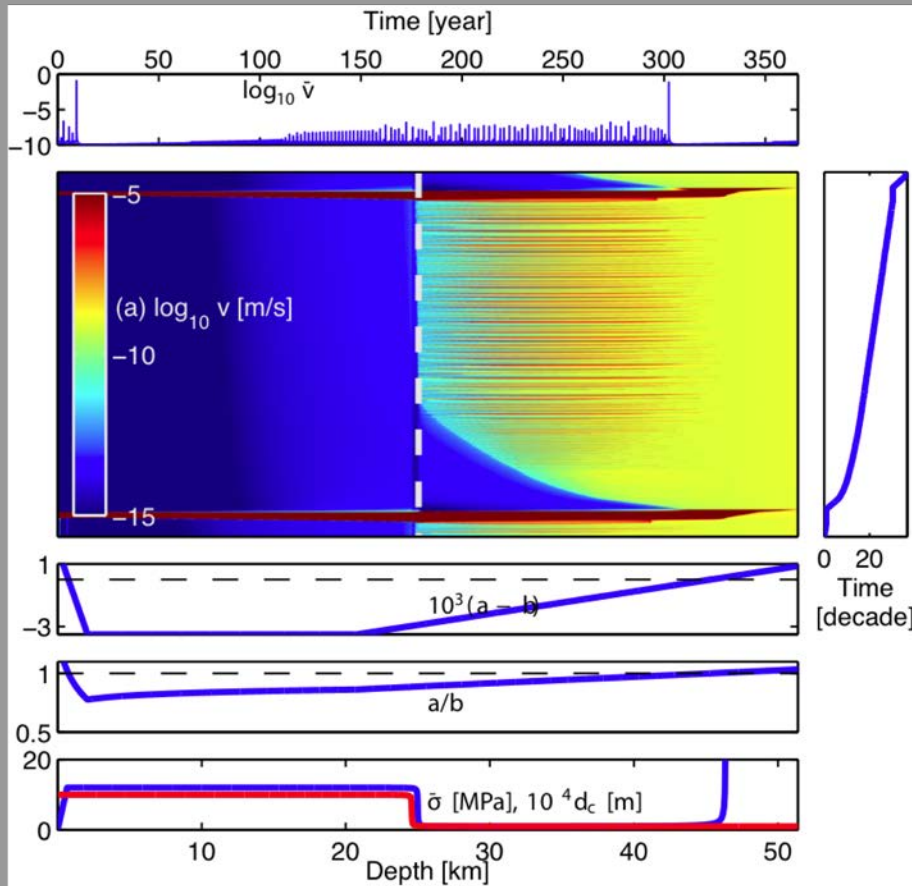


FDRA — Fault Dynamics with Radiation damping Approximation



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Segall and Bradley, Journal of Applied Mechanics MAY 2012, Vol. 79 (Jim Rice Volume)

Momentum Balance Equation

$$\tau_0 + \frac{\mu}{2\pi(1-\nu)} \int_{-\infty}^{\infty} \frac{\partial \delta / \partial \xi}{\xi - x} d\xi - f(v, \theta)[\sigma - p(y=0)] = \frac{\mu}{2v_s} v \quad (1)$$

Options for Elastic Interaction:

- Fourier Transform (2D symmetric)
- Boundary Element (DDM; 2D and 3D)
 - H-matrix Compression: *hmmvp*
 - variable mesh size w/ 2nd order accuracy: *dc3dm*
- Spring Slider
- User Defined Input

Pore-fluid and Heat Transport

$$\frac{\partial T}{\partial t} = c_{\text{th}} \frac{\partial^2 T}{\partial y^2} + \frac{\tau \dot{\gamma}}{\rho c}$$

$$\dot{\gamma}(x, y, t) = \frac{v(x, t)}{h} g(y), \quad \text{where} \quad \int g(y) dy = h$$

Or for infinitesimal fault thickness

$$\frac{\partial T}{\partial t} = c_{\text{th}} \frac{\partial^2 T}{\partial y^2}, \quad \left. \frac{\partial T}{\partial y} \right|_{y=0} = -\frac{\tau v}{2c\rho c_{\text{th}}}$$

Pore pressure

$$\frac{\partial p}{\partial t} = \frac{1}{\eta\beta} \frac{\partial}{\partial y} \left(\kappa \frac{\partial p}{\partial y} \right) + \Lambda \frac{\partial T}{\partial t} - \frac{1}{\beta} \frac{\partial \phi}{\partial t}$$

- Note 1D diffusion normal to fault
- Also “membrane diffusion”

Constitutive Equations

Friction:

$$f(\theta, v) = a \operatorname{arcsinh} \left[\frac{v}{2v_0} \exp \left(\frac{f_0 + b \ln(\theta v_0 / d_c)}{a} \right) \right]$$

$$\begin{aligned} \frac{d\theta}{dt} &= 1 - \frac{\theta v}{d_c} \\ \frac{d\theta}{dt} &= -\frac{\theta v}{d_c} \ln \left(\frac{\theta v}{d_c} \right) \end{aligned}$$

Segall Rice (1995) Dilatancy

$$\frac{d\phi}{dt} = -\frac{v}{d_c} (\phi - \phi_{ss})$$

$$\phi_{ss} = \phi_0 + \varepsilon \ln \left(\frac{v}{v_0} \right)$$

$$\delta\phi = -\varepsilon \ln \left(\frac{v_0 \theta}{d_c} \right)$$

$$\frac{d\phi}{dt} = -\varepsilon \frac{d}{dt} \ln \left(\frac{v_0 \theta}{d_c} \right) = -\frac{\varepsilon}{\theta} \frac{d\theta}{dt}$$

Also: Flash heating
“V-cutoff” friction

Non-Uniform Discretization of Diffusion Equations

$$\dot{T}(y, t) = (c_{th} T(y, t))_y \quad \text{for } y \in (0, y_\infty)$$

$$T(0, t)_y = -\frac{\tau V}{2\rho c_p c_{th}} \quad \text{and} \quad T(y_\infty, t) = 0.$$

Non-Uniform Mesh in Fault Normal Coordinate $z(y) = \log(c + y)$:

$$\dot{T} = e^{-z} (c_{th} e^{-z} T_z)_z \quad \text{for } y \in (0, y_\infty)$$

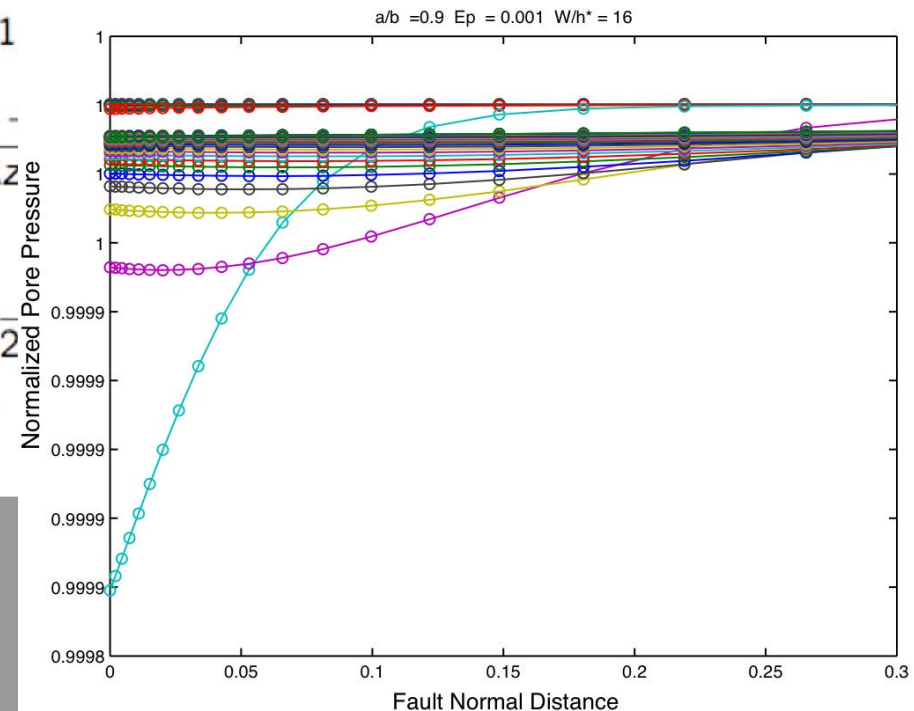
$$e^{-z} T_z = -\frac{\tau V}{2\rho c_p c_{th}} \quad \text{at } y = 0$$

Spatial Discretization: $\delta \equiv \Delta z/2$, and $k \in 1$

$$\dot{T} = e^{-z_k} \left(\frac{-c_{th} (z_k - \delta) e^{-(z_k - \delta)} (T_k - T_{k-1})}{\Delta z} \right)$$

$$e^{-z_0} \frac{T_1 - T_{-1}}{2\Delta z} = -\frac{\tau V}{2\rho c_p c_{th}}$$

$$e^{-z_K} T_K = 0.$$



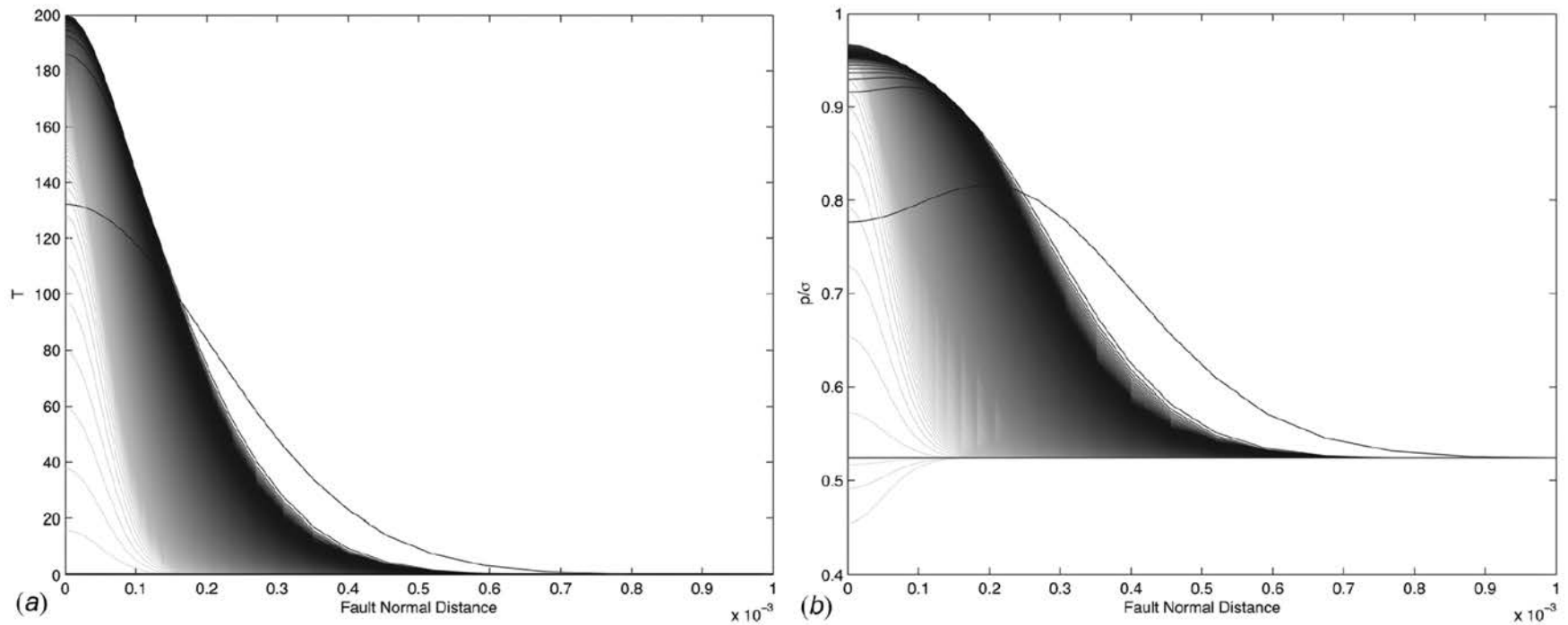


Fig. 8 (a) Temperature ($^{\circ}\text{C}$) and (b) pore pressure profiles as a function of distance normal to the fault during a dynamic slip event. In this calculation $h = 100 \mu\text{m}$, $d_c = 10 \mu\text{m}$. In this simulation the effective d_c for dilatancy was scaled by $h/h_c = 1/10$. $\sigma - p^{\infty} = 100 \text{ MPa}$, $W/h^* = 30$.

Implicit-Explicit Time-Stepping

Explicit Time Step - $\dot{\delta} = v$ and $\dot{\theta} = \text{slip or aging law}$.

Implicit Time Step - A_T : Finite difference operator; bc_T : bound. cond.

$$\frac{T^{n+1} - T^n}{\Delta t} = A_T T^{n+1} + bc_T$$
$$(I - \Delta t A_T) T^{n+1} = T^n + \Delta t bc_T.$$

For each fault node given $(v^{n-1}, p^{n-1}, T^{n-1})$ at time t^{n-1} , we determine (v^n, p^n, T^n) at time $t^n = t^{n-1} + \Delta t$ by solving

$$\begin{pmatrix} (I - \Delta t A_T) T^n - T^{n-1} - \Delta t bc_T^n \\ (I - \Delta t A_p) p^n - p^{n-1} - \Lambda(T^n - T^{n-1}) - \Delta t bc_p^n \\ \tau_{\text{elastic}}^n - (\sigma - p_0^n) f(v^n, \theta^n) - \eta v^n \end{pmatrix} = 0.$$

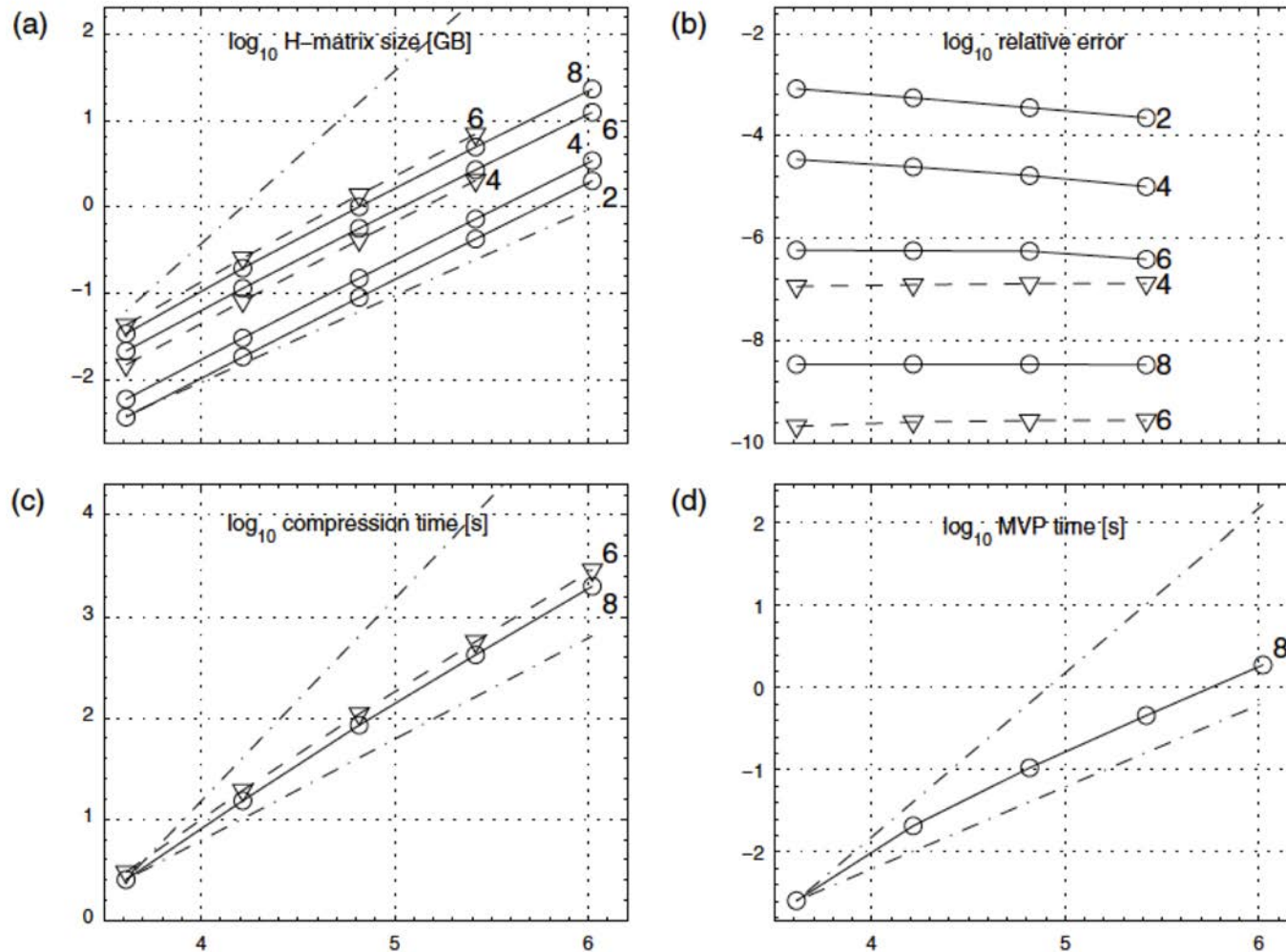
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Some other capabilities:

- Mixed ODE/DAE approach
- fdra uses OpenMP for parallelization.
- hmmvp supports OpenMP and MPI.

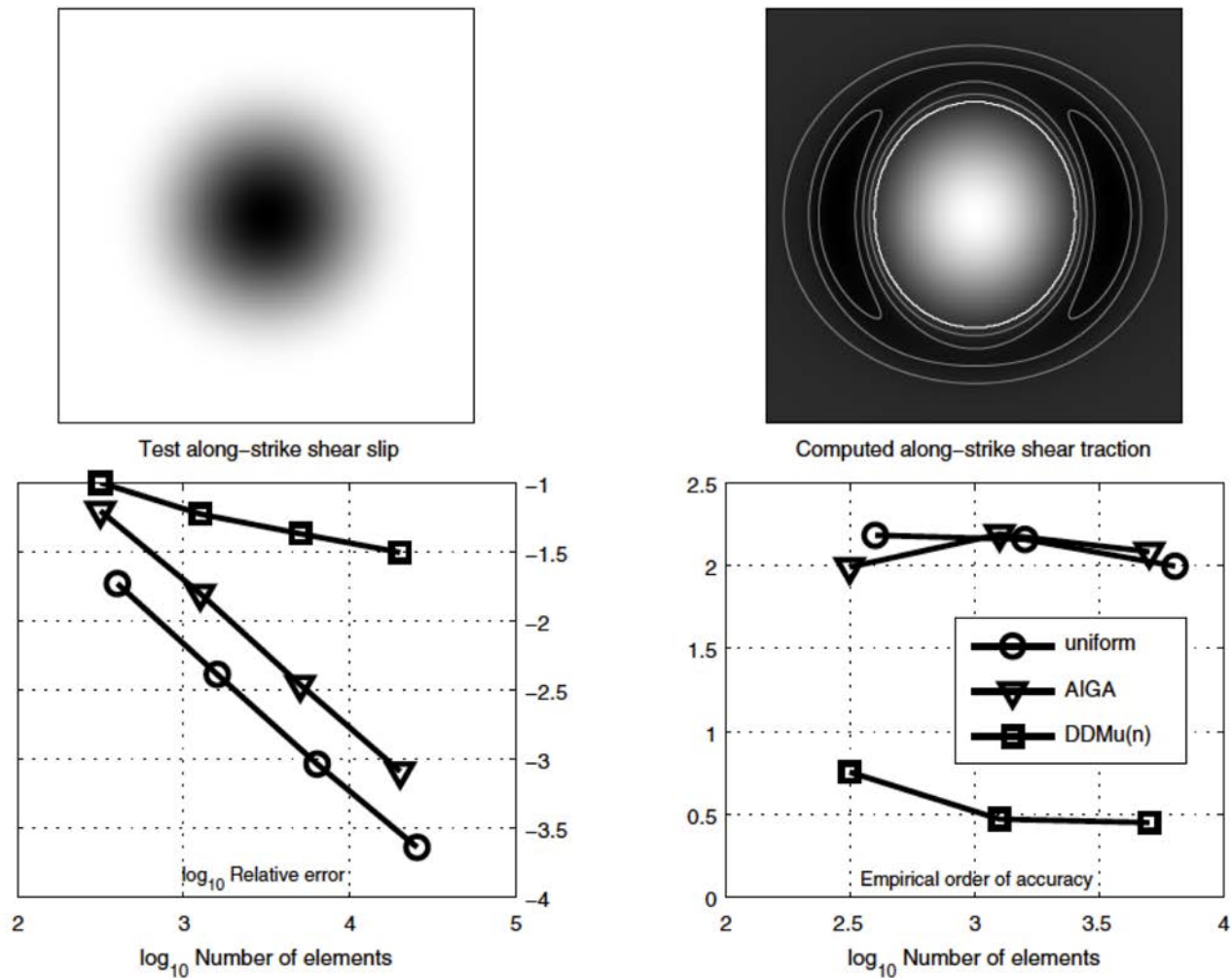
HMMVP: Hierarchical Matrix Vector Product

SRL Early Edition



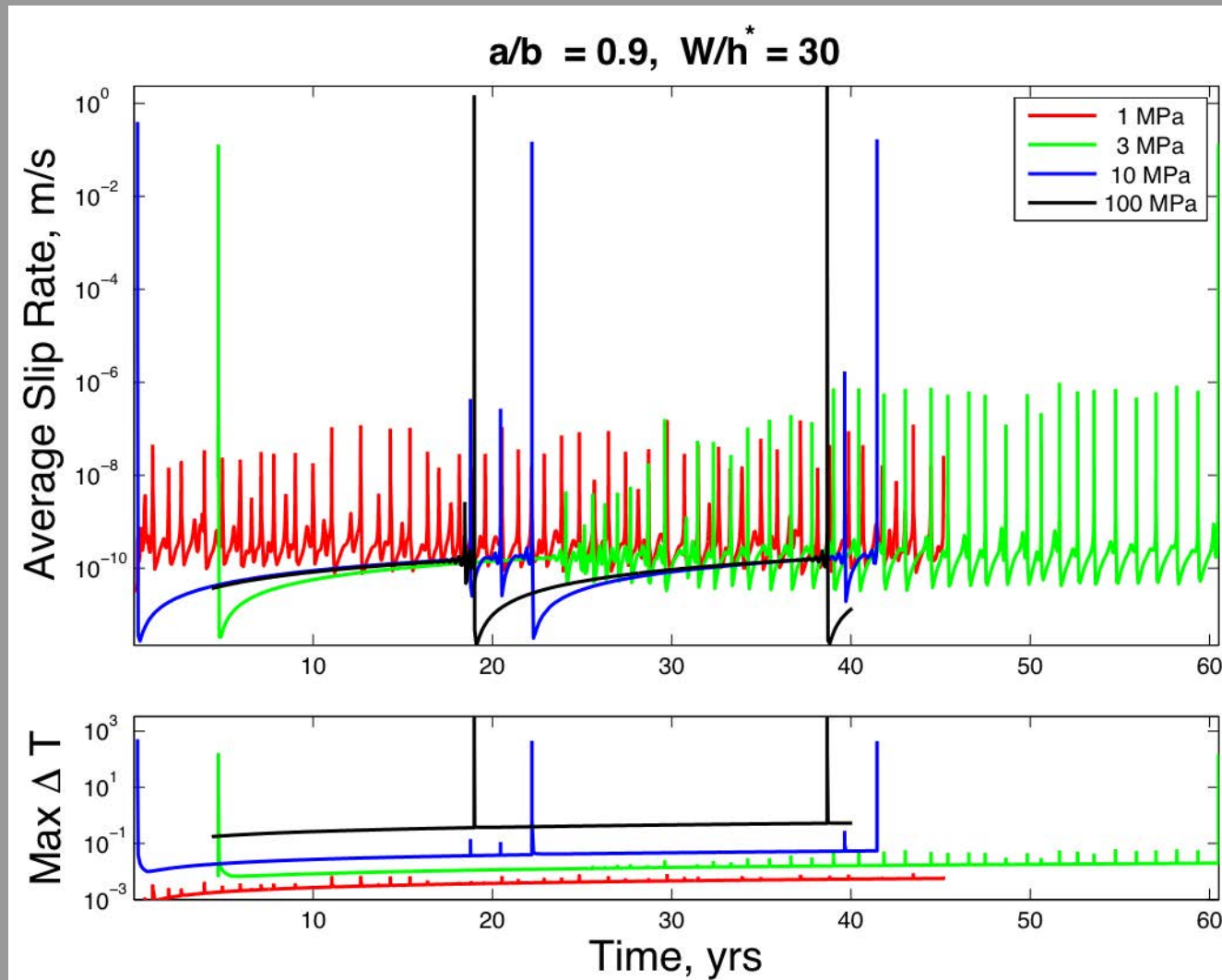
▲ **Figure 1.** Results for *hmmvp* numerical experiment. The x axis is shown as $\log_{10} N$, in which N is the number of elements in fault mesh; the y axis is indicated by plot titles. Solid curves are for method M; dashed lines are for method B; and dash-dotted lines are the reference lines of $O(N)$ and $O(N^2)$. Numbers k indicate tolerance 10^{-k} .

DC3DM



▲ Figure 3. Results for *dc3dm* numerical experiment. (Top) Plots for the test of along-strike shear slip function and the resulting computed plots of along-strike shear traction. In the slip image, white is zero, and dark is positive. In the traction image, the white contour separates the negative (inside) from the positive (outside), and gray contours are in the positive region only. On each edge is a zero-velocity boundary condition. (Bottom) The relative error in traction and empirical OOA.

Dilatancy Stabilization at Low Effective Stress



Consistent with inferences of low effective stress in ETS zones

Model SSE Ultimately Nucleate Dynamic Events

