Multiaxial Constitutive and Numerical Modeling in Geo-mechanics within Critical State Theory

Mahdi Taiebat[1] and Yannis F. Dafalias[2]

[1] Associate Professor, Department of Civil Engineering, The University of British Columbia, Vancouver, BC, Canada

[2] Distinguished Professor, Department of Civil and Environmental Engineering, University of California, Davis, CA, USA, and Department of Mechanics, National Technical University of Athens, Zographou, Greece

SCEC Workshop on 3D Site Effects in Physics-Based Ground Motion Simulations
Los Angeles, CA . May 5, 2015

ACKNOWLEDGEMENT:
National Science and Engineering Research Council of Canada, and
National Science Foundation of United States

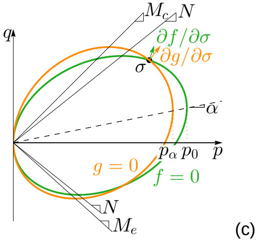
Outline

1. Constitutive modeling for clays
    - SANICLAY class
    - Model Performance
2. Constitutive modeling for sands
    - SANISAND class
    - Model Performance
3. Application in numerical modeling
    - Model implementations
    - Nonlinear effective stress seismic site response analysis
4. Discussion related to the SCEC workshop

SANICLAY: Simple ANIsotropic CLAY plasticity model

- SANICLAY

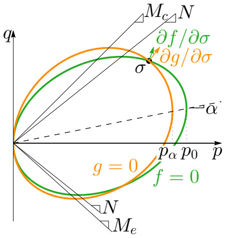
  (Dafalias et al., 2006)



SANICLAY: Simple ANIsotropic CLAY plasticity model

- SANICLAY

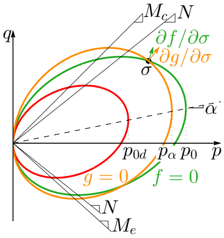
  (Dafalias et al., 2006)



(c)

- SANICLAY-D

  (Taiebat et al., 2010a)

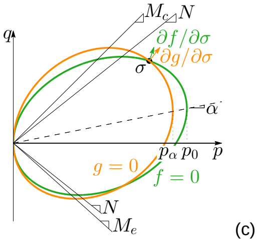


(d)

SANICLAY: Simple ANIsotropic CLAY plasticity model

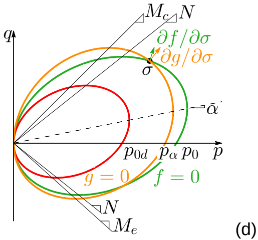
- SANICLAY

  (Dafalias et al., 2006)



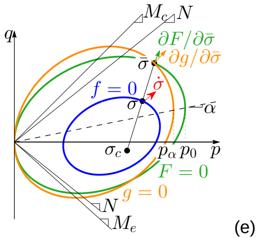
- SANICLAY-D

  (Taiebat et al., 2010a)



- SANICLAY-B

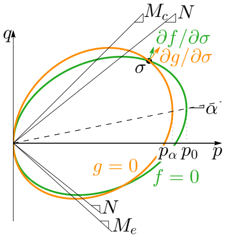
  (Seidalinov and Taiebat, 2014)



SANICLAY: Simple ANIsotropic CLAY plasticity model

- SANICLAY

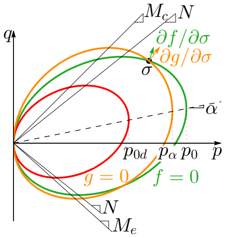
  (Dafalias et al., 2006)



(c)

- SANICLAY-D

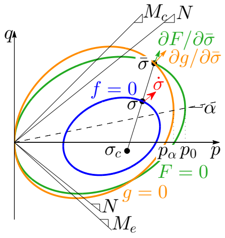
  (Taiebat et al., 2010a)



(d)

- SANICLAY-B

  (Seidalinov and Taiebat, 2014)



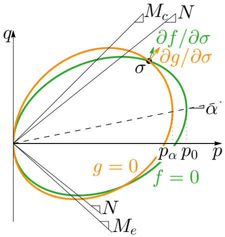
(e)

- Systematic tensorial extension to multiaxial stress space

SANICLAY: Simple ANIsotropic CLAY plasticity model

- SANICLAY

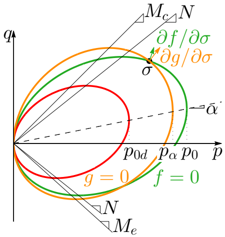
  (Dafalias et al., 2006)



(c)

- SANICLAY-D

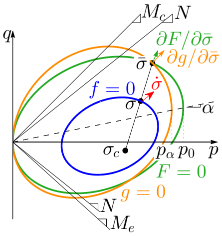
  (Taiebat et al., 2010a)



(d)

- SANICLAY-B

  (Seidalinov and Taiebat, 2014)



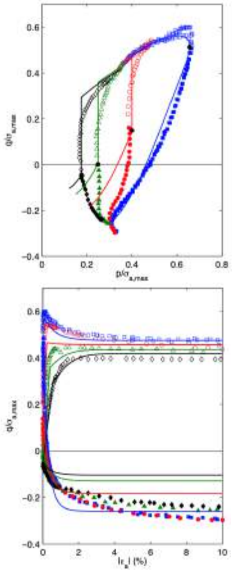
(e)

- Systematic tensorial extension to multiaxial stress space
- Relatively straightforward calibration process

	Model constant category	Designation	Georgia kaolin	Cloverdale	Ariake
MCC	Elasticity	κ	0.03	0.037	0.05
		ν	0.2	0.2	0.2
	Critical state	λ	0.21	0.121	0.41
		M_c, M_e	1.29, 1.27	0.87, 0.86	1.68, 1.65
SANICLAY	Yield surface	N	1	0.8	1.68
	Rotational hardening	C	3	3	15
		x	1.73	1.69	1.76
SANICLAY-D	Destructuration	k_i	2	0	0
SANICLAY-B	Bounding surface	h_0	550	50	1600
		a_d	68	7	80

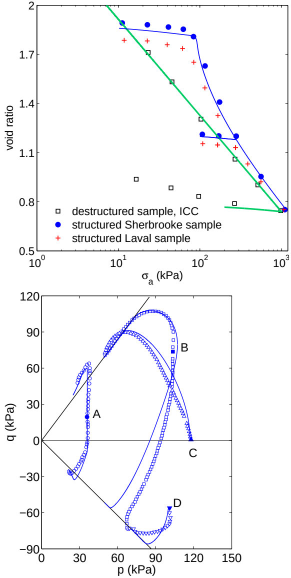
Monotonic loading (iso. comp., oedometer, undrained triaxial comp./ext.)

Lower Cromer Till



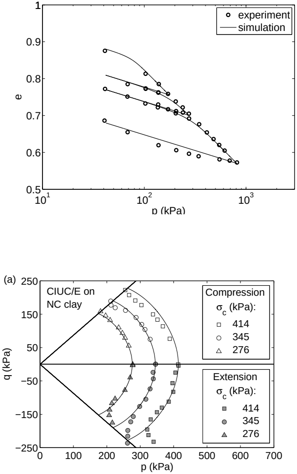
Data: after Gens (1982)

Bothkennar clay



Data: after Smith et al. (1992)

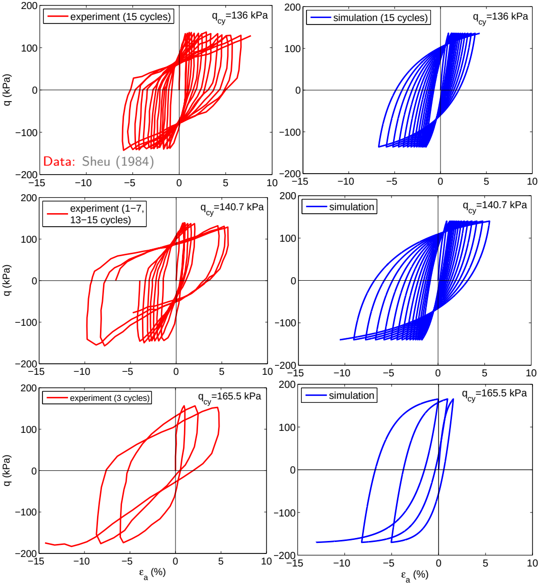
Georgia kaolin clay



Data: after Sheu (1984)

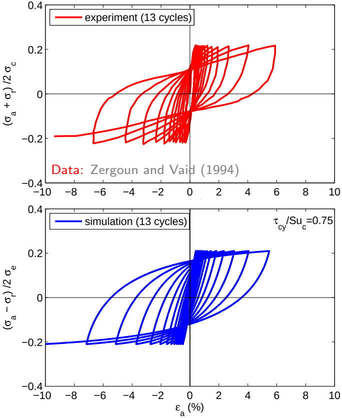
Undrained cyclic triaxial tests

- Georgia kaolin clay (reconstituted)



Data: Sheu (1984)

- Cloverdale clay (structured)

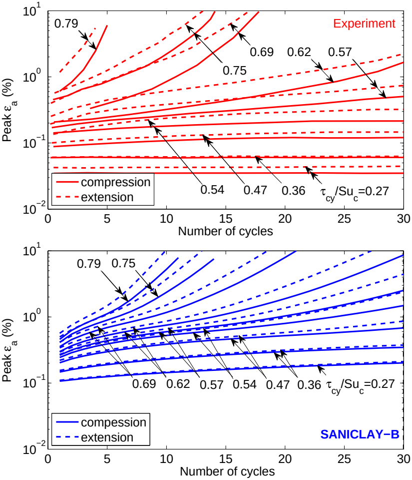


Data: Zergoun and Vaid (1994)

Simulations: Seidalinov and Taiebat (2014)

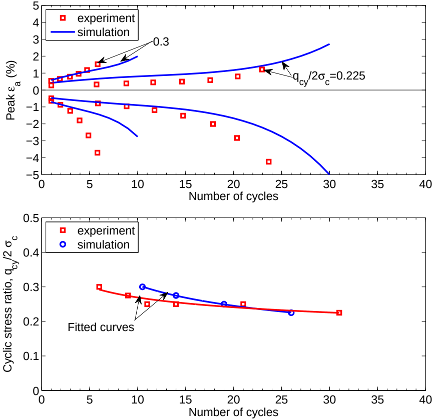
Undrained cyclic triaxial tests

- Cloverdale clay (structured)



Data: Zergoun and Vaid (1994)

- Ariake clay (reconstituted)



Data: Yasuhara et al. (1992)

Simulations: Seidalinov and Taiebat (2014)

SANISAND: Simple ANIsotropic SAND plasticity model

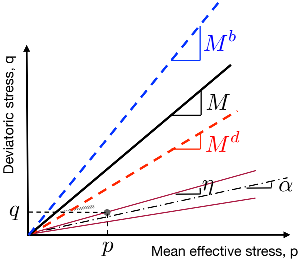
Dafalias, Manzari, Li, Papadimitriou, Taiebat (1997-2012)

- Formulation in triaxial stress space

$$f = |\eta - \alpha| - m = 0 \qquad \eta = \frac{q}{p}$$

$$\dot{\varepsilon}_q^p = \frac{\dot{\eta}}{hb} \qquad b = (M^b - M)$$

$$\dot{\varepsilon}_v^p = A_d d |\dot{\varepsilon}_q^p| \qquad d = (M^d - M)$$



SANISAND: Simple ANIsotropic SAND plasticity model

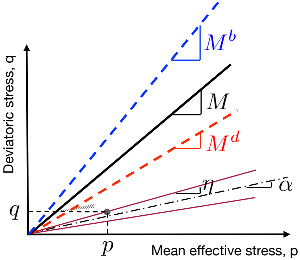
Dafalias, Manzari, Li, Papadimitriou, Taiebat (1997-2012)

- Formulation in triaxial stress space

$$f = |\eta - \alpha| - m = 0 \qquad \eta = \frac{q}{p}$$

$$\dot{\varepsilon}_q^p = \frac{\dot{\eta}}{hb} \qquad b = (M^b - M)$$

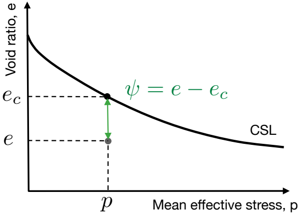
$$\dot{\varepsilon}_v^p = A_d d |\dot{\varepsilon}_q^p| \qquad d = (M^d - M)$$



- Dependence on state parameter

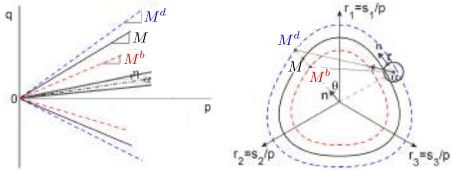
$$M^b = M \exp(-n^b \psi)$$

$$M^d = M \exp(n^d \psi)$$



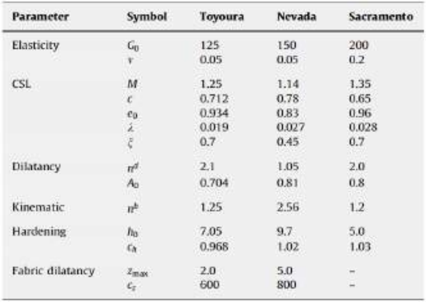
Generalization and model constants

- Systematic tensorial extension to multiaxial stress space



- Relatively straightforward calibration process

Taiebat et al. (2010b)



Parameter	Symbol	Toyoura	Nevada	Sacramento
Elasticity	G_0	125	150	200
	ν	0.05	0.05	0.2
CSL	M	1.25	1.14	1.35
	c	0.712	0.78	0.65
	e_0	0.934	0.83	0.96
	λ	0.019	0.027	0.028
	ξ	0.7	0.45	0.7
Dilatancy	n^d	2.1	1.05	2.0
	A_0	0.704	0.81	0.8
Kinematic	n^b	1.25	2.56	1.2
Hardening	h_0	7.05	9.7	5.0
	c_h	0.968	1.02	1.03
Fabric dilatancy	z_{max}	2.0	5.0	–
	c_z	600	800	–

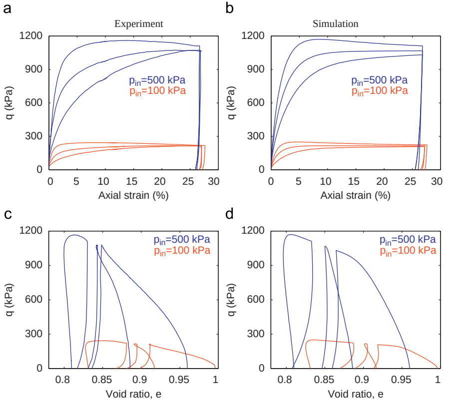
- Soil-specific set of constants for different densities & confining pressures.

- Constitutive ingredients to account for
    - ψ-dependent dilatancy stress-ratio (Manzari and Dafalias, 1997; Li and Dafalias, 2000)
    - evolving fabric anisotropy (Dafalias and Manzari, 2004)
    - inherent fabric anisotropy (Dafalias et al., 2004)
    - plastic strains under const. stress-ratio & particle crushing (Taiebat and Dafalias, 2008)
    - anisotropic critical state (Li and Dafalias, 2012)

Triaxial loading and unloading on Toyoura sand

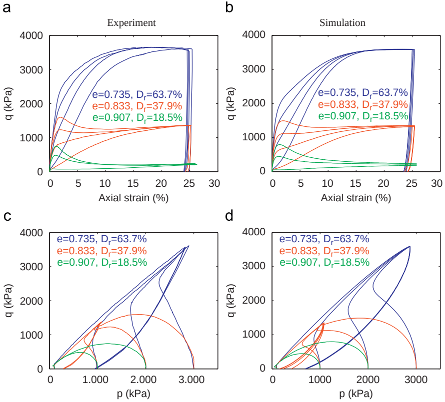
- Drained triaxial tests

Experiment Simulation



- Undrained triaxial tests

Experiment Simulation

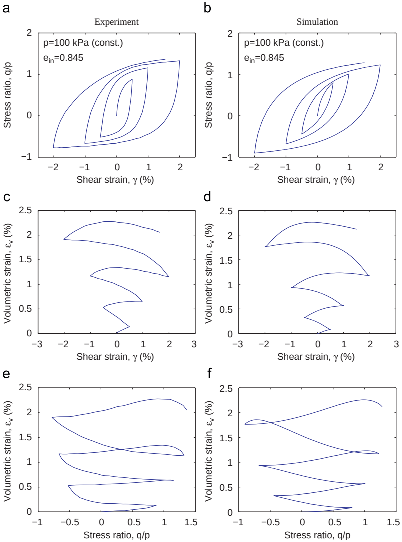


Data: Verdugo and Ishihara (1996); Simulations: Taiebat et al. (2010b)

Constant-p cyclic triaxial on Toyoura sand

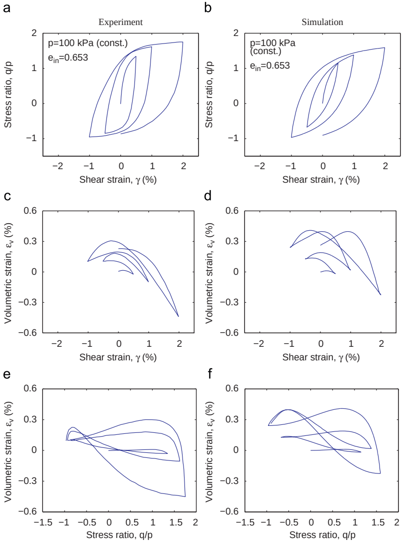
- Loose sample

Experiment Simulation



- Dense sample

Experiment Simulation



Data: Pradhan et al. (1989); Simulations: Taiebat et al. (2010b)

Finite Element platform and model implementation

- OpenSees: The Open System for Earthquake Engineering Simulation
  - Fully coupled nonlinear dynamic finite element program
  - Open-source: `http://opensees.berkeley.edu`
  - Variety of relevant element types for continuum modeling of soil medium
    - 2D (quad) and 3D (brick)
    - single phase (solid, u) and double phase (solid and pore fluid, u–p)
  - Variety of analysis types, integration schemes, and solvers

Finite Element platform and model implementation

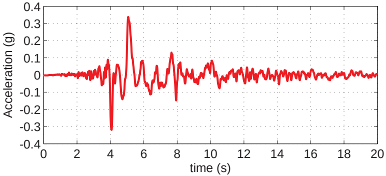
- OpenSees: The Open System for Earthquake Engineering Simulation
  - Fully coupled nonlinear dynamic finite element program
  - Open-source: `http://opensees.berkeley.edu`
  - Variety of relevant element types for continuum modeling of soil medium
    - 2D (quad) and 3D (brick)
    - single phase (solid, u) and double phase (solid and pore fluid, u–p)
  - Variety of analysis types, integration schemes, and solvers

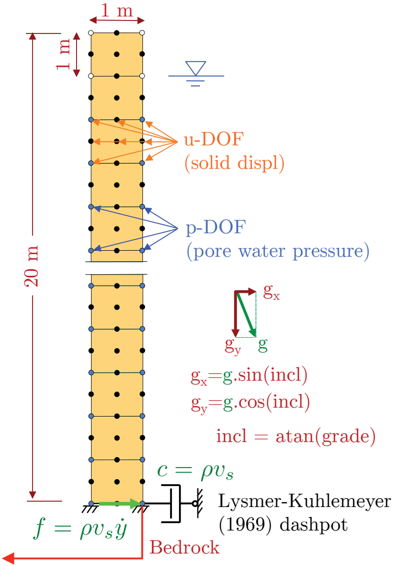
- SANICLAY and SANISAND implementation:
  - SANICLAY: Refined explicit integration scheme with automatic sub-stepping and error control (Seidalinov and Taiebat, 2014)
  - SANISAND: Various explicit and implicit integration schemes (Ghofrani and Arduino, 2014)
  - All implementations are in full tensorial forms of stresses and strains (3D)

Application of SANICLAY models

Modeling of infinite slope subjected to earthquake excitation:

- 20 m deep (2% grade) deposit of NC clay
  - SANICLAY & SANICLAY-B models
  - 1 m water table, permeability: 10^{-8} m/s
  - Periodic BCs to emulate 1D analysis
- Modeling
  - 9-node quad u–p element (Biot's theory)
  - Periodic BCs to emulate 1D analysis
  - Base dashpot to account for the finite rigidity of the underlying elastic medium
  - Velocity time history $\dot{u}(t)$ and high $V_{s,\mathrm{base}}$
- Imperial Valley record scaled to PGA=0.35g





$g_x = g.\sin(\mathrm{incl})$

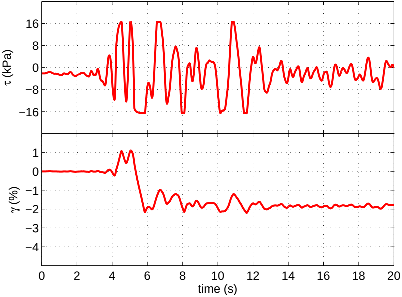
$g_y = g.\cos(\mathrm{incl})$

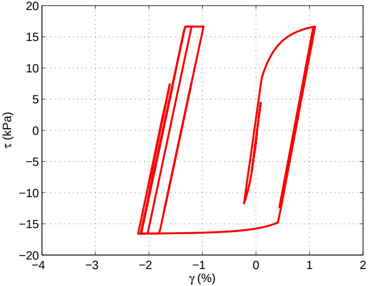
$\mathrm{incl} = \mathrm{atan}(\mathrm{grade})$

Adopted from McGann and Arduino (2013)

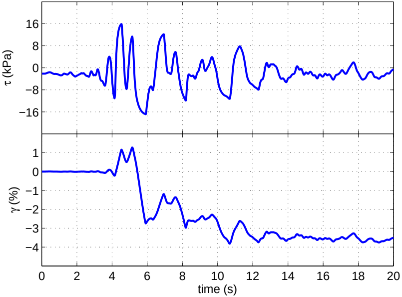
Results: shear stress (τ) & shear strain (γ) at depth of 5.5 m

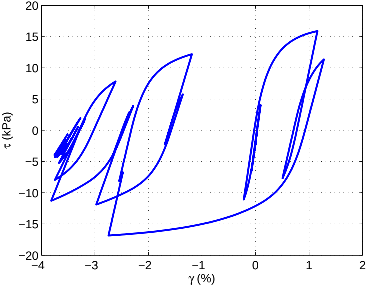
- SANICLAY





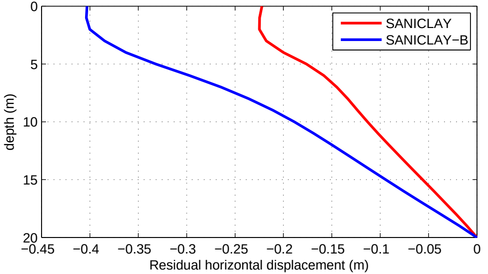
- SANICLAY-B



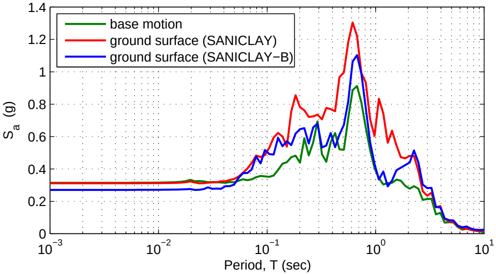


Results: Displacement profile and spectral accelerations

- Horizontal displacement profiles at the end of shaking



- Spectral acceleration at the base and top of soil columns

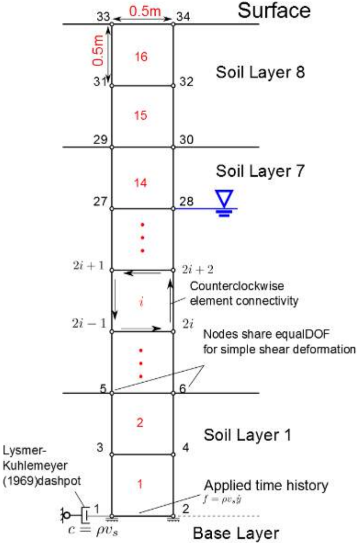


Application of SANISAND model in PRENOLIN

Modeling of free field soil column subjected to earthquake excitation in Sendai:

- 8 m deep deposit of sand
    - 0–7 m: SANISAND, and 7–8 m elastic
    - 1.5 m water table, permeability: 10^{-5} m/s
    - Periodic BCs to emulate 1D analysis
- Modeling
    - SSPquadUP element (Biot's theory)
    - Periodic BCs to emulate 1D analysis
    - Base dashpot to account for the finite rigidity of the underlying elastic medium
    - Velocity time history $\dot{u}(t)$ and high $V_{s,\text{base}}$
- Several motions from downhole arrays at Sendai site

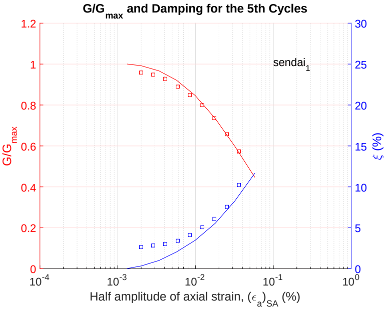
[Collaborative study with UW]

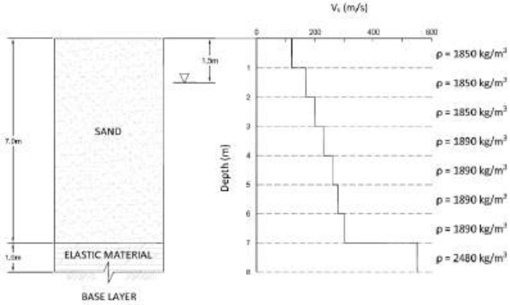


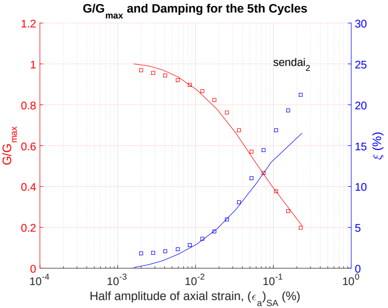
Adopted from McGann and Arduino (2013)

Soil properties

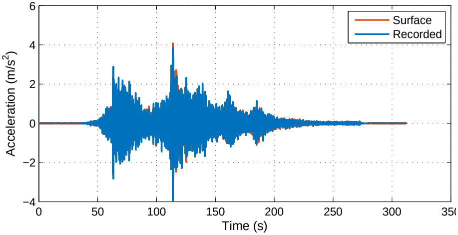
- Calibration based on data of
  - drained monotonic triaxial tests at three different confining pressures
  - undrained cyclic triaxial tests on two frozen samples at depths of 3.5 and 5.5 m, resulting in plots of $G/G_{\max}$ and ξ vs. $(\varepsilon_a)_{SA}$
- Stiffness adjusted based on profile of V_s

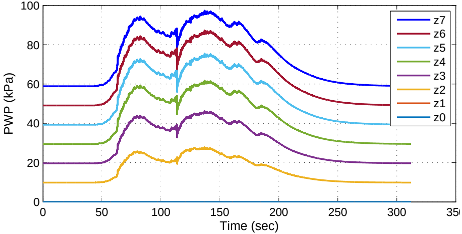


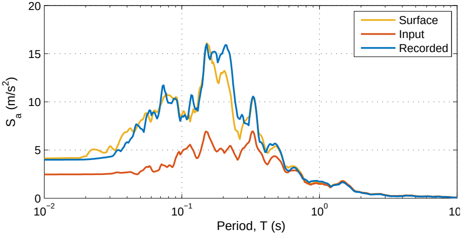


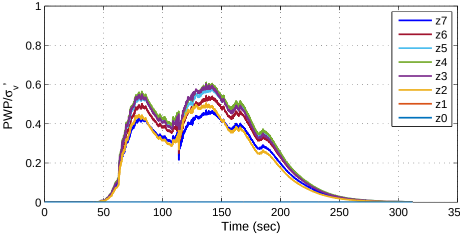


Analysis results for one of the ground motions









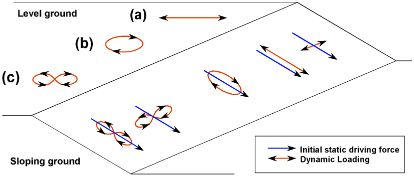
Challenges for 3D seismic site response?!

- Moving from 1D to 3D in regional-scale simulations:
    - Our models have always been 3D ✓
    - 3D is the same in any scale and our scale is that of continuum ✓

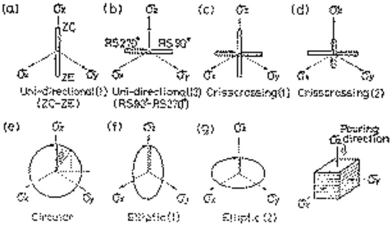
Challenges for 3D seismic site response?!

- Moving from 1D to 3D in regional-scale simulations:
  - Our models have always been 3D ✓
  - 3D is the same in any scale and our scale is that of continuum ✓

- Further works in constitutive modeling
  - Fabric-related strongly anisotropic response (next slide)
  - Constitutive modeling of intermediate soils …
  - Validating the models for multiaxial loading …



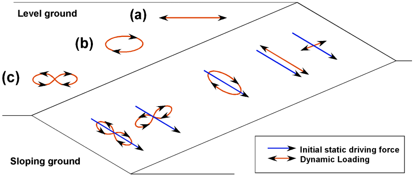
Kammerer (2002)



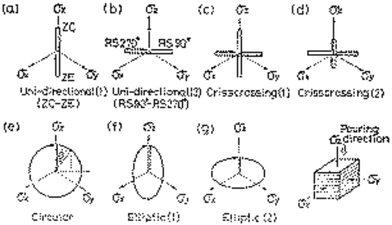
Yamada and Ishihara (1983)

Challenges for 3D seismic site response?!

- Moving from 1D to 3D in regional-scale simulations:
  - Our models have always been 3D ✓
  - 3D is the same in any scale and our scale is that of continuum ✓
- Further works in constitutive modeling
  - Fabric-related strongly anisotropic response (next slide)
  - Constitutive modeling of intermediate soils ...
  - Validating the models for multiaxial loading ...



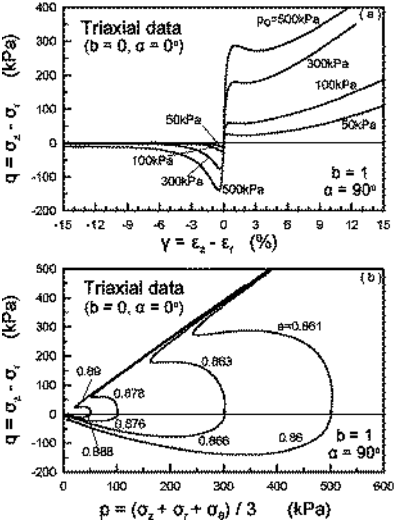
Kammerer (2002)

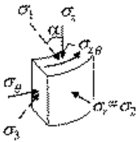


Yamada and Ishihara (1983)

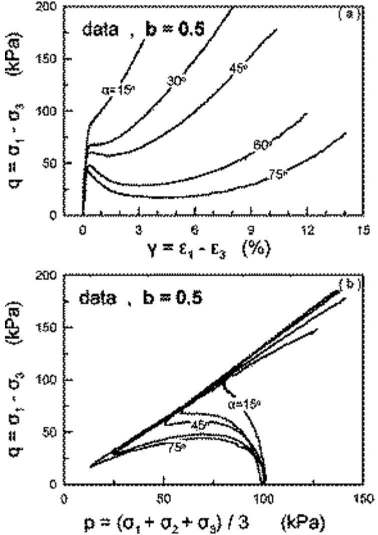
- Calibration and simulation
  - State parameters including internal variables from in-situ testing results?!
  - Statistical methods to deal with scarce and sparse input parameters?!
  - Professional programming and use of HPC techniques?!

Fabric-related strongly anisotropic response





$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}$$



Yoshimine et al. (1998)

THANK YOU!

Acknowledgments:

Collaborators:
- Prof. Pedro Arduino (UW)

Students:
- Mr. Gaziz Seidalinov (UBC)
- Mr. Graeme McAllister (UBC)
- Mr. Alborz Ghofrani (UW)
- Mr. Long Chen (UW)

Bibliography I

Dafalias, Y. F. and Manzari, M. T. (2004), ‘Simple plasticity sand model accounting for fabric change effects’, *ASCE Journal of Engineering Mechanics* **130**(6), 622–634.

Dafalias, Y. F., Manzari, M. T. and Papadimitriou, A. G. (2006), ‘SANICLAY: simple anisotropic clay plasticity model’, *Int’l Journal for Numerical and Analytical Methods in Geomechanics* **30**(12), 1231–1257.

Dafalias, Y. F., Papadimitriou, A. G. and Li, X. S. (2004), ‘Sand plasticity model accounting for inherent fabric anisotropy’, *Journal of Engineering Mechanics* **130**(11), 1319–1333.

Gens, A. (1982), Stress–strain and strength of a low plasticity clay, Ph.D. thesis, Imperial College, London University. 856 pages.

Kammerer, A. M. (2002), Undrained Response of Monterey 0/30 Sand Under Multidirectional Cyclic Simple Shear Loading Conditions, PhD thesis, University of California, Berkeley.

Li, X. S. and Dafalias, Y. F. (2000), ‘Dilatancy for cohesionless soils’, *Géotechnique* **54**(4), 449–460.

Li, X. S. and Dafalias, Y. F. (2012), ‘Anisotropic critical state theory: role of fabric’, *Journal of Engineering Mechanics* **138**(3), 263–275.

Manzari, M. T. and Dafalias, Y. F. (1997), ‘A critical state two–surface plasticity model for sands’, *Géotechnique* **47**(2), 255–272.

McGann, C. and Arduino, P. (2013), ‘Effective stress site response analysis of a layered soil column. `http://opensees.berkeley.edu/wiki/index.php/Effective_Stress_Site_Response_Analysis_of_a_Layered_Soil_Column`’.

Pradhan, T. B., Tatsuoka, F. and Sato, Y. (1989), ‘Experimental stress–dilatancy relations of sand subjected to cyclic loading’, *Soils and Foundations* **29**(1), 45–64.

Seidalinov, G. and Taiebat, M. (2014), ‘Bounding surface SANICLAY plasticity model for cyclic clay behavior’, *International Journal for Numerical and Analytical Methods in Geomechanics* **38**(7), 702–724.

Bibliography II

Sheu, W. (1984), Modeling of stress-strain-strength behavior of a clay under cyclic loading, Ph.D. dissertation, Univeristy of Colorado, Boulder, Colorado, USA.

Smith, P. R., Jardine, R. J. and Hight, D. W. (1992), 'Yielding of bothkennar clay', *Géotechnique* **42**(2), 257–274.

Taiebat, M. and Dafalias, Y. F. (2008), 'SANISAND: simple anisotropic sand plasticity model', *International Journal for Numerical and Analytical Methods in Geomechanics* **32**(8), 915–948.

Taiebat, M., Dafalias, Y. F. and Peek, R. (2010a), 'A destructuration theory and its application to SANICLAY model', *International Journal for Numerical and Analytical Methods in Geomechanics* **34**(10), 1009–1040.

Taiebat, M., Jeremić, B., Dafalias, Y. F., Kaynia, A. M. and Cheng, Z. (2010b), 'Propagation of seismic waves through liquefied soils', *Soil Dynamics and Earthquake Engineering* **30**(4), 236–257.

Verdugo, R. and Ishihara, K. (1996), 'The steady state of sandy soils', *Soils and Foundations* **36**(2), 81–91.

Vucetic, M. and Dobry, R. (1991), 'Effect of soil plasticity on cyclic response', *Journal of Geotechnical Engineering* **117**(1), 89–107.

Wood, D. M. (1974), Some Aspects of the Mechanical Behaviour of Kaolin under Truly Triaxial Conditions of Stress and Strain., PhD thesis, University of Cambridge.

Yamada, Y. and Ishihara, K. (1983), 'Undrained deformation characteristics of sand in multi-directional shear', *Soils and Foundations* **23**(1), 61–79.

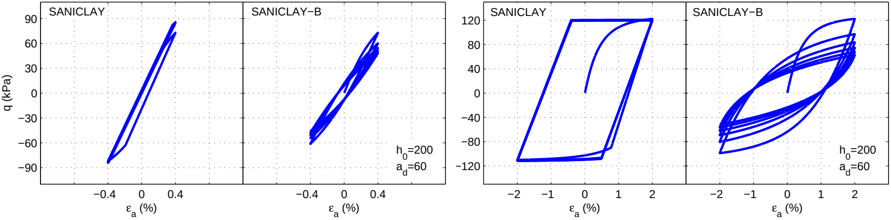
Yasuhara, K., Hirao, K. and Hyde, A. (1992), 'Effects of cyclic loading on undrained strength and compressibility of clay', *Soils and Foundations* **32**(1), 100–116.

Yoshimine, M., Ishihara, K. and Vargas, W. (1998), 'Effects of principal stress direction and intermediate principal stress on drained shear behavior of sand', *Soils and Foundations* **38**(3), 177—186.

Zergoun, M. and Vaid, Y. (1994), 'Effective stress response of clay to undrained cyclic loading', *Canadian Geotechnical Journal* **31**(5), 714–727.

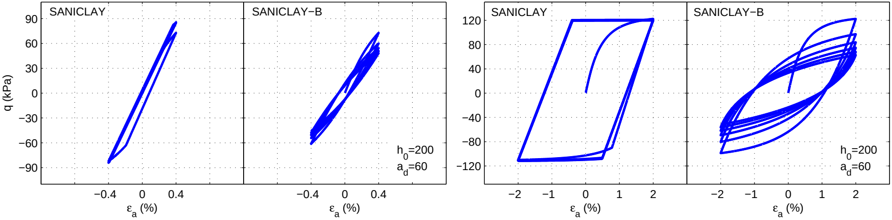
Qualitative comparison between SANICLAY and SANICLAY-B

- Stress-strain simulations in undrained cyclic triaxial test

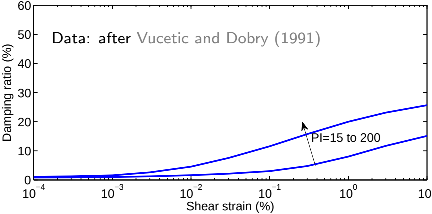


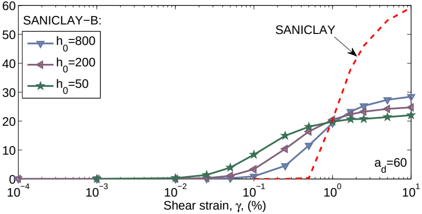
Qualitative comparison between SANICLAY and SANICLAY-B

- Stress-strain simulations in undrained cyclic triaxial test

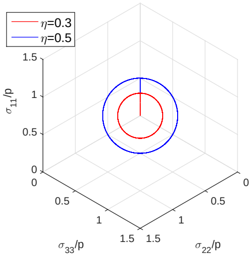


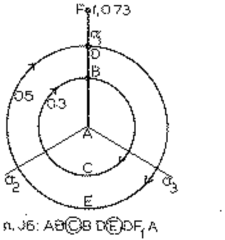
- Damping ratio vs. shear strain simulations in undrained cyclic simple shear test

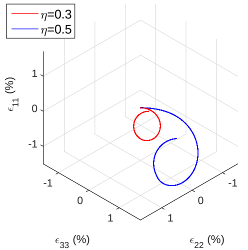


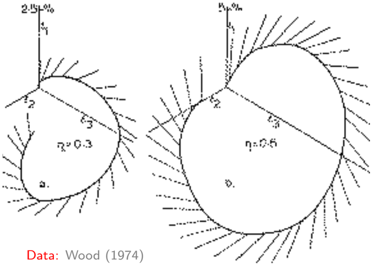


Constant-p circular stress path in π-plane



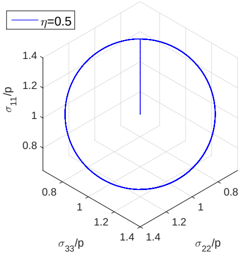


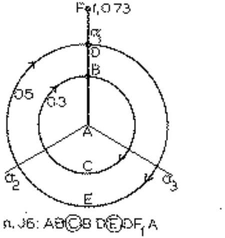


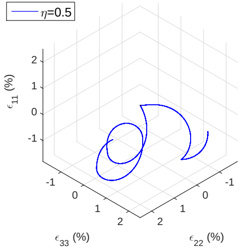


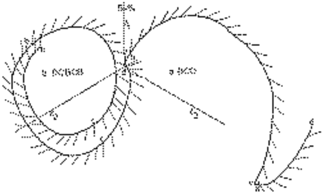
Data: Wood (1974)

Constant-p circular stress path in π-plane









Data: Wood (1974)

Undrained circular stress path in π-plane