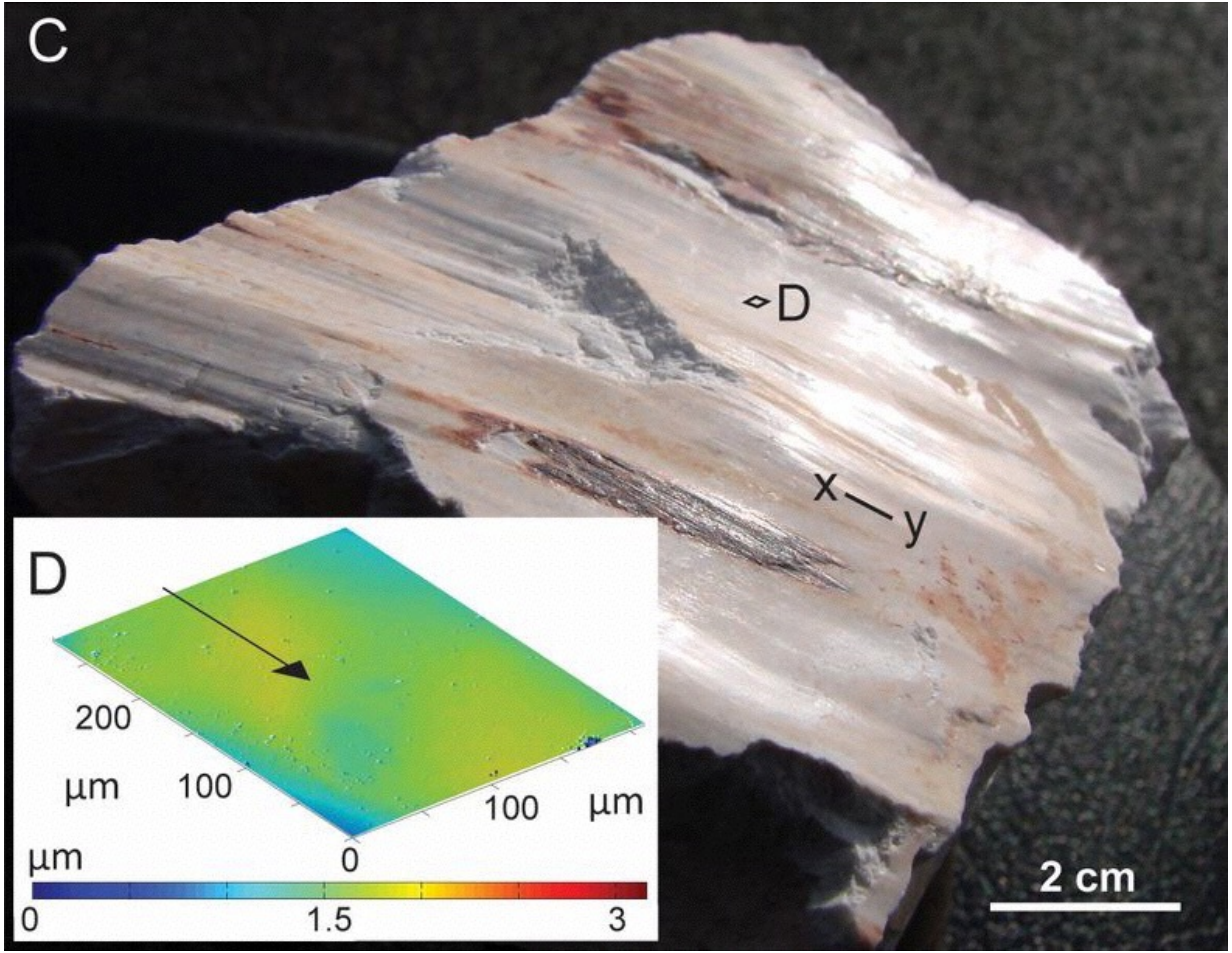

Moab fault, Utah
heavily damaged host rock
slip planes
Haakon Fossen (via google images)


Moab fault, Utah
heavily damaged host rock
R’ shear
Haakon Fossen (via google images)



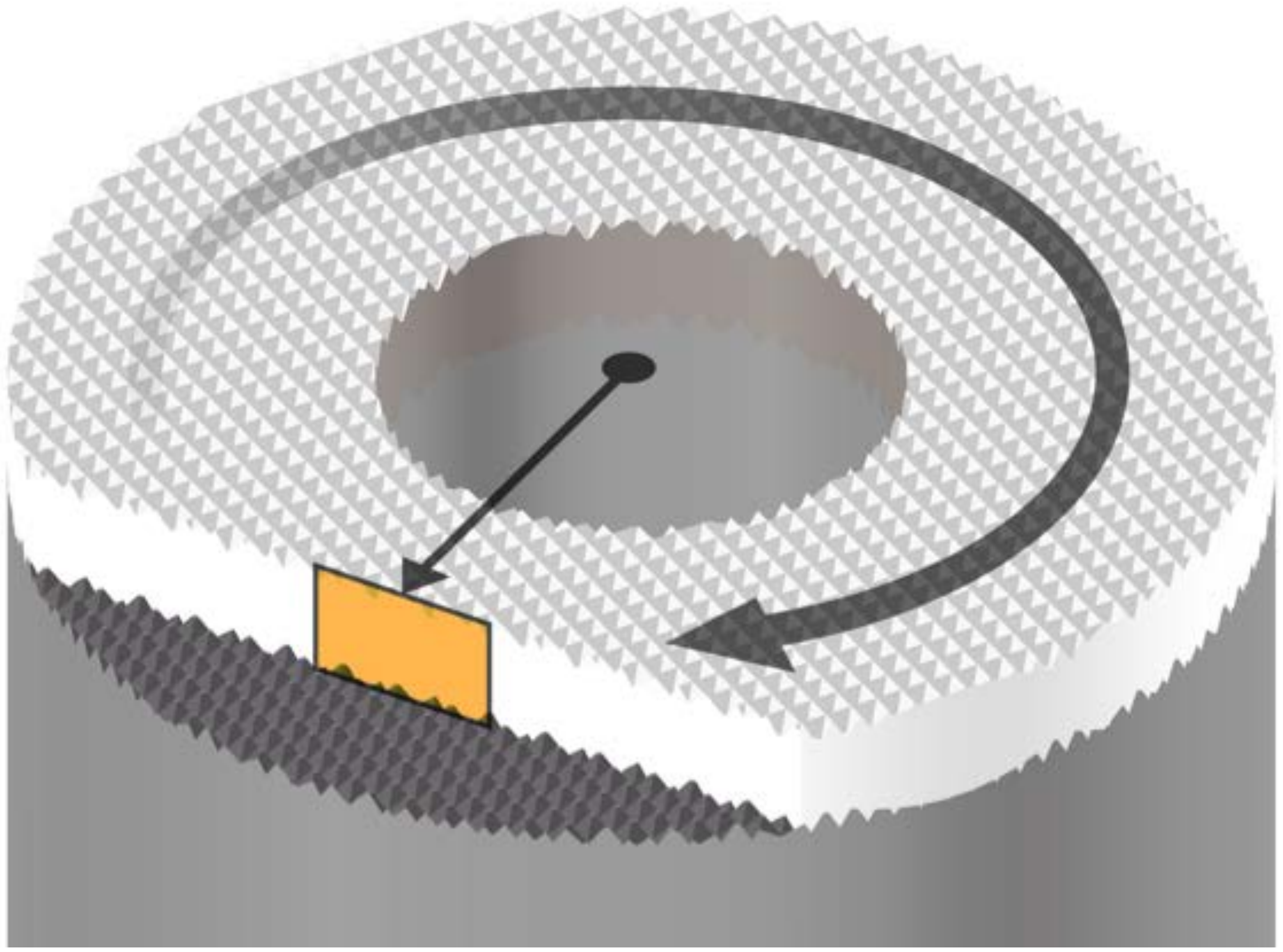
Kfar Giladi quarry, Israel

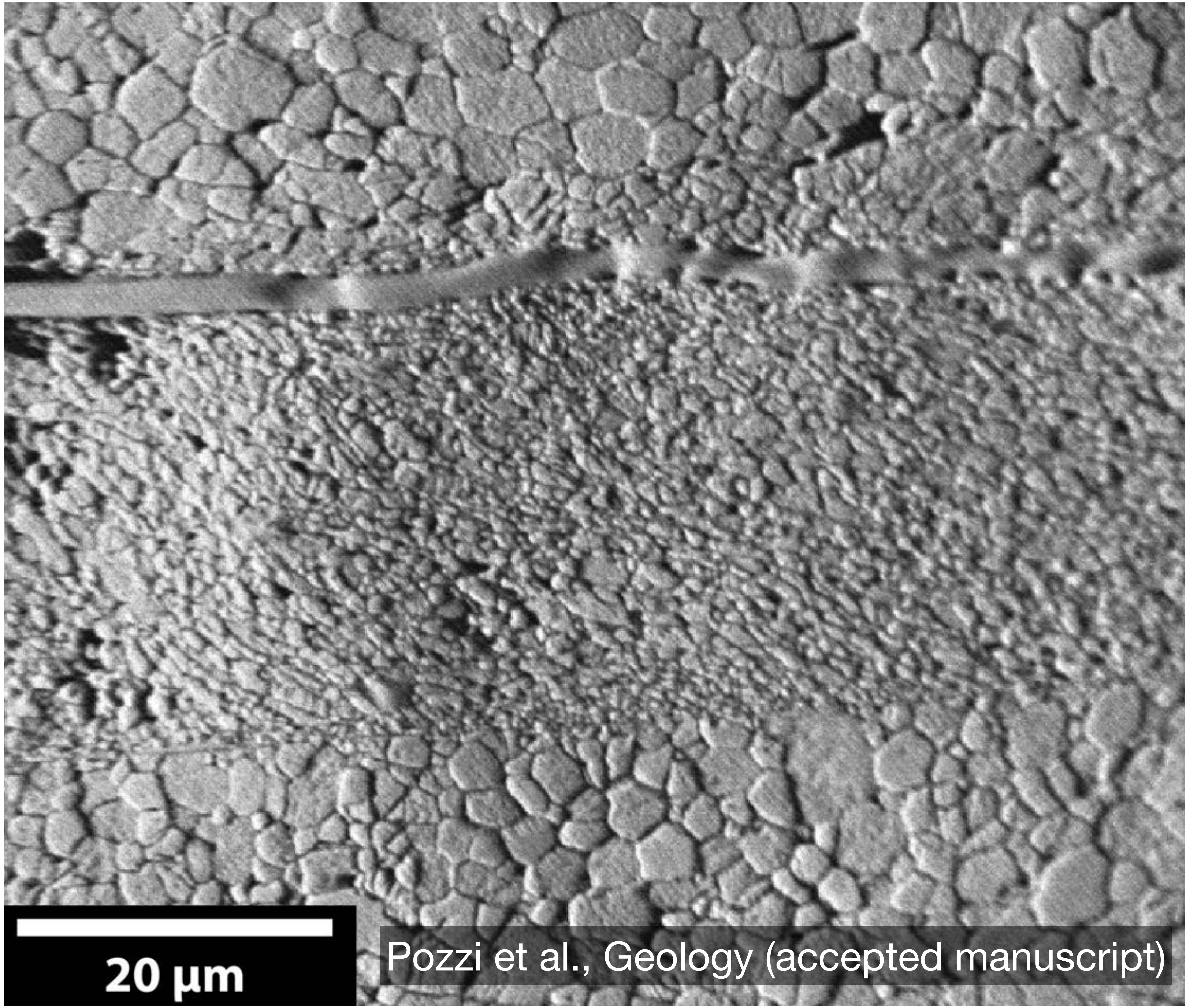
(Siman-Tov et al., 2013, Geology)



Foiana Line (Italian Southern Alps)

(Fondriest et al., 2013, Geology)

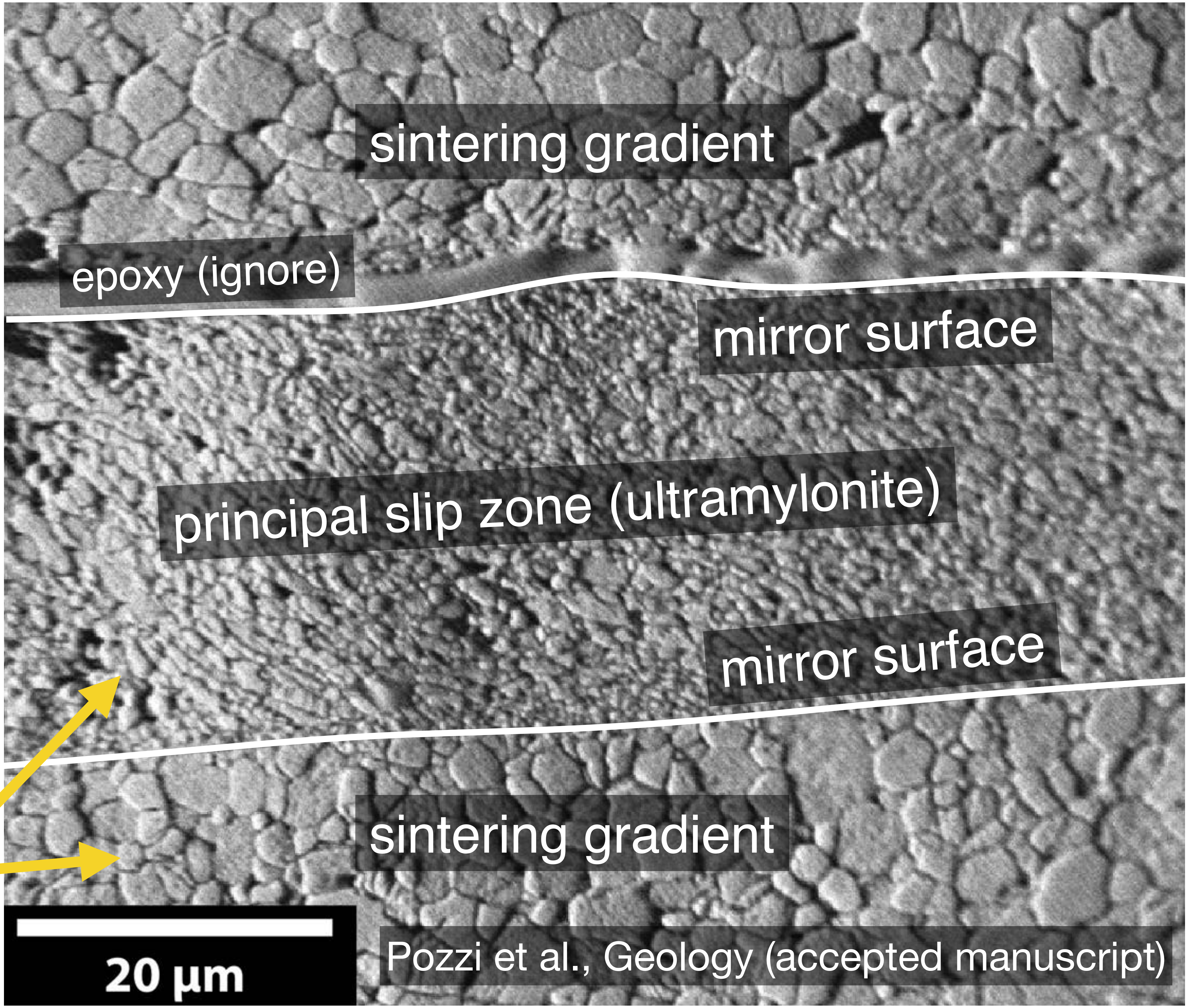



20 µm
Pozzi et al., Geology (accepted manuscript)

Factbox

- ✤ calcite (other materials too)
- ✤ pressure: 20 MPa (~1 km)
- (i) initial powder GS: 63-90 μm
- (ii) cataclastic band GS: ~100 nm
- (iii) sintered GS: > 2 μm
- (iii) ultramylonite GS: ~700 nm
- ✤ duration: < 1 s
- ✤ slip rate: 1.4 m s^{-1}
- ✤ thickness: 30 μm
- ✤ strain rate: ~10^4 s^{-1}

grain size sensitive rheological bifurcation!



Observations

- ✤ Fault zones are thin compared to the scale of the crust
  - ↳ Modelling seismic/aseismic slip transients as interfacial processes makes sense!

    (Especially when off-fault brittle damage generation is included.)
- ✤ Fault zone physics can be complicated bulk rock processes
- ✤ Faults evolve spontaneously in response to a changing stress environment
- ✤ Faults live in nonlinear media (porous flow & several creep mechanisms)

↳ Frictional plasticity provides a consistent way to model these features!

Newtonian viscous rheology
Drucker-Prager plasticity: $\tau_y = C + \mu P$
U_0
U_0
$\eta_2 >> \eta_1$
$\eta_1 << \eta_2$
after Spiegelman, May, Wilson (AGU, 2014; GGG, 2016)
strain rate (10^{-15} s^{-1})
2.5
5
7.5
0
10

stress-dependent
rheological bifurcation!

Nonlinear visco-elastic rheology

Drucker-Prager plasticity: $\tau_y = C + \mu\, P$

Spontaneous evolution from incipient subduction over Ma

Van Dinther (pers. comm.)

Nonlinear visco-elastic rheology

Drucker-Prager plasticity: $\tau_y = C + \mu P$

Spontaneous evolution from incipient subduction over Ma

Viscous creep at work!

Van Dinther (pers. comm.)

Nonlinear visco-elastic rheology

Drucker-Prager plasticity: $\tau_y = C + \mu P$

Spontaneous evolution from incipient subduction over Ma

Bending-induced normal faulting!

Van Dinther (pers. comm.)

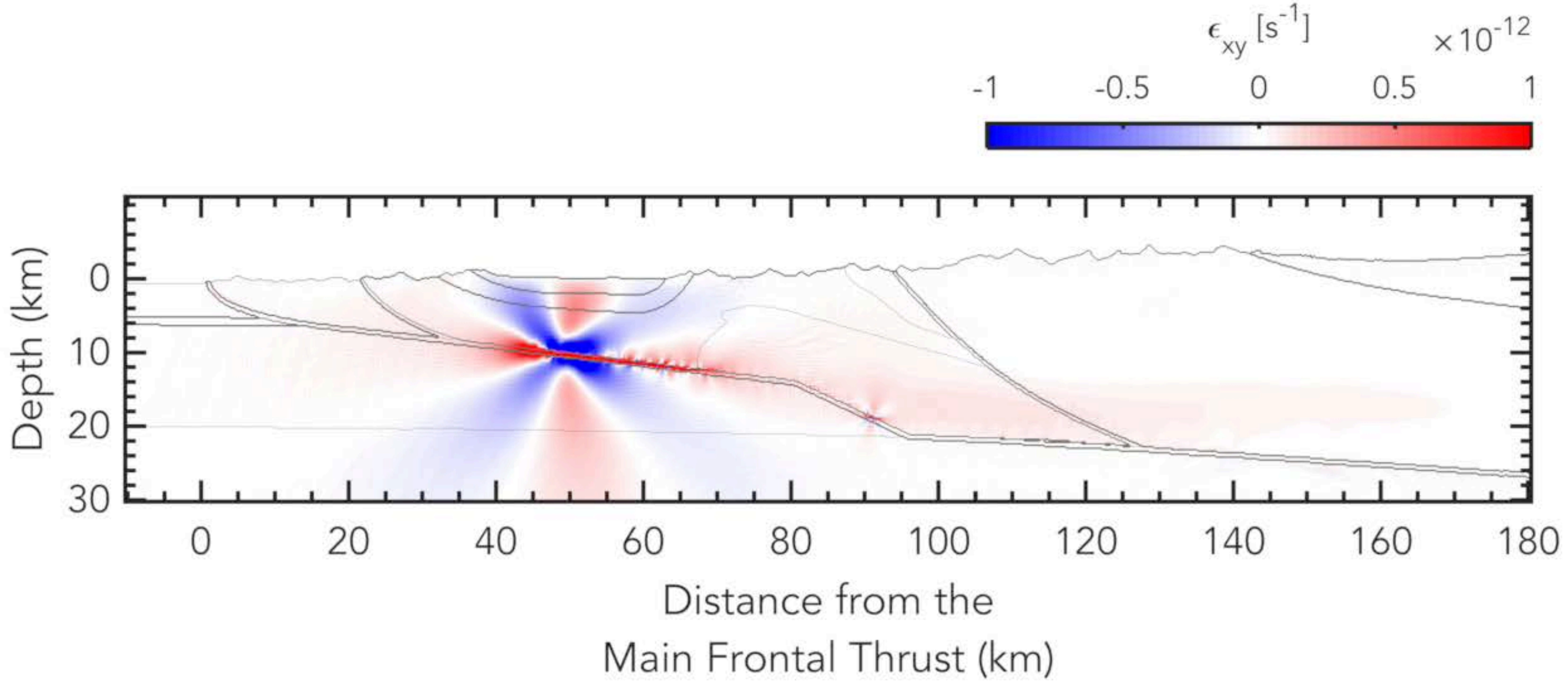
Nonlinear visco-elastic rheology

Drucker-Prager plasticity: $\tau_y = C + \mu P$

Spontaneous evolution from incipient subduction over Ma

Splay faulting and extensive failure of the wedge!

(movie)



Dal Zilio et al. (submitted)

Postulations

- The rate-and-state friction formulation is sufficiently general to include bulk rock processes

- We extend RSF to …

  - $\dot{\epsilon} \;=\; \dot{\epsilon}_e + \dot{\epsilon}_v + \dot{\epsilon}_p$

  - $V \longrightarrow 2\,D\,\dot{\epsilon}^{p}_{eff}$

  - $\dot{\epsilon}^{p}_{eff} \;=\; \mathrm{II}(\dot{\epsilon}_p)$

  - $\tau_y\left(P,\, \dot{\epsilon}^{p}_{eff},\, \Theta\right) = a\,P\,\sinh^{-1}\left(\frac{1}{2}\;\frac{\dot{\epsilon}^{p}_{eff}}{\dot{\epsilon}^{p}_{0}}\;\exp\left(\frac{1}{a}\;\left(\mu_0 + b\,\log\left(\frac{2\,D\,\dot{\epsilon}^{p}_{0}\,\Theta}{L}\right)\right)\right)\right)$

  - $\dot{\Theta} = 1 - \frac{2\,D\,\dot{\epsilon}^{p}_{eff}}{L}\,\Theta$

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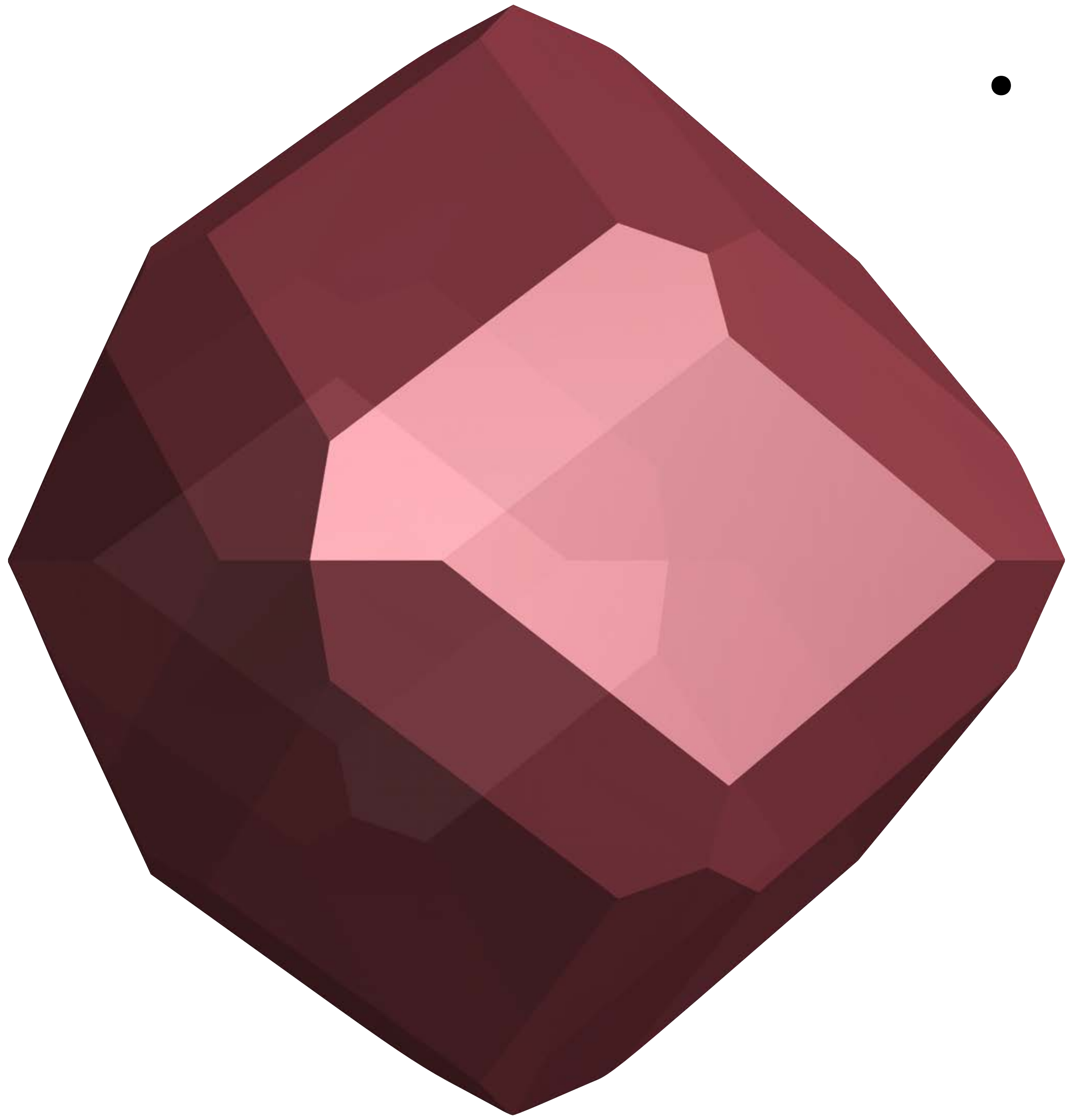
  - $\dot{\Theta} = 1 - \frac{2\,D\,\dot{\epsilon}^p_{eff}}{L}\,\Theta$

implications:

- a-b < 0: complete localisation in finite time
- @ full localisation: D = dx recovers discrete behaviour
- @ full localisation: unstable slip transients
- approximation errors: stress invariant, grid orthotropy
- stress = strength everywhere, self-organising rheology

- GARNET is a header-only, modern C++(14) library
- Implements basic concepts from continuum mechanics:
    - tensor- and scalar-valued fields
    - differential (and other) operators
        - tensor product, staggered grid FD (but deformable)
            - time-integrators, preconditioners, (nonlinear) solvers (PETSc)
- Library implements *all* numerics, *no* physics (ideally)



GARNET

EARTH SCIENCES

GENERAL	REGULAR GRID	NEWTON-KRYLOV	TIME-DEPENDENT
	RESIDUAL-BASED	NONLINEAR	TIGHTLY-COUPLED
ALGORITHM	ROOT-FINDING	N-DIMENSIONAL	TOOLBOX

- GARNET is completely generic w.r.t. physics and problem dimensionality
- User declares model space and defines objective functions
- Jacobian is automatically finite-differenced from the objective function: O(N) performance
- Matrix-free Newton-Krylov solvers for solving coupled nonlinear EQ
- Field-split, geometric/algebraic assembled preconditioners —> O(N) performance
- MPI-3 shared memory + point-to-point communication, aggressive inlining
- Paper *in prep.*, code freely available in the future.

GARNET

Features (Wishlist)

Physics

- rate-and-state friction
- acoustic & seismic WP
- visco-elasto-plasticity
- inertial dynamics
- PVE 2-phase flow
- tsunami propagation
- thermal adv.-diff.
- shear heating
- radiation damping

Solution strategies

- PC MF NK solver
- adaptive timestepping
- back-tracking advection
- marker-based advection
- explicit time marching

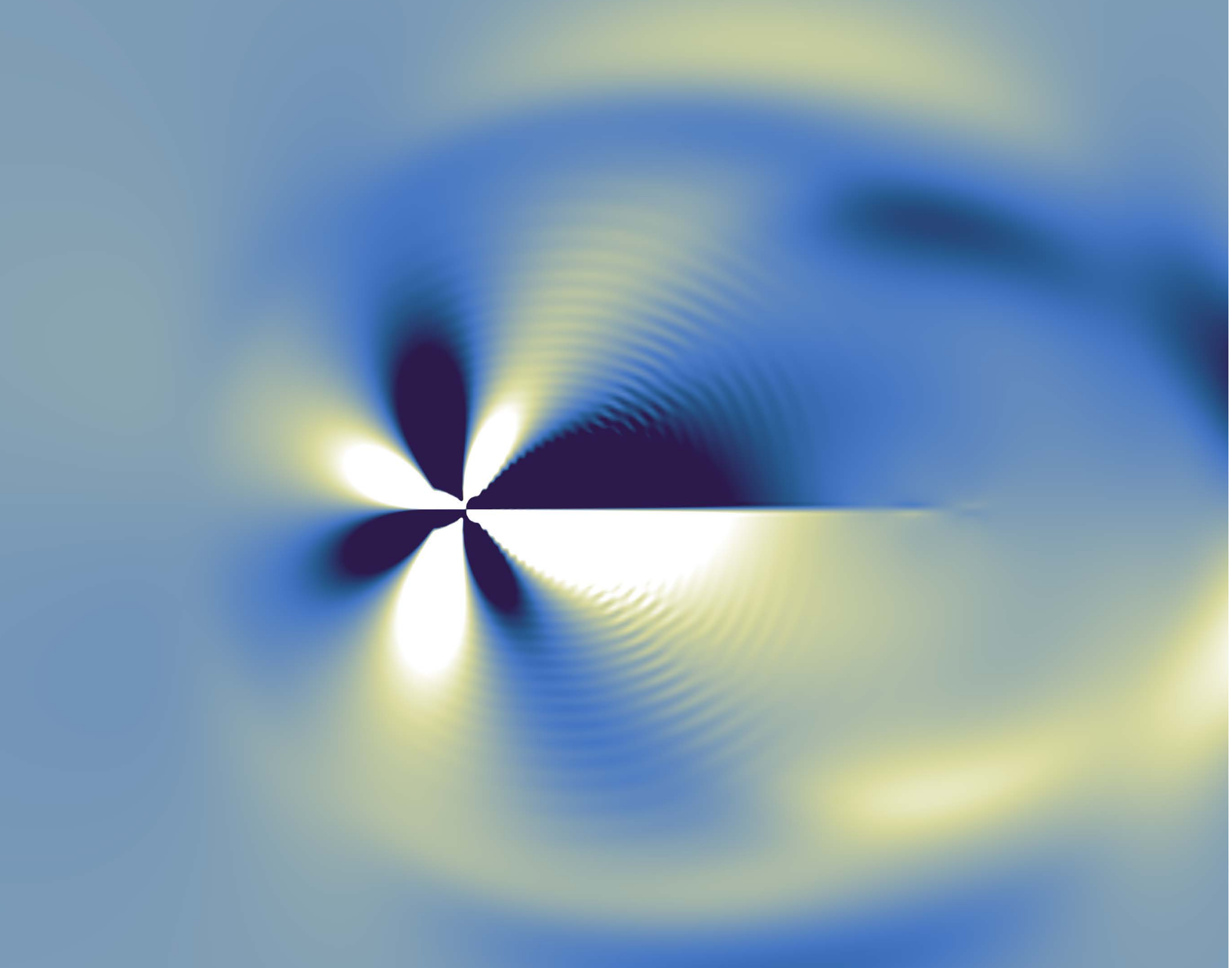
Performance

- MPI-3 shared memory
- MPI P2P communication
- aggressive inlining
- GPU where possible

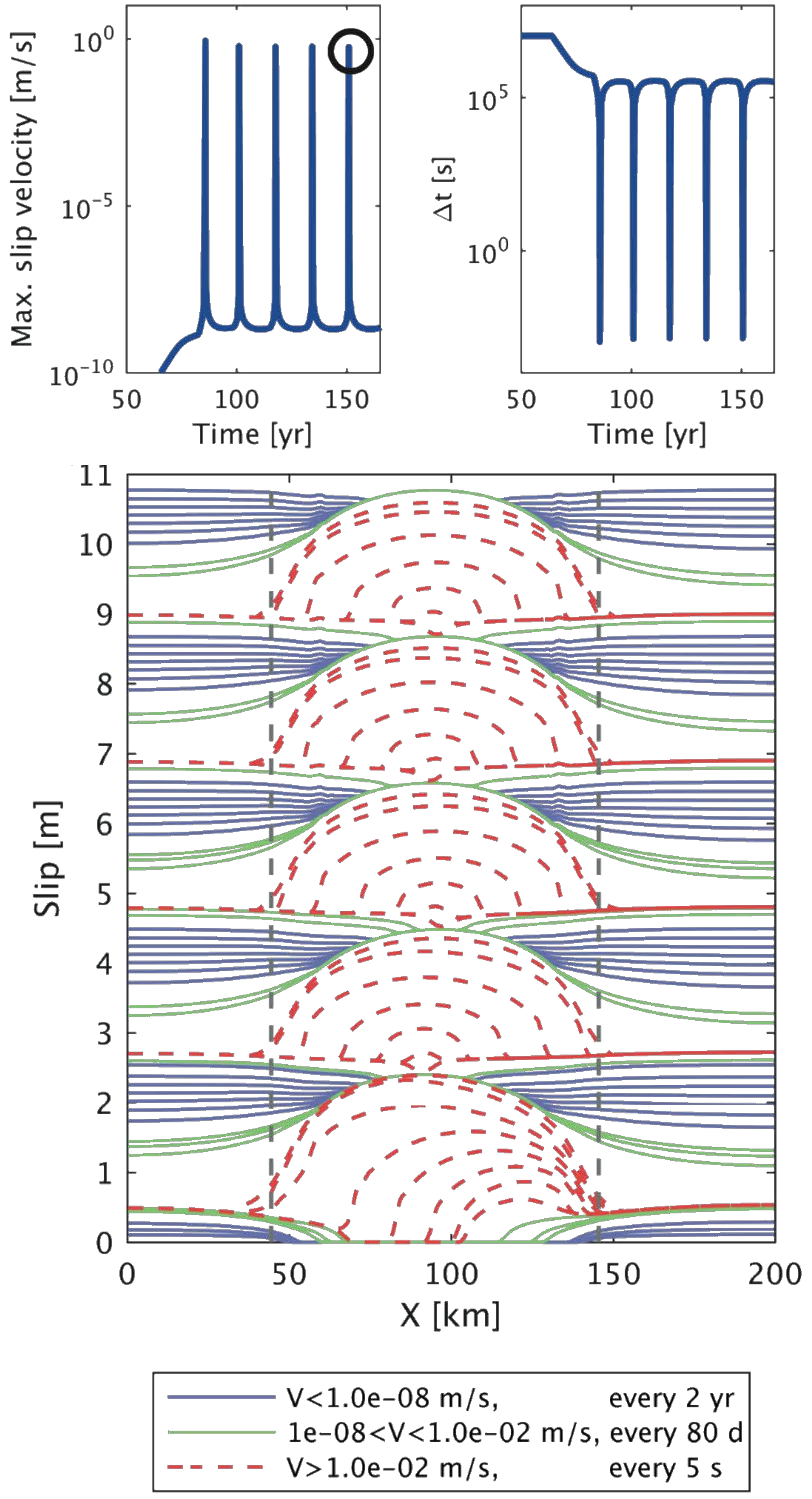
Miscellaneous

- 1D / 2D / 3D
- absorbing BC
- deformable grids
- non-local plasticity
- out-of-plane modelling
- checkpointing

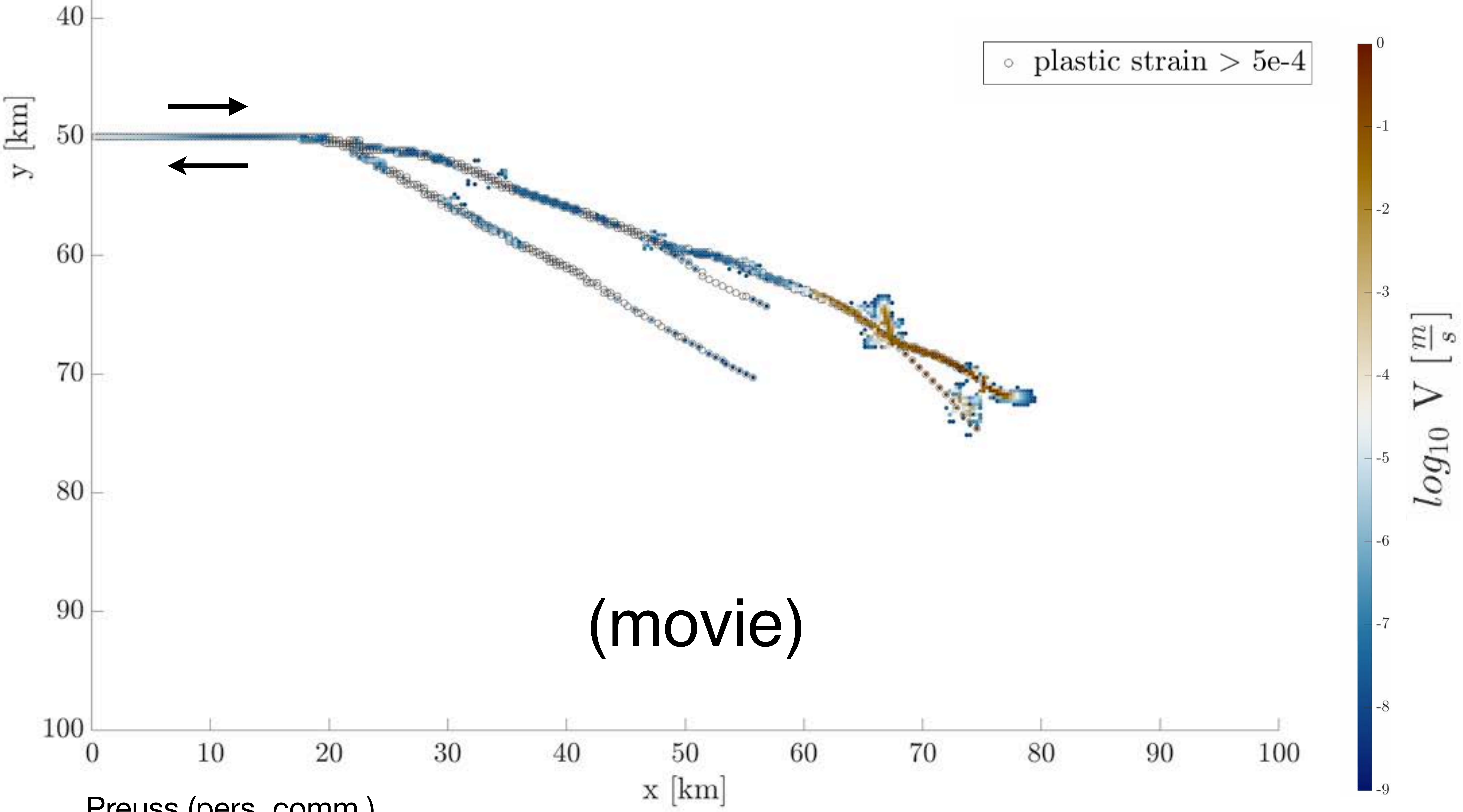
GARNET



(GARNET)



Herrendoerfer et al. (submitted)



(movie)

Preuss (pers. comm.)

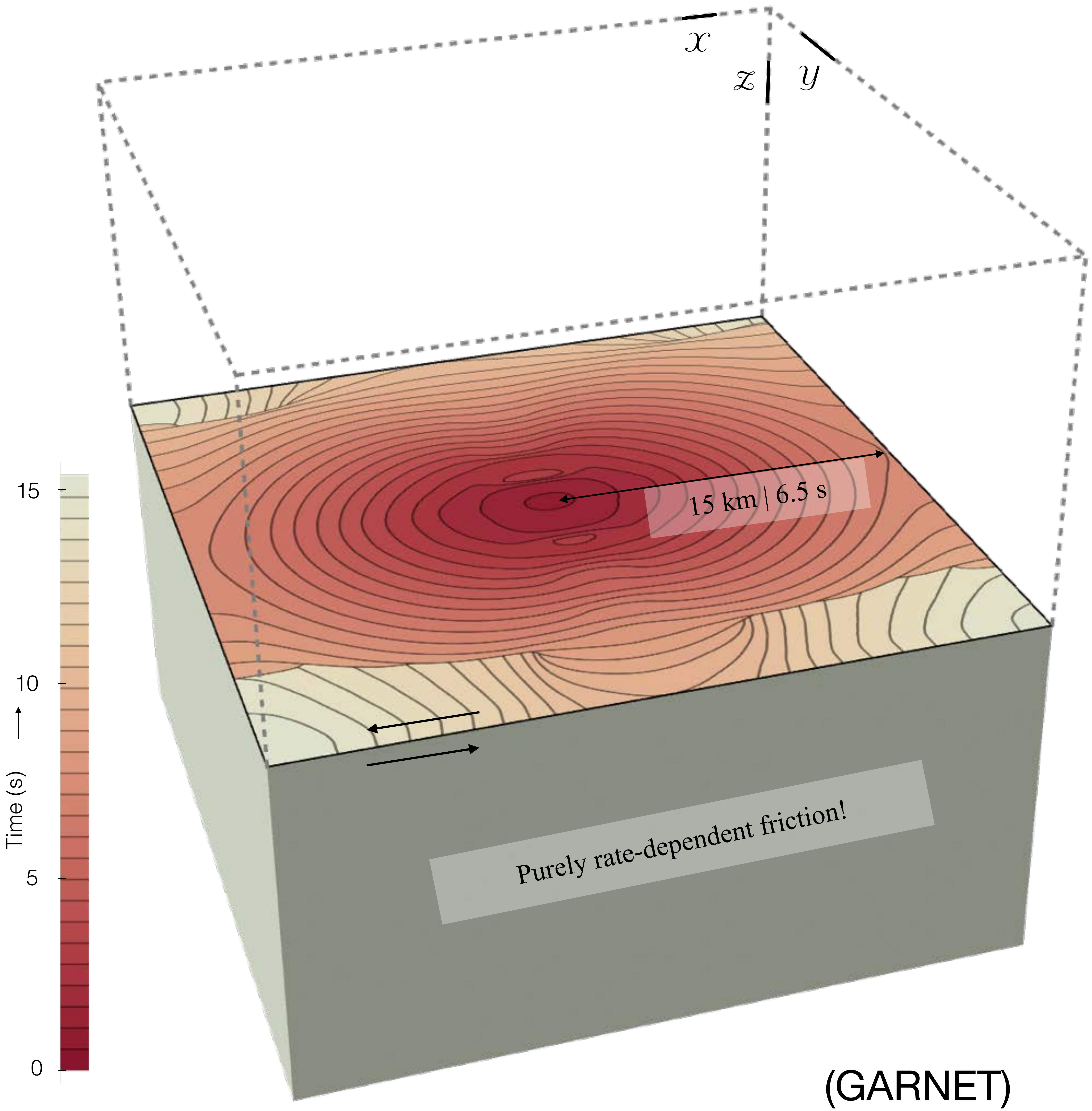
Conclusing remarks

- ✤ Frictional Plastic shear bands potentially enable long-term seismic cycle modelling with spontaneous fault evolution.
- ✤ Our group is making progress, but the technique is not mature yet.
- ✤ Being the odd one out, the SEAS effort is very interesting to us!

<u>Future directions:</u>

  - ✤ More physics-based approach to fault evolution, two-phase formalism.
  - ✤ Towards natural setup (3D), subduction zone seismicity.

additional slides



(GARNET)

```
auto SlipRate  = [&]( double Λ, double τII )
    { return 2 * Ω.h[1] * Λ * τII; };

auto YieldFunc = [&]( double τII, double V, double P, double Φ, double b )
    { return τII - a * P * std::asinh( 0.5 * V/V0 * std::exp((μ0+b*std::log(V0*Φ/L))/a) ); };

auto MomentumBalance = [&]( auto& rhs )
{
    θ(); ϵ().RemoveTrace();

    // dτ/dt = 2 G (-Λ τ + dϵ/dt)
    τ.TrivialSolve<BDF<2>>();

    // dP/dt = - K θ
    P.TrivialSolve<BDF<2>>();

    rhs.Set( [&]( double Δτ, double ΔP ) { return (Δτ-ΔP) / ρ; }, Δτ(), ΔP() );
};

auto Residual = [&]( auto& Rv, auto& f )
{
    v. template Residual<BDF<2>>(Rv);

    V.Set( SlipRate, Λ, II(τ[0]) );

    // dΦ/dt = 1 - VΦ/L
    Φ.TrivialSolve<BDF<2>>();

    f.Set( YieldFunc, τII, V, P, Φ[0], b );
};
```

GARNET